Title: Quantum Matter Lecture

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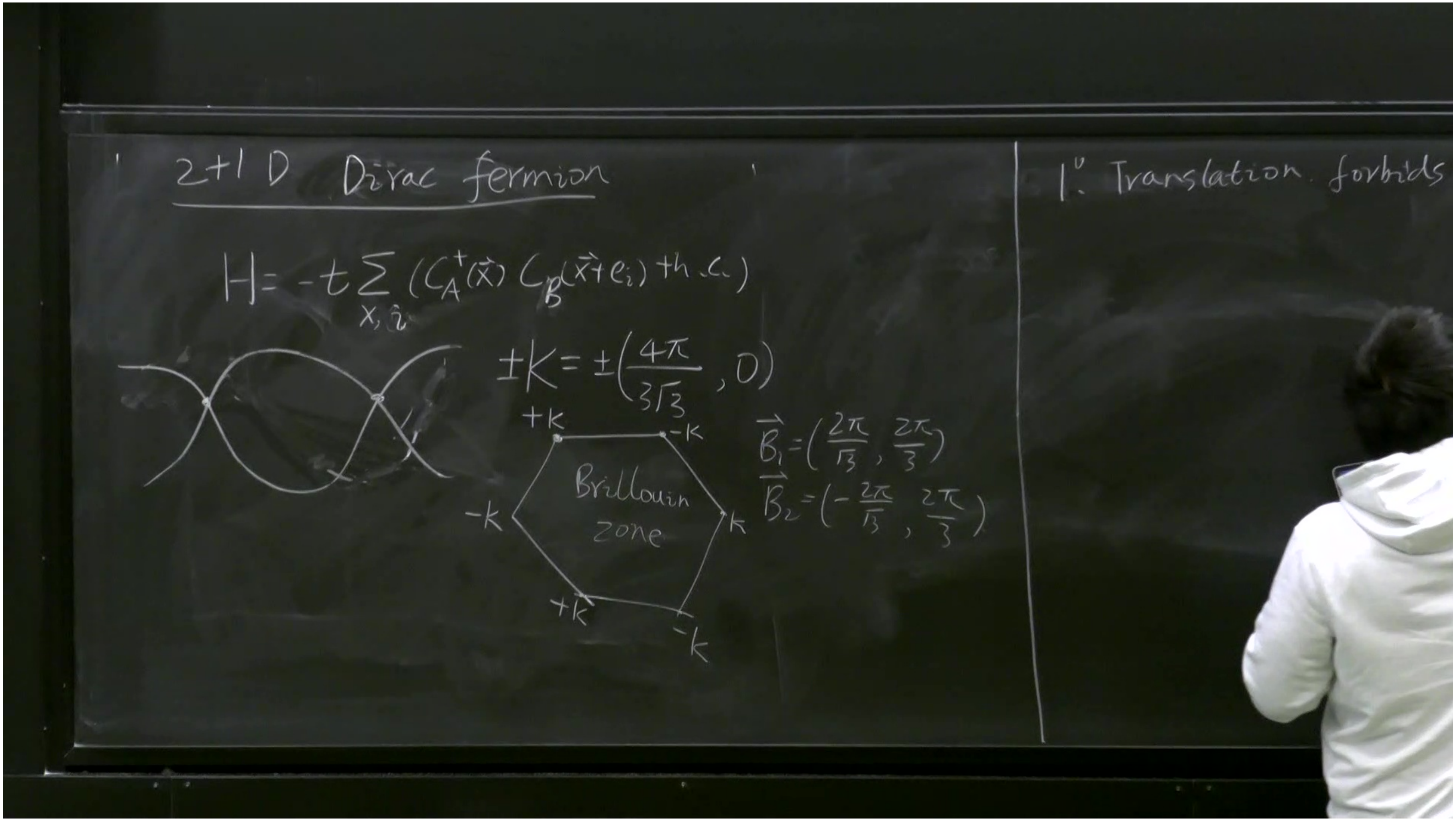

