Title: Quantum Matter Lecture

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2. Conformal symm.

$H = -\sum X_i X_{i+1} + \sum Z_i$

scaling symm.

$$\langle X_i X_j \rangle \sim \frac{1}{|i-j|^{2\Delta_\sigma}}, \quad \Delta_\sigma = \frac{1}{8}$$

RG: fixed point $\Longrightarrow$ scaling symm.

conformal symm.: angle preserving transf.

$X_{i+1} \mp \sum Z_i$

deform metric: $\delta g_{\mu\nu} = c(x)\,\delta_{\mu\nu}$

Invariant under RG transf.

Scaling symm.

Infinitesimal : $\tilde{X}^{\mu} = X^{\mu} + \epsilon^{\mu}$ $\delta_{\alpha\beta}$

$$\tilde{g}_{\mu\nu}(\tilde{X}) = \frac{\partial X^{\alpha}}{\partial \tilde{X}^{\mu}} \cdot \frac{\partial X^{\beta}}{\partial \tilde{X}^{\nu}}\, g_{\alpha\beta}(X).$$

…eserving transf.

$$\delta g_{\mu\nu} = (\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu) \equiv c(x)\,\delta_{\mu\nu}$$

conformal Killing equation.

$: \quad \delta g_{\mu\nu} = C(x)\, \delta_{\mu\nu}$

der RG transf.

$$\tilde{x}^{\mu} = x^{\mu} + \epsilon^{\mu} \qquad \delta_{\alpha\beta}$$

$$\frac{\partial x^{\alpha}}{\partial \tilde{x}^{\mu}} \cdot \frac{\partial x^{\beta}}{\partial \tilde{x}^{\nu}}\, g_{\alpha\beta}(x)$$

$$\epsilon_{\nu} + \partial_{\nu}\epsilon_{\mu}) \equiv C(x)\, \delta_{\mu\nu}$$

onformal Killing equation.

sol. generic d:

1° $\epsilon^{\mu} = \text{const}$, $C(x) = 0$, translation

2°. $\epsilon^{\mu} = x^{\nu} \omega^{\mu}{}_{\nu}$, $\omega_{\mu\nu} = -\omega_{\nu\mu}$, $C(x) = 0$

rotation

3°. $\epsilon^{\mu} = \lambda x^{\mu}$, $C(x) = 2\lambda$. Scale transf.

dilatation

4°. $\epsilon^{\mu} = 2(\vec{a}\cdot\vec{x})x^{\mu} - x^{2}a^{\mu}$, $C(x) = \vec{a}\cdot\vec{x}$

special conformal transf. (SCT)

SCT: $\tilde{x}^\mu = \dfrac{x^\mu - a^\mu x^2}{1 - 2(\vec{a}\cdot\vec{x}) + a^2 x^2}$

