Title: QFT III Lecture

Speakers: Jaume Gomis

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Diffeo: $x \to x + \xi(x)$

$$\delta_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

that can be undone by a Weyl transformation $\delta_\sigma g_{\mu\nu} = 2\sigma\, g_{\mu\nu}$

$$\delta_\xi g_{\mu\nu} + \delta_\sigma g_{\mu\nu} = 0$$

- ∞'al conformal transformations generated by vector field:

$$\xi^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + b^\mu x^2 - 2x^\mu b\cdot x$$

P_μ $M_{\mu\nu}$ D K_μ $\Longrightarrow$ $SO(2,D)$ Lie algebra

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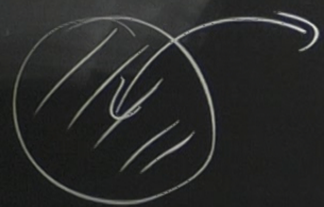
$$\uparrow P_\mu \qquad \uparrow M_{\mu\nu} \qquad \uparrow \hat{D} \qquad \uparrow K_\mu \Longrightarrow SO(2,D) \text{ Lie algebra}$$

- Discrete conformal transformation

$I : x^\mu \to x^\mu / x^2$

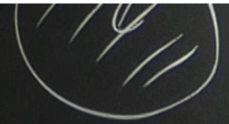
1)



2) $\mathbb{Z}_2$ transformation $I^2 = 1$

3) Reverses orientation of spacetime

$$\delta_\xi g_{\mu\nu} + \delta_\sigma g_{\mu\nu} = 0 \Longrightarrow \text{CKV equation.}$$



3) Reserves orientation of spacetime

$$I P_\mu I = K_\mu$$
$$I K_\mu I = P_\mu$$
$$I M_{\mu\nu} I = \hat{M}_{\mu\nu}$$
$$I D I = -\hat{D}$$

2) $\mathbb{Z}_2$ transformation $I^2 = 1$

$$I e^{a^\mu P_\mu} I = I \cdot P_\mu \left(\frac{x^\mu}{x^2}\right) = I \cdot \left(x^\mu / x^2 + a^\mu\right) = \frac{x^\mu / x^2 + a^\mu}{\left(\frac{x^\mu}{x^2} + a^\mu\right)^2}$$

