Title: Quantum Matter Lecture

Speakers: Yin-Chen He

Collection: Quantum Matter 2023/24

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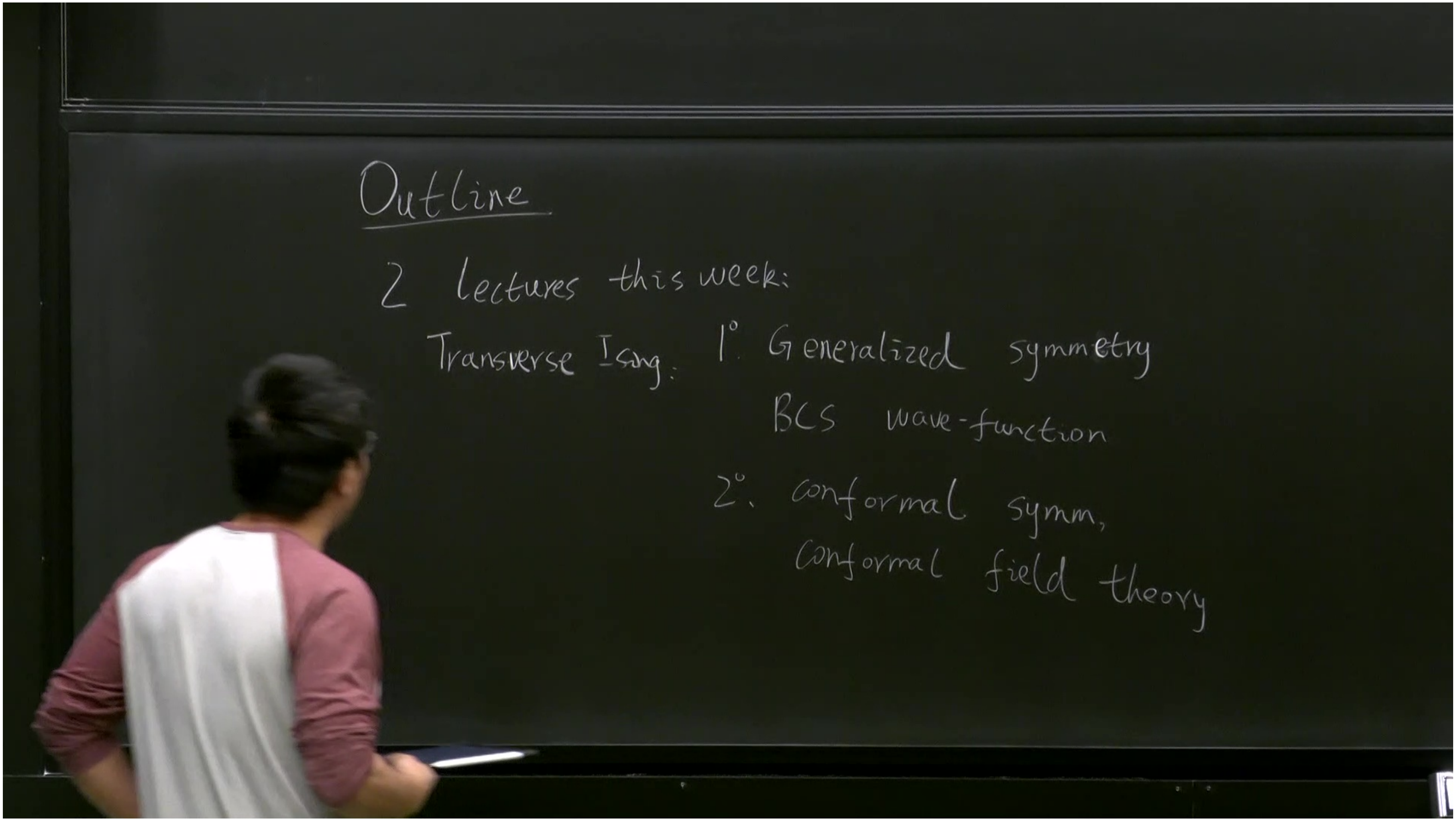
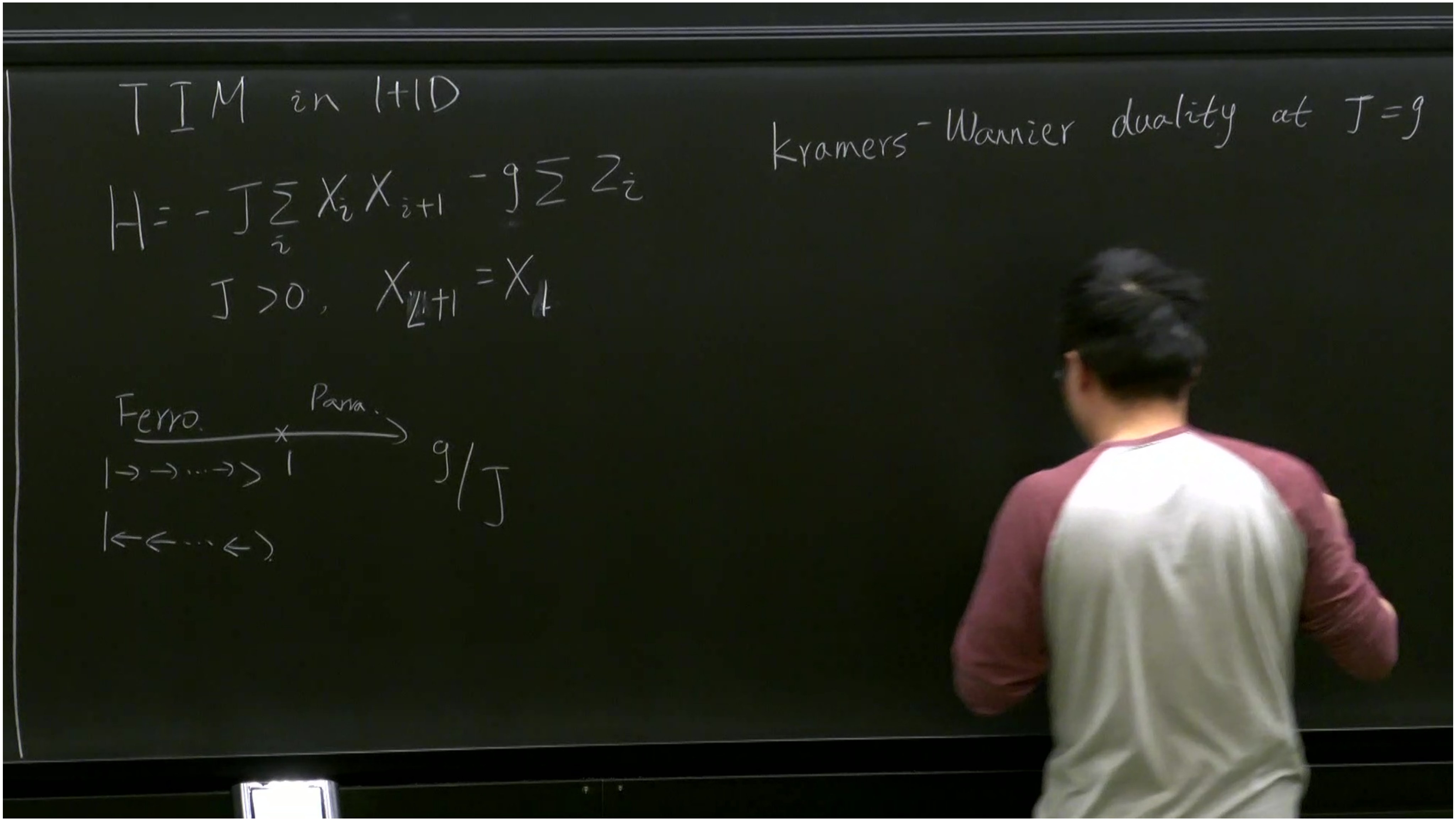
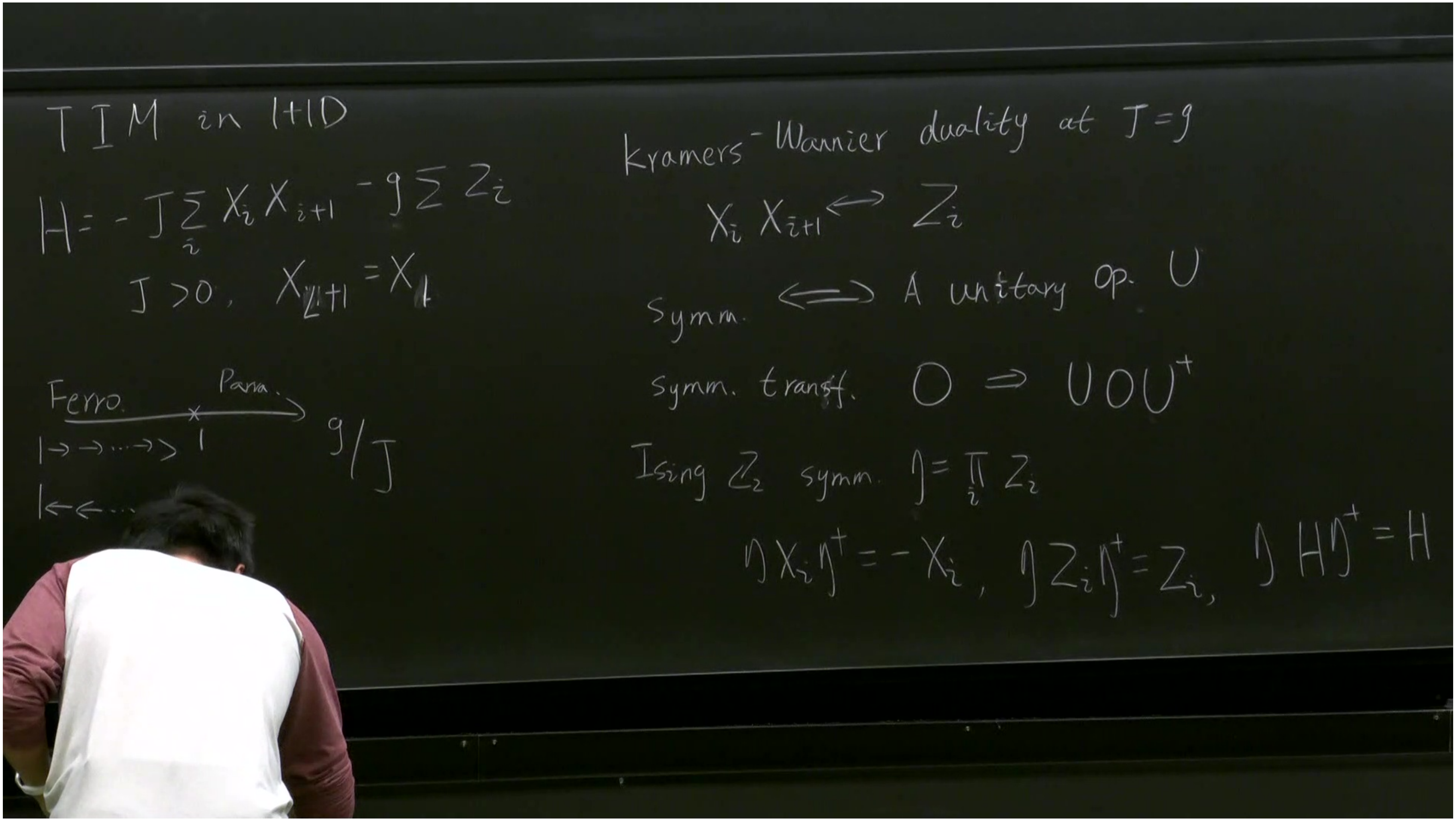