2+1 D Dirac fermion
$H = -t \sum_{x,i} (C_A^\dagger(\vec{x}) C_B(\vec{x}+e_i) + h.c.)$
$\pm K = \pm\left(\frac{4\pi}{3\sqrt{3}}, 0\right)$
+K
-K
Brillouin zone
-K
K
+K
-K
$\vec{B}_1 = \left(\frac{2\pi}{\sqrt{3}}, \frac{2\pi}{3}\right)$
$\vec{B}_2 = \left(-\frac{2\pi}{\sqrt{3}}, \frac{2\pi}{3}\right)$
1°. Translation forbids

1°. Translation forbids $\Psi^\dagger \tau^{x,y} \sigma^z \Psi$: $\psi_+^\dagger \sigma^z \psi_-$

2°. Inversion, $A \leftrightarrow B$, $(x, y) \to (-x, -y)$.

$$c_{A, K+k} \longrightarrow c_{B, -K-k}$$

$$\psi_+(k) = \begin{pmatrix} c_{A, K+k} \\ c_{B, K+k} \end{pmatrix} \longrightarrow \begin{pmatrix} c_{B, -K-k} \\ c_{A, -K-k} \end{pmatrix} = \sigma^z \begin{pmatrix} c_{B, -K-k} \\ -c_{A, -K-k} \end{pmatrix} = \sigma^z \psi_-(-k)$$

$$\psi_-(k) \to \sigma^z \psi_+(-k), \quad \Psi(k) \to \tau^x \sigma^z \Psi(-k)$$

3. Time reversal symm. $c_A \to c_A$, $c_B \to c_B$, $i \to -i$

$$c_{A,k} = \sum e^{i\vec{k}\cdot\vec{x}} c_A(\vec{x})$$

$$c_{A,K+k} \longrightarrow c_{A,-K-k}$$

$$\psi_+(k) \longrightarrow -i\sigma^y \psi_-(-k)$$

$$\Psi(k) \longrightarrow -i\tau^x \sigma^y \Psi(-k)$$

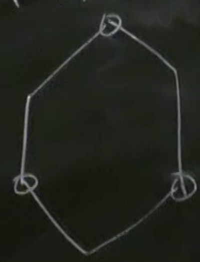
$\Psi^\dagger \sigma^z \Psi$ is forbidden

$c_B \to c_B$, $i \to -i$

Physics of Dirac mass

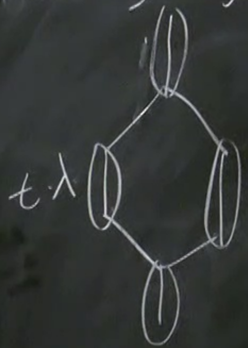
1°. $\Psi^\dagger \tau^z \sigma^z \Psi$ breaks inversion, preserves translation

$$\mu c_A^\dagger c_A - \mu c_B^\dagger c_B$$



$$\begin{pmatrix} \mu & f(k) \\ f^*(k) & -\mu \end{pmatrix}$$

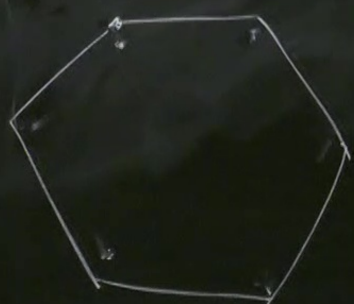
2°: C_3 symm.: Is Dirac cone stable?



$$H_k = -t\begin{pmatrix} 0 & \lambda + e^{-i\vec{k}\cdot\vec{A}_1}+e^{-i\vec{k}\cdot\vec{A}_2} \\ \lambda+e^{i\vec{k}\cdot\vec{A}_1}+e^{i\vec{k}\cdot\vec{A}_2} & 0\end{pmatrix}$$

