Title: QFT III Lecture

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Exact computation of correl

Last time: path integral $\rightsquigarrow$ operators & Hilbert space $\mathcal{H}$

- Construct states out of primaries $|\phi\rangle = \phi(0,0)|0\rangle$ $\in \mathbb{CP}^1 = \mathbb{C}\cup\{\infty\}$

  $\langle\phi| = \lim_{z,\bar{z}\to\infty} z^{2h}\bar{z}^{2\bar{h}}\langle 0|\phi(z,\bar{z})$,

- Operators can be expanded in modes: $\phi(z,\bar{z}) = \sum_{n,m\in\mathbb{Z}} z^{-n-h}\bar{z}^{-m-\bar{h}}\phi_{n,m}$

  $\phi^\dagger_{n,m} = \phi_{-n,-m}$

- To match corr. func. in path integral we need radial quantization

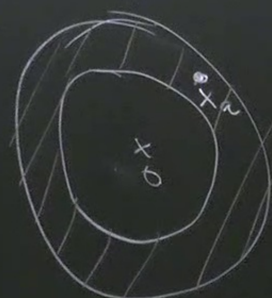
$$R\,\phi_1(z)\phi_2(w) = \begin{cases} \phi_1(z)\phi_2(w) & \text{if } |z|>|w| \\ \phi_2(w)\phi_1(z) & \text{if } |w|>|z| \end{cases}$$

...lation function from algebra!]

- It is important when we compute action of generators of symmetries:

If we have Ward id.: $\partial_\mu \langle j^\mu(y) X \rangle = \sum_a \delta(y - x_a)\, i G_a \langle X \rangle$

Take integral $\int d^2x$: $\int_{\partial\Omega} dn_\mu \langle j^\mu(y) X \rangle = i G_a \langle X \rangle$



$= \Omega$:

$$\Downarrow$$

$$\langle \dots [Q, \phi_a] \dots \rangle = \langle \dots i G_a \phi_a(x_a) \dots \rangle$$

$$Q = \oint_{|z|=R} dn_\mu j^\mu$$

Last time we applied this to conformal symmetry:

$$Q = \oint_{|z|=R} dz\, \epsilon(z)\, T(z) = \sum_{k\in\mathbb{Z}} \epsilon_k L_k \qquad T(z) = \sum_{k\in\mathbb{Z}} z^{-k-2} L_k$$

$\xrightarrow{\text{using}}$ OPEs:

$$\boxed{[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^2-1)n\,\delta_{n+m,0} \quad \text{— Virasoro algebra}}$$

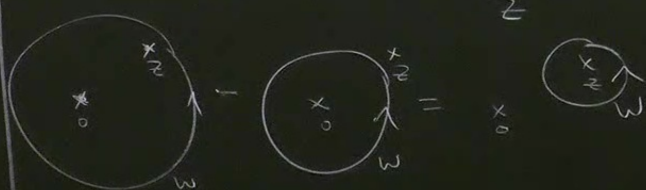
- We can interpret generators $L_0, \bar{L}_0$ in a useful way:

$$\begin{cases} l_0 = -z\frac{\partial}{\partial z} \\ \bar{l}_0 = -\bar{z}\frac{\partial}{\partial \bar{z}} \end{cases} \quad \begin{matrix} z = e^{\frac{2\pi}{L}(t+ix)} \\ \bar{z} = e^{\frac{2\pi}{L}(t-ix)} \end{matrix} \quad \rightsquigarrow \begin{cases} l_0 + \bar{l}_0 \sim \frac{\partial}{\partial t} \\ l_0 - \bar{l}_0 \sim \frac{\partial}{\partial x} \end{cases} \Rightarrow \begin{cases} L_0 + \bar{L}_0 = H \\ i(L_0 - \bar{L}_0) = P \end{cases}$$

So how does the symmetry acts on primary fields?

$$[L_n, \phi(z,\bar z)] = \oint_z \frac{dw}{2\pi i} w^{n+1} T(w)\phi(z) = \oint_z \frac{dw}{2\pi i} w^{n+1}\left(\frac{h\,\phi(z)}{(w-z)^2} + \frac{\partial\phi(z)}{w-z}\right) =$$

$$= h(n+1) z^{n}\phi(z) + z^{n+1}\partial\phi(z)$$



or in terms of modes: $[L_n, \phi_m] = (h(n-1)-m)\phi_{n+m}$

· What about action of L_n on assymptotic states?

$$|h,\bar h\rangle = \lim_{z\to 0}\phi(z,\bar z)|0\rangle = \lim_{z\to 0}\sum_{n,m\in\mathbb{Z}} z^{-n-h}\,\bar z^{-m-\bar h}\,\phi_{n,m}|0\rangle = \left|\begin{array}{l}\phi_{n,m}|0\rangle = 0 \\ \text{if } n>-h \\ \quad\;\; m>-\bar h\end{array}\right| = \phi_{-h,-\bar h}|0\rangle$$

