Title: QFT III Lecture

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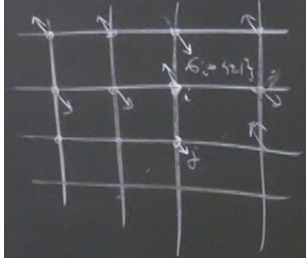
2d Statistical models and their critical points



N sites, 2^N configurations

$$\mathbb{P}(s) = \frac{1}{Z} e^{-\frac{E(s)}{T}}$$

Partition function

$$Z = \sum_s e^{-\frac{E(s)}{T}}$$

$$\Rightarrow \sum_s \mathbb{P}(s) = 1$$

Ising model: $E(s) = -h \sum_i \sigma_i - k \sum_{<ij>} \sigma_i \sigma_j$

Thermodynamical parameter: (S, T)

(V, P)

(M, h)

extensive intensive

$Z \sim$

$$Z \sim \exp\left(S - \frac{U = \langle E \rangle}{T}\right), \quad \langle Q \rangle = \sum_{s} \mathbb{P}(s)\, Q(s)$$

$$= e^{-F/T} \Rightarrow \boxed{F = -T \log Z = -TS + U}$$

$$\boxed{T\,dS = dU + p\,dV}$$

$$\left.\right\} \quad p = -\frac{\partial F}{\partial V}, \quad S = -\frac{\partial F}{\partial T}$$

Ideal gas: $p = n k_B T$

$Q(s)$

$$p = -\frac{\partial F}{\partial V}$$

$$S = -\frac{\partial F}{\partial T}$$

Ideal gas: $p = n k_B T$

$Q(s)$

$$\boxed{p = -\frac{\partial F}{\partial V}}$$

$$S = -\frac{\partial F}{\partial T}$$

$$M = \frac{\partial F}{\partial h}$$

Q(S)

$$\boxed{p = -\frac{\partial F}{\partial V}}$$

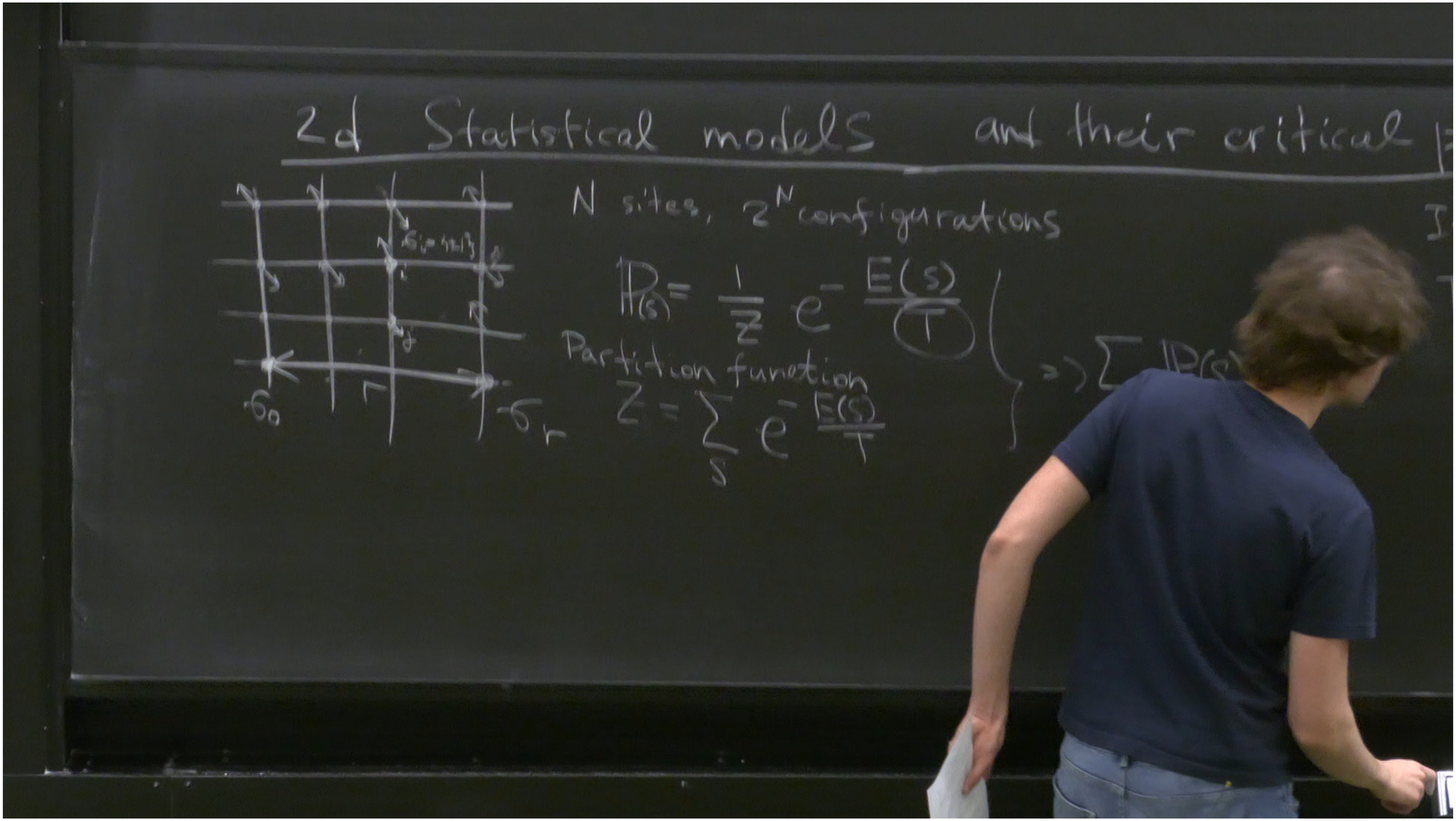
Ideal gas: $p = n k_B T$

$$S = -\frac{\partial F}{\partial T}$$

$$M = \frac{\partial F}{\partial h}$$

Two point correlation functions (at large distances):

$$G(r) = \langle \sigma_r \sigma_0 \rangle - \langle \sigma_r \rangle \langle \sigma_0 \rangle$$


2d Statistical models and their critical
N sites, 2^N configurations
$\mathbb{P}(s) = \frac{1}{Z} e^{-\frac{E(s)}{T}}$
Partition function
$Z = \sum_s e^{-\frac{E(s)}{T}}$

$Q(s)$

$$\boxed{p = -\frac{\partial F}{\partial V}}$$

Ideal gas: $p = n k_B T$

$$S = -\frac{\partial F}{\partial T}$$

$$M = \frac{\partial F}{\partial h}$$

Two point correlation functions (at large distances):

$$G(r) = \langle \sigma_r \sigma_0 \rangle - \langle \sigma_r \rangle \langle \sigma_0 \rangle$$

$$r \longrightarrow \infty \quad G(r) = r^{-\frac{d-1}{2}}\, g(r/\xi)$$

$$g \propto e^{-r/\xi}$$

ξ - correlation lengts.