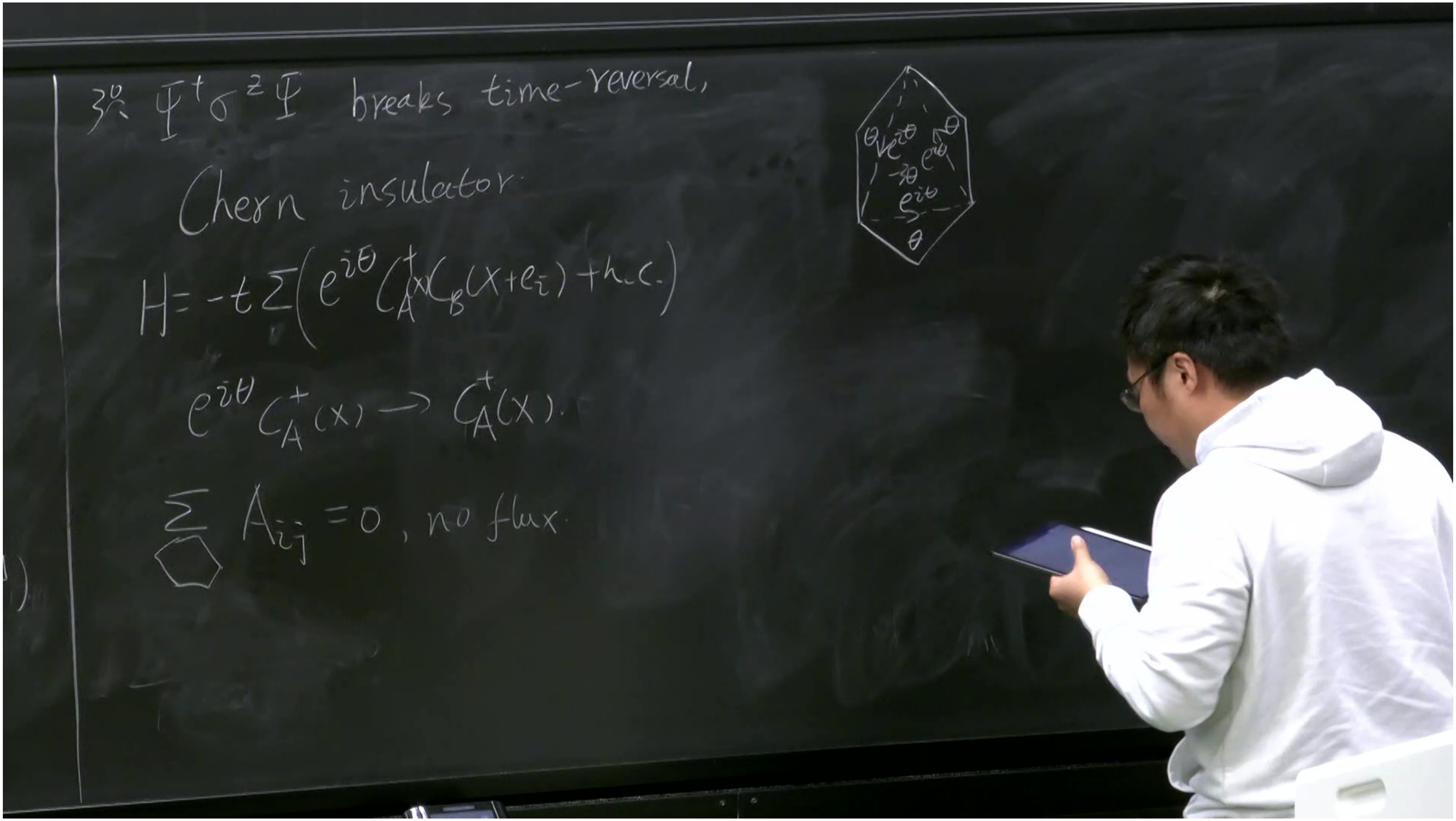
Dirac cones still exist,

but their positions are moved.



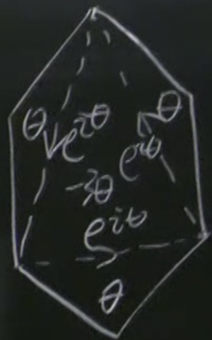
$$(\lambda-1)(P_x\sigma^x+P_y\sigma^y)+(k_x\sigma^x+k_y\sigma^y)$$

