Title: Quantum Matter Lecture

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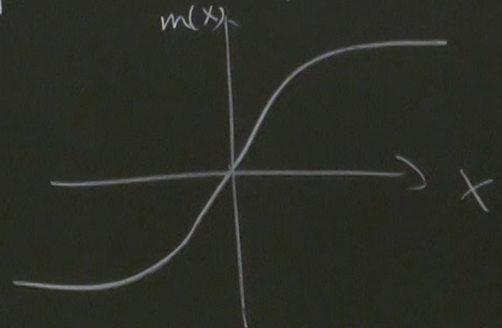
1+1D Dirac fermion

$$H = -\sum (c_j^{+} c_{j+1} + h.c.) + \sum \phi_j (-1)^j c_j^{+} c_{j+1} + h.c.$$

SSH

$$\phi_j = \begin{cases} m \\ -m \end{cases}$$

$$H(x) = \sigma^z(-i\partial_x) + m(x)\sigma^x$$



$$\psi(x) \sim \begin{pmatrix} i \\ 1 \end{pmatrix} e^{-m(x)x}$$

$$m(x) = \begin{cases} m_1, & x > 0 \\ -m_2, & x < 0 \end{cases}$$

$$H(x)\psi(x) = \begin{pmatrix} -i\partial_x & m(x) \\ m(x) & i\partial_x \end{pmatrix} \begin{pmatrix} i \\ 1 \end{pmatrix} e^{-m(x)x}$$

$$= \begin{pmatrix} im(x) & m(x) \\ m(x) & -im(x) \end{pmatrix} \begin{pmatrix} i \\ 1 \end{pmatrix} e^{-m(x)x} = 0$$

$$i \Big) e^{-m(x)x}$$

$$\begin{pmatrix} i \\ 1 \end{pmatrix} e^{-m(x)x} = 0$$

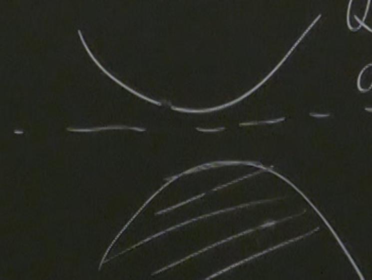
A few remarks.

1°. The zero mode is topological

Atiyah-Singer index theorem

2°. The solution has fractional charge $\frac{1}{2}$

Normal case



$Q|vac\rangle = 0$

$a_E^\dagger|vac\rangle$

$Q=1$

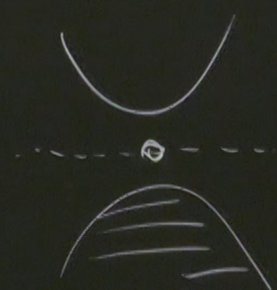
$\downarrow C$

$a_{-E}|vac\rangle$

$Q=-1$

$$\epsilon_k = \pm\sqrt{k^2+m^2}$$

Soliton

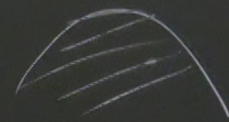


$|vac,-\rangle$, $|vac,+\rangle$

$q \xrightarrow{C} q+1$

$q = -\frac{1}{2}$

$$\begin{pmatrix} \cdots(x) & m(x) \\ \cdots(x) & -im(x) \end{pmatrix} \begin{pmatrix} i \\ 1 \end{pmatrix} e^{-m(x)x} = 0$$



$\epsilon_k = \pm\sqrt{k^2+m^2}$

$Q=1$

$\downarrow c$

$a_{-E}|vac\rangle$

$Q=-1$

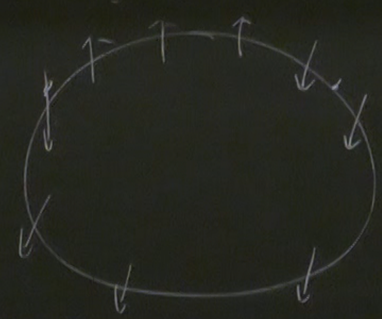
$-q=q+1$

$|vac,-\rangle, \ |vac,+\rangle$

$q \xrightarrow{c} q+1$

$q=-\frac{1}{2}$

3. Realized in materials
   Organic polymer

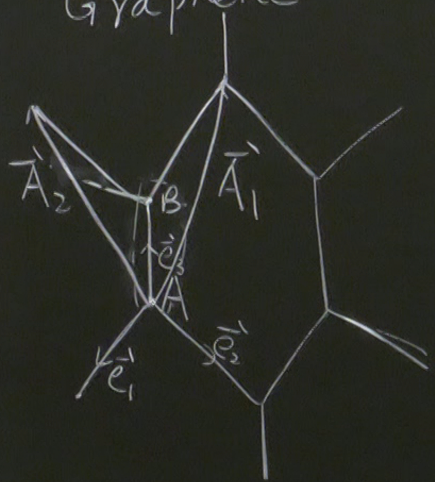


4. Zero mode is related to Chiral anomaly

Heeger, Kivelson, Schriffer, Su
Rev. Mod. Phys. 60, 781 (1988)