$Q(s)$

$$\boxed{p = -\frac{\partial F}{\partial V}}$$

$$S = -\frac{\partial F}{\partial T}$$

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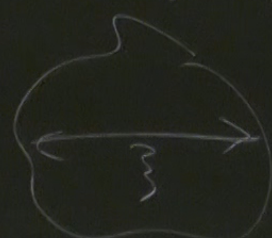


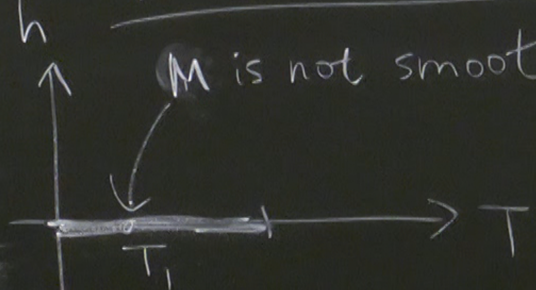
$$g \propto e^{-r/\xi}$$

ξ - correlation lengts.

Phase transitions : when F is not "smooth enough"

• First order phase trans: firs

M is not smooth



$$M(T_1; h=+0) = -M(T_1; h=-0)$$

F is not "smooth enough"

- First order phase trans: first derivatives of F are not smooth.
- Second order: second derivatives.

  Correlation length - diverges.

$$\chi = \frac{\partial^2 F}{\partial h^2} = \sum_{i,j}$$

$$C = \frac{\partial^2 F}{\partial T^2}$$

-0)

$Q(s)$

$$\boxed{p = -\frac{\partial F}{\partial V}} \longrightarrow \text{Ideal gas: } p = n k_B T$$

$$S = -\frac{\partial F}{\partial T}$$

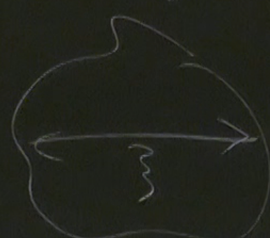
$$M = \frac{\partial F}{\partial h}$$

Two point correlation functions (at large distances):

$$G_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle$$

$$G(r) = \langle \sigma_r \sigma_0 \rangle - \langle \sigma_r \rangle \langle \sigma_0 \rangle$$

$$r \longrightarrow \infty \quad G(r) = r^{-\frac{d-1}{2}} g(r/\xi)$$



$$g \propto e^{-r/\xi}$$

ξ - correlation lengts.

F is not "smooth enough"

- First order phase trans: first derivatives of F are not smooth.
- Second order: — $\|$ — second derivatives. – – –

$r/\xi = 1$

Correlation length diverges.

$$\chi = \frac{\partial^2 F}{\partial h^2} = \sum_{i,j} G(|i-j|) \propto V \int_0^{+\infty} r^{d-1} G(r)\,dr = V \int_0^{+\infty} r^{d-1} \cdot r^{-\frac{d-1}{2}} g(r/\xi)\,dr \sim \xi^{\frac{d+1}{2}}$$

$h = -0)$

$$C = \frac{\partial^2 F}{\partial T^2}$$

$\boxed{\text{Magnetic suc. diverges with corr. length!}}$

$$t = \frac{T - T_c}{T_c}$$

h

$$C\Big|_{h=0} \propto |t|^{-\alpha}$$

$$M\Big|_{h=+0} \propto (-t)^{\beta} \qquad M\Big|_{t=0} \propto |h|^{1/\delta}$$

$$\chi\Big|_{h=0} \propto |t|^{-\gamma}$$

$$t = \frac{T - T_c}{T_c}$$

h

$\alpha, \beta, \gamma, \delta$ -

- thermodynamic critical exponents.

$$C\big|_{h=0} \propto |t|^{-\alpha}$$

$$M\big|_{h=+0} \propto (-t)^{\beta}$$

$$\chi\big|_{h=0} \propto |t|^{-\gamma}$$

$$M\big|_{t=0} \propto |h|^{1/\delta}$$

$$\xi \propto |$$

$$\xi \propto |t|^{-\nu}$$

$$G\left(r,\ \substack{t=0,\\ h=0}\right) \propto 1/r^{d-2+\eta}$$

$$t = \frac{T - T_c}{T_c}$$

h

$\alpha, \beta, \gamma, \delta$ -

- thermodynamic critical exponents.

ν, η - "correlation function" critical exponents.