In $d=2$, $z = x_1 + i x_2$, $\bar{z} = x_1 - i x_2$

$z \to f(z)$ is conformal if $f(z)$ analytical

$ds^2 \to |f'(z)|^2 ds^2$

$z^1, z^2, \cdots$

alytical

<u>Conformal generators</u> $SO(d+1,1)$

1°. Translation: $P_\mu = \partial_\mu$

2°. Rotation: $M_{\mu\nu} = x_\mu \partial_\nu - x_\nu \partial_\mu$

3°. Dilatation: $D = x^\mu \partial_\mu$

4°. SCT: $K_\mu = 2x_\mu (x^\nu \partial_\nu) - x^2 \partial_\mu$

$1)$

$\nu)\, \partial_\mu$

$\nu) - x^2 \partial_\mu$

Algebra

$$[M_{\mu\nu}, D] = 0$$

$$[D, P_\mu] = P_\mu$$

$$[D, K_\mu] = -K_\mu$$

$$[P_\mu, K_\nu] = 2\delta_{\mu\nu} D - 2M_{\mu\nu}$$

$[R \cdot H, P_\mu] = P_\mu$

$H|\phi\rangle = (E_0 + \frac{\Delta}{R})|\phi\rangle$

$H P_\mu|\phi\rangle = (E_0 + \frac{\Delta+1}{R}) P_\mu|\phi\rangle$

$H K_\mu|\phi\rangle = (E_0 + \frac{\Delta-1}{R}) K_\mu|\phi\rangle$

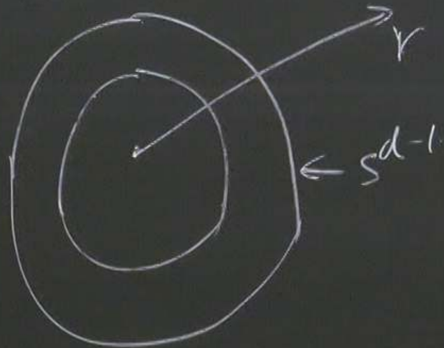
Primary state : $K_\mu|\Phi\rangle = 0$

descendant state : $P_{\mu_1} \cdots P_{\mu_n}|\Phi\rangle$

$E_0 + \frac{\Delta_\Phi + n}{R}$

$K_\mu|0\rangle = 0, \quad P_\mu|0\rangle = 0$

$\mathbb{R}^d$ (classical)



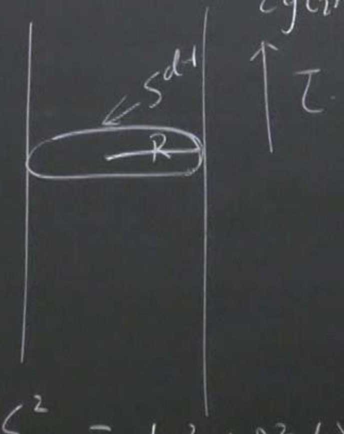
$\vec{n} = \vec{n}'$

$\tau = R \log r$

$\xrightarrow{\hspace{3em}}$

Weyl transf.

