Title: QFT III Lecture

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Conformal Symmetry fixes position dependence of 2 and 3 point functions:

$$\langle O_I(x_1)\ O_J(x_2)\rangle = \frac{\delta_{IJ}}{x_{12}^{2\Delta_I}} \qquad x_{ij} = |x_i - x_j|$$

$$\langle O_I(x_1)\ O_J(x_2)\ O_k(x_3)\rangle = \frac{C_{IJk}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3}\ x_{23}^{\Delta_2+\Delta_3-\Delta_1}\ x_{13}^{\Delta_1+\Delta_3-\Delta_2}}.$$

CFT data: $\{(\Delta_I, S_I),\ C_{IJk}\}$

structure constants

Conformal Symmetry fixes position dependence of 2 and 3 point functions

under dilations $\mathcal{O}(x) = \lambda^{-\Delta}\, \mathcal{O}(\lambda^{-1} x)$

$$\langle O_I(x_1)\ O_J(x_2)\rangle = \frac{\delta_{IJ}}{x_{12}^{2\Delta_I}} \qquad x_{ij} = |x_i - x_j|$$

$$\langle O_I(x_1)\, O_J(x_2)\, O_K(x_3)\rangle = \frac{C_{IJK}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3}\ x_{23}^{\Delta_2+\Delta_3-\Delta_1}\ x_{13}^{\Delta_1+\Delta_3-\Delta_2}}$$

CFT data: $\{(\Delta_I, S_I),\ C_{IJK}\}$

structure constants

ns:

under dilations $\hat{O}(x) = \lambda^{-\Delta} O(\lambda^{-1}x)$

Ward identities:

$$\langle \hat{O}(x_1) \cdots \hat{O}(x_n) \rangle = \langle O(x_1) \cdots O(x_n) \rangle.$$

∞'al

$$\sum_{i=1}^{n} \langle O(x_1) \cdots \delta O(x_i) \cdots O(x_n) \rangle = 0$$

$\Delta_1 + \Delta_3 - \Delta_2.$

$$\delta\theta = \xi^\mu \partial_\mu \theta + \Omega_{\mu\nu} L^{\mu\nu}\cdot\theta + \Delta\sigma(x)\,\theta(x)$$

$$\xi^\mu = x^2$$

$$\sigma(x) = \cdot + b\cdot x$$

$$\Omega_{\mu\nu} = b_\mu x_\nu - b_\nu x_\mu$$

$$\delta(\partial_\mu\theta)$$

structure constants

4 pt functions in D spacetime dimensions

$$x_1 \quad x_2 \quad x_3 \quad x_4 \longrightarrow 0 \quad x_2 \quad (1,0,\dots,0) \quad \infty$$

$$\big\downarrow K_\mu$$

$$x_1 \quad x_2 \quad x_3 \quad \infty$$

$$\big\downarrow P_\mu$$

$$0 \quad x_2 \quad x_3 \quad \infty$$

$$\big\downarrow \hat{D}$$

$$0 \quad x_2 \quad \vec{n} \quad \infty$$

e constants

time dimensions

$0 \quad x_2 \quad (1\,0 \dots 0) \quad \infty \qquad M_{\mu\nu}$

$M_{12}, M_{13} \dots M_{1D}$

$M_{23}, \dots M_{2D}$

$0 \quad (a, b, 0 \dots 0) \quad (1 \dots 0) \quad \infty$

$D=3$ all M's have been used

$\Rightarrow$ 4 pt function depends on 2 coordinates (a,b)

(1 0 ... 0) ∞

$M_{23}, \ldots M_{2D}$

... 0) (1 ... 0) ∞

M's have been used

depends on 2 coordinates (a, b)

# of parameters in N-point function.

$$ND - \frac{(D+2)(D+1)}{2}.$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \quad \xleftrightarrow{1 \leftrightarrow 3} \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$X_2 \quad (1\,0 \cdots 0) \quad \infty$

$M_{23}, \ldots M_{2D}$

$z = a + ib$

$(a, b, 0 \cdots 0) \quad (1 \cdots 0) \quad \infty$

$D=3$ all M's have been used

4 pt function depends on 2 coordinate (a, b)

