Title: QFT III Lecture

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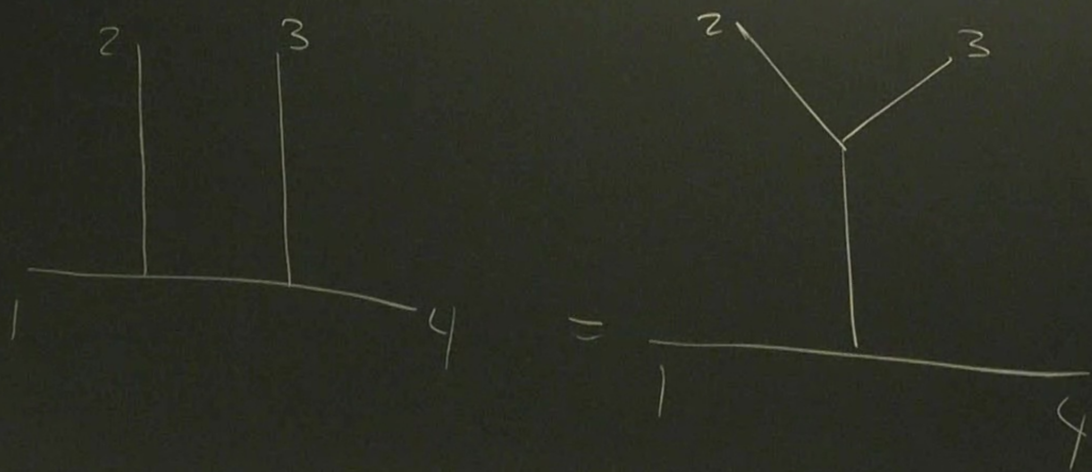
Recap:

- CFT data: $\{ (\Delta_I, S_I) ; C_{IJK} \}$

  2 + 3pt functions

- 4pt. function. Associativity of OPE data is constrained

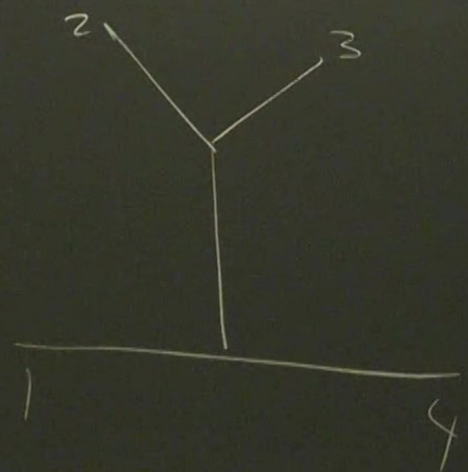


$$g(u,v) - \left(\frac{u}{v}\right)^{\Delta\phi} g(v$$

$$g(u,v) = \sum_{O} C^2_{\phi\phi O}\, g$$

$$\{ (\Delta_I, S_I) \, ; \, C_{IJK} \}.$$

sociativity of OPE data is constrained $\langle \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4) \rangle$



$$g(u,v) - \left(\frac{u}{v}\right)^{\Delta\phi} g(v,u) = 0$$

$$g(u,v) = \sum_{O} C^2_{\phi\phi O} \; \underset{\substack{\downarrow \\ \text{conformal block.}}}{g_{\Delta_O, \ell_O}}(u,v)$$

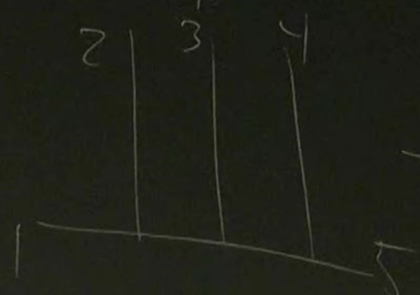
1 4

conformal block.

$$g_{\Delta_\theta,\ell_\theta}(u,v) = x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}\, C_a(x_{12},\partial_2)\, C_b(x_{34},y_4,\partial_4)\, \frac{I^{ab}(x_{34})}{x_{34}^{2\Delta_\theta}}$$

- Even spacetime dimensions explicitly known.