$$C\big|_{h=0} \propto |t|^{-\alpha}$$

$$M\big|_{h=+0} \propto (-t)^{\beta}$$

$$\chi\big|_{h=0} \propto |t|^{-\gamma}$$

$$M\big|_{t=0} \propto |h|^{1/\delta}$$

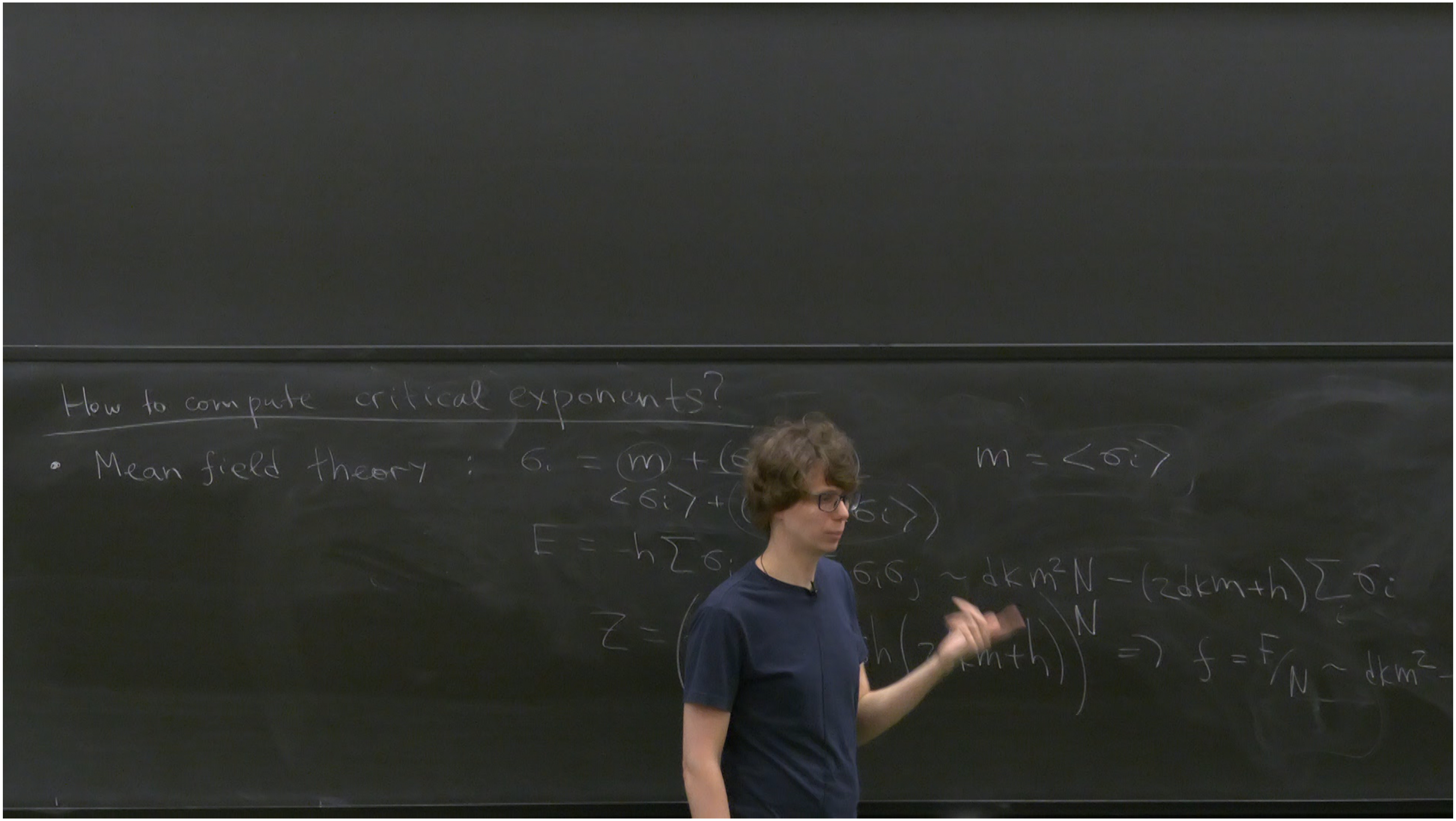
$\xi \propto$

$G(r,$

$$\xi \propto |t|^{-\nu}$$

$$G(r, \underset{h=0}{t=0}) \propto 1/r^{d-2+\eta}$$

$$\langle \phi_\Delta(x)\, \phi_\Delta(0) \rangle = \frac{1}{x^{2\Delta}}$$

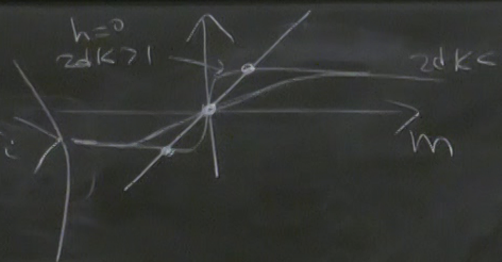

How to compute critical exponents?
• Mean field theory :
m = <σi>

exponents?

$$\sigma_i = m + (\sigma_i - m)$$

$$\langle \sigma_i \rangle + (\sigma_i - \langle \sigma_i \rangle)$$

$$m = \langle \sigma_i \rangle$$



$$m = \tanh(\underbrace{2dk}_{2dK}m + h)$$

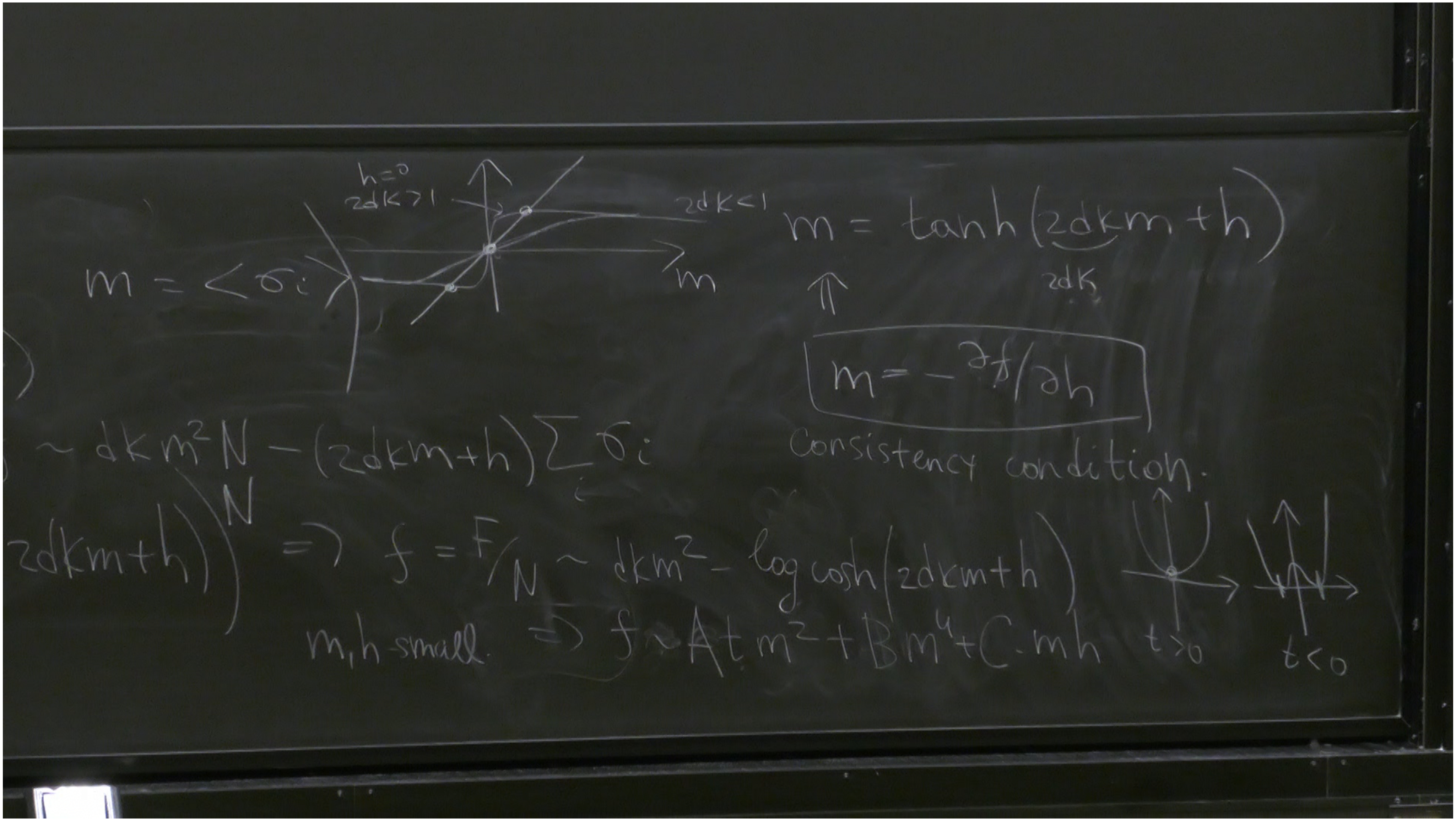
$$\Uparrow$$

$$\boxed{m = -\partial f / \partial h}$$

Consistency condition.

