

Title: Quantum Matter Lecture

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Collection: Quantum Matter 2023/24

Date: March 07, 2024 - 10:15 AM

URL: <https://pirsa.org/24030041>

2. Conformal symm.

scaling symm.

$$H = -\sum X_i X_{i+1} + \sum Z_i$$

$$\langle X_i X_j \rangle \sim \frac{1}{|i-j|^{2\Delta_\sigma}}, \quad \Delta_\sigma = \frac{1}{8}$$

RG : fixed point \Rightarrow scaling symm.

conformal symm.: angle preserving transf.

$$X_{i+1} = \sum Z_i$$

deform metric: $\delta g_{\mu\nu} = C(x) \delta_{\mu\nu}$

Invariant under RG transf.

Infinitesimal: $\tilde{X}^\mu = X^\mu + \epsilon^\mu \delta_{\alpha\beta}$

$$\tilde{g}_{\mu\nu}(\tilde{X}) = \frac{\partial X^\alpha}{\partial \tilde{X}^\mu} \cdot \frac{\partial X^\beta}{\partial \tilde{X}^\nu} g'_{\alpha\beta}(x).$$

$$\delta g_{\mu\nu} = (\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu) \equiv C(x) \delta_{\mu\nu}$$

conformal Killing equation.

Scaling symm.

reserving transf.

$$\delta g_{\mu\nu} = C(x) \delta_{\mu\nu}$$

der RG transf.

$$\tilde{x}^\mu = x^\mu + \epsilon^\mu \quad \delta_{\alpha\beta}$$

$$\frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \cdot \frac{\partial x^\beta}{\partial \tilde{x}^\nu} g'_{\alpha\beta}(x)$$

$$\epsilon_\nu + \partial_\nu \epsilon_\mu \equiv C(x) \delta_{\mu\nu}$$

conformal Killing equation.

sol. generic d.

$$1^0 \epsilon^\mu = \text{const}, \quad C(x) = 0, \quad \text{translation}$$

$$2^0 \epsilon^\mu = x^\nu W_{\nu}^{\mu}, \quad W_{\mu\nu} = -W_{\nu\mu}, \quad C(x) = 0$$

rotation.

$$3^0 \epsilon^\mu = \lambda x^\mu, \quad C(x) = 2\lambda, \quad \text{scale transf.}$$

dilatation

$$4^0 \epsilon^\mu = 2(\vec{a} \cdot \vec{x}) x^\mu - x^2 a^\mu, \quad C(x) = \vec{a} \cdot \vec{x}$$

special conformal transf. (SCT)

$$\text{SCT: } \tilde{X}^\mu = \frac{X^\mu - a^\mu X^2}{1 - 2(\vec{a} \cdot \vec{X}) + a^2 X^2}$$

$$\text{In } d=2, \quad z = X_1 + iX_2, \quad \bar{z} = X_1 - iX_2$$

$z \rightarrow f(z)$ is conformal if $f(z)$ analytical

$$ds^2 \rightarrow |f'(z)|^2 ds^2$$

$$z', z'', \dots$$

Conformal generators $SO(d+1, 1)$

1^o. Translation: $P_\mu = \partial_\mu$

2^o. Rotation: $M_{\mu\nu} = X_\mu \partial_\nu - X_\nu \partial_\mu$

3^o. Dilatation: $D = X^\mu \partial_\mu$

4^o. SCT: $K_\mu = 2X_\mu (X^\nu \partial_\nu) - X^2 \partial_\mu$

alytical

1)

