Title: QFT III Lecture

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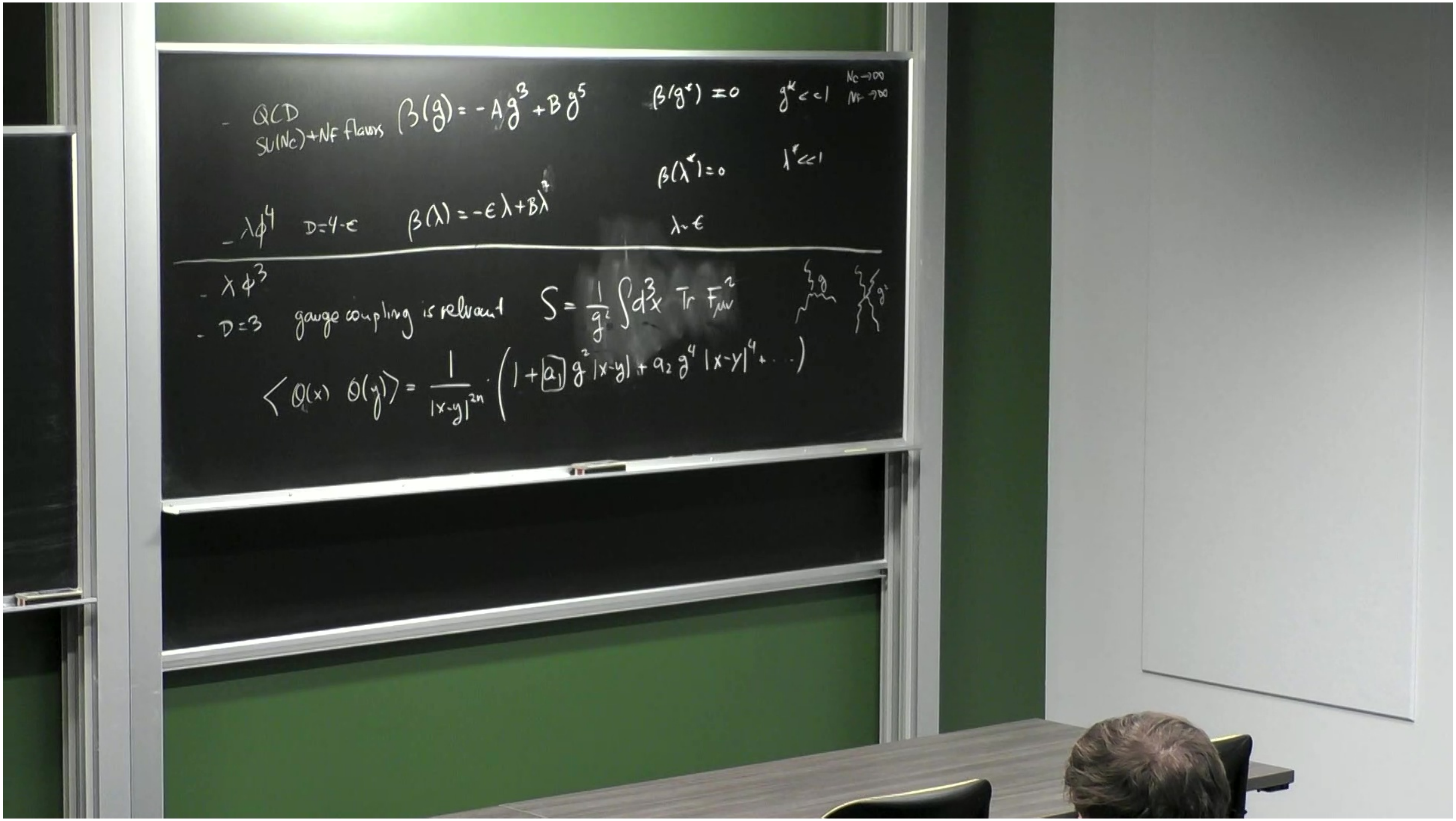

- QCD
SU(Nc)+Nf flavors β(g) = - A g^3 + B g^5
β(g*) = 0
g* << 1
Nc → ∞
Nf → ∞
β(λ*) = 0
λ* << 1
- λφ^4 D=4-ε β(λ) = -ελ+Bλ^2
λ~ε
- λφ^3
- D=3 gauge coupling is relevant S = 1/g^2 ∫d^3x Tr F_μν^2
⟨O(x) O(y)⟩ = 1/|x-y|^2n · (1 + a1 g^2 |x-y| + a2 g^4 |x-y|^4 + . . .)