2+1D Dirac fermions

Graphene



$$H = -t \sum_{\vec{x}, i} \left(c_A^{\dagger}(\vec{x})\, c_B(\vec{x} + \vec{e}_i) + h.c. \right)$$

$$\vec{A}_1 = \vec{e}_3 - \vec{e}_1 = \left(\frac{\sqrt{3}}{2}, \frac{3}{2} \right)$$

$$\vec{A}_2 = \vec{e}_3 - \vec{e}_2 = \left(-\frac{\sqrt{3}}{2}, \frac{3}{2} \right)$$

$$H = \sum_{\vec{k}} -t \left(c_{Ak}^{\dagger} \;\; c_{Bk}^{\dagger} \right) \begin{pmatrix} 0 & 1 + e^{-i\vec{k}\cdot\vec{A}_1} + e^{-i\vec{k}\cdot\vec{A}_2} \\ 1 + e^{i\vec{k}\cdot\vec{A}_1} + e^{i\vec{k}\cdot\vec{A}_2} & 0 \end{pmatrix} \Big($$

$$\mathcal{E}_k = \pm t\sqrt{\left(1+\cos(\vec k\cdot\vec A_1)+\cos(\vec k\cdot\vec A_2)\right)^2+\left(\sin(\vec k\cdot\vec A_1)+\sin(\vec k\cdot\vec A_2)\right)^2}$$

$$-t\sum_{\vec x, i}\left(c_A^\dagger(\vec x)\,c_B(\vec x+\vec e_i)+h.c.\right)$$

Dirac points

$$\pm K = \pm\left(\frac{4\pi}{3\sqrt3}, 0\right)$$

$$\vec A_1 = \vec e_3 - \vec e_1 = \left(\frac{\sqrt3}{2}, \frac32\right)$$

$$\vec A_2 = \vec e_3 - \vec e_2 = \left(-\frac{\sqrt3}{2}, \frac32\right)$$

$$\sum_{\vec k} -t\left(c^\dagger_{Ak}\ \ c^\dagger_{Bk}\right)\begin{pmatrix} 0 & \underbrace{1+e^{-i\vec k\cdot\vec A_1}+e^{-i\vec k\cdot\vec A_2}}_{=f(k)} \\ 1+e^{i\vec k\cdot\vec A_1}+e^{i\vec k\cdot\vec A_2} & 0\end{pmatrix}\begin{pmatrix}c_{Ak}\\ c_{Bk}\end{pmatrix}$$

$$)+\cos(k\cdot\vec{A}_2))^2+(\sin(\vec{k}\cdot\vec{A}_1)+\sin(\vec{k}\cdot\vec{A}_2))^2$$

Dirac points

$$\pm K = \pm\left(\frac{4\pi}{3\sqrt{3}}, 0\right)$$

$$\vec{A}_2) \begin{pmatrix} C_{Ak} \\ \\ C_{Bk} \end{pmatrix}$$

$$f(\pm\vec{K}+\vec{k}) = \frac{3}{2}t(\pm k_x - ik_y) + O(k^2)$$

$$\psi_\pm(k) = \begin{pmatrix} C_{A,\pm K+k} \\ \\ C_{B,\pm K+k} \end{pmatrix}$$

$$H = \frac{3t}{2}\sum_{\vec{k}} \psi_+^\dagger(k)(\sigma^x k_x+\sigma^y k_y)\psi_+(k)$$

$$+\frac{3t}{2}\sum_{\vec{k}} \psi_-^\dagger(k)(-\sigma^x k_x+\sigma^y k_y)\psi_-(k)$$

$$i\sigma^y\psi_- \longrightarrow \psi_-$$

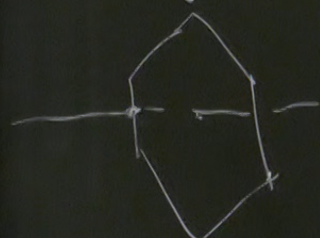
$$\frac{3t}{2}\sum_{\vec{k},f=\pm} \psi_f^\dagger(\vec{k})(\sigma^x k_x+\sigma^y k_y)\psi_f(\vec{k})$$

$\psi_+(k) = \begin{pmatrix} c_{A,K+k} \\ c_{B,K+k} \end{pmatrix}$, $\psi_-(k) = \begin{pmatrix} c_{B,-K+k} \\ -c_{A,-K+k} \end{pmatrix}$, $\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$

$M \Psi^\dagger \tau^\alpha \sigma^z \Psi$, $\tau^\alpha \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$

Stability

Symm.



1°. Translation. $c_{A,B}(\vec{x}) \to c_{A,B}(\vec{x} + \vec{A}_{1,2})$

2°. C_3 rotation around A or B

3°. Inversion, $A \leftrightarrow B$, $(x,y) \to (-x,-y)$

4°. Reflection, $A \leftrightarrow B$, $(x,y) \to (x,-y)$

5°. Time-reversal: $c_{A,B} \to c_{A,B}$, $i \to -i$

Translati

Translation

$$c_{A;K+k} \xrightarrow{T_{\vec{A}_1}} e^{-i(\vec{K}+\vec{k})\cdot\vec{A}_1} c_{A,K+k}$$

$$\Psi \to \begin{pmatrix} e^{-i(K+k)\cdot\vec{A}_1} & \\ & e^{-i(-K+k)\cdot\vec{A}_1} \end{pmatrix} \Psi$$

$$= e^{-i\vec{k}\cdot\vec{A}_1} \underbrace{\begin{pmatrix} e^{-i\frac{2\pi}{3}} & \\ & e^{i\frac{2\pi}{3}} \end{pmatrix}}_{-\frac{1}{2}\tau^0 + i\frac{\sqrt{3}}{2}\tau^z} \Psi$$

$\Psi^\dagger \tau^{x,y} \sigma^z \Psi$ is forbidden

$\Psi^\dagger \sigma^z \Psi$, $\Psi \tau^z \sigma^z \Psi$ are allowed.