- $g_{\Delta,0}(u,v) = u^{\Delta/2}(1+\dots)$



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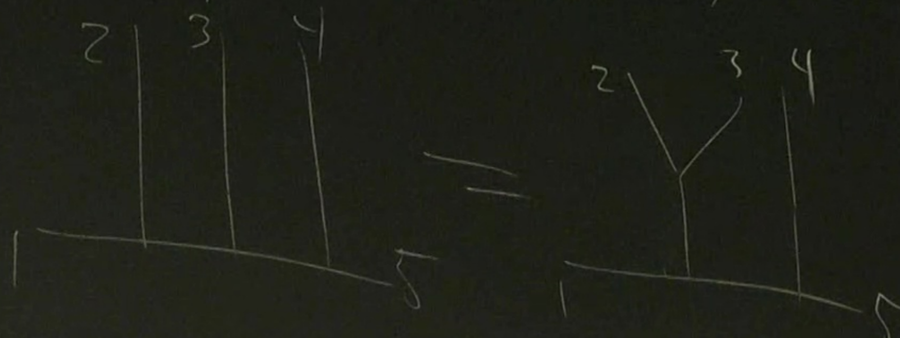


Anomalies

- Symmetry: if symmetry is continuous, $\partial^\mu j_\mu = 0$ (Noether's theorem).