Kinematics of CFTs

1. Kinematics
2. Dynamics.

- Geometry of conformal transf.
- Derive the Lie algebra of conformal transf.

symmetry: $\exists\ G$

For a continous G, the small transf. are determined by a Lie algebra $\mathcal{G}$

$$\underset{\substack{\cap \\ \text{Group}}}{g(\alpha)} \cdot g(\beta) = g(f(\alpha,\beta))$$

$$g(\alpha) = 1 + \alpha^a T^a + \ldots$$

$$f^a(\alpha,\beta) = \alpha^a + \beta^a + \ldots$$

$$\boxed{[T^a, T^b] = i f^{ab}_{\ \ c} T^c}$$

symmetry: $\exists\, G$

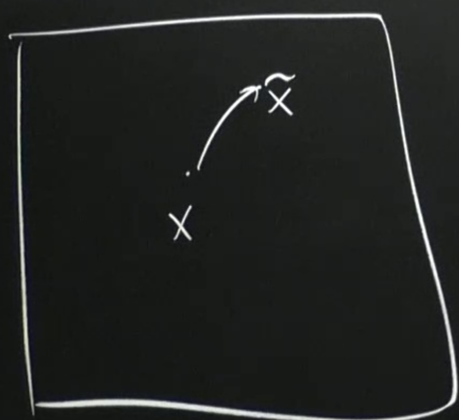
$$[T^a, T^b] = i f^{ab}{}_c T^c$$

For a continous G, the small transf. are determined by a Lie algebra $\mathcal{G}$

Conformal Transformations.

$x^\mu \to \tilde{x}^\mu(x)$

$\mathbb{R}^{1,D-1}$ = Minkowski



1. Preserves angles.
2. Preserves the lightcones.

$$ds^2 \to e^{2\omega(x)} ds^2$$

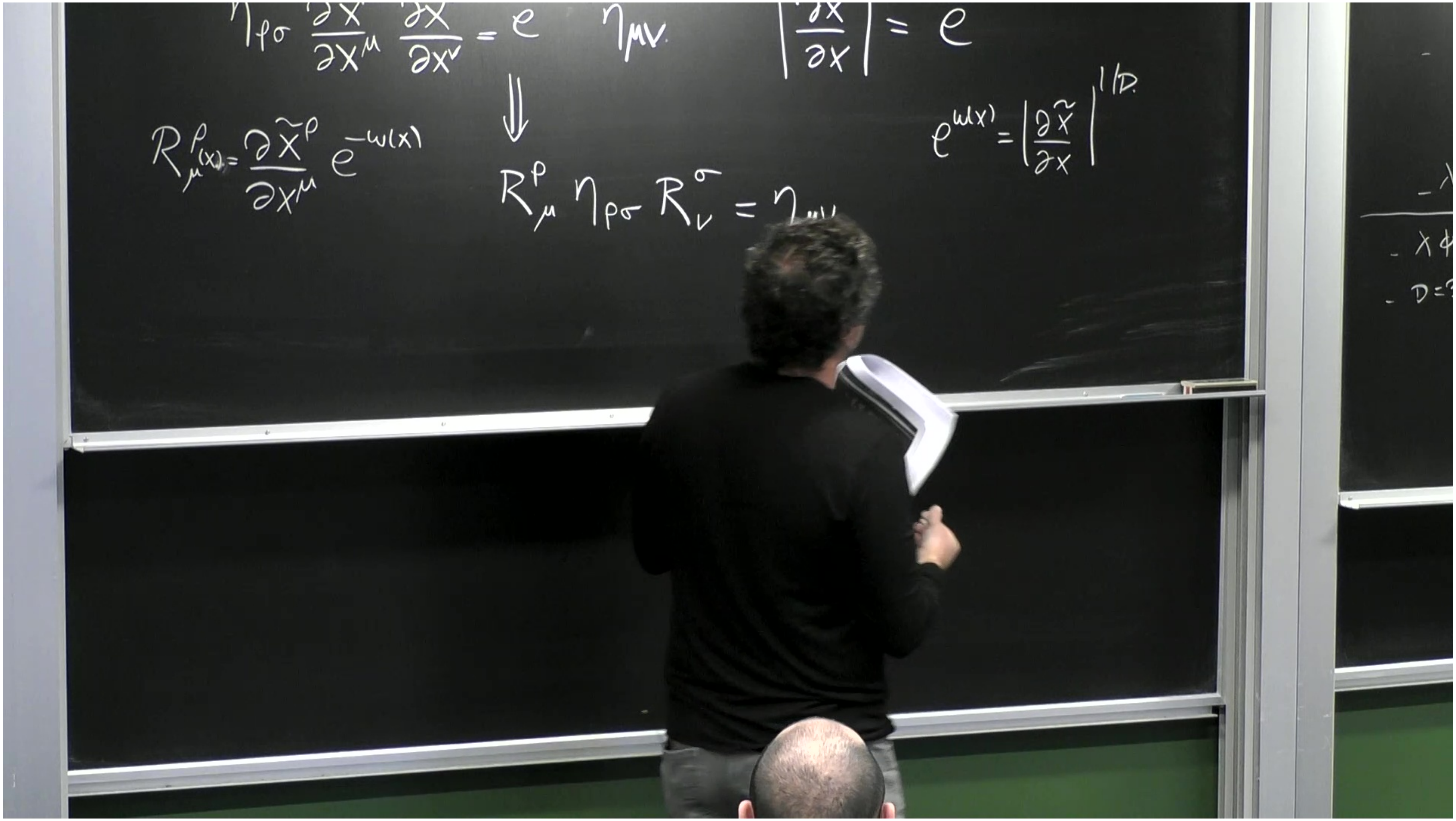
$$\eta_{\rho\sigma}\, d\tilde{x}^\rho d\tilde{x}^\sigma = e^{2\omega(x)}\, \eta_{\mu\nu}\, dx^\mu dx^\nu$$