# of parameters in N-point function.

$$ND - \frac{(D+2)(D+1)}{2}$$

$$u = \frac{X_{12}^2 X_{34}^2}{X_{13}^2 X_{24}^2} \quad \xleftrightarrow{1 \leftrightarrow 3} \quad v = \frac{X_{14}^2 X_{23}^2}{X_{13}^2 X_{24}^2}$$

$$u = z\bar{z} \qquad v = (1-z)(1-\bar{z})$$

$0 \quad x_2 \quad \vec{y} \quad \infty$

4pt function:

$1 \leftrightarrow 3$

$$\langle \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2\Delta_\phi}\,x_{34}^{2\Delta_\phi}} = \frac{g(v,u)}{x_{12}^{2\Delta_\phi}\,x_{3}^{2\Delta_\phi}}\left(\frac{u}{v}\right)^{\Delta_\phi}$$

Δ_ϕ

- $g(u,v)$ is not an independent quantity.
- constraint $\{\Delta_I, C_{IJK}\}$.

x_2 $\vec{n}$ ∞

4pt function. $1 \leftrightarrow 3$ Constraint

$$\langle \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}} = \frac{g(v,u)}{x_{12}^{2\Delta_\phi}\, x_{3}^{2\Delta_\phi}} \left(\frac{u}{v}\right)^{\Delta_\phi}$$

Δ_ϕ

$g(u,v)$ is not an independent quantity.

constraint $\{\Delta_I, C_{IJK}\}$.

$\mathcal{O}_i(x_1)$

$\mathcal{O}_j(x_2)$ =

Constraint CFT data by using associativity of OPE

$$\mathcal{O}_1(\mathcal{O}_2\mathcal{O}_3) = (\mathcal{O}_1\mathcal{O}_2)\,\mathcal{O}_3.$$