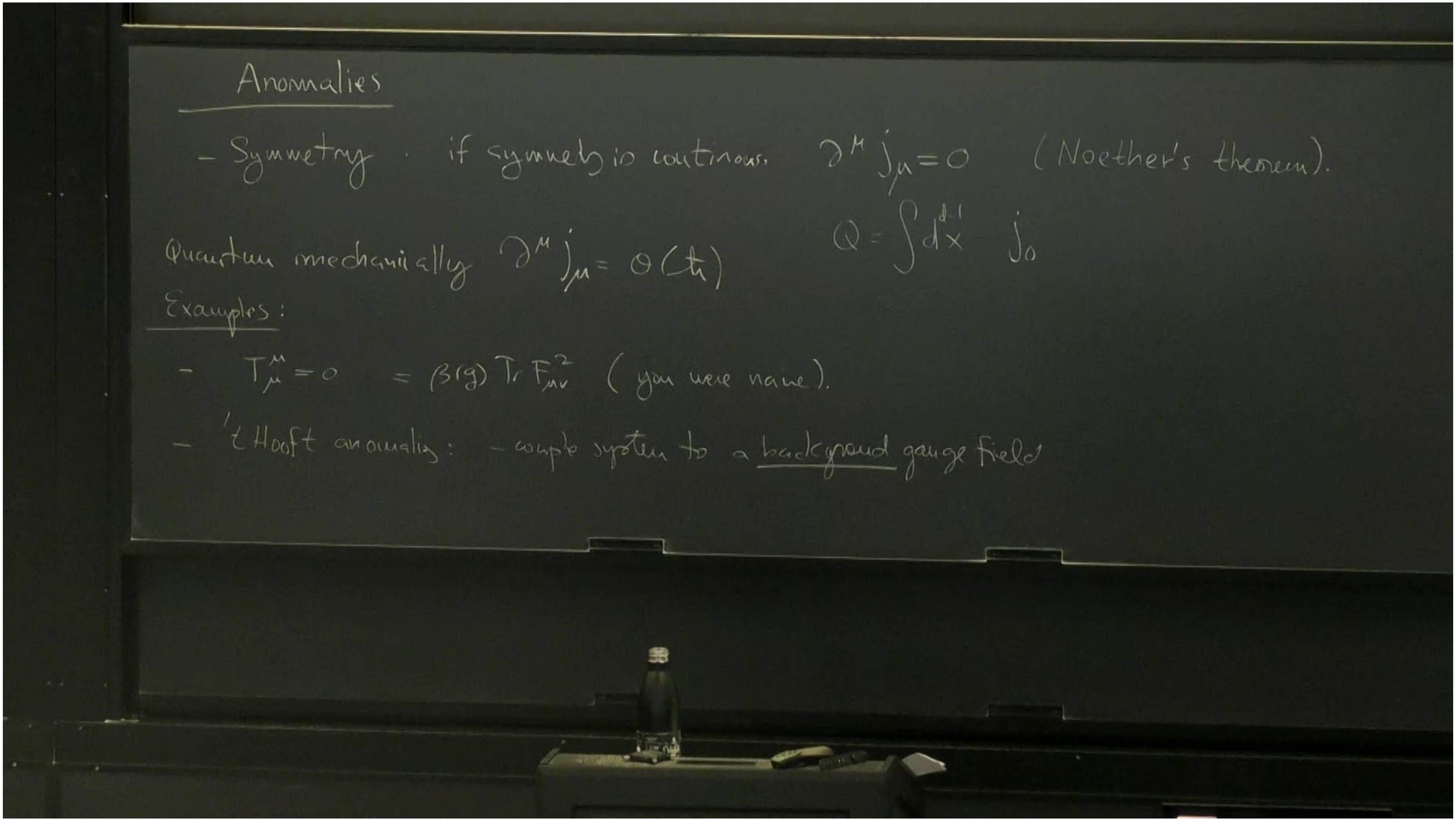
$$Q = \int d^{d-1}x \; j_0$$

Quantum mechanically $\partial^\mu j_\mu = O(\hbar)$

Examples:

- $T^\mu_\mu = 0 \quad = \beta(g)\, \mathrm{Tr}\, F_{\mu\nu}^2$ (you were naive).

- 't Hooft anomaly


Anomalies
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Examples:
- $T^\mu_\mu = 0$ $= \beta(g)\, \mathrm{Tr}\, F_{\mu\nu}^2$ (you were naive).
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Anomalies

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Examples:

- $T^\mu_\mu = 0 \quad = \beta(g)\, \mathrm{Tr}\, F_{\mu\nu}^2$ (you were naive).
- 't Hooft anomaly: - couple system to a background gauge field $\int dx\, A_\mu J^\mu$

anomaly coefficient ← $\partial^\mu j_\mu = \boxed{K} f(A_\mu)$ ← topological invariant

$F = dA$

$D=2$

$D=4 \quad \mathrm{Tr}(F \wedge F)$

UV

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = (\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})F_1(p^2) + \eta_{\mu\nu}\eta_{\rho\sigma}F_2(p^2) + \cdots + p_\mu p_\nu p_\rho p_\sigma F_5(p^2)$$

- Demand that $p^\mu \langle \quad \rangle = 0$ $\qquad \partial^\mu T_{\mu\nu} = 0 \Rightarrow p^\mu T_{\mu\nu} = 0$

3 relations between $F_i(p^2)$.

$$\Pi_{\mu\nu} = (p_\mu p_\nu - \eta_{\mu\nu} p^2)$$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = \left[\Pi_{\mu\rho}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\Pi_{\nu\rho} - \frac{2}{D-1}\Pi_{\mu\nu}\Pi_{\rho\sigma}\right]F(p^2) + \Pi_{\mu\nu}\Pi_{\rho\sigma}G(p^2)$$

$$\tilde{P}_\mu = \epsilon_{\mu\nu} p^\nu$$

$$\epsilon_{\mu\alpha}\,\epsilon_{\nu\beta} = \eta_{\mu\nu}\,\eta_{\alpha\beta} - \eta_{\mu\beta}\,\eta_{\alpha\nu}$$

$$\Pi_{\mu\nu} = \tilde{P}_\mu \tilde{P}_\nu$$

In $D=2$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = \tilde{P}_\mu \tilde{P}_\nu \tilde{P}_\rho \tilde{P}_\sigma\, G(p^2) = c\,\frac{\tilde{P}_\mu \tilde{P}_\nu \tilde{P}_\rho \tilde{P}_\sigma}{p^2}$$