$T(z)|0\rangle$ is non-singular at $z\to 0$

$\Rightarrow$ $L_n|0\rangle = 0$ for $n \geq -1$

$l_0 = -z\frac{\partial}{\partial z}$ $\quad z = e^{\frac{-i}{L}(t-ix)}$ $\quad l_0 - \bar{l}_0 \sim \frac{\partial}{\partial x}$ $\Rightarrow$ $i(L_0 - \bar{L}_0) = P$

$|h,\bar h\rangle$: $\quad L_0|h,\bar h\rangle = h|h,\bar h\rangle \qquad L_n|h,\bar h\rangle = 0$ for $n \geq 1$

$\bar L_0|h,\bar h\rangle = \bar h|h,\bar h\rangle \qquad \bar L_n|h,\bar h\rangle = 0$

$L_n \phi_{-h,-\bar h}|0\rangle = (\phi_{-h,-\bar h} L_n + (n(h-$

In mathematical literature $V_{h,c}$ - Verma moduls, in physics — conformal f

· Important formula $L_0|\lambda,h\rangle = (h + \sum_{i=1}^{n}\lambda_i)|\lambda,h\rangle \Leftarrow [L_0, L_n] = -$

· Sets λ are called Young diagrams in math!

$\lambda = \{1\}$ $\qquad \lambda = \{1,1\}$ $\qquad \lambda = \{3,1\}$

$\Rightarrow \quad L_n|0\rangle = 0 \quad \text{for} \quad n \geq -1$

for $n \geq 1$

$$V_{h,c} = \langle \bar{L}_{-\mu_m} \cdots \bar{L}_{-\mu_1} \underbrace{L_{-\lambda_n} \cdots L_{-\lambda_1} |h,\bar{h}\rangle}_{|\lambda, h\rangle} \mid \begin{matrix} \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n > 0 \\ \mu_1 \geq \cdots\cdots \geq \mu_m > 0 \end{matrix} \rangle$$

descendant states.

$\phi_{-h,-\bar{h}} L_n + (n(h-1)+h)\phi_{-h+n,\bar{h}})|0\rangle$

conformal family.

$[L_0, L_n] = -n L_n$

$\lambda = \{3,1\}$

$|\lambda, h\rangle = L_{-1} L_{-2} |h\rangle$

$\Rightarrow \quad L_n|0\rangle = 0 \quad \text{for} \quad n \geq -1$

for $n \geq 1$

$$V_{h,c} = \langle \bar{L}_{-\mu_m} \cdots \bar{L}_{-\mu_1} \underbrace{L_{-\lambda_n} \cdots L_{-\lambda_1} |h,\bar{h}\rangle}_{|\lambda, h\rangle} \mid \begin{array}{l} \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n > 0 \\ \mu_1 \geq \cdots\cdots \geq \mu_m > 0 \end{array} \rangle$$

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conformal family.

$[L_0, L_n] = -n L_n$

$\lambda = \{3,1\}$

$|\lambda, h\rangle = L_{-1} L_{-2} |h\rangle$

"How many" states there are in $V_{h,c}$?

$$\mathrm{tr}_{V_{h,c}}(q^{L_0}) = q^h \sum_{N \geq 0} \#_N q^N =$$

$$= q^h(1 + q + 2q^2 + \ldots$$

$(L_{-1})^2 \qquad L_{-2}$

for $n \geq 1$

$$V_{h,c} = \langle L_{-\mu_m} \dots L_{-\mu_1} \underbrace{L_{-\lambda_n} \dots L_{-\lambda_1} |h, \bar{h}\rangle}_{|\lambda, h\rangle} \;\Big|\; \begin{matrix} \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n > 0 \\ \mu_1 \geq \dots\dots \geq \mu_m > 0 \end{matrix} \rangle$$

descendant states

$$\left(\phi_{-h,-\bar{h}} L_n + (n(h-1)+h)\phi_{-h+n,\bar{h}}\right)|0\rangle$$

— conformal family.

$$= [L_0, L_n] = -n L_n$$

$$|\lambda, h\rangle = L_{-1} L_{-2} |h\rangle$$

$\lambda = \{3, 1\}$

"How many" states there are in $V_{h,c}$?

$$\mathrm{tr}_{V_{h,c}}(q^{L_0}) = q^h \sum_{N \geq 0} \#_N q^N =$$

$$= q^h(1 + q + 2q^2 + 3q^3 + \dots) = q^h \prod_{n>0} \frac{1}{1-q^n}$$

$(L_{-1})^2$, L_{-2} ; L_{-3}, $L_{-2}L_{-1}$, $(L_{-1})^3$

Hilbert space is of CFT in 2d : $\mathcal{H} = \bigoplus_{h, \bar{h}} V_{h, \bar{h}, c}$: so the OPEs in general

Theory-dependent data :
- set of primaries $(h, \bar{h})$
- central charge(s) $c, \bar{c}$
- Three-point functions C_{h_1, h_2, h_3}