$$\eta_{\rho\sigma} \frac{\partial \tilde{x}^\rho}{\partial x^\mu} \frac{\partial \tilde{x}^\sigma}{\partial x^\nu} = e^{2\omega(x)}\, \eta_{\mu\nu}$$

$$\Downarrow$$

$$R^\rho_\mu(x) = \frac{\partial \tilde{x}^\rho}{\partial x^\mu}\, e^{-\omega(x)}$$

$$R^\rho_\mu\, \eta_{\rho\sigma}\, R^\sigma_\nu = \eta_{\mu\nu}$$


$R_\mu^{\ \rho}(x) = \frac{\partial \tilde{x}^\rho}{\partial x^\mu} e^{-\omega(x)}$
$R_\mu^{\ \rho} \eta_{\rho\sigma} R_\nu^{\ \sigma} = \eta_{\mu\nu}$
$e^{\omega(x)} = \left|\frac{\partial \tilde{x}}{\partial x}\right|^{1/D}$

$$\left|\frac{\partial \tilde{x}}{\partial x}\right|^{\frac{2}{D}} = 1 + \frac{2}{D}\,\partial\cdot\xi$$

$$\det A = e^{\mathrm{Tr}\log A}$$

∞' transformations:

$$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu$$

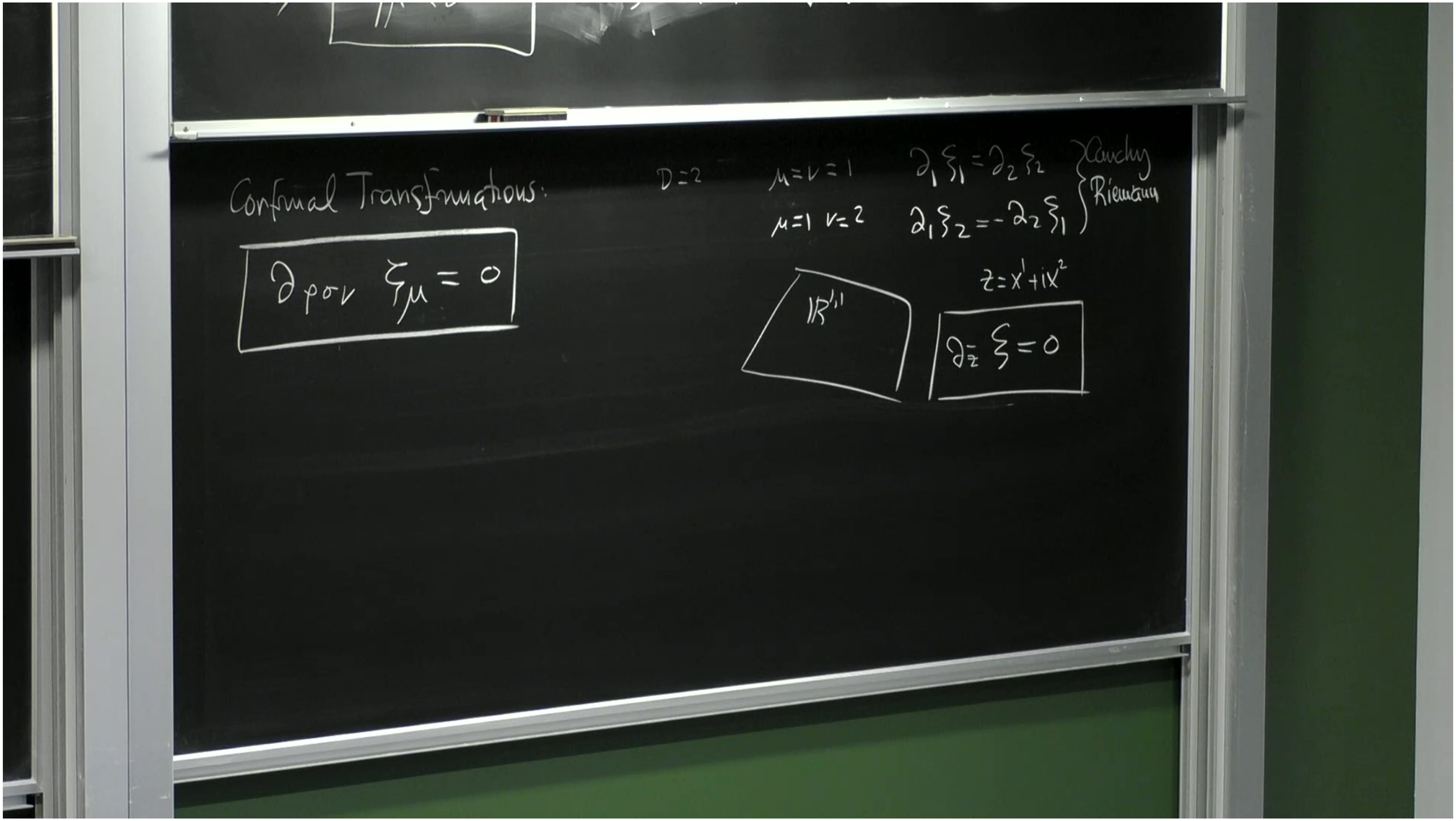
ξ^μ generates a vector field

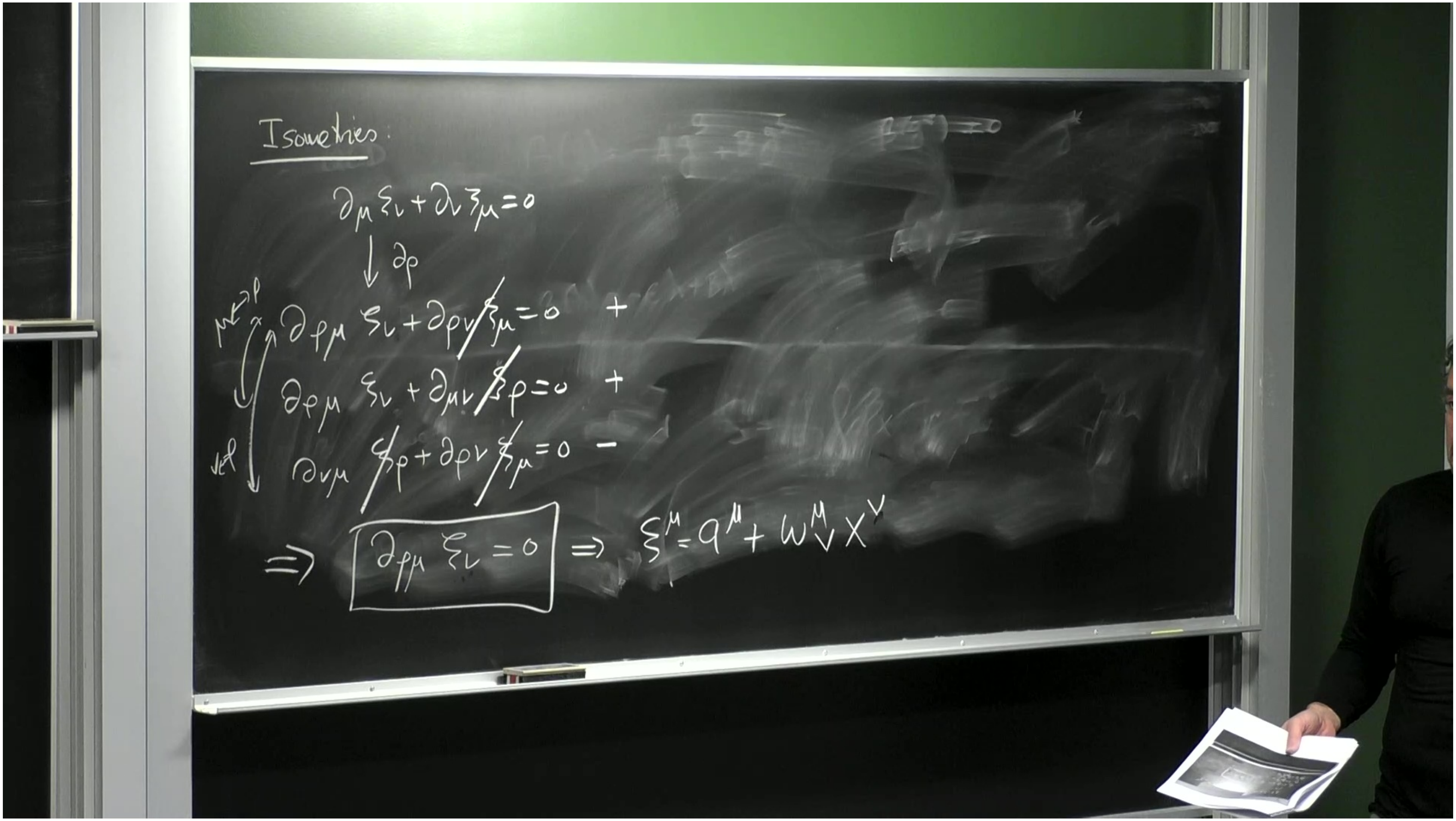
$$\xi = \xi^\mu \partial_\mu.$$

$$\eta_{\rho\sigma}\left(\delta^\rho_\mu + \partial_\mu \xi^\sigma\right)\left(\delta^\sigma_\nu + \partial_\nu \xi^\sigma\right) = \left(1 + \frac{2}{D}\,\partial\cdot\xi\right)\eta_{\mu\nu}$$