- Scale symmetry.

$$\tilde{T}_{\mu\nu} = \int d^D x\, e^{ip\cdot x}\, T_{\mu\nu}(x)$$

$$\tilde{P}_\mu = \epsilon_{\mu\nu} p^\nu$$

$$\epsilon_{\mu\alpha}\epsilon_{\nu\beta} = \eta_{\mu\nu}\eta_{\alpha\beta} - \eta_{\mu\beta}\eta_{\alpha\nu}$$

$$\Pi_{\mu\nu} = \tilde{P}_\mu \tilde{P}_\nu$$

In $\underline{D=2}$

$$T_{\mu\nu}(p)\, T_{\rho\sigma}(q)$$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = \tilde{P}_\mu \tilde{P}_\nu \tilde{P}_\rho \tilde{P}_\sigma\, G(p^2) = c\,\frac{\tilde{P}_\mu \tilde{P}_\nu \tilde{P}_\rho \tilde{P}_\sigma}{p^2}$$

- Scale symmetry. $\tilde{T}_{\mu\nu} = \int d^D x\, e^{ip\cdot x}\, T_{\mu\nu}(x)$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = c\, \frac{\hat{P}_\mu \hat{P}_\nu \hat{P}_\rho \tilde{P}_\sigma}{p^2}$$

$$\eta^{\mu\nu}\tilde{P}_\mu \hat{P}_\nu = p^2$$

Naively $T^\mu_\mu = 0$

$$\langle T^\mu_\mu(p)\, T_{\rho\sigma}(-p)\rangle = c\, \hat{P}_\rho \tilde{P}_\sigma$$ conformal anomaly

$$\langle T_{\mu\nu}(x)\, T_{\rho\sigma}(0)\rangle =$$

$$\langle T^{\mu}_{\mu}(0)\rangle_h = \langle T^{\mu}_{\mu}(0)\, e^{\int h_{\rho\sigma} T^{\rho\sigma} d^2x} \rangle$$

$$= \int d^2x \boxed{\langle T^{\mu}_{\mu}(0)\; T_{\rho\sigma}(x)\rangle} h^{\rho\sigma}(x)$$

$$R\big|_{g=\eta+h}$$

$$= c \boxed{(\partial_\rho \partial_\sigma - \Box\, \eta_{\rho\sigma}) h^{\rho\sigma}} = c \cdot R|_h$$

$$\boxed{T^{\mu}_{\mu} = c R}$$

- In odd spacetime dimension, $\nexists$ conformal anomalies

$$\boxed{T^\mu_\mu = 0}$$

- $D=4$

$$a_{UV} > a_{IR}$$

RG monotonicity.