Outline
2 lectures this week:
Transverse Ising: 1°. Generalized symmetry
BCS wave-function
2°. conformal symm,
Conformal field theory

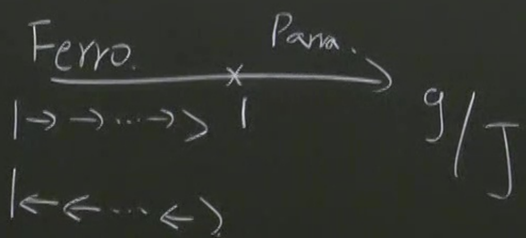

TIM in 1+1D
$H = -J \sum_i X_i X_{i+1} - g \sum Z_i$
$J > 0, \quad X_{L+1} = X_1$
Ferro.
Para.
1
g/J
$|\rightarrow \rightarrow \cdots \rightarrow\rangle$
$|\leftarrow \leftarrow \cdots \leftarrow\rangle$
Kramers-Wannier duality at $J=g$


TIM in 1+1D
$H = -J\sum_i X_i X_{i+1} - g\sum Z_i$
$J > 0$, $X_{L+1} = X_1$
Ferro. Para.
$|\rightarrow\rightarrow\cdots\rightarrow\rangle$ 1 g/J
$|\leftarrow\leftarrow\cdots$
Kramers-Wannier duality at $J=g$
$X_i X_{i+1} \longleftrightarrow Z_i$
Symm. $\Longleftrightarrow$ A unitary op. U
Symm. transf. $O \Rightarrow UOU^\dagger$
Ising $\mathbb{Z}_2$ symm. $\eta = \prod_i Z_i$
$\eta X_i \eta^\dagger = -X_i$, $\eta Z_i \eta^\dagger = Z_i$, $\eta H \eta^\dagger = H$

TIM in 1+1D

$$H = -J \sum_i X_i X_{i+1} - g \sum Z_i$$

$$J > 0, \quad X_{L+1} = X_1$$



Kramers-Wannier duality at $J = g$

$$X_i X_{i+1} \longleftrightarrow Z_i$$

Symm. $\Longleftrightarrow$ A unitary op. U

Symm. transf. $O \Rightarrow U O U^\dagger$

Ising $\mathbb{Z}_2$ symm. $\eta = \prod_i Z_i$

$$\eta X_i \eta^\dagger = -X_i, \quad \eta Z_i \eta^\dagger = Z_i, \quad \eta H \eta^\dagger = H$$

Assume there is U. $U Z_i U^\dagger = X_i X_{i+1}$

$$U \eta U^\dagger = U\Big(\prod_{j=1}^{L} Z_j\Big)U^\dagger = \prod_{j=1}^{L} (U Z_j U^\dagger) = \prod_{j=1}^{L} (X_j X_{j+1}) = 1$$

$$\eta = 1$$

old symm.: $U O_1 U^\dagger = O_2$ $\Rightarrow$ Generalized symm.: $U O_1 = O_2 U$

$U^{-1} = U^\dagger$

U^{-1} doesn't exist.

Non-invertible symm.