Algebra

$$[M_{\mu\nu}, D] = 0$$

$$[D, P_\mu] = P_\mu$$

$$[D, K_\mu] = -K_\mu$$

$$[P_\mu, K_\nu] = 2S_{\mu\nu} D - 2M_{\mu\nu}$$

∂_μ

$x^\nu \partial_\mu$

$$[R \cdot H, P_\mu] = P_\mu$$

$$H|\phi\rangle = (E_0 + \frac{\Delta}{R})|\phi\rangle$$

$$H P_\mu |\phi\rangle = (E_0 + \frac{\Delta+1}{R}) P_\mu |\phi\rangle$$

$$H K_\mu |\phi\rangle = (E_0 + \frac{\Delta-1}{R}) K_\mu |\phi\rangle$$

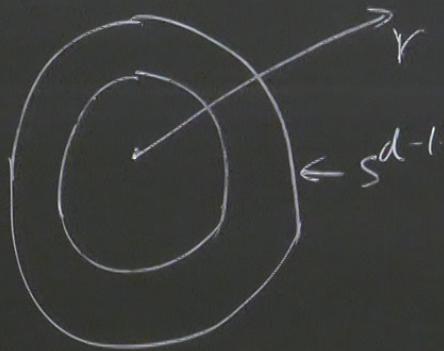
Primary state: $K_\mu |\Phi\rangle = 0$

descendant state: $P_{\mu_1} \dots P_{\mu_n} |\Phi\rangle$

$$E_0 + \frac{\Delta + n}{R}$$

$$K_\mu |0\rangle = 0, \quad P_\mu |0\rangle = 0$$

\mathbb{R}^d (classical)



$$\vec{\pi} = \vec{\pi}$$
$$\mathcal{L} = R \log r$$

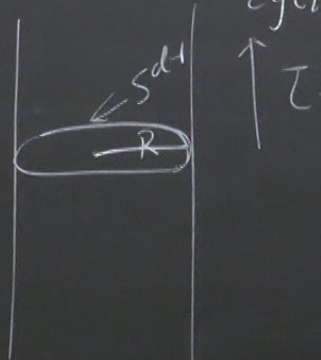
Weyl transf.

$$dS_{\text{flat}}^2 = dr^2 + r^2 d\vec{\pi}^2$$

$$D: r \rightarrow e^\lambda \cdot r$$

$$\langle \hat{O}(x), \hat{O}(y) \rangle = \frac{1}{|x-y|^{2\Delta}}$$

$S^{d-1} \times \mathbb{R}$ (quantum)
cylinder



$$dS_{\text{cyl}}^2 = d\tau^2 + R^2 d\vec{\pi}^2 = \frac{R^2}{r^2} dS_{\text{flat}}^2$$

$$R.H: \tau \rightarrow \tau + R\lambda$$

H: Hamiltonian

$|\Phi\rangle$

$$[H, P_\mu] = P_\mu$$

$$|\phi\rangle = (E_0 + \frac{\Delta}{R}) |\phi\rangle$$

$$P_\mu |\phi\rangle = (E_0 + \frac{\Delta+1}{R}) P_\mu |\phi\rangle$$

$$K_\mu |\phi\rangle = (E_0 + \frac{\Delta-1}{R}) K_\mu |\phi\rangle$$

eg. CFT $\Delta_\sigma = \frac{1}{8}$

Z_2 odd, X_i $\Delta_\epsilon = 1$

Z_2 even, Z_i

Primary state: $K_\mu |\Phi\rangle = 0$

descendant state: $P_{\mu_1} \dots P_{\mu_n} |\Phi\rangle$
 $E_0 + \frac{\Delta_\Phi + n}{R}$

$$K_\mu |0\rangle = 0, \quad P_\mu |0\rangle = 0$$

1st excited: $E_1 - E_0 \sim \frac{\#}{L}$

2nd excited: $E_2 - E_0 \sim \frac{\#}{L}$

$$(E_2 - E_0) = 8(E_1 - E_0)$$

$$= dr^2 + r^2 d\vec{n}^2$$

$$\rightarrow e^\lambda \cdot r$$

$$\langle x | \hat{O}(y) \rangle = \frac{1}{|x-y|^{2\Delta}}$$

$$dS_{\text{cyl}}^2 = dt^2 + R^2 d\vec{n}^2 = \frac{R^2}{r^2} dS_{\text{flat}}^2$$

$$R \cdot H : \mathbb{Z} \rightarrow \mathbb{Z} + R\lambda$$

H: Hamiltonian.

$$|\Phi\rangle$$

2D Ising CFT.

$\sigma = \mathbb{Z}_2$ odd, X_i

$\epsilon = \mathbb{Z}_2$ even, Z_i

$z \rightarrow z^\lambda$ Virasoro primary

$$L_n, \bar{L}_n, n \in \mathbb{Z}$$

$$L_{n>0} |\phi\rangle = \bar{L}_{n>0} |\phi\rangle = 0$$

$$L_1 |\phi\rangle = 0$$

$$P_\mu + K_\mu = \sum e^{i\frac{2\pi}{L} \cdot j} H_j$$

$$H_j = -X_j X_{j+1} - Z_j$$