$$E = -h\sum \sigma_i - k \sum \sigma_i \sigma_j \sim dkm^2 N - (2dkm+h)\sum_i \sigma_i$$

$$Z = \left(2e^{-dkm^2}\cosh(2dkm+h)\right)^N \Rightarrow f = F/N \sim dkm^2 - \log\cosh(2dkm+h)$$



h=0
2dK>1
2dK<1
m = <σ_i>
m
m = tanh(2dKm + h)
2dK
⇑
m = −∂f/∂h
~ dKm²N − (2dKm+h) Σ_i σ_i
Consistency condition.
2dKm+h))^N ⇒ f = F/N ~ dKm² − log cosh(2dKm+h)
m, h-small ⇒ f ~ At m² + B m⁴ + C·mh
t>0
t<0

How to compute critical exponents?

- Mean field theory :

$\alpha=0,\ \beta=1/2,\ \gamma=1,\ \delta=3$

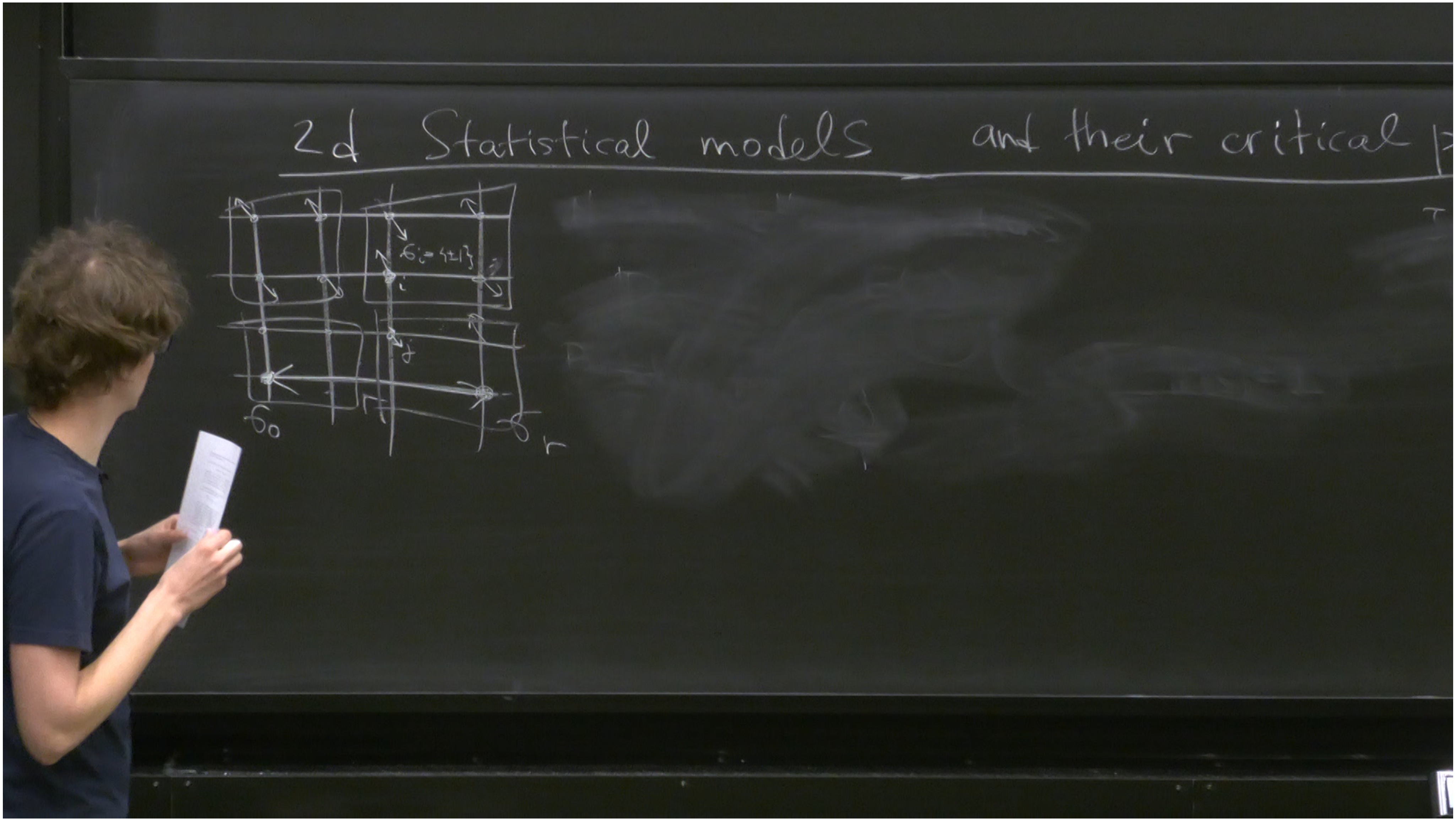
$\nu=1/2,\ \eta=0$