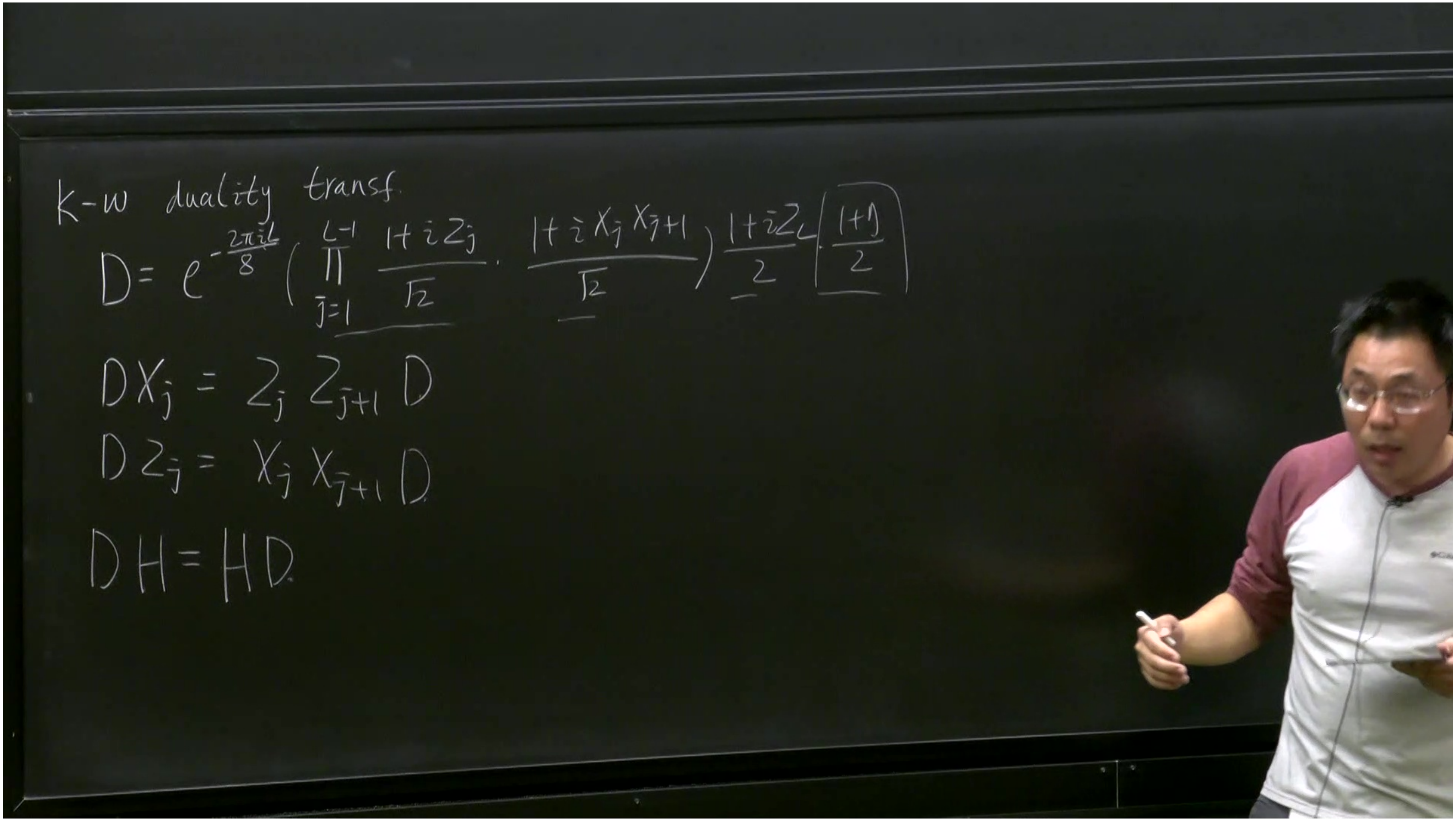
K-W duality transf.

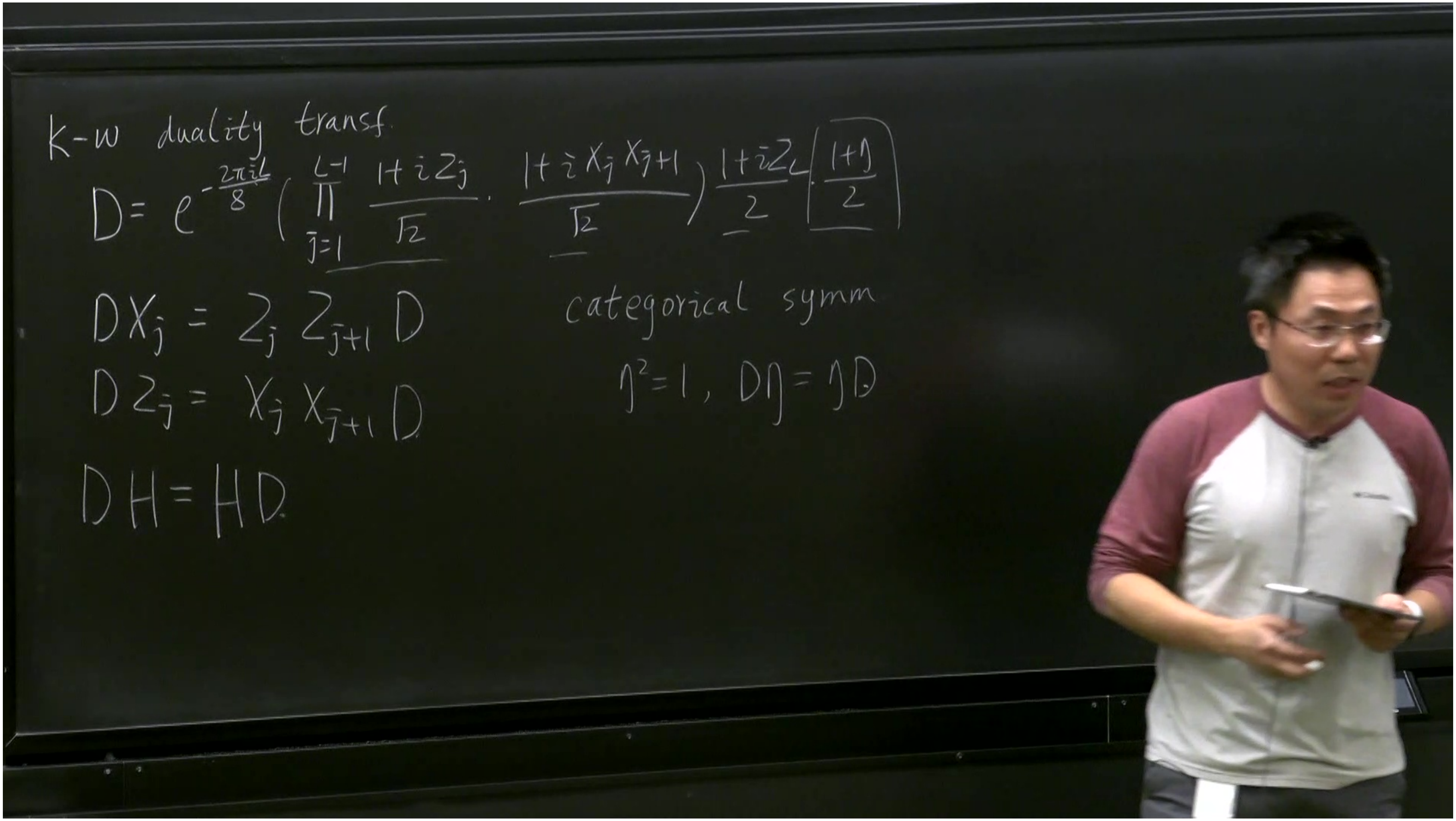
$$D = e^{-\frac{2\pi i L}{8}} \left(\prod_{j=1}^{L-1} \frac{1+iZ_j}{\sqrt{2}} \cdot \frac{1+iX_jX_{j+1}}{\sqrt{2}} \right) \frac{1+iZ_L}{2} \cdot \frac{1+\eta}{2}$$