$O(\xi^0)$: ✓

$O(\xi)$: $\boxed{\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{D}\,\eta_{\mu\nu}(\partial\cdot\xi)}$ conformal Killing equation


Conformal Transformations:
$\partial_{\rho\sigma\nu}\,\xi_\mu = 0$
$D=2$
$\mu=\nu=1$ $\partial_1\xi_1=\partial_2\xi_2$
$\mu=1$ $\nu=2$ $\partial_1\xi_2=-\partial_2\xi_1$
Cauchy Riemann
$z=x^1+ix^2$
$\mathbb{R}^{1,1}$
$\partial_{\bar z}\,\xi=0$


Isometries:
$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = 0$
$\downarrow \partial_\rho$
$\mu \leftrightarrow \rho$
$\partial_{\rho\mu} \xi_\nu + \partial_{\rho\nu} \cancel{\xi_\mu} = 0 \quad +$
$\partial_{\rho\mu} \xi_\nu + \partial_{\mu\nu} \cancel{\xi_\rho} = 0 \quad +$
$\nu \leftrightarrow \rho$
$\partial_{\nu\mu} \cancel{\xi_\rho} + \partial_{\rho\nu} \cancel{\xi_\mu} = 0 \quad -$
$\Rightarrow \boxed{\partial_{\rho\mu} \xi_\nu = 0} \Rightarrow \xi^\mu = a^\mu + \omega^\mu{}_\nu x^\nu$

Most general CKV

$$\xi^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + b^\mu x^2 - 2 x^\mu\, b\cdot x$$