3°: $\Psi^\dagger\sigma^z\Psi$ breaks ti


3°. $\Psi^\dagger \sigma^z \Psi$ breaks time-reversal,
Chern insulator
$H = -t \sum \left(e^{i\theta} c_A^\dagger(x) c_B(x+e_i) + h.c. \right)$
$e^{i\theta} c_A^\dagger(x) \longrightarrow c_A^\dagger(x)$.
$\sum_{\hexagon} A_{ij} = 0$, no flux.

ersal,

.c.)



$$H = -t \sum_{x,i} c_A^\dagger(\vec{x})\, c_B(\vec{x}+\vec{e}_i) + h.c.$$

$$- t' \sum_{x,i} e^{i\theta} c_A^\dagger(\vec{x})\, c_A(\vec{x}+\vec{A}_i) + h.c.$$

$$+ \ A \leftrightarrow B \ )$$

Haldane model, PRL 61, 2015 (1988).

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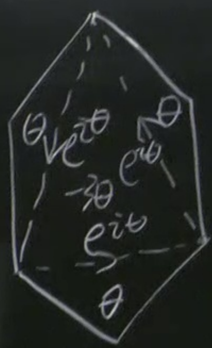

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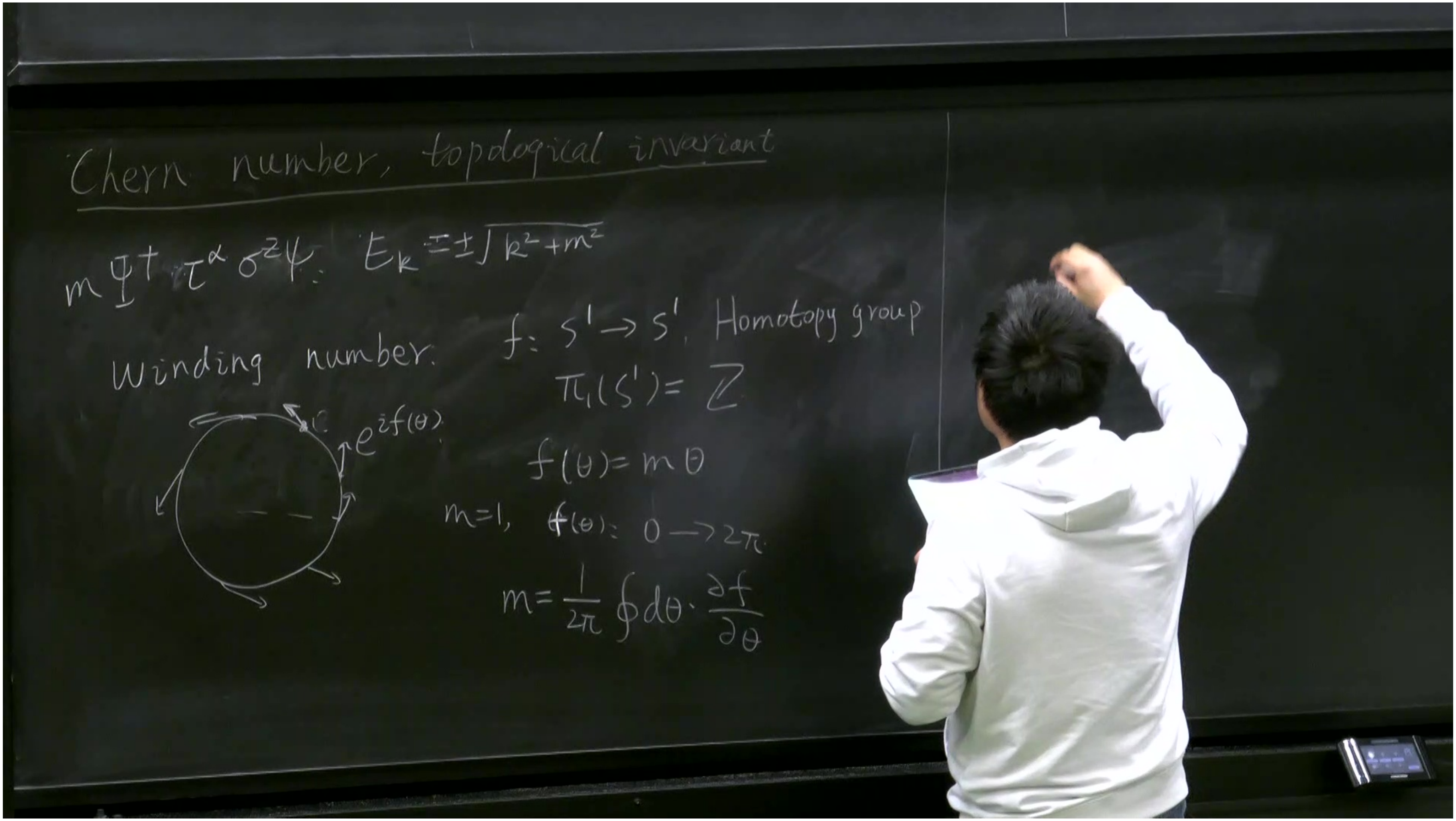
Haldane model, PRL 61, 2015 (1988).