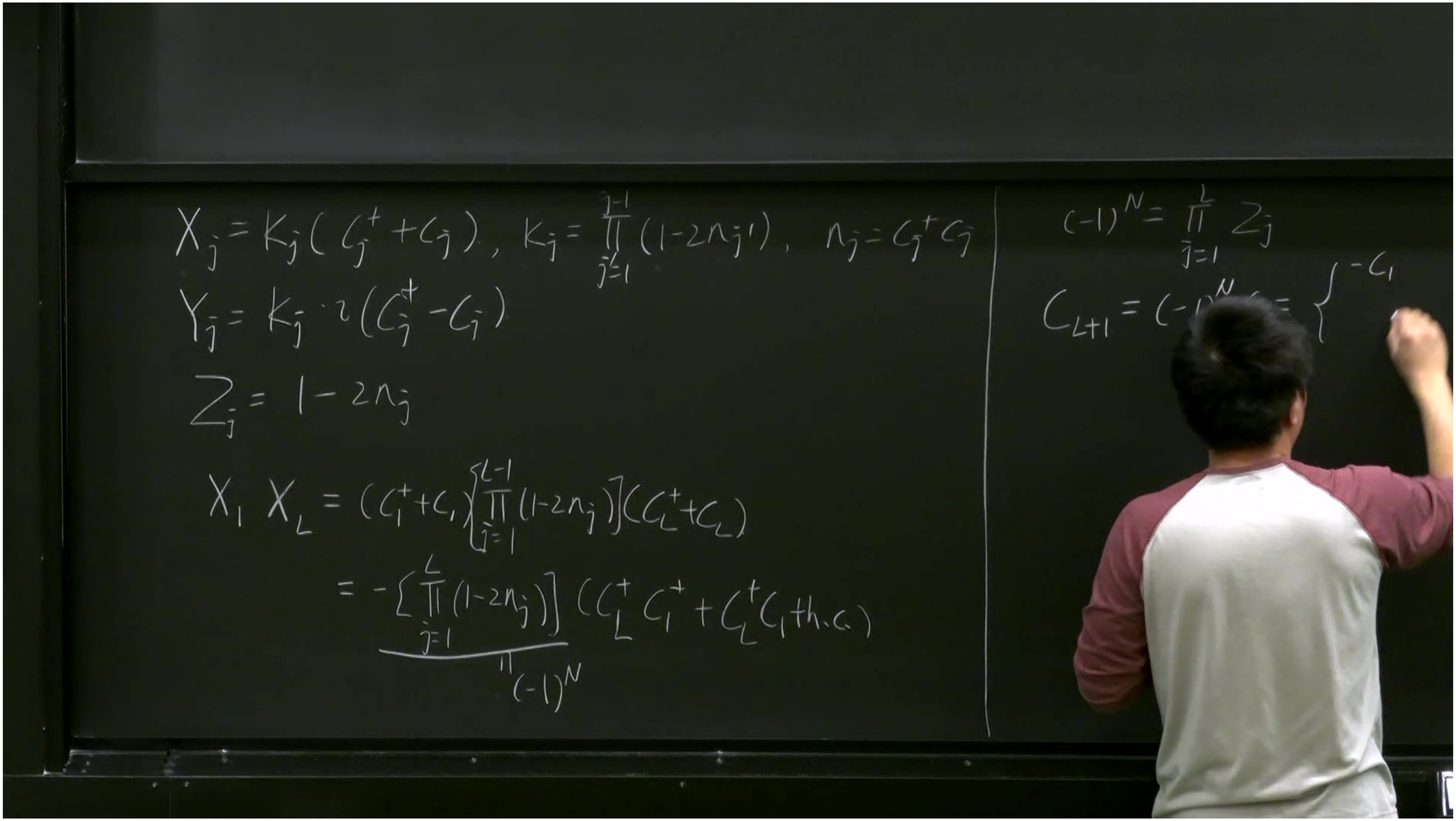
$$D X_j = Z_j Z_{j+1} D$$

$$D Z_j = X_j X_{j+1} D$$

$$DH = HD$$


K-W duality transf.
$D = e^{-\frac{2\pi i L}{8}} \left(\prod_{j=1}^{L-1} \frac{1+iZ_j}{\sqrt{2}} \cdot \frac{1+iX_jX_{j+1}}{\sqrt{2}} \right) \frac{1+iZ_L}{2} \cdot \frac{1+\eta}{2}$
$DX_j = Z_j Z_{j+1} D$
$DZ_j = X_j X_{j+1} D$
$DH = HD$


K-W duality transf.
$D = e^{-\frac{2\pi i L}{8}} \left(\prod_{j=1}^{L-1} \frac{1+iZ_j}{\sqrt{2}} \cdot \frac{1+iX_j X_{j+1}}{\sqrt{2}} \right) \frac{1+iZ_L}{2} \cdot \frac{1+\eta}{2}$
$D X_j = Z_j Z_{j+1} D$
$D Z_j = X_j X_{j+1} D$
$DH = HD$
categorical symm
$\eta^2 = 1, \; D\eta = \eta D$



$X_j = K_j(c_j^\dagger + c_j)$, $K_j = \prod_{j'=1}^{j-1}(1-2n_{j'})$, $n_j = c_j^\dagger c_j$
$Y_j = K_j \cdot i(c_j^\dagger - c_j)$
$Z_j = 1 - 2n_j$
$X_1 X_L = (c_1^\dagger + c_1)\left[\prod_{j=1}^{L-1}(1-2n_j)\right](c_L^\dagger + c_L)$
$= -\left[\prod_{j=1}^{L}(1-2n_j)\right](c_L^\dagger c_1^\dagger + c_L^\dagger c_1 + h.c.)$
$(-1)^N$
$(-1)^N = \prod_{j=1}^{L} Z_j$
$c_{L+1} = (-1)^N c_1 = \{ -c_1$