P^μ $\quad M_{\mu\nu}$ $\quad \hat{D}$ (Dilation) $\quad K_\mu$ (Special conformal Transformation)

$$\eta_{\rho\sigma}\frac{\partial\tilde{x}^\rho}{\partial x^\mu}\frac{\partial\tilde{x}^\sigma}{\partial x^\nu} \; [\text{illegible}] \; \eta_{\mu\nu}. \qquad \left|\frac{\partial\tilde{x}}{\partial x}\right| = e$$

$$R^\rho_\mu(x) = \frac{\partial\tilde{x}^\rho}{\partial x^\mu}e^{-\omega(x)}$$

$$\Downarrow$$

$$e^{\omega(x)} = \left|\frac{\partial\tilde{x}}{\partial x}\right|^{1/D}$$

$$R^\rho_\mu\,\eta_{\rho\sigma}\,R^\sigma_\nu = \eta_{\mu\nu}$$

$$\left|\frac{\partial\tilde{x}}{\partial x}\right|^{\frac{2}{D}} = 1 + \frac{2}{D}\,\partial\cdot\xi$$

$$\det A = e^{\mathrm{Tr}\log A}$$

$x^\mu \to x'^\mu = x^\mu + \xi^\mu$ $\quad$ ξ^μ generates a vector field

$$\left(\xi = \xi^\mu\partial_\mu\right)$$

$$\eta_{\rho\sigma}\left(\delta^\rho_\mu + \partial_\mu\xi^\rho\right)\left(\delta^\sigma_\nu + \partial_\nu\xi^\sigma\right) = \left(1 + \frac{2}{D}\partial\cdot\xi\right)\eta_{\mu\nu}$$