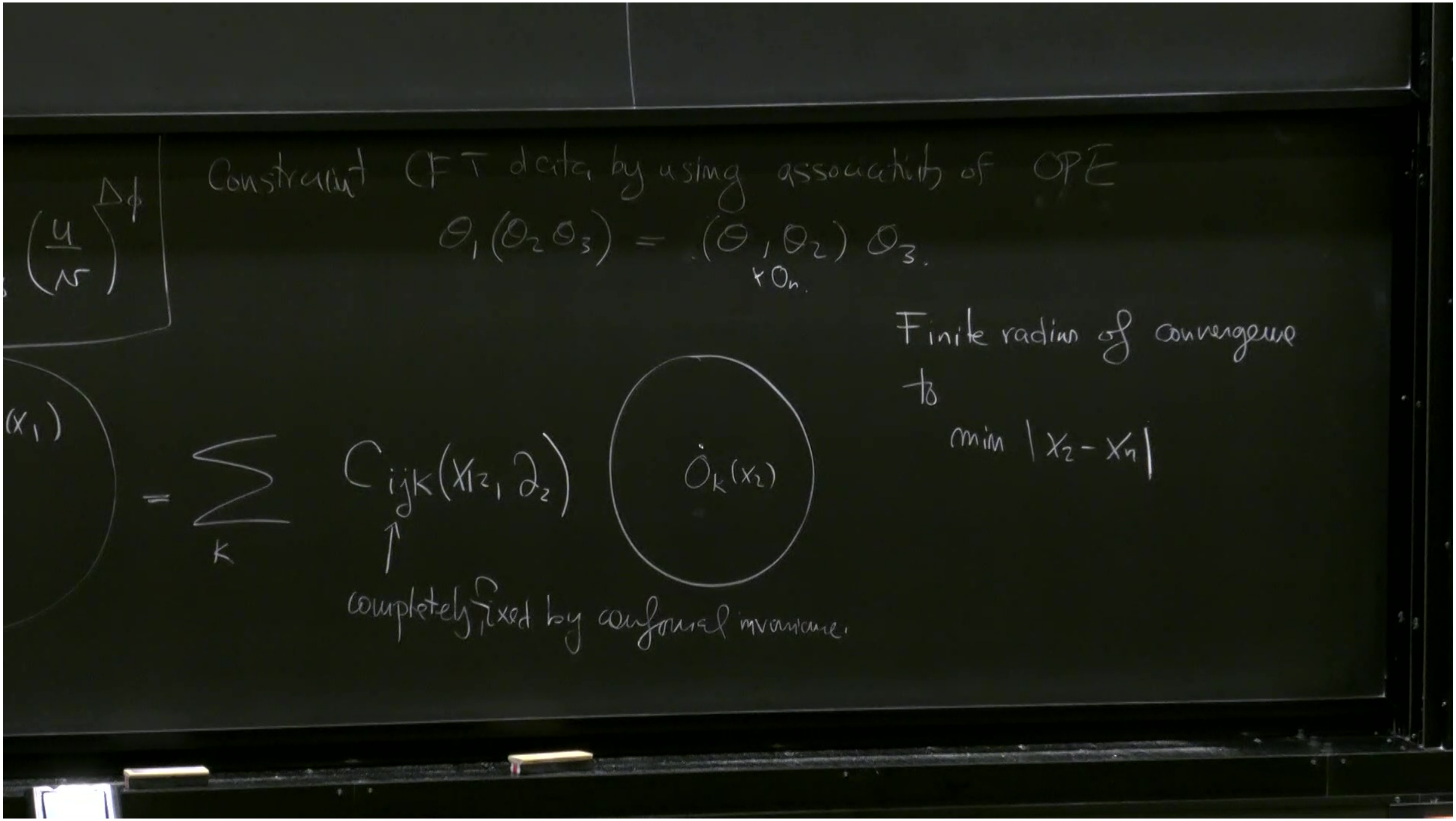
$+\,\mathcal{O}_n.$

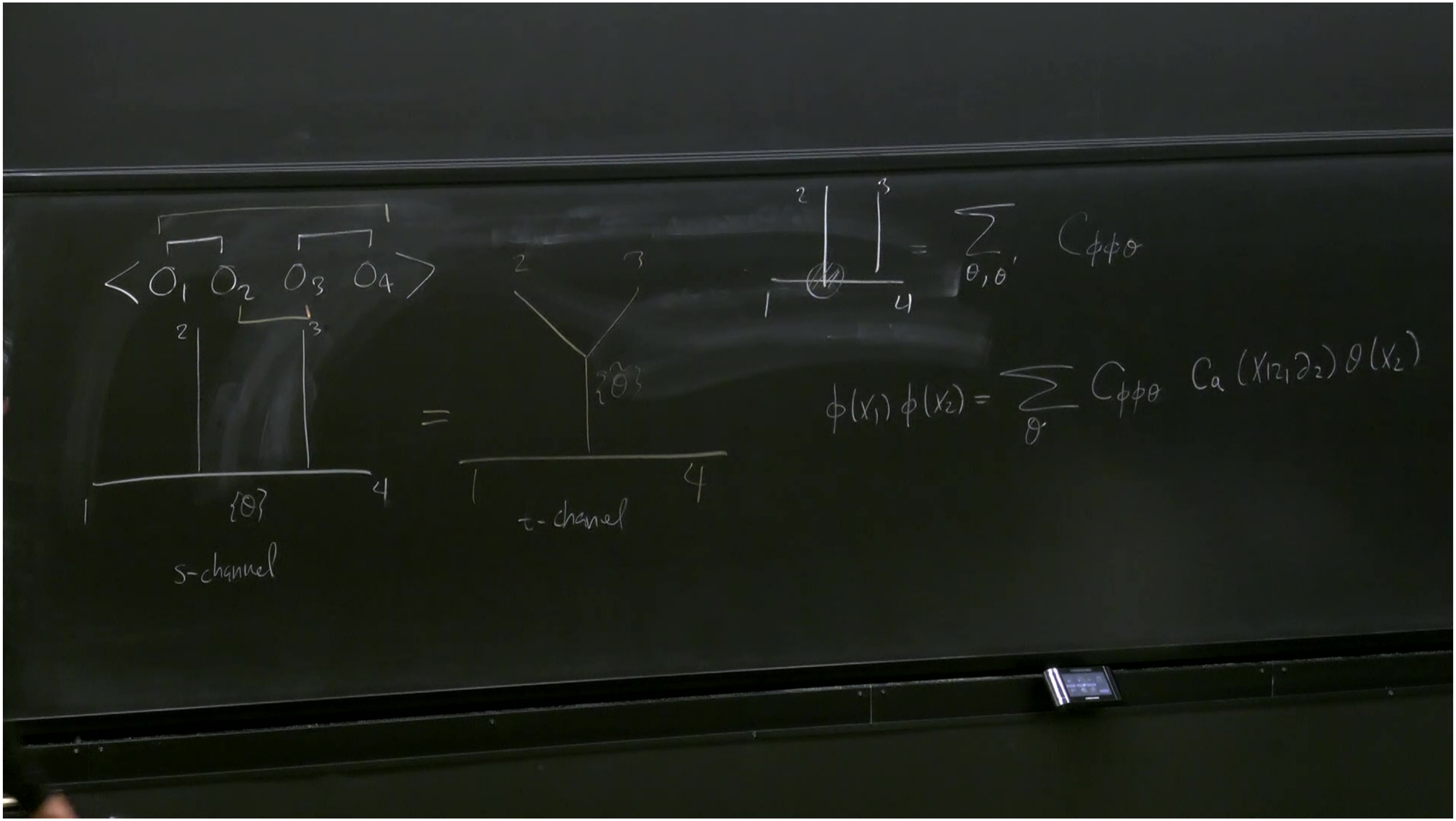
$$\left(\frac{u}{v}\right)^{\Delta_\phi}$$