...s in general $\phi_{h_1}(z)\,\phi_{h_2}(w) = \sum_{h_3} \sum_{\lambda,\mu} C^{h_3,\lambda,\mu}_{h_1,h_2}\,(z-w)^{h_3-h_1-h_2+|\lambda|}\,(\bar{z}-\bar{w})^{\bar{h}_3-\bar{h}_1-\bar{h}_2+|\mu|}$

$\cdot\,(L_{-\lambda}\bar{L}_{-\mu}\phi_{h_3})(w)$

$(L_{-1})^{\lambda}\,(\bar{L}_{-1})^{\mu}$

$\to h_3$ over all primaries. in theory

this constants can be algorithmically computed

$$C^{h_3,\lambda,\mu}_{h_1,h_2} = C_{h_1,h_2,h_3}\cdot\beta^{\lambda}(h_1,h_2,h_3)\,\bar{\beta}^{\mu}(\bar{h}_1;\bar{h}_2;\bar{h}_3)$$

theory-dependent constant

universal functions.

$$C_{h_1,h_2,h_3} = \langle h_1|\phi_{h_2}(1)|h_3\rangle$$

...he OPEs in general

$$\phi_{h_1}(z)\,\phi_{h_2}(w) = \sum_{h_3}\sum_{\lambda,\mu} C^{h_3,\lambda,\mu}_{h_1,h_2}\,(z-w)^{h_3-h_1-h_2+|\lambda|}\,(\bar z-\bar w)^{\bar h_3 \ldots}\cdot\big(L_{-\lambda}\bar L_{-\mu}\phi_{h_3}\big)(w)$$

$$L_{-\lambda} = (L_{-1})^{\lambda}, \quad \bar L_{-\mu} = (\bar L_{-1})^{\mu}$$

h_3: over all primaries in theory

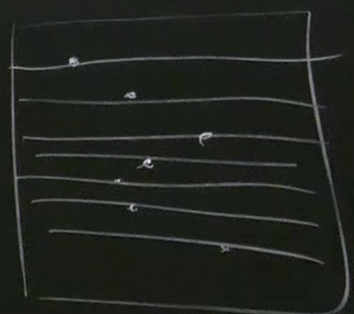
this constants can be algorithmically computed

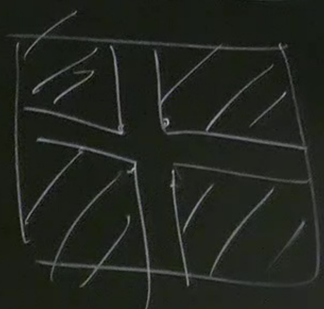
$$C^{h_3,\lambda,\mu}_{h_1,h_2} = C_{h_1,h_2,h_3}\cdot\beta^{\lambda}(h_1,h_2,h_3)$$

theory-dependent constant

universal fu...

$$C_{h_1,h_2,h_3} = \langle h_1|\phi_{h_2}(1)|h_3\rangle$$





$\ldots,h_2,h_3$

$$\langle\phi_{h_1}\phi_{h_2}\phi_{h_3}\phi_{h_4}\rangle = \sum C^{h,\lambda}_{h_1h_2}\langle L_{-\lambda}\phi_h\,\phi_{h_3}\,\phi_{h_4}\rangle = \sum C^{h,\lambda}_{h_1h_2}\,C^{h,\mu}_{h_3h_4}\,\mathcal{F}$$

$, c$

es $(h, \bar{h})$

s) $C, \bar{C}$

nctions C_{h_1,h_2,h_3}

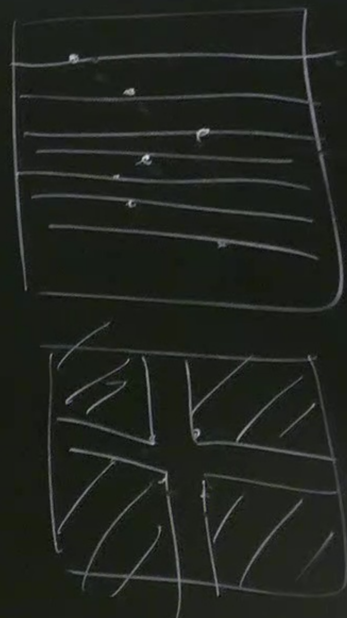
; so the OPEs in general $\phi_{h_1}(z)\,\phi_{h_2}(w) = \sum_{h_3} \sum_{\lambda,\mu} C^{h_3,\lambda,\mu}_{h_1,h_2} (z-w)^{h_3-}$

$\cdot (L_-$

over all primaries in theory

this constants can be algorithmic

$C^{h_3,\lambda,\mu}_{h_1,h_2} = C_{h_1,h}$



$$\langle \phi_{h_1}\phi_{h_2}\phi_{h_3}\phi_{h_4}\rangle = $$

$$= \sum C^{h,\lambda}_{h_1 h_2} \langle L_{-\lambda}\phi_h \phi_{h_3}\phi_{h_4}\rangle$$

$$= \sum C^{h,\lambda}_{h_1 h_2} C^{h,\mu}_{h_3 h_4} F$$

theory-dependen

$C_{h_1,h_2,h_3} =$