$+C_j$

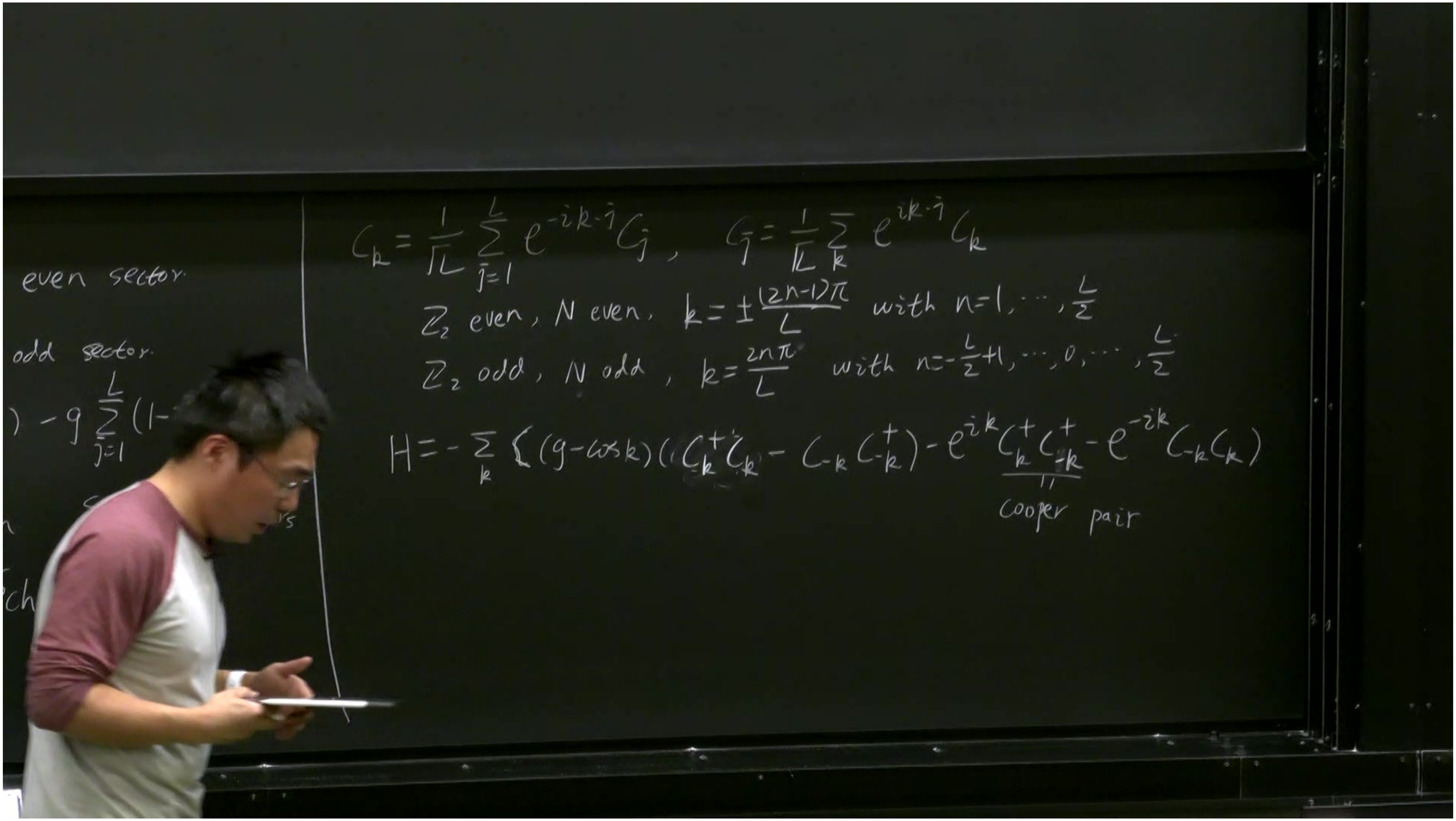
$$(-1)^N = \prod_{j=1}^{L} Z_j$$

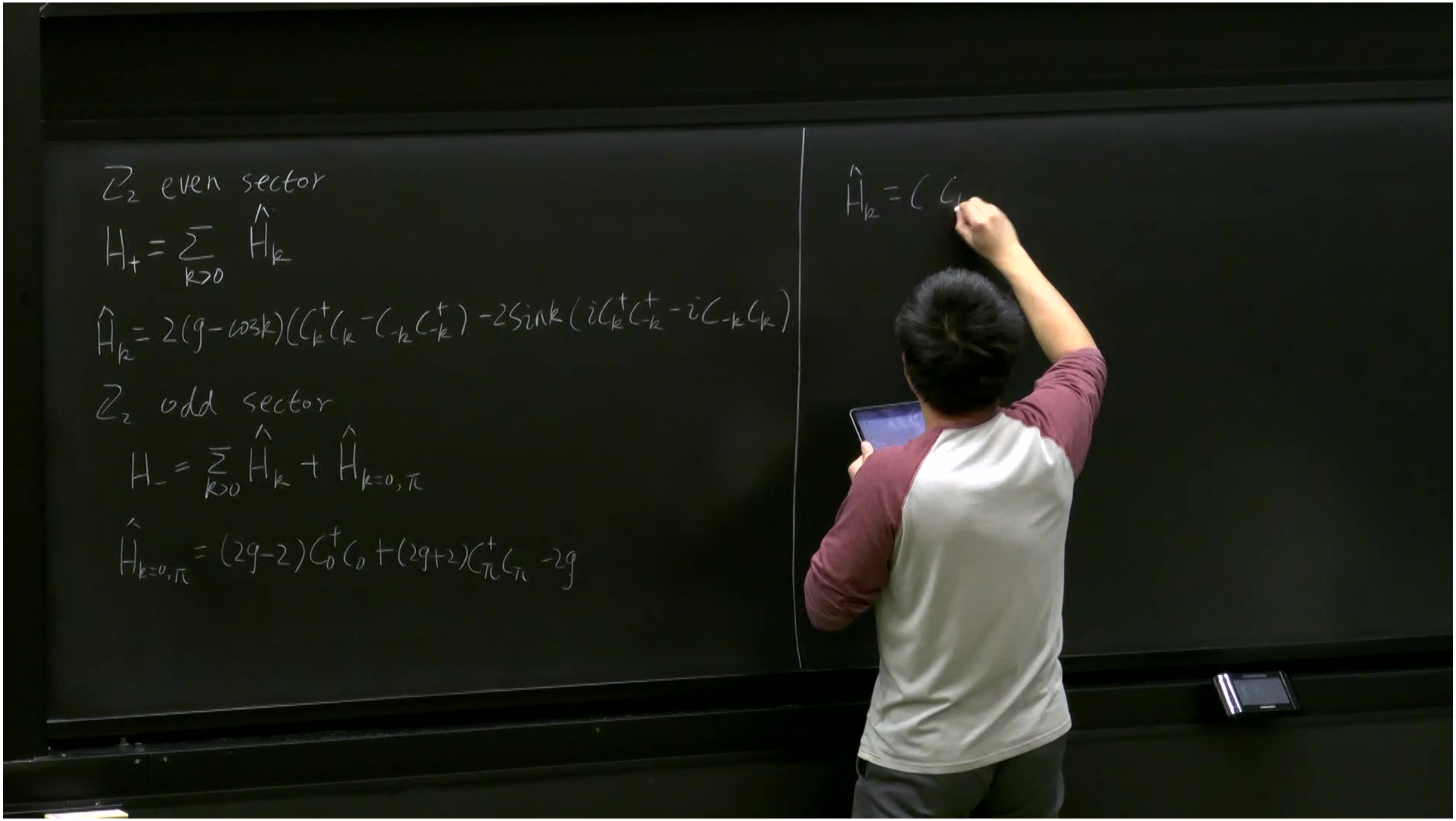
$$C_{L+1} = (-1)^N C_1 = \begin{cases} -C_1, & \mathbb{Z}_2 \text{ even sector} \\ C_1, & \mathbb{Z}_2 \text{ odd sector} \end{cases}$$

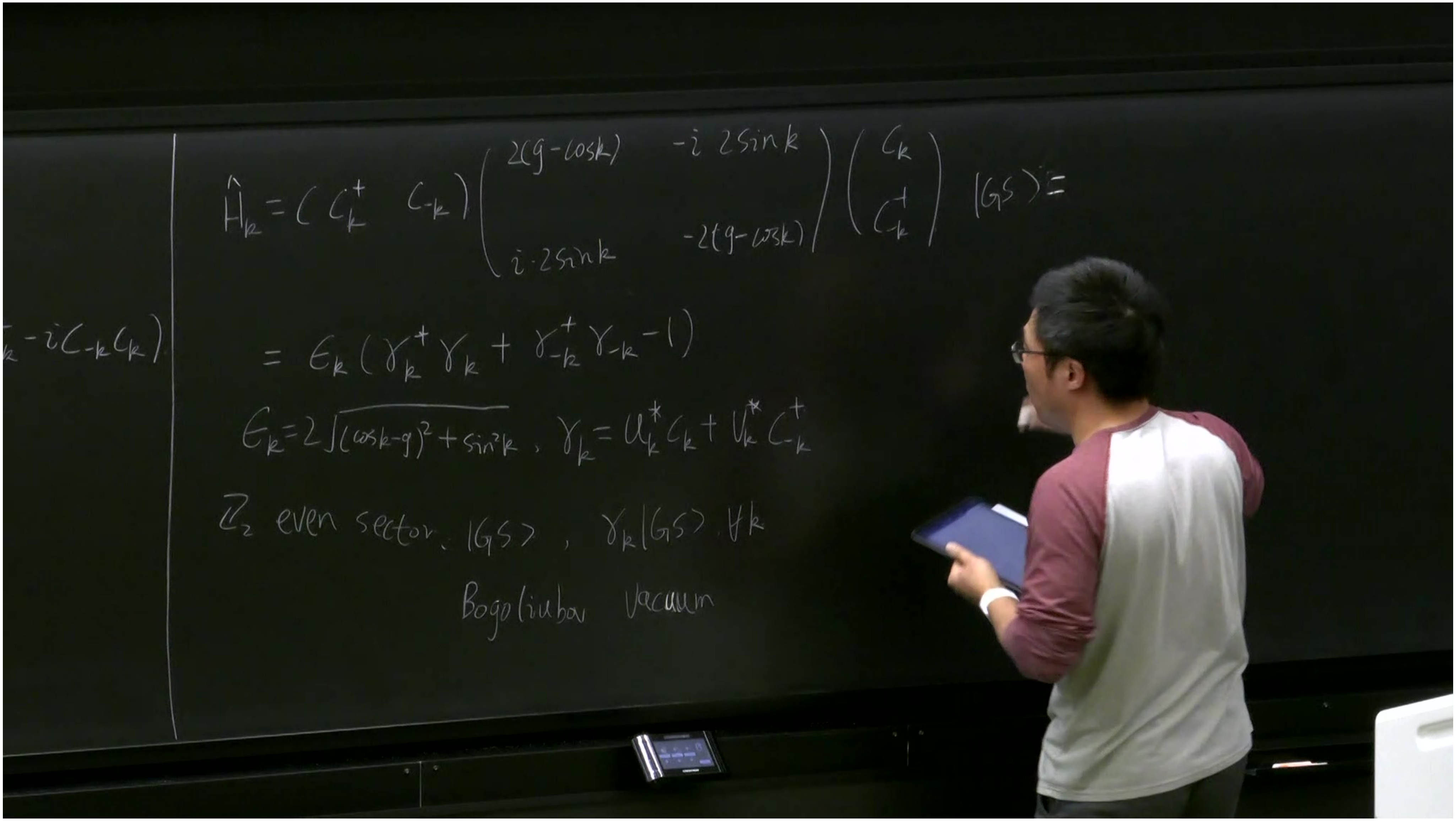
$$H = -\sum_{j=1}^{L} \left(C_j^{+} C_{j+1}^{+} + C_j^{+} C_{j+1} + h.c. \right) - g \sum_{j=1}^{L} \left(1 - 2 C_j^{+} C_j \right)$$