$$T^\mu_\mu = a \,\text{Euler} + c \,\text{Weyl}^2$$

$$\sum_{i=0}^{3} (-1)^i b_i = \chi = \int_{M_4} \text{Euler}$$

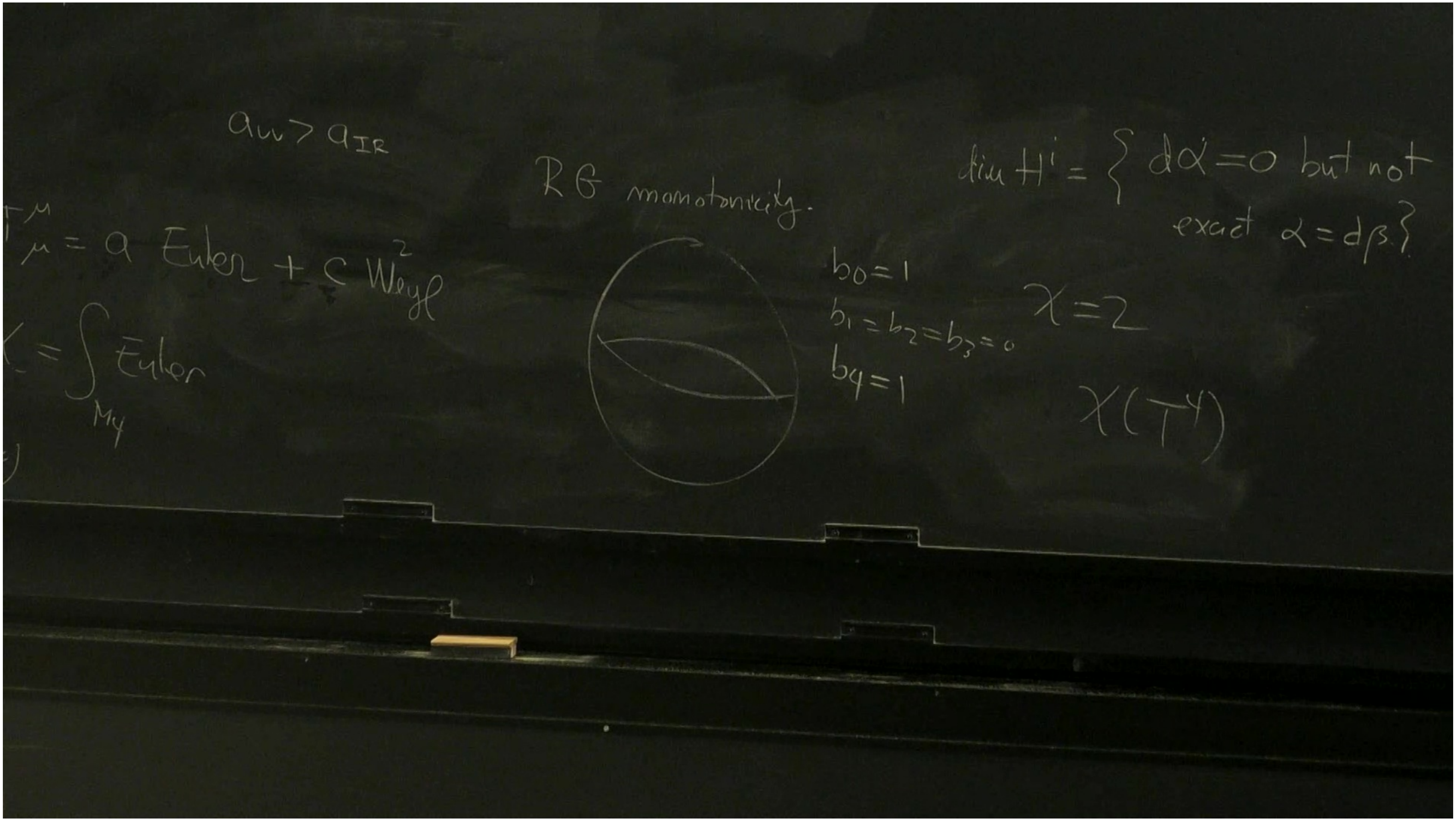
A conformal anomalies

a_{IR}

RG monotonicity.

$\dim H^1 = \{ d\alpha = 0$ but not exact $\alpha = d\beta \}$

$c \; W^2_{Weyl}$


$a_{UV} > a_{IR}$
RG monotonicity.
$T^{\mu}_{\mu} = a$ Euler $+ c$ Weyl^2
$\chi = \int_{M_4}$ Euler
$\dim H^i = \{ d\alpha = 0$ but not exact $\alpha = d\beta \}$
$b_0 = 1$
$b_1 = b_2 = b_3 = 0$
$b_4 = 1$
$\chi = 2$
$\chi(T^4)$