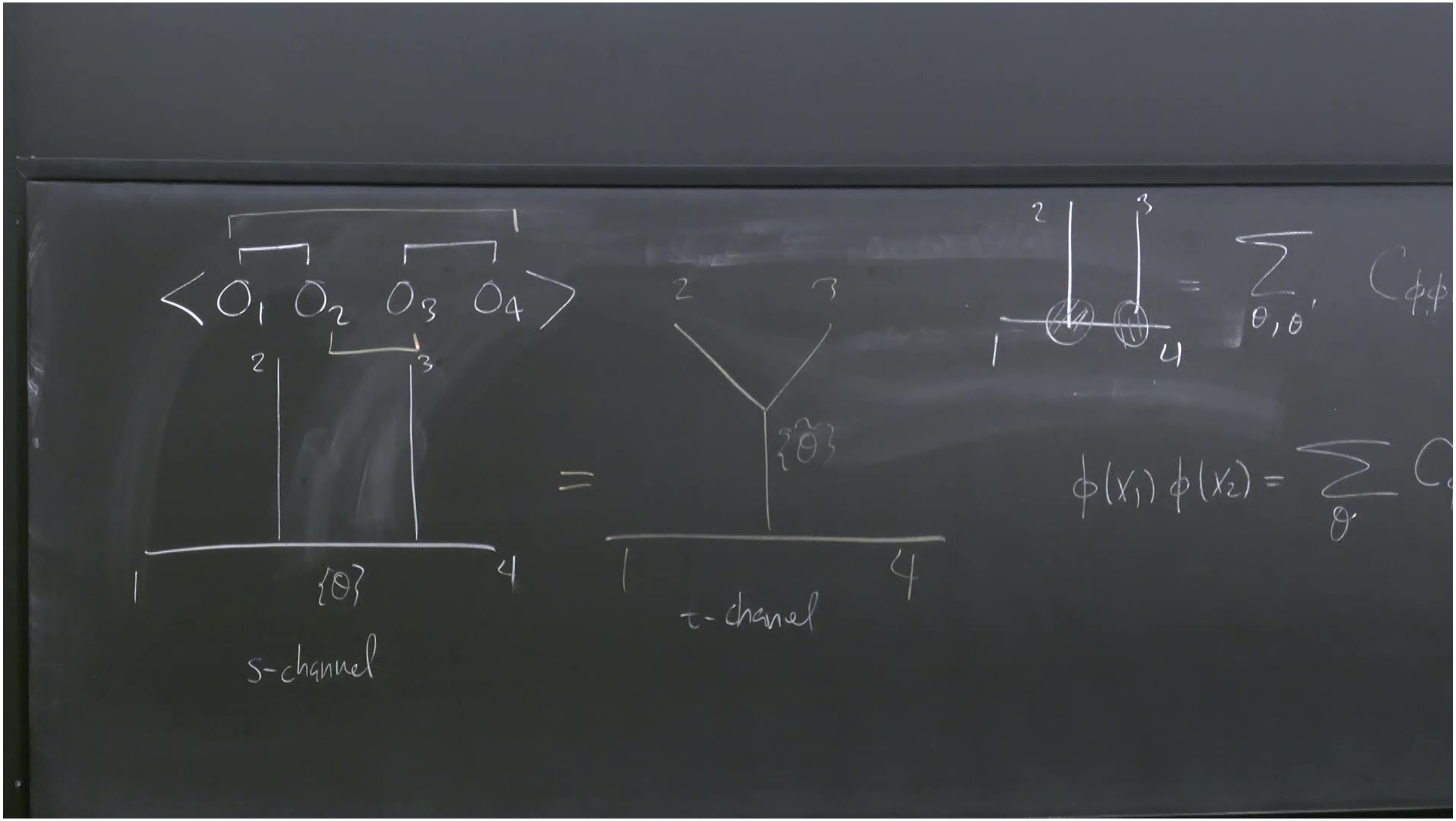
$$(x_1) = \sum_k C_{ijk}(x_{12}, \partial_2)\; \mathcal{O}_k(x_2)$$