$O(\xi^0)$: ✓

$O(\xi)$: $\boxed{\partial_\mu\xi_\nu + \partial_\nu\xi_\mu = \frac{2}{D}\,\eta_{\mu\nu}(\partial\cdot\xi)}$ $\quad$ conformal Killing equation

(Dilation) (Special conformal Transformation)

$D \quad \frac{D(D-1)}{2} \quad 1 \quad D \quad$ dim of conformal algebra $= \frac{(D+2)(D+1)}{2}$

$$[\xi_1, \xi_2] = \xi_3.$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = i\,\eta^{\mu\rho} M^{\nu\sigma} + \begin{smallmatrix}\mu\leftrightarrow\nu \\ \rho\leftrightarrow\sigma\end{smallmatrix}$$

$$[M^{\mu\nu}, P^\rho] = i(\eta^{\mu\rho} P^\nu - \eta^{\nu\rho} P^\mu)$$

$$[M^{\mu\nu}, K^\rho] = i(\eta^{\mu\rho} K^\nu - \eta^{\nu\rho} K^\mu).$$

$$[M^{\mu\nu}, D] = 0$$

$$[D, P^\mu] = -iP^\mu$$

$$[D, K^\mu] = iK^\mu$$

$$[P^\mu, K^\nu] = -2i\left(\eta^{\mu\nu} D + M^{\mu\nu}\right).$$

$$X^M = (\boxed{X^\mu}, X^D, X^{D+1}) \qquad L_{MN} = -L_{NM} \qquad S$$

$$\parallel$$

Minkowski

$$[L_{MN}, L_{PQ}] = i\,\hat{\eta}_{MP} L_{NP} + \dots$$

$$\hat{\eta}_{MN} = (\underbrace{\cancel{-}\ +}_{x_\mu}, +, -)$$

$$[D, P^\mu] = -iP^\mu$$

$$[D, K^\mu] = iK^\mu$$

$$[P^\mu, K^\nu] = -2i(\eta^{\mu\nu} D + M^{\mu\nu}).$$

$$X^M = (\boxed{X^\mu}, X^D, X^{D+1}) \qquad L_{MN} = -L_{NM} \qquad S$$

X^μ || Minkowski

$$[L_{MN}, L_{PQ}] = i\hat{\eta}_{MP} L_{NP} + \ldots$$

$$\hat{\eta}_{MN} = (\underbrace{- +}_{x_\mu}, +, -) \qquad SO(2,D)$$

Minkowski:

$$\hat{\eta}_{MN} = (\underbrace{-+,+}_{x_\mu},+,-)$$

$$[L_{MN}, L_{PQ}] = i\,\hat{\eta}_{MP}\, L_{NP} + \dots$$

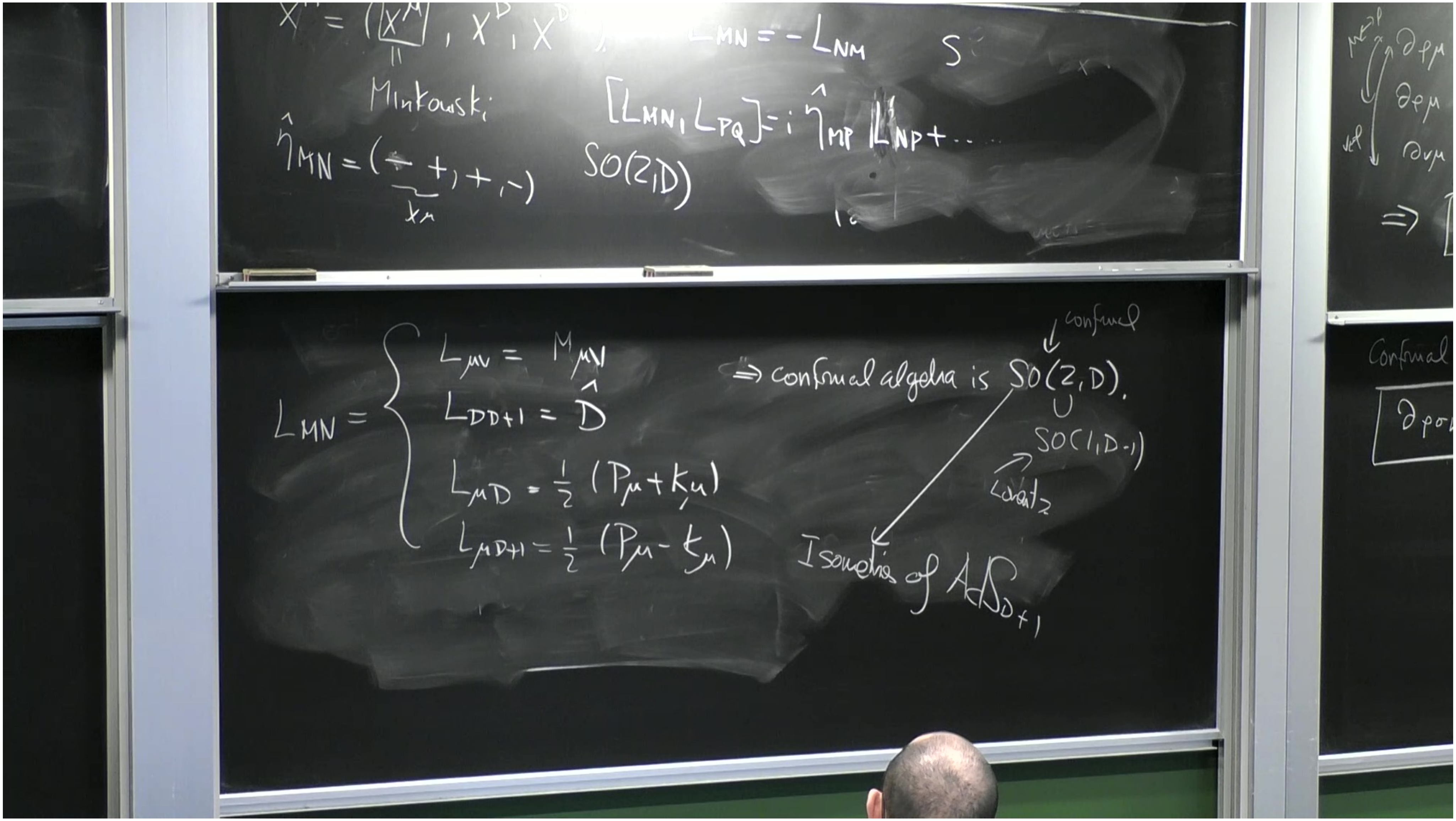
$$SO(2,D)$$

$$L_{MN} = \begin{cases} L_{\mu\nu} = M_{\mu\nu} \\ L_{D\,D+1} = \hat{D} \\ L_{\mu D} = \frac{1}{2}(P_\mu + K_\mu) \\ L_{\mu\, D+1} = \frac{1}{2}(P_\mu - K_\mu) \end{cases}$$

$\Rightarrow$ conformal algebra is $SO(2,D)$. (conformal)

$$\cup$$

$SO(1,D-1)$ (Lorentz)


$X^M = (\boxed{X^\mu}, X^D, X^{D+1})$
Minkowski
$L_{MN} = -L_{NM}$
$[L_{MN}, L_{PQ}] = i\,\hat\eta_{MP} L_{NP} + \dots$
$\hat\eta_{MN} = (\underbrace{-,+}_{x_\mu},+,-)$
$SO(2,D)$
$L_{MN} = \begin{cases} L_{\mu\nu} = M_{\mu\nu} \\ L_{D\,D+1} = \hat{D} \\ L_{\mu D} = \frac{1}{2}(P_\mu + K_\mu) \\ L_{\mu\,D+1} = \frac{1}{2}(P_\mu - K_\mu) \end{cases}$
$\Rightarrow$ conformal algebra is $SO(2,D)$.
conformal
$\cup$
$SO(1,D-1)$
Lorentz
Isometries of AdS_{D+1}
Conformal

$$\Rightarrow \boxed{\partial_{\rho\mu}\xi_\nu = 0} \Rightarrow \xi^\mu = \underset{P^\mu}{a^\mu} + \underset{M_{\mu\nu}}{\omega^\mu{}_\nu} x^\nu$$

Conformal Transformations:

$$\boxed{\partial_{\rho\sigma\nu}\xi_\mu = 0}$$

$D=2$

$\mu=\nu=1$ $\quad \partial_1\xi_1 = \partial_2\xi_2$

$\mu=1\ \nu=2$ $\quad \partial_1\xi_2 = -\partial_2\xi_1$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ Cauchy Riemann