$S^{d-1} \times \mathbb{R}$ (quantum)
cylinder



$$ds^2_{flat} = dr^2 + r^2 d\vec{n}^2$$

$$D: r \to e^{\lambda} \cdot r$$

$$\langle \hat{O}(x)\, \hat{O}(y) \rangle = \frac{1}{|x-y|^{2\Delta}}$$

$$ds^2_{cyl} = d\tau^2 + R^2 d\vec{n}^2 = \frac{R^2}{r^2} ds^2_{flat}$$

$$R \cdot H : \tau \to \tau + R\lambda$$

H: Hamiltonian

$|\Phi\rangle$

$H, P_\mu] = P_\mu$

$|\phi\rangle = (E_0 + \frac{\Delta}{R})|\phi\rangle$

$P_\mu|\phi\rangle = (E_0 + \frac{\Delta+1}{R}) P_\mu|\phi\rangle$

$K_\mu|\phi\rangle = (E_0 + \frac{\Delta-1}{R}) K_\mu|\phi\rangle$

ng. CFT. $\Delta_\sigma = \frac{1}{8}$

Z_2 odd, X_i $\Delta_\varepsilon = 1$

Z_2 even. Z_i

Primary state: $K_\mu|\Phi\rangle = 0$

descendant state: $P_{\mu_1} \cdots P_{\mu_n}|\Phi\rangle$

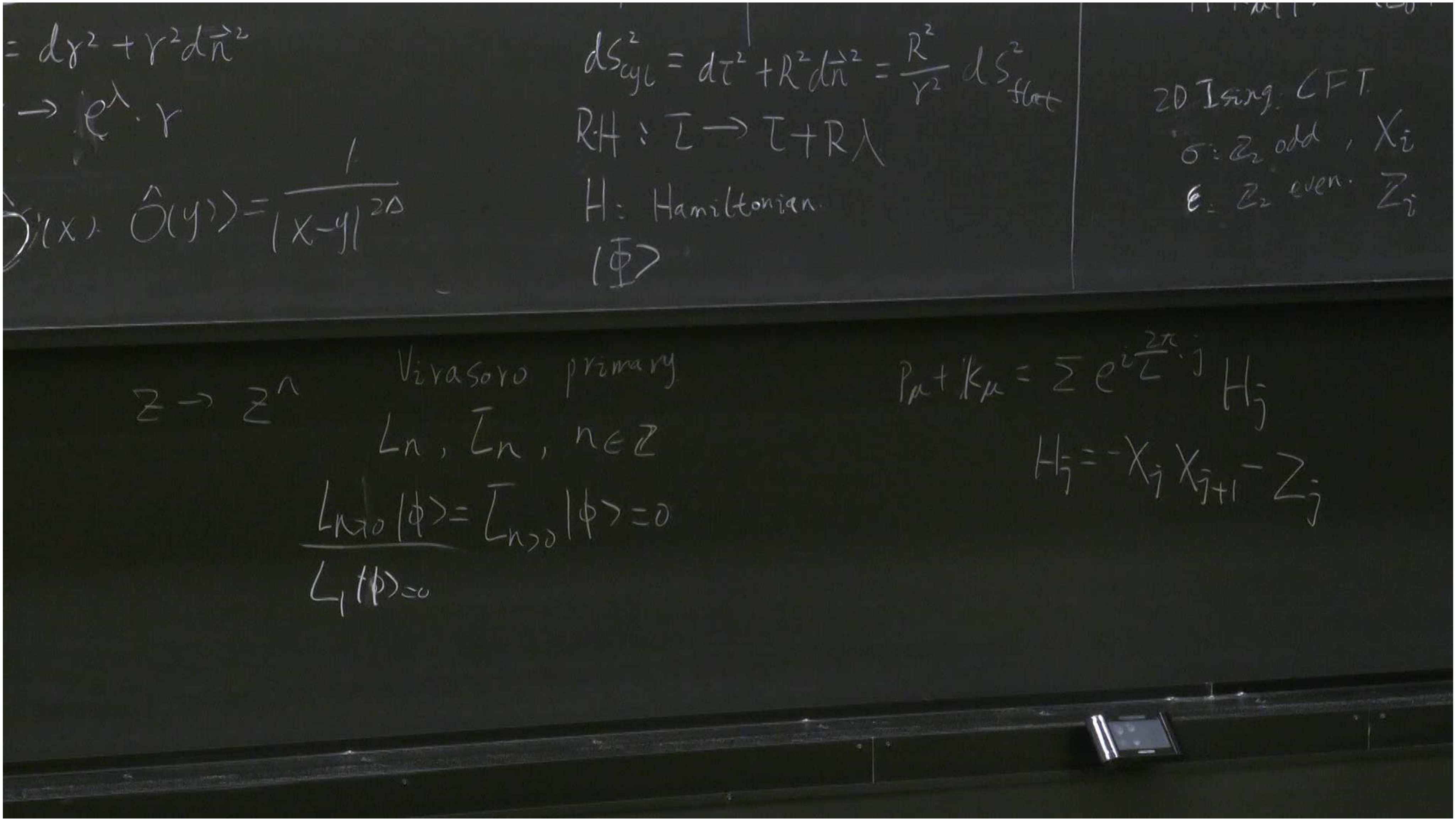
$E_0 + \frac{\Delta_\Phi + n}{R}$

$K_\mu|0\rangle = 0$, $P_\mu|0\rangle = 0$

1st excited: $E_1 - E_0 \sim \frac{\#}{L}$

2st excited: $E_2 - E_0 \sim \frac{\#}{L}$

$(E_2 - E_0) = 8(E_1 - E_0)$


= dr^2 + r^2 dn^2
→ e^λ · r
Ô(x) Ô(y)> = 1/|x-y|^{2Δ}
ds_cyl^2 = dτ^2 + R^2 dn^2 = R^2/r^2 ds_flat^2
RH : τ → τ + Rλ
H: Hamiltonian
|Φ>
2D Ising CFT
σ: Z_2 odd, X_i
ε: Z_2 even. Z_i
z → z^λ
Virasoro primary
L_n, L̄_n, n ∈ Z
L_{n>0}|φ> = L̄_{n>0}|φ> = 0
L_1|φ> = 0
P_μ + K_μ = Σ e^{i 2π/L · j} H_j
H_j = -X_j X_{j+1} - Z_j