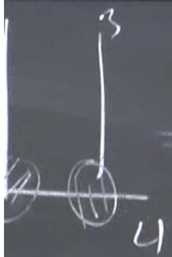
Finite radius of convergence to

$\min |x_2 - x_n|$


Constraint CFT data by using associativity of OPE
$\mathcal{O}_1(\mathcal{O}_2\mathcal{O}_3) = (\mathcal{O}_1\mathcal{O}_2)\mathcal{O}_3$
$+\mathcal{O}_n$
$\Delta\phi$
$\left(\frac{u}{v}\right)$
(x_1)
$= \sum_k C_{ijk}(x_{12}, \partial_2)\, \mathcal{O}_k(x_2)$
completely fixed by conformal invariance.
Finite radius of convergence
to
$\min |x_2 - x_n|$


⟨O1 O2 O3 O4⟩
{O}
s-channel
=
{O}
t-chanel
= Σ θ,θ' Cφφθ
φ(x1) φ(x2) = Σ θ Cφφθ Ca (x12, ∂2) θ(x2)

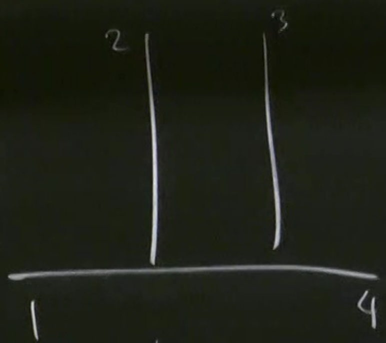

$\langle O_1\, O_2\, O_3\, O_4 \rangle$
{θ}
s-channel
=
{θ̃}
t-chanel
$= \sum_{\theta,\theta'}$
$\phi(x_1)\,\phi(x_2) = \sum_{\theta}$



$$= \sum_{\theta,\theta'} C_{\phi\phi\theta}\, C_{\phi\phi\theta'}\, C_a(x_{12},\partial_2)\, C_b(x_{34},\partial_4) \left\langle \mathcal{O}_a(x_2)\, \mathcal{O}_b(x_4) \right\rangle$$

$$\left\langle \mathcal{O}_a(x_2)\, \mathcal{O}_b(x_4) \right\rangle = \frac{I_{ab}(x_{24})}{x_{24}^{2\Delta_\theta}}\, \delta_{\theta,\theta'}$$

$$\phi(x_1)\,\phi(x_2) = \sum_{\theta} C_{\phi\phi\theta}\, C_a(x_{12},\partial_2)\, \mathcal{O}_a(x_2)$$



$$\sim \frac{1}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}} \sum_{\theta} C^2_{\phi\phi\theta}\, g_{\Delta_\theta, \ell_\theta}(x) = \frac{\boxed{g(u,v}}{x_{12}^{2\Delta_\phi}\, x_{34}^{2}}$$

$$\| \quad x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}\, C_a(x_1, x_2)$$

$$\boxed{g(u,v) - \left(\frac{u}{v}\right)^{\Delta_\phi} g(v,u) = 0}$$

crossing equations

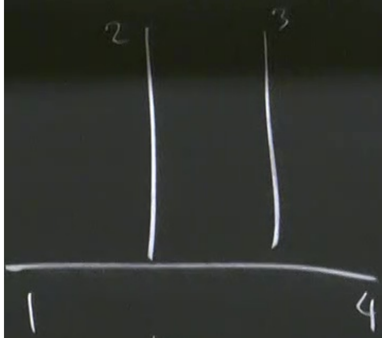
$$\sum_{\theta} C^2_{\phi\phi\theta}\, g_{\Delta_\theta, \ell_\theta}(x) = \frac{\boxed{g(u,v)}}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}}$$

$$\| $$

$$x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}\, C_a(x_{12}, \partial_2)\, C_b(x_{34}, \partial_4)\, \frac{I^{ab}(x_{24})}{x_{24}^{2\Delta_\phi}}$$

$= 0$

1) Completely solve $\{\Delta_i, C_{ijk}\}$ in $D=2$.