Kane, Mele, PRL 95, 226801 (2005)

Topological insulator.


Chern number, topological invariant
$m \Psi^\dagger \tau^\alpha \sigma^z \psi$: $E_k = \pm\sqrt{k^2+m^2}$
Winding number. $f: S^1 \to S^1$ Homotopy group
$\pi_1(S^1) = \mathbb{Z}$
$e^{if(\theta)}$
$f(\theta) = m\theta$
$m=1$, $f(\theta): 0 \to 2\pi$
$m = \frac{1}{2\pi}\oint d\theta \cdot \frac{\partial f}{\partial \theta}$

$$H = \sin k_x\, \sigma^x + \sin k_y\, \sigma^y + (m + \cos k_x + \cos k_y)\, \sigma^z$$

$$\pm \epsilon_k = \pm\sqrt{\sin^2 k_x + \sin^2 k_y + (m + \cos k_x + \cos k_y)^2}$$

$m=0$, two Dirac cones. $(0,\pi)$, $(\pi,0)$

Define unit vector. $\vec{n} = \left(\frac{\sin k_x}{\epsilon_k}, \frac{\sin k_y}{\epsilon_k}, \frac{m + \cos k_x + \cos k_y}{\epsilon_k} \right)$

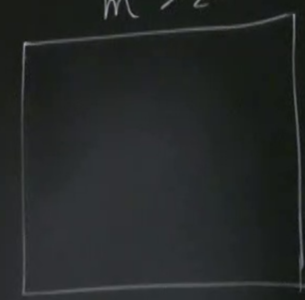
$f: T^2 \longrightarrow S^2$ Skyrmion number

$k_x, k_y \in [0, 2\pi]$

$$C = \frac{1}{4\pi} \int \vec{n} \cdot \left(\frac{\partial \vec{n}}{\partial k_x} \times \frac{\partial \vec{n}}{\partial k_y} \right) dk_x\, dk_y$$

$$C = \begin{cases} -1, & 0 < m < 2 \\ 1, & -2 < m < 0 \\ 0, & |m| > 2 \end{cases}$$