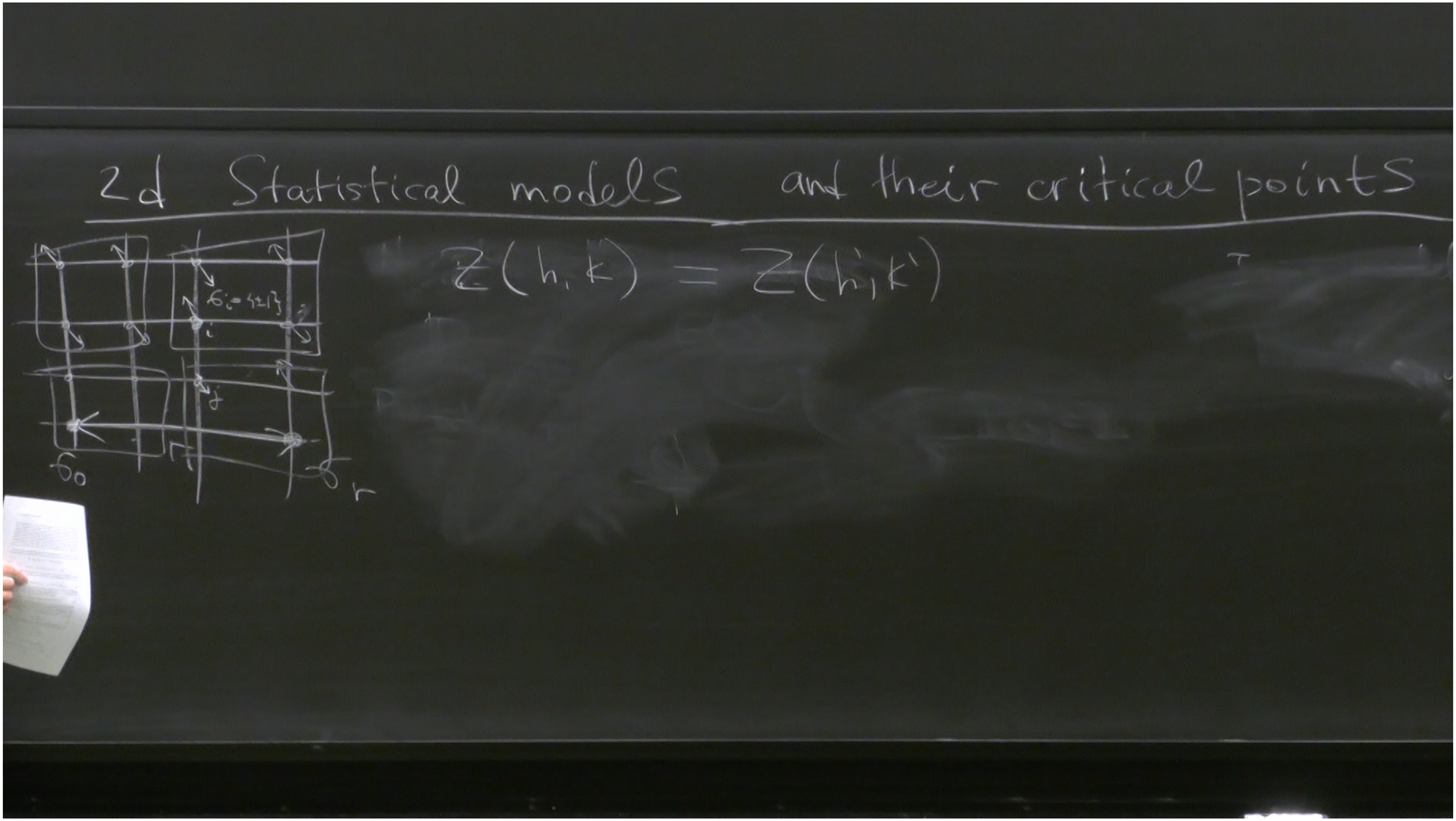
$\sigma_i = m + (\sigma_i - m)$

$\langle\sigma_i\rangle + (\sigma_i - \langle\sigma_i\rangle)$

$E = -h\sum \sigma_i - k\sum \sigma_i\sigma_j$

$Z = \Big(2e^{-dkm^2}\cosh(2$


2d Statistical models and their critical
σi = {±1}
i
j
σ0
σr


2d Statistical models and their critical points
Z(h, k) = Z(h', k')
σi = {±1}
i
j
σ0
σr

cal models and their critical points

$$Z(h,k) = Z(h',k')$$

$$\frac{E(S)}{T} = -K_0 - K_1\sum_i \sigma_i - K_2\sum_{i,j}\sigma_i\sigma_j - K_3\sum_{i,j,k}\sigma_i\sigma_j\sigma_k - \cdots$$