$$= \frac{1}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}} \sum_{\mathcal{O}} C^2_{\phi\phi\mathcal{O}}\, g_{\Delta_\mathcal{O}, l_\mathcal{O}}(x) = \frac{\boxed{g(u,v)}}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}}$$

$$\| $$

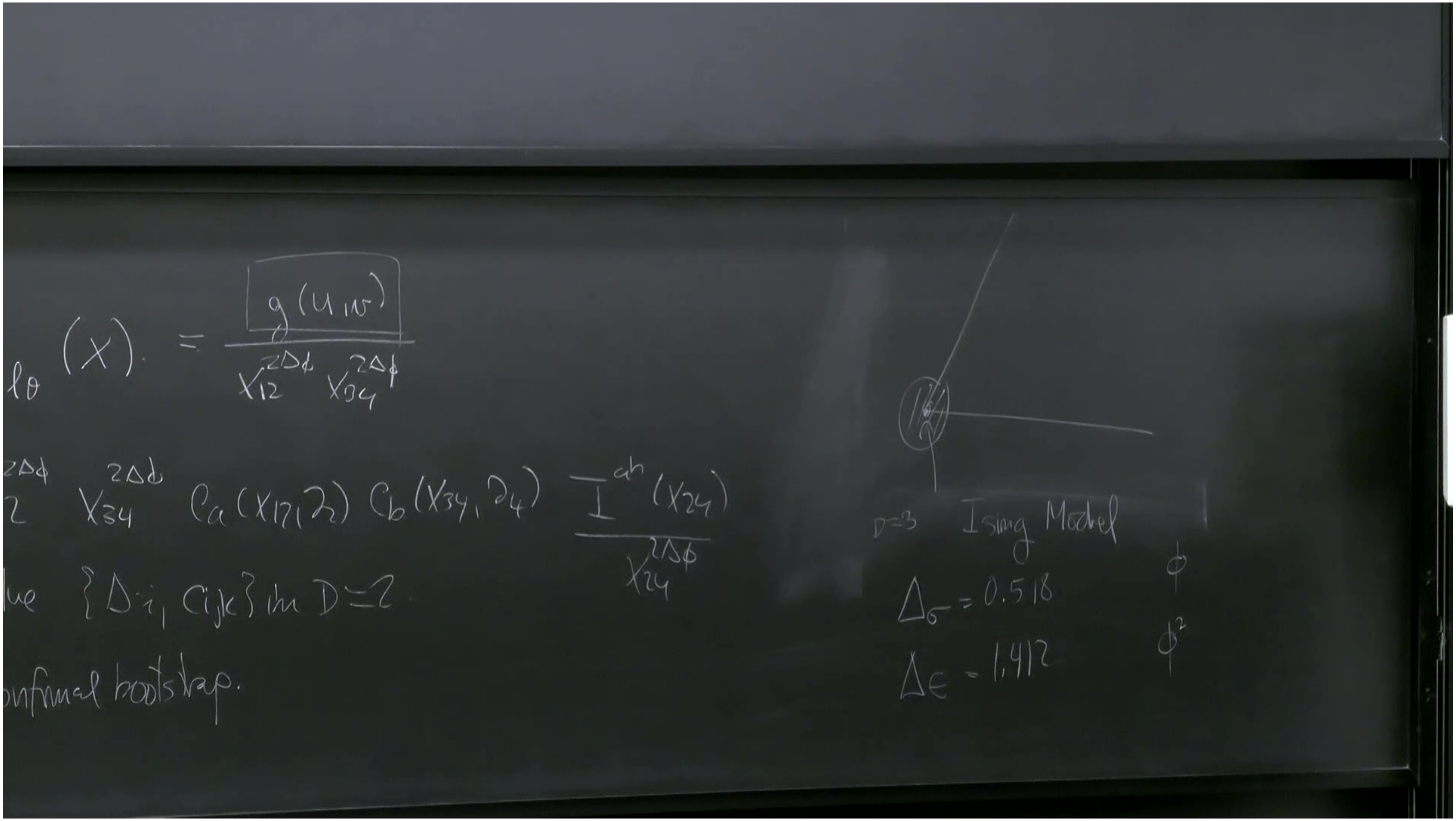
$$x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}\, C_a(x_{12}, z_2)\, C_b(x_{3}$$