BCS Hamiltonian superconductors

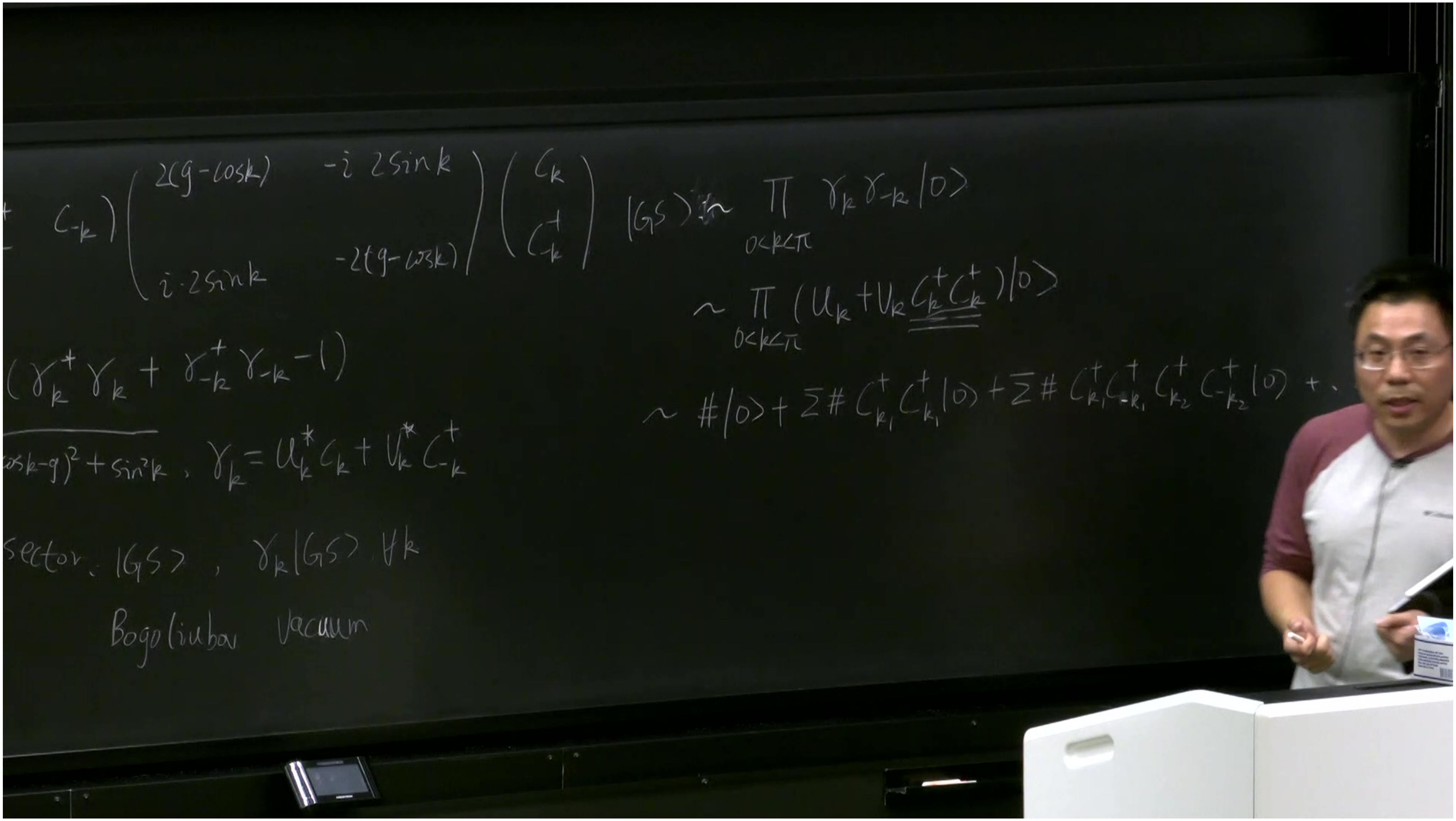
Bardeen – Cooper – Schrieffer

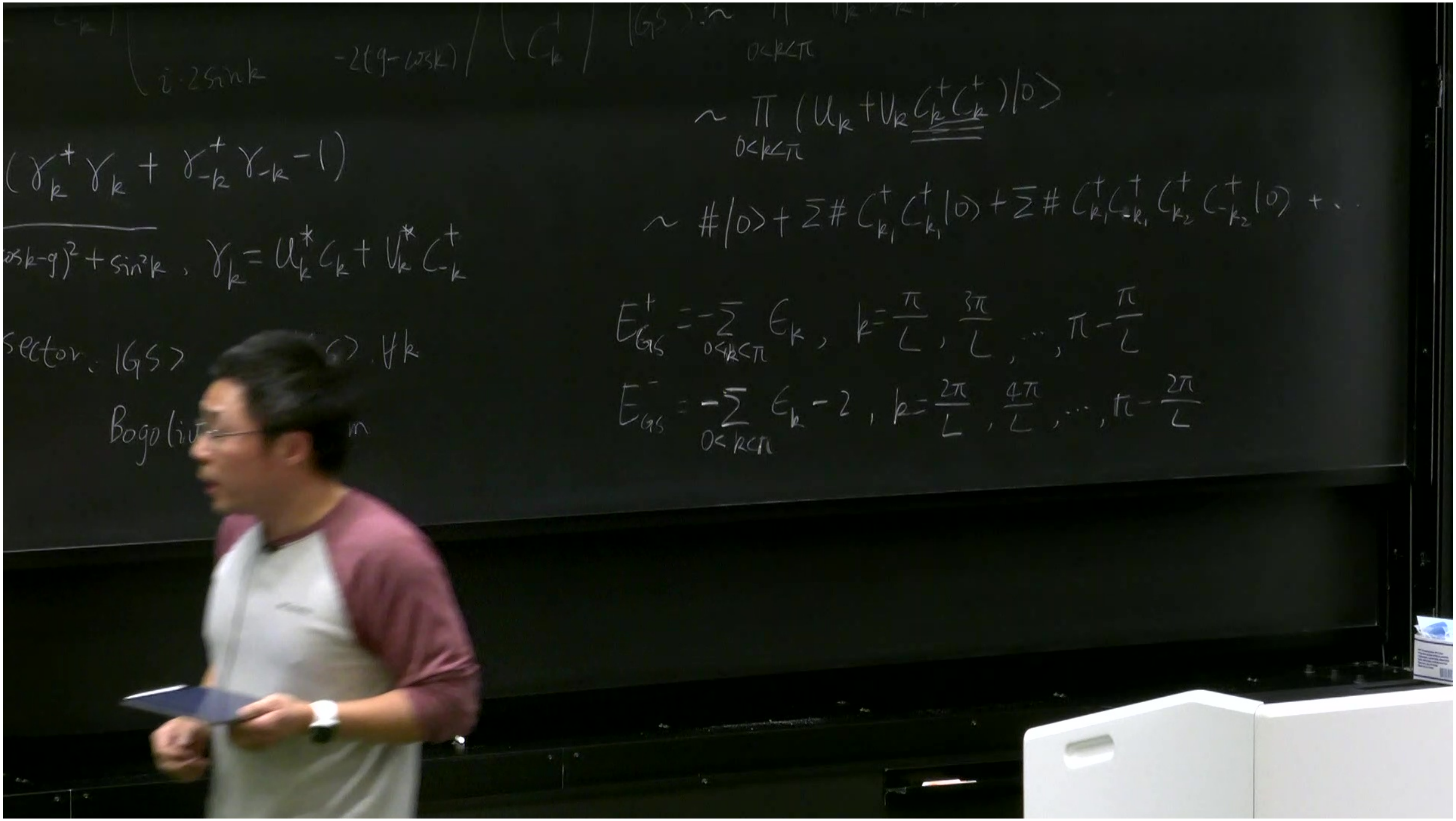

even sector
odd sector
$c_k = \frac{1}{\sqrt{L}} \sum_{j=1}^{L} e^{-ik\cdot j} c_j, \quad c_j = \frac{1}{\sqrt{L}} \sum_k e^{ik\cdot j} c_k$
$\mathbb{Z}_2$ even, N even, $k = \pm \frac{(2n-1)\pi}{L}$ with $n = 1, \cdots, \frac{L}{2}$
$\mathbb{Z}_2$ odd, N odd, $k = \frac{2n\pi}{L}$ with $n = -\frac{L}{2}+1, \cdots, 0, \cdots, \frac{L}{2}$
$H = -\sum_k \{ (g - \cos k)(c_k^+ c_k - c_{-k} c_{-k}^+) - e^{ik} c_k^+ c_{-k}^+ - e^{-ik} c_{-k} c_k)$
cooper pair