$$Z(\{K\}) = Z(\{K'\}) \Rightarrow K' = RG(K)$$

$$N' = b^{-d}N$$

$$\xi' = b^{-1}\xi$$

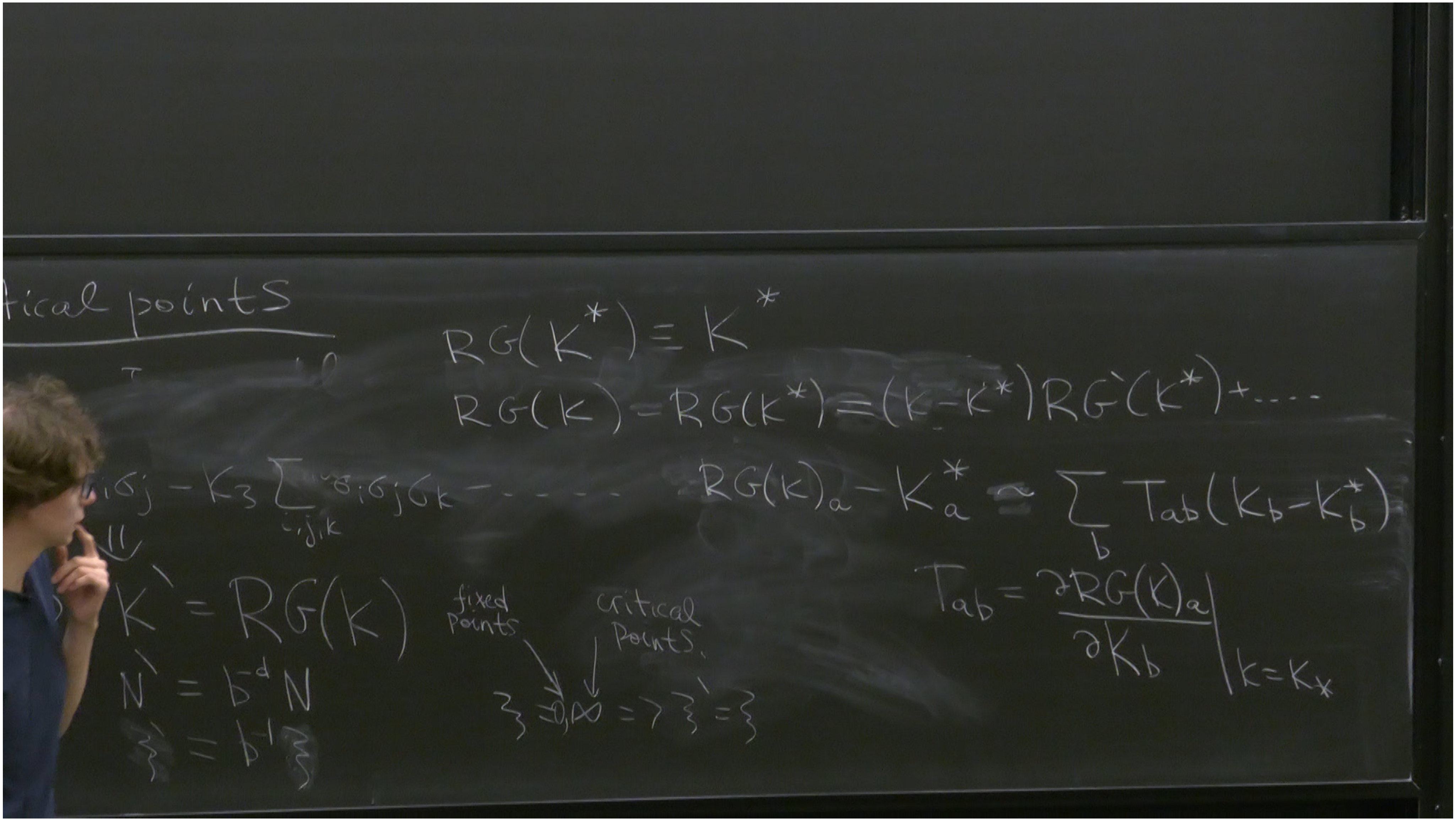
$$RG(K^*) = K^*$$

$$RG(K) = RG(K^*$$

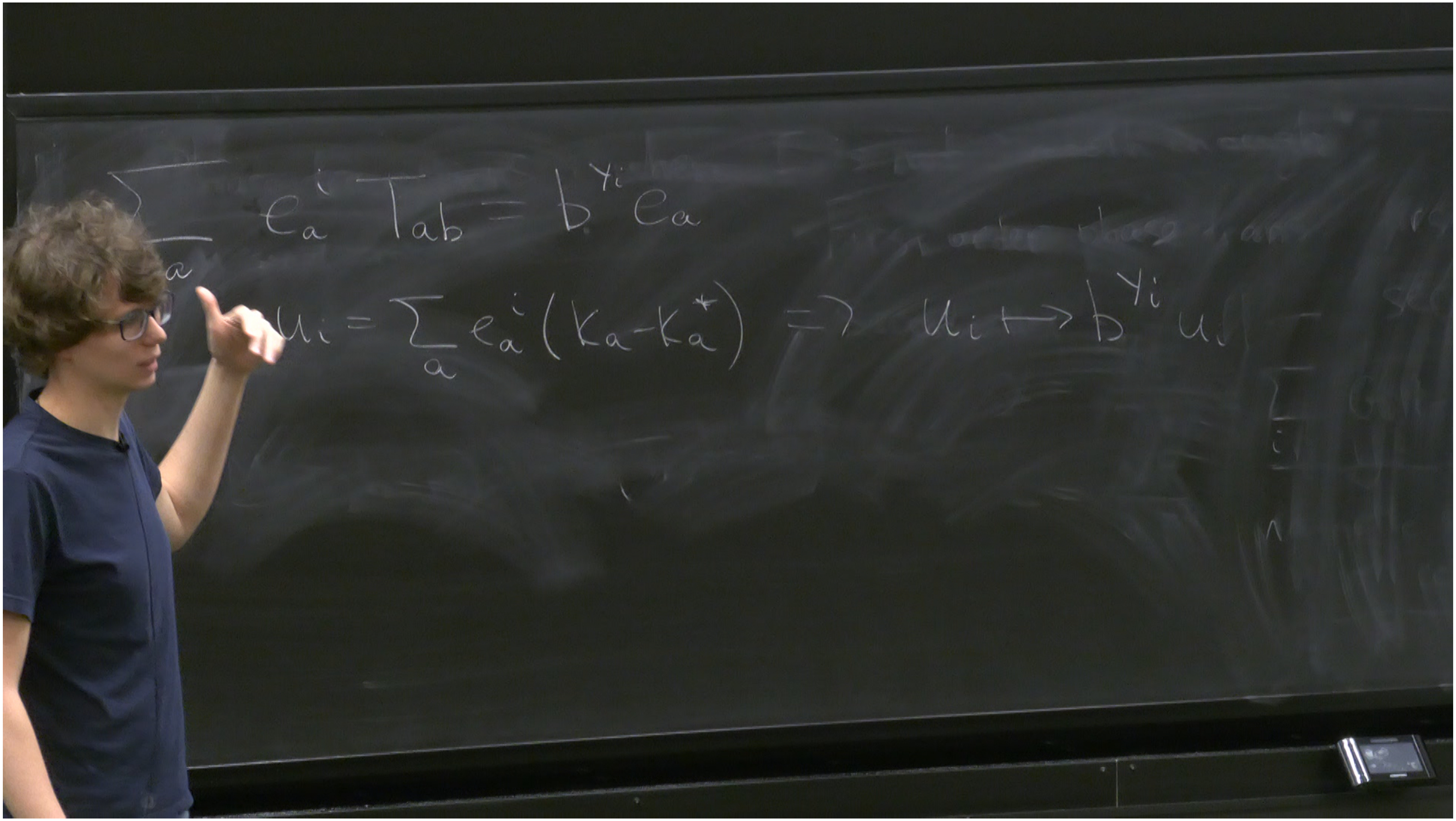
fixed points

critical points

$$\xi = 0, \infty \Rightarrow \xi' = \xi$$


tical points
$RG(K^*) = K^*$
$RG(K) - RG(K^*) = (K - K^*) RG'(K^*) + \ldots$
$RG(K)_a - K_a^* \approx \sum_b T_{ab}(K_b - K_b^*)$
$T_{ab} = \frac{\partial RG(K)_a}{\partial K_b}\Big|_{K=K_*}$
$K' = RG(K)$
$N' = b^{-d} N$
$\xi' = b^{-1}\xi$
fixed points
critical points
$\xi = 0, \infty \Rightarrow \xi' = \xi$