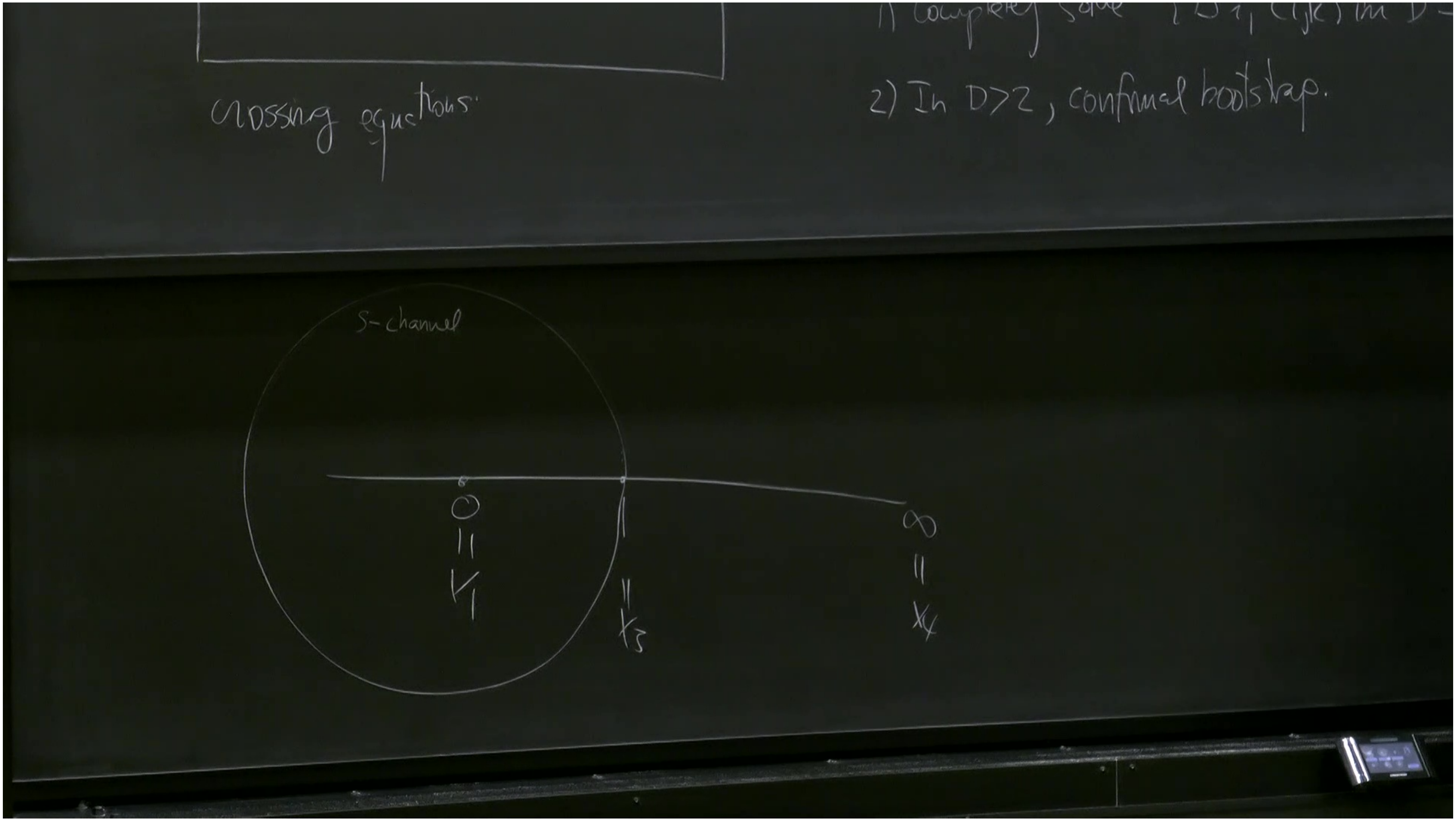
$$\boxed{g(u,v) - \left(\frac{u}{v}\right)^{\Delta_\phi} g(v,u) = 0}$$