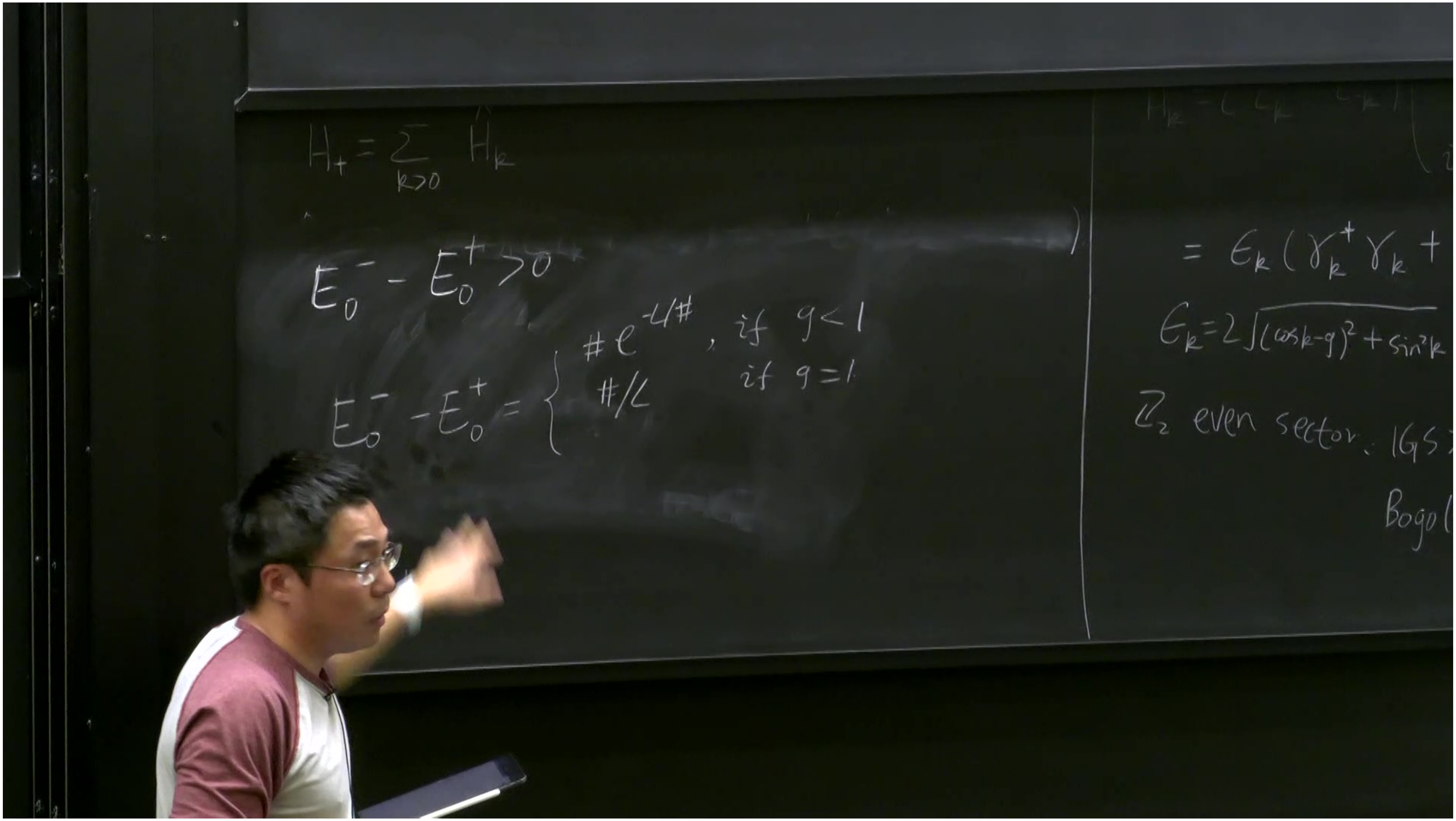

Z_2 even sector
$H_+ = \sum_{k>0} \hat{H}_k$
$\hat{H}_k = 2(g-\cos k)(C_k^\dagger C_k - C_{-k} C_{-k}^\dagger) - 2\sin k\,(i C_k^\dagger C_{-k}^\dagger - i C_{-k} C_k)$
Z_2 odd sector
$H_- = \sum_{k>0} \hat{H}_k + \hat{H}_{k=0,\pi}$
$\hat{H}_{k=0,\pi} = (2g-2) C_0^\dagger C_0 + (2g+2) C_\pi^\dagger C_\pi - 2g$
$\hat{H}_k = (C$