$m > 2$



$\ldots k_y \sigma^y + (m + \cos k_x + \cos k_y)\sigma^z$

$\ldots \sin^2 k_y + (m + \cos k_x + \cos k_y)^2$

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Skyrmion number

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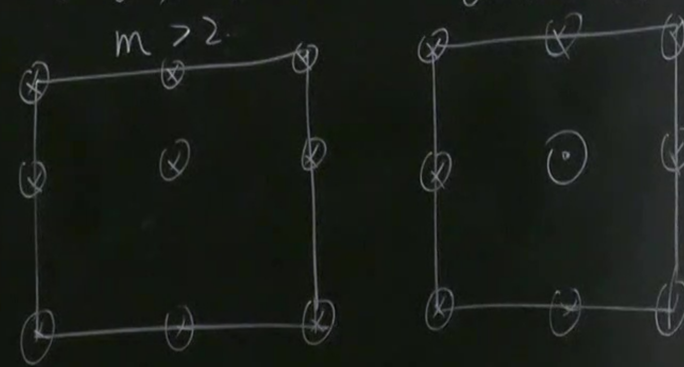
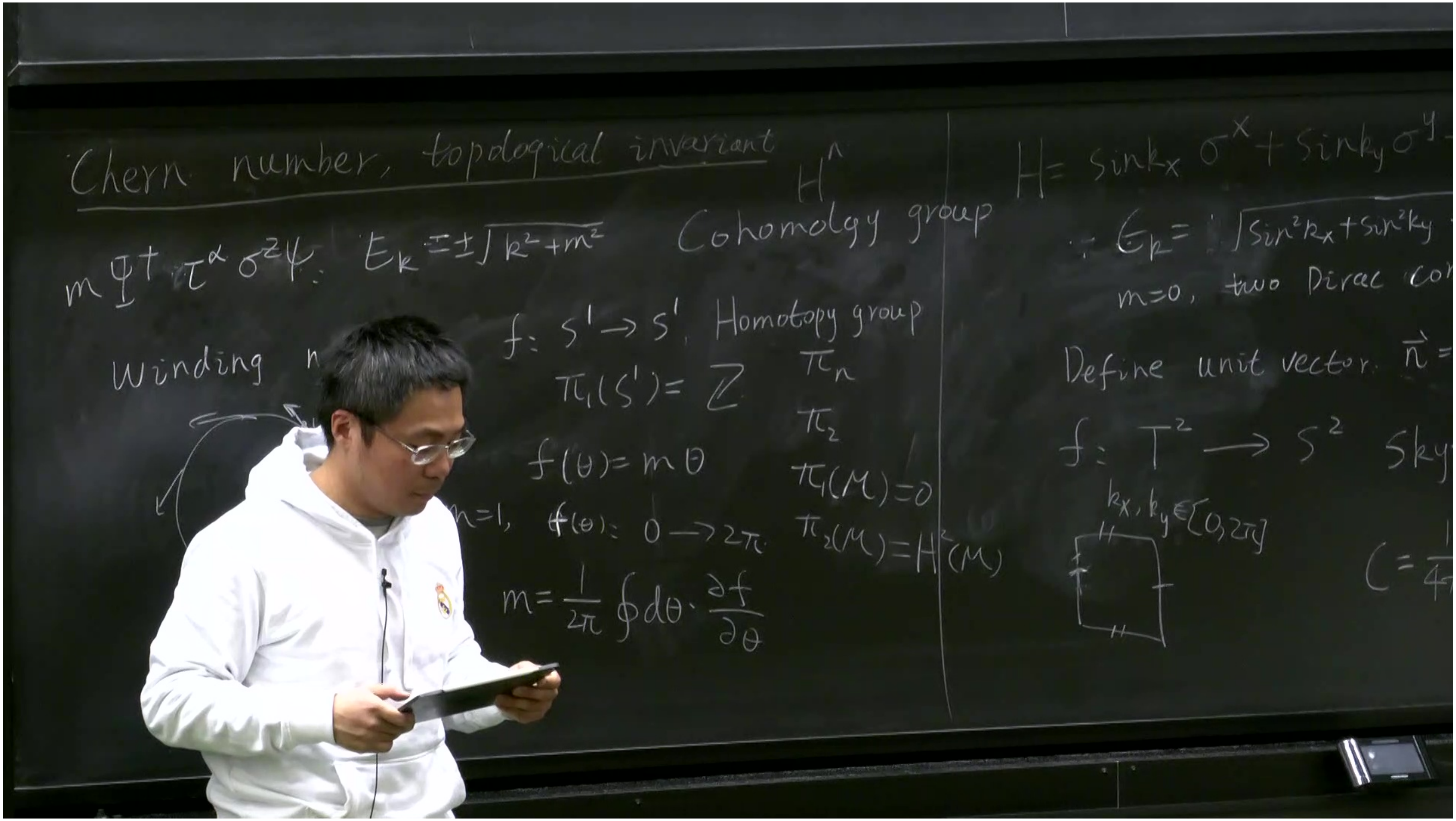
$m > 2$ $\qquad$ $0 < m < 2$




Chern number, topological invariant
Cohomolgy group
f: S¹ → S¹ Homotopy group
Winding n
π₁(S¹) = Z
f(θ) = mθ
Define unit vector
f: T² → S²