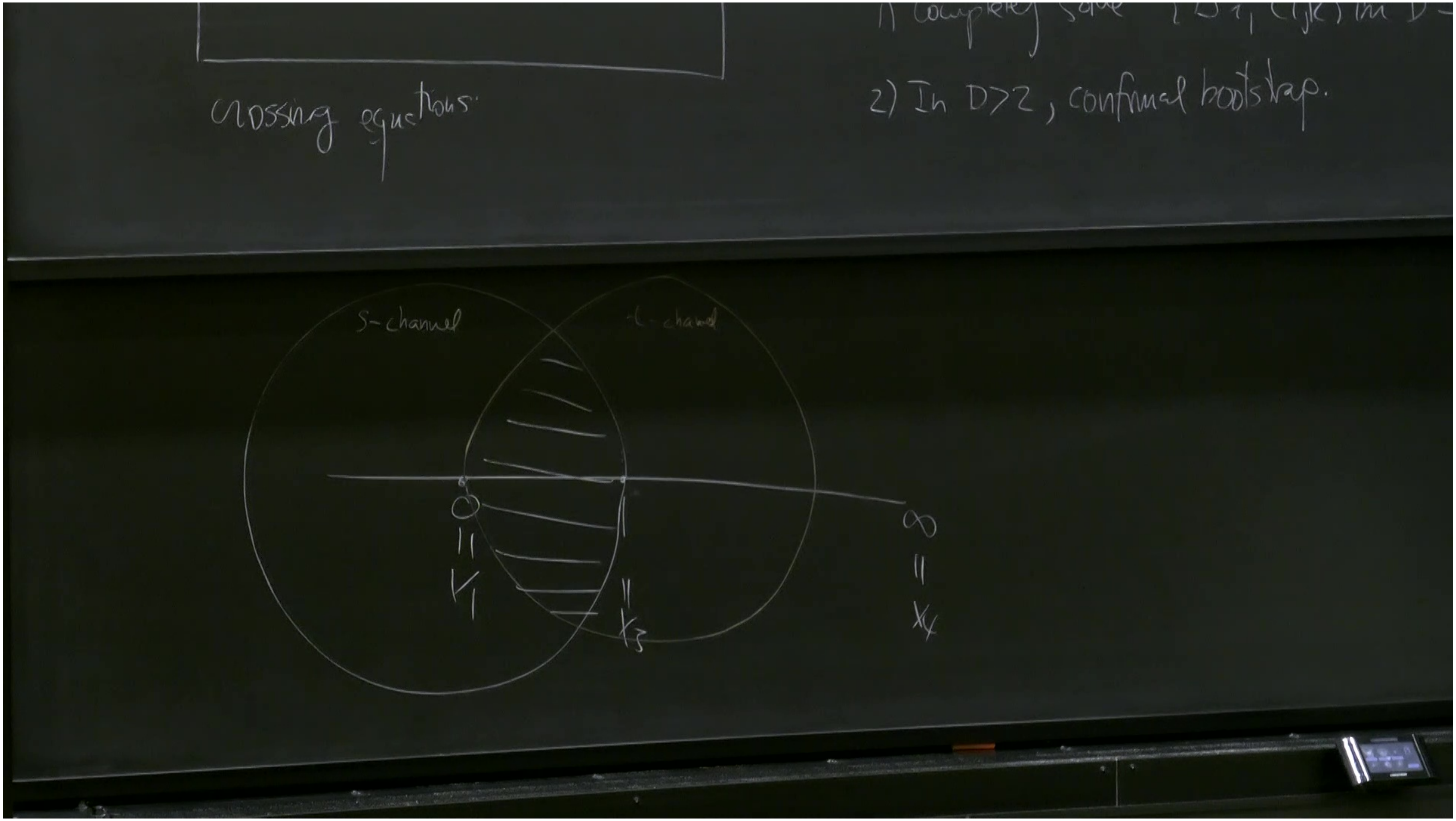
crossing equations

1) Completely solve $\{\Delta_i, C_{ijk}\}$ in $D=2$

2) In $D>2$, conformal bootstrap.


$(x) = \frac{g(u,v)}{x_{12}^{2\Delta\phi}\, x_{34}^{2\Delta\phi}}$
$x_{34}^{2\Delta\phi}\, C_a(x_{12}, \partial_2)\, C_b(x_{34}, \partial_4)\, \frac{I^{ab}(x_{24})}{x_{24}^{2\Delta\phi}}$
$\{\Delta_i, C_{ijk}\}$ in $D=2$
onformal bootstrap.
D=3 Ising Model
$\Delta_\sigma = 0.518$
ϕ
$\Delta_\epsilon = 1.412$
ϕ^2


crossing equations:
2) In D>2, conformal bootstrap.
s-channel
0 = x1
1 = x3
∞ = x4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