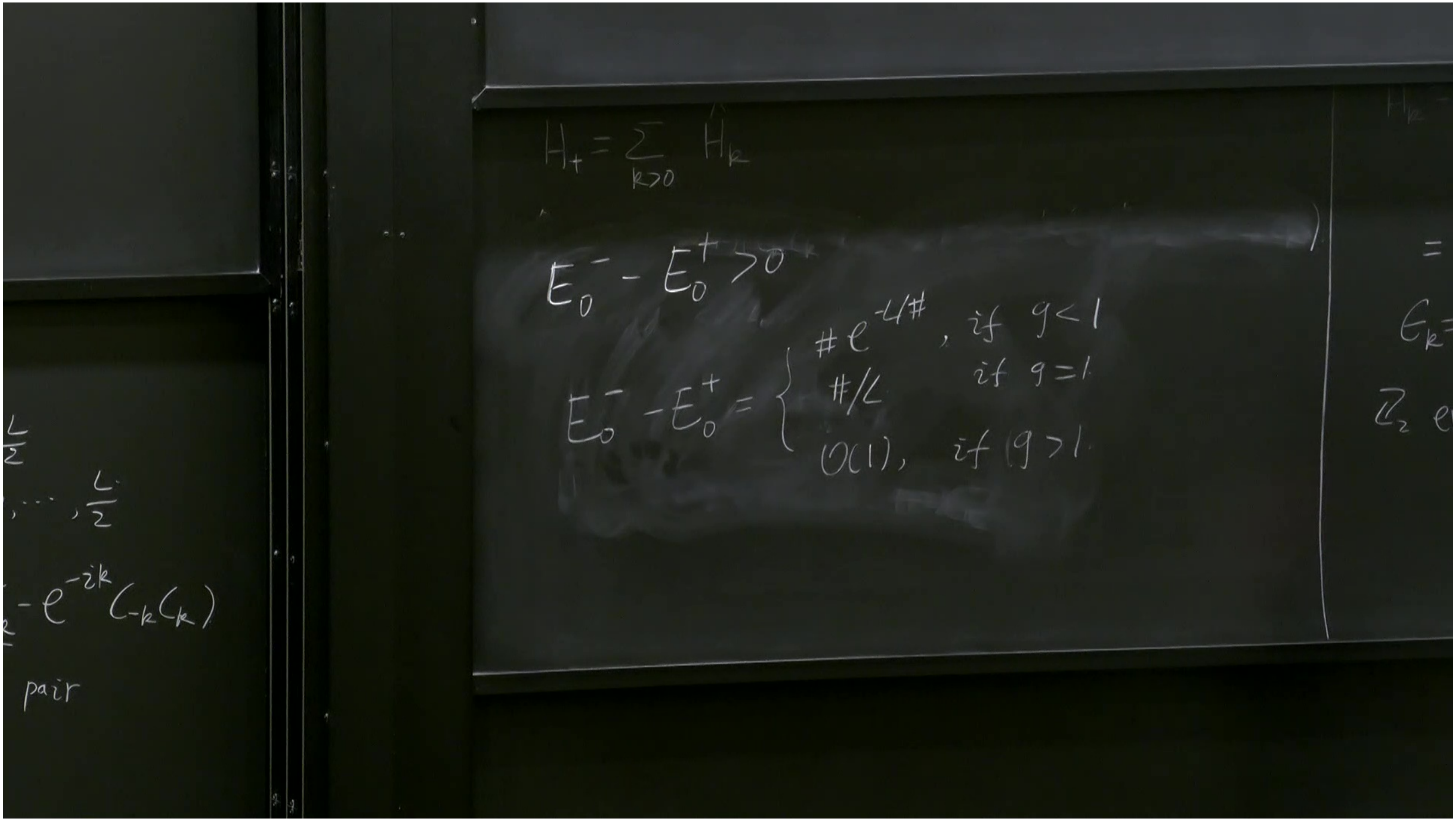


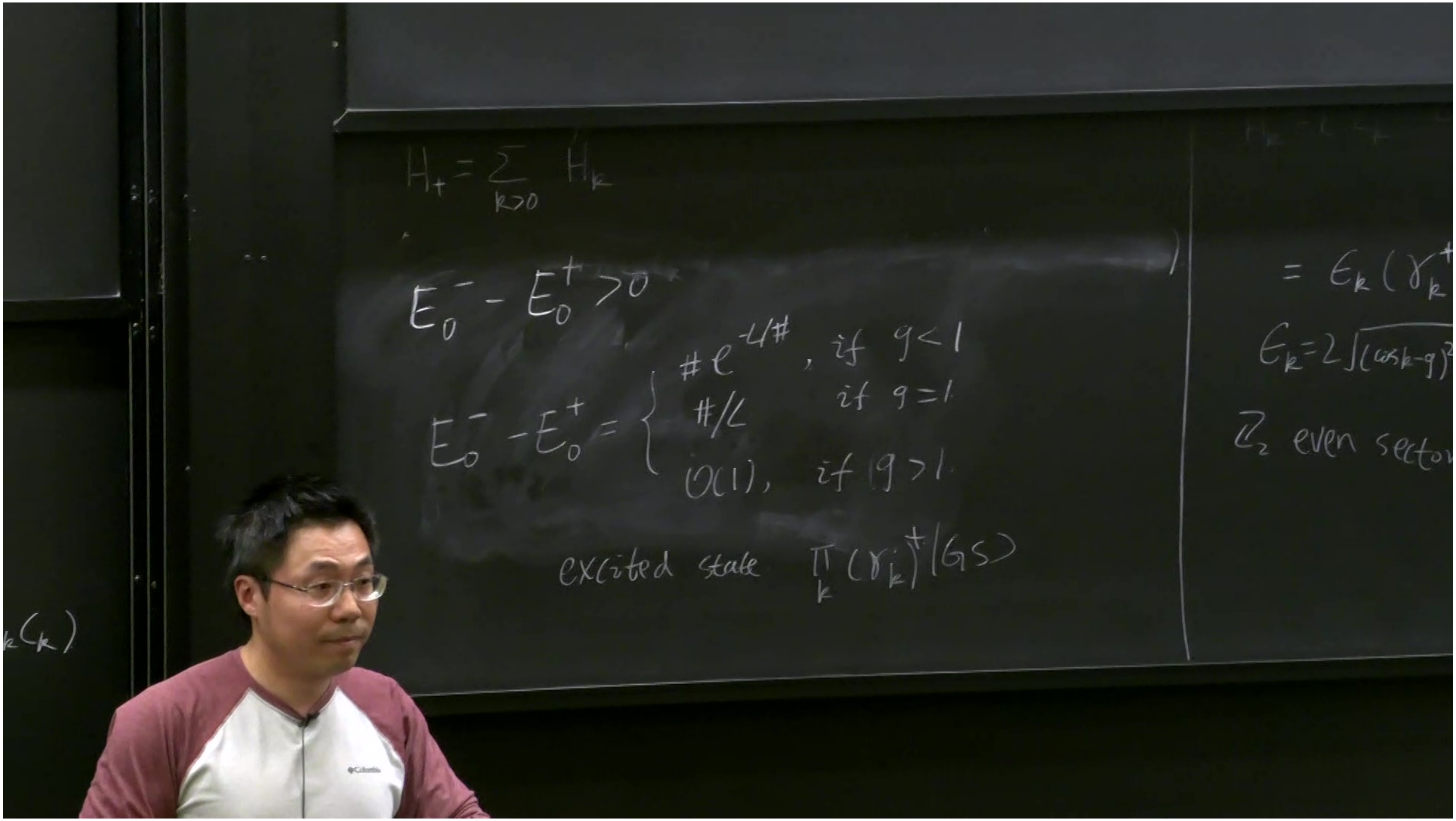
$-iC_{-k}C_k)$
$\hat{H}_k = (C_k^\dagger \;\; C_{-k}) \begin{pmatrix} 2(g-\cos k) & -i\,2\sin k \\ i\cdot 2\sin k & -2(g-\cos k) \end{pmatrix} \begin{pmatrix} C_k \\ C_{-k}^\dagger \end{pmatrix}$ $|GS\rangle =$
$= \epsilon_k (\gamma_k^\dagger \gamma_k + \gamma_{-k}^\dagger \gamma_{-k} - 1)$
$\epsilon_k = 2\sqrt{(\cos k - g)^2 + \sin^2 k}$, $\gamma_k = U_k^* C_k + V_k^* C_{-k}^\dagger$
$\mathbb{Z}_2$ even sector: $|GS\rangle$, $\gamma_k |GS\rangle$ $\forall k$
Bogoliubov Vacuum


2(g-cosk)   -i 2sink
i·2sink   -2(g-cosk)
|GS> ~ Π_{0<k<π} γ_k γ_{-k} |0>
~ Π_{0<k<π} (U_k + V_k C_k^+ C_{-k}^+) |0>
~ # |0> + Σ # C_{k_1}^+ C_{-k_1}^+ |0> + Σ # C_{k_1}^+ C_{-k_1}^+ C_{k_2}^+ C_{-k_2}^+ |0> + ...
(γ_k^+ γ_k + γ_{-k}^+ γ_{-k} - 1)
γ_k = U_k^* C_k + V_k^* C_{-k}^+
sector: |GS> , γ_k |GS> ∀k
Bogoliubov vacuum


~ ∏_{0<k<π} (U_k + V_k c_k^† c_{-k}^†)|0>
~ #|0> + Σ # c_{k1}^† c_{-k1}^† |0> + Σ # c_{k1}^† c_{-k1}^† c_{k2}^† c_{-k2}^† |0> + ...
(γ_k^† γ_k + γ_{-k}^† γ_{-k} − 1)
γ_k = U_k^* c_k + V_k^* c_{-k}^†
E_GS^+ = −Σ_{0<k<π} ε_k , k = π/L, 3π/L, ..., π − π/L
E_GS^- = −Σ_{0<k<π} ε_k − 2 , k = 2π/L, 4π/L, ..., π − 2π/L
sector: |GS>
Bogo


H+ = Σ k>0 Ĥk
E0− − E0+ > 0
E0− − E0+ = { # e^(−L/#), if g<1
#/L if g=1
= εk (γk† γk +
εk = 2√((cos k − g)² + sin²k
Z2 even sector: |GS
Bogo(


$H_+ = \sum_{k>0} \hat{H}_k$
$E_0^- - E_0^+ > 0$
$E_0^- - E_0^+ = \begin{cases} \# e^{-L/\#}, & \text{if } g<1 \\ \#/L & \text{if } g=1 \\ O(1), & \text{if } g>1 \end{cases}$
$- e^{-ik} c_{-k} c_k)$
pair


H+ = Σ Hk
k>0
E0⁻ − E0⁺ > 0
E0⁻ − E0⁺ = { # e^{-L/#}, if g<1 ; #/L if g=1 ; O(1), if g>1
excited state Π_k (γk)† |GS⟩
= εk (γk†
εk = 2 √(cos k − g)²
Z2 even sector