$$= \frac{x^\mu + a^\mu x^2}{\left(1 + 2 a \cdot x + a^2 x^2\right)}$$

Conformal Group

$SO(2, D)$

4 disconnected components

P, T , I is conjugate P

- Any local QFT has a canonical operator : $T_{\mu\nu}$

1. Translations : $\partial^\mu T_{\mu\nu} = 0$ (Noether's theorem)

2. Lorentz : $T_{\mu\nu} = T_{\nu\mu}$.

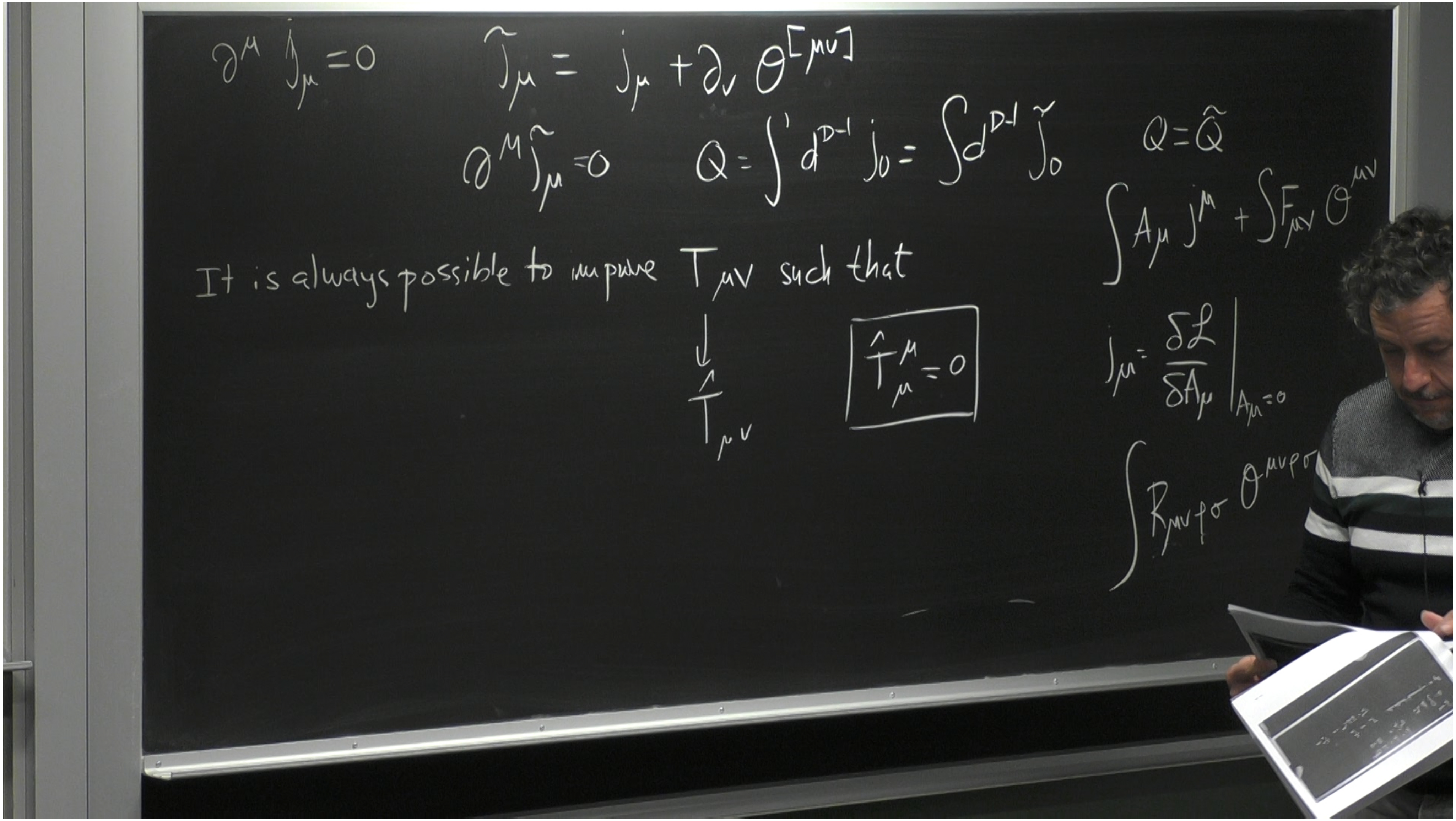
$$\delta S = \frac{1}{2}\int d^Dx \; T_{\mu\nu}\, \delta_\xi g^{\mu\nu} = \frac{1}{2}\int d^Dx \; T_{\mu\nu}\, (\partial^\mu \xi^\nu + \partial^\nu \xi^\mu).$$