$$\sum_a e_a^i T_{ab} = b^{\gamma i} e_a$$

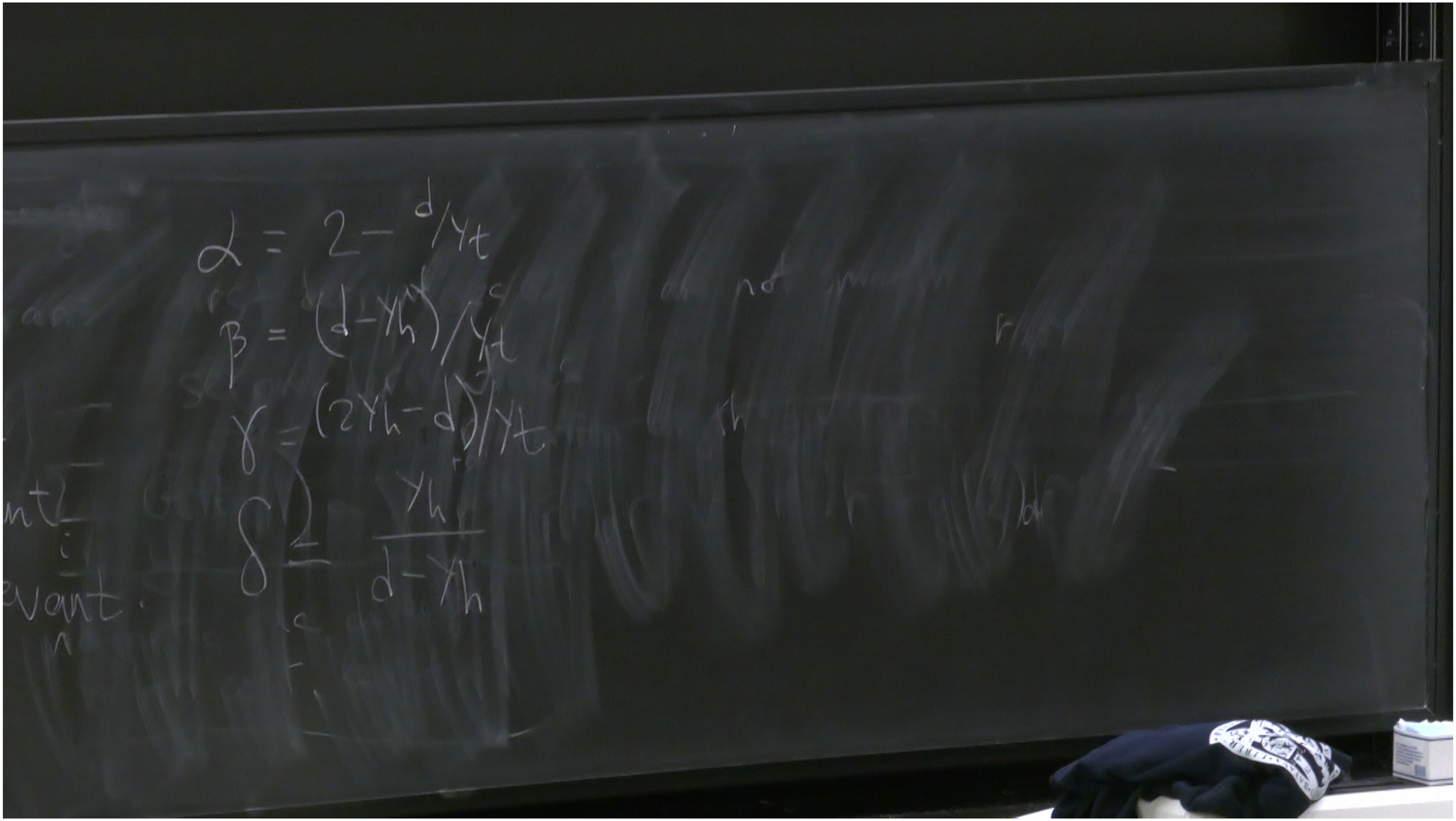

$\sum_a e_a^i T_{ab} = b^{y_i} e_a$
$u_i = \sum_a e_a^i (K_a - K_a^*) \Rightarrow u_i \mapsto b^{y_i} u_i$

$$\sum_a e_a^i T_{ab} = b^{y_i} e_a$$

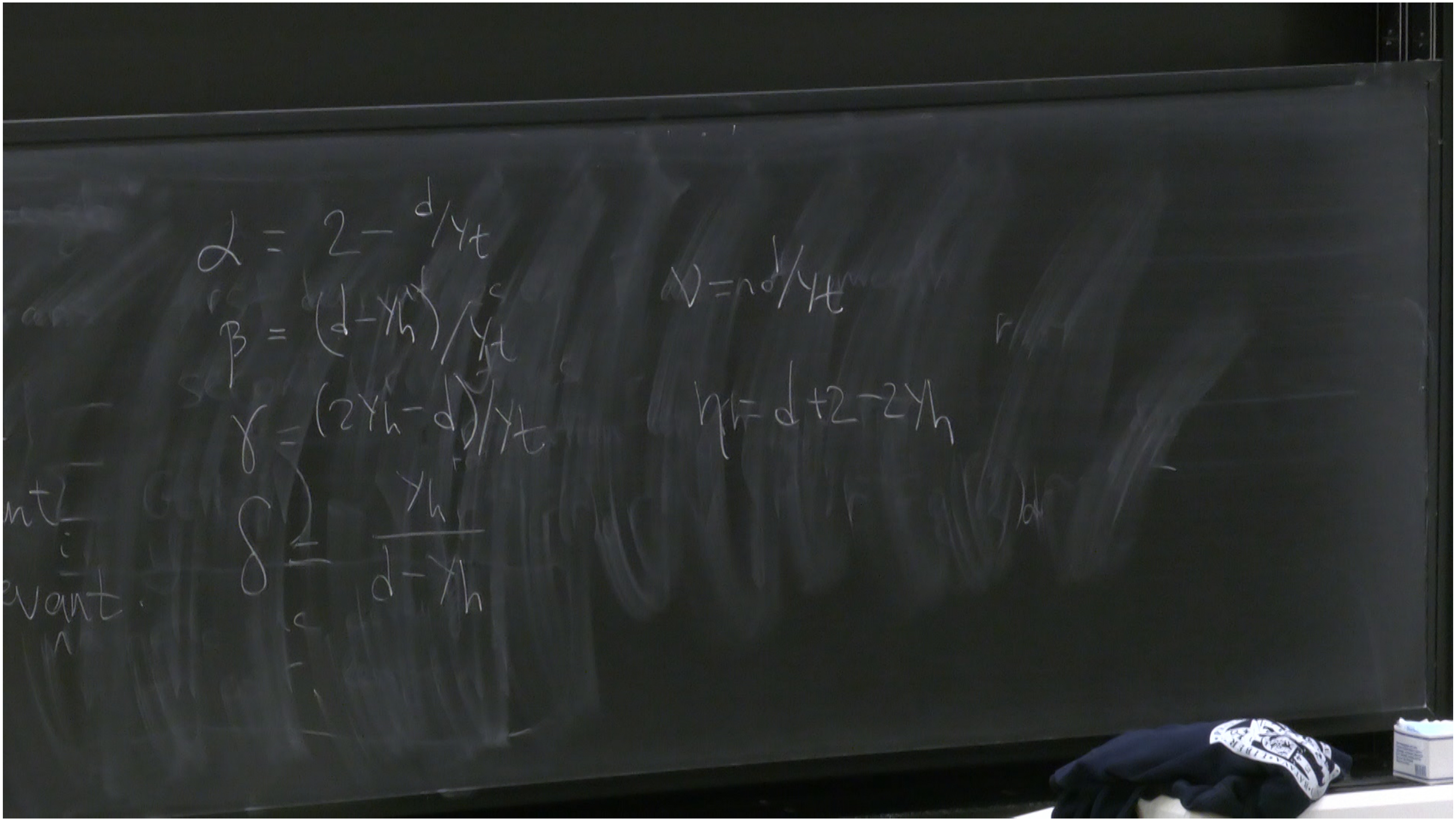
$$u_i = \sum_a e_a^i (K_a - K_a^*) \Rightarrow u_i \mapsto b^{y_i} u_i$$