$\mathbb{R}^{1,1}$

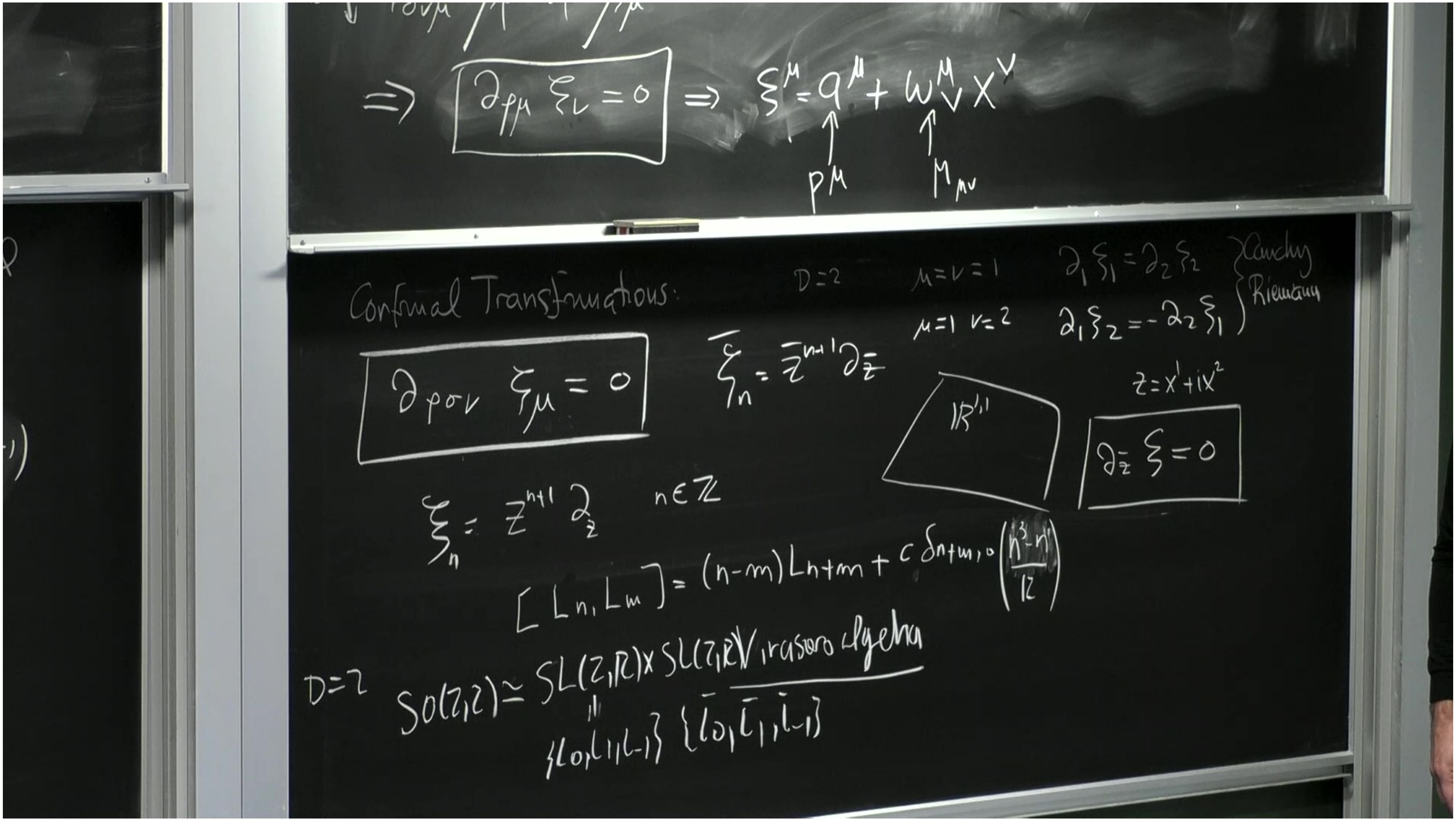
$z = x^1 + ix^2$

$$\boxed{\partial_{\bar z}\xi = 0}$$

$$\xi_n = z^{n+1}\partial_z \qquad n\in\mathbb{Z}$$

$$[L_n, L_m] = (n-m)L_{n+m} + c\,\delta_{n+m,0}\left(\frac{n^3-n}{12}\right)$$

Virasoro algebra


⇒ ∂_(ρμ ξ_ν) = 0 ⇒ ξ^μ = a^μ + ω^μ_ν x^ν
P^μ M_μν
Conformal Transformations:
∂_(ρ σ_ν ξ_μ) = 0
ξ_n = z̄^(n+1) ∂_z̄
D=2 μ=ν=1 ∂_1 ξ_1 = ∂_2 ξ_2
μ=1 ν=2 ∂_1 ξ_2 = −∂_2 ξ_1
Cauchy Riemann
z = x^1 + i x^2
∂_z̄ ξ = 0
ξ_n = z^(n+1) ∂_z n ∈ ℤ
[L_n, L_m] = (n−m) L_n+m + c δ_n+m,0 (n^3−n)/12
D=2 SO(2,2) ≃ SL(2,R) × SL(2,R) Virasoro algebra
{L_0, L_1, L_−1} {L̄_0, L̄_1, L̄_−1}