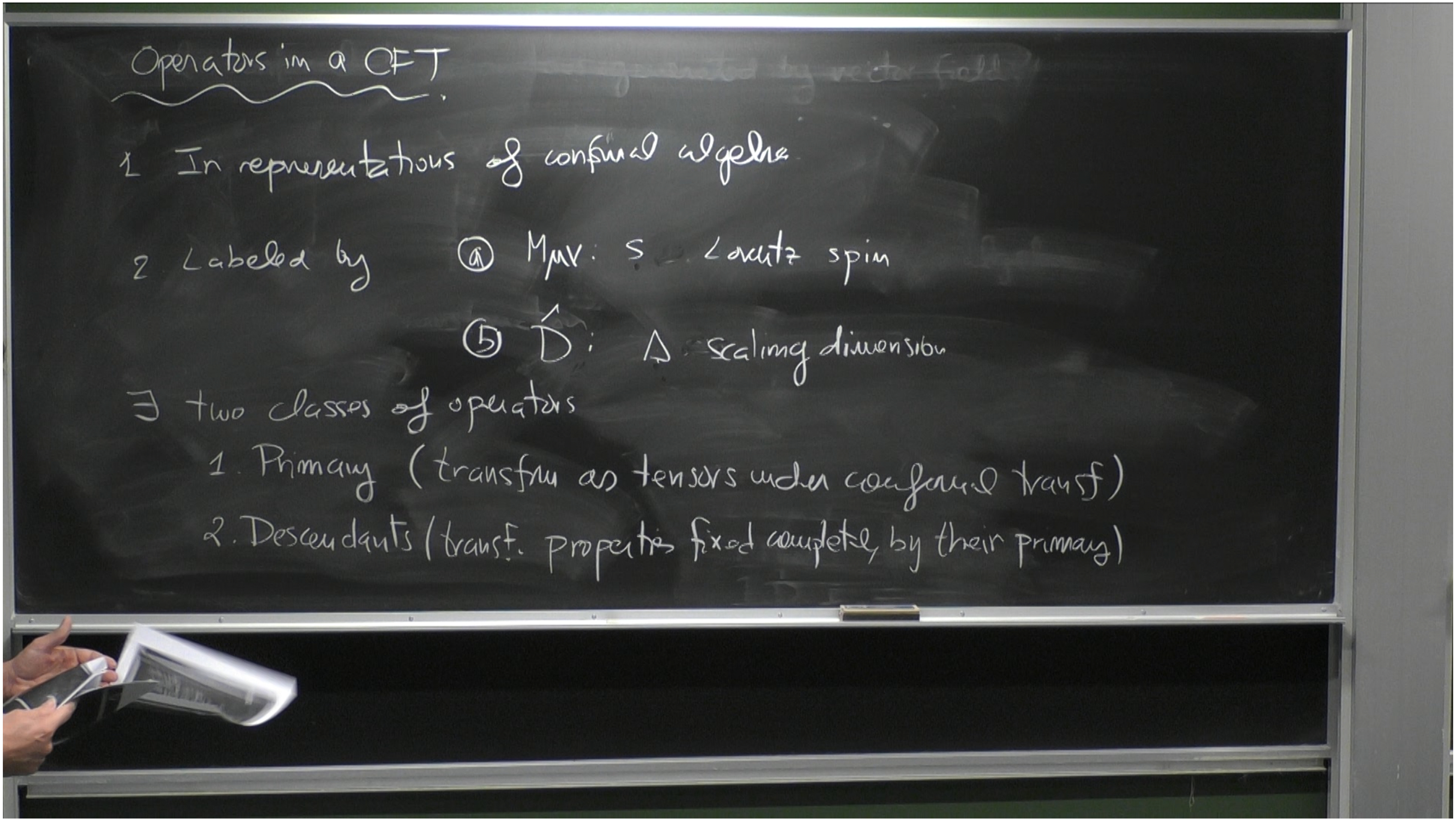
$$\partial^\mu \xi^\nu + \partial^\nu \xi^\mu = \frac{2}{D}\,\eta^{\mu\nu}(\partial\cdot\xi)$$

$$\delta S \propto \int d^Dx \; T^\mu_\mu (\partial\cdot\xi)$$

1) Scale invariance : $\boxed{T^\mu_\mu = \partial^\mu L_\mu}$

2) Special conformal $\boxed{T^\mu_\mu = \partial_\mu \partial_\nu L^{\mu\nu}}$


$\partial^\mu j_\mu = 0$
$\tilde{j}_\mu = j_\mu + \partial_\nu \theta^{[\mu\nu]}$
$\partial^\mu \tilde{j}_\mu = 0$
$Q = \int d^{D-1} j_0 = \int d^{D-1} \tilde{j}_0$
$Q = \tilde{Q}$
$\int A_\mu j^\mu + \int F_{\mu\nu} \theta^{\mu\nu}$
It is always possible to improve $T_{\mu\nu}$ such that
$\downarrow$
$\hat{T}_{\mu\nu}$
$\boxed{\hat{T}^\mu_\mu = 0}$
$j_\mu = \frac{\delta \mathcal{L}}{\delta A_\mu}\Big|_{A_\mu=0}$
$\int R_{\mu\nu\rho\sigma} \theta^{\mu\nu\rho\sigma}$


Operators in a CFT
1 In representations of conformal algebra
2 Labeled by ⓐ $M_{\mu\nu}$: s Lorentz spin
ⓑ $\hat{D}$: Δ scaling dimension
∃ two classes of operators
1. Primary (transfm as tensors under conformal transf)
2. Descendants (transf. properties fixed complete, by their primary)

(5) $\hat{D}$: Δ scaling dimension

$\exists$ two classes of operators

1. Primary (transfm as tensors under conformal transf)
2. Descendants (transf. properties fixed completely by their primary)

$$[\hat{D}, \mathcal{O}] = \Delta\, \mathcal{O}$$

$$[\hat{D}, [P_\mu, \mathcal{O}]] = (\Delta+1)\, [P_\mu, \mathcal{O}] \quad P_\mu \uparrow$$

$$[\hat{D}, [K_\mu, \mathcal{O}]] = (\Delta-1)\, [K_\mu, \mathcal{O}] \quad K_\mu \downarrow$$