$b^{y_i} > 1$ - relevant

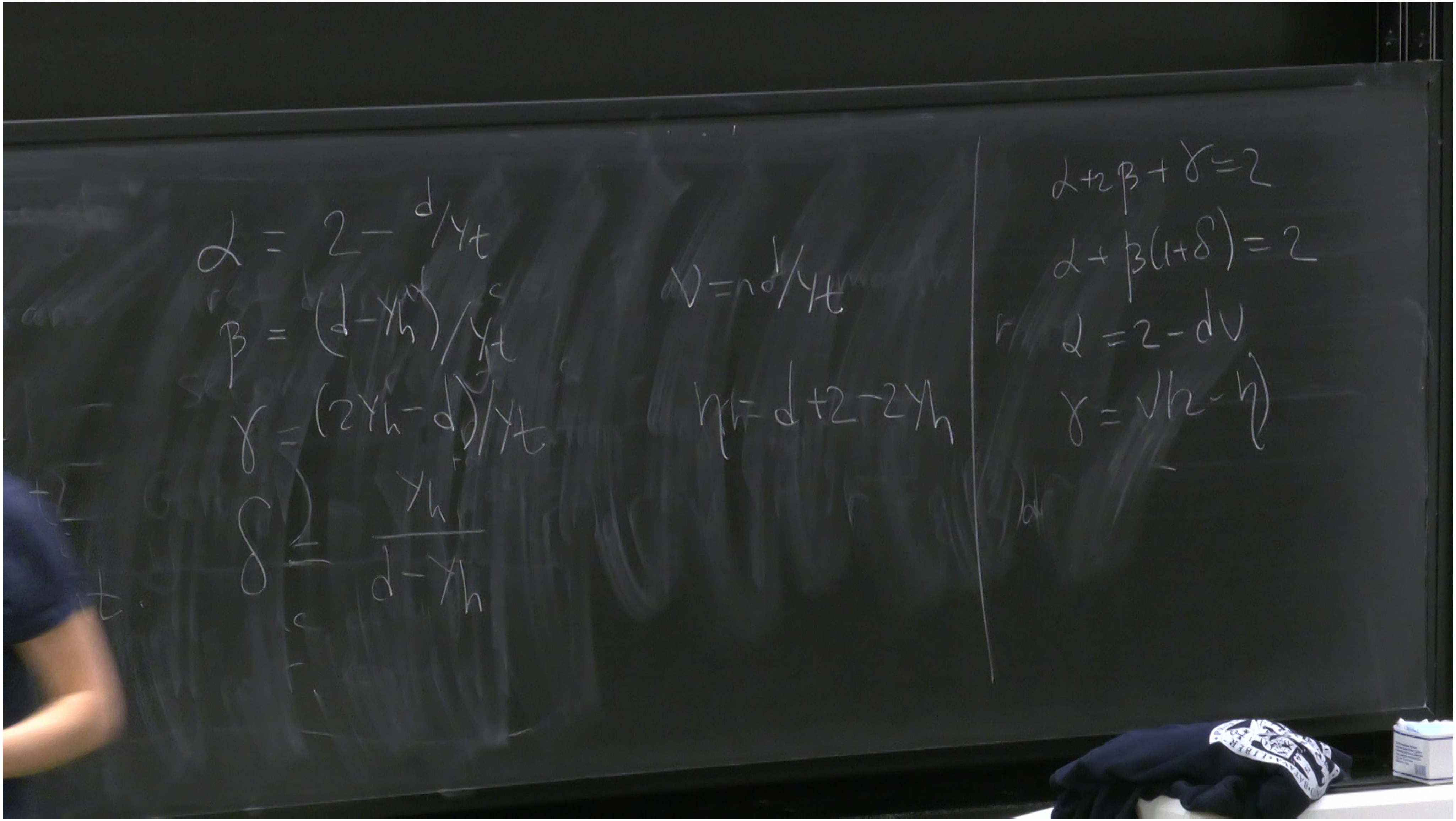
$b^{y_i} < 1$ - irrelevant.



$\alpha = 2 - d/y_t$
$\beta = (d - y_h)/y_t$
$\gamma = (2y_h - d)/y_t$
$\delta = \frac{y_h}{d - y_h}$



$\alpha = 2 - d/y_t$
$\nu = 1/y_t$
$\beta = (d - y_h)/y_t$
$\gamma = (2y_h - d)/y_t$
$\eta = d + 2 - 2y_h$
$\delta = \frac{y_h}{d - y_h}$


$\alpha = 2 - d/y_t$
$\beta = (d - y_h)/y_t$
$\gamma = (2y_h - d)/y_t$
$\delta = \frac{y_h}{d - y_h}$
$\nu = 1/y_t$
$\eta = d + 2 - 2y_h$
$\alpha + 2\beta + \gamma = 2$
$\alpha + \beta(1+\delta) = 2$
$\alpha = 2 - d\nu$
$\gamma = \nu(2 - \eta)$

$$\sum_a e_a^i T_{ab} = b^{y_i} e_b^i$$

$$u_i = \sum_a e_a^i (K_a - K_a^*) \Rightarrow u_i \mapsto b^{y_i} u_i$$

$b^{y_i} > 1$ - relevant

$b^{y_i} < 1$ - irrelevant.

α

β

γ

δ