⑤ $\hat{D}$: Δ scaling dimension

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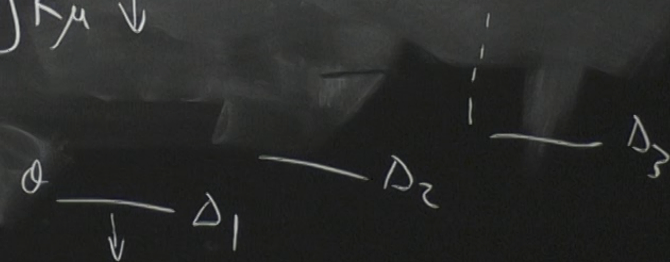
Primary operator: $[K_\mu, \mathcal{O}] = 0$

$\parallel$

Lowest weight state

Representation

$\mathcal{O}$	Primary	Δ
$P_\mu \mathcal{O}$		$\Delta+1$
$P_{\mu_1} P_{\mu_2} \mathcal{O}$		$\Delta+2$



$$x^\mu \to \tilde{x}^\mu = (g\cdot x)^\mu \qquad g(a,\lambda,\omega,b)$$

Primaries:

$$\mathcal{O}_{A,\Delta}(x) \longrightarrow \tilde{\mathcal{O}}_{A,\Delta} = \left|\frac{\partial x}{\partial \tilde{x}}\right|^{\Delta/D} L^A{}_B(R)\, \mathcal{O}_B(g^{-1}x)$$

A: Lorentz spin

Δ: scaling dimension

$$R^\mu{}_\nu = \frac{\partial \tilde{x}^\mu}{\partial x^\nu} e^{-\omega(x)}$$

$$R^T \eta R = \eta$$

dimension $\quad R^{\mu}{}_{\nu} = \frac{\partial \tilde{x}}{\partial x^{\nu}}\, e$

$$R^T \eta R = \eta$$

Under infinitesimal $\tilde{x} = x + \xi$.

$$\delta \mathcal{O}_{A,\Delta}(x) = \hat{\mathcal{O}}_{A,\Delta}(x) - \mathcal{O}_{A,\Delta}(x) = -\xi^{\mu}\partial_{\mu}\mathcal{O}_{A,\Delta}(x) + \frac{1}{2}\Omega_{\mu\nu}(x)\left(M^{\mu\nu}_{R}\right)\mathcal{O}_{B,\Delta}(x) - \Delta\omega(x)\,\mathcal{O}_{A,\Delta}$$

$$\begin{cases} \Omega_{\mu\nu} = \omega_{\mu\nu} - 2(x_{\mu}b_{\nu} - x_{\nu}b_{\mu}) \\ \omega(x) = \lambda - 2x\cdot b \end{cases}$$

$$R^T \eta R = \eta$$

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$$- \Delta \omega(x)\, \mathcal{O}_{A,\Delta}$$

$$\begin{cases} \Omega_{\mu\nu} = \omega_{\mu\nu} - 2(x_\mu b_\nu - x_\nu b_\mu) \\ \omega(x) = \lambda - 2x \cdot b \end{cases}$$

$$[\delta_{\xi_1}\theta, \delta_{\xi_2}]\theta = \delta_{[\xi_1, \xi_2]}\theta$$

$$\delta_{\xi_1}\delta_{\xi}\theta - \delta_{\xi_2}\delta_{\xi_1}\theta$$

Constraint Observables

- Position dependence of 2 and 3 pt functions is completely fixed

in a CFT ✓

$$\langle \mathcal{O}(x_1)\ \mathcal{O}(x_2)\rangle = f(|x_1 - x_2|)$$

$$\langle \mathcal{O}_{\Delta_1}(x_1)\ \mathcal{O}_{\Delta_2}(x_2)\rangle = \frac{C}{|x_1 - x_2|^{2\Delta_1}}\ \delta_{\Delta_1 \Delta_2}$$

$$\langle \mathcal{O}_{\Delta_1}(x_1)\ \mathcal{O}_{\Delta_2}(x_2)\ \mathcal{O}_{\Delta_3}(x_3)\rangle = \frac{C_{123}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3}\ |x_{23}|^{\Delta_2+\Delta_3-\Delta_1}\ |x_{13}|^{\Delta_1+\Delta_3-\Delta_2}}$$