Title: Advanced General Relativity - 240306 (afternoon)

Speakers: Eric Poisson

Collection: Advanced General Relativity (PHYS7840)

Date: March 06, 2024 - 1:30 PM

URL: https://pirsa.org/24030004

Gauss–Codazzi

$${}^4R_{\mu\alpha\beta\gamma}\, n^\mu e^\alpha_a e^\beta_b e^\gamma_c = D_c K_{ab} - D_b K_{ac}$$

$${}^4R_{\alpha\beta\gamma\delta}\, e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d = R_{abcd} - \varepsilon(K_{ac}K_{bd} - K_{ad}K_{bc})$$

$${}^4G_{\mu\nu}\, n^\mu n^\nu = -\frac{1}{2}(\varepsilon R + K_{ab}K^{ab} - K^2)$$

$${}^4G_{\mu\alpha}\, n^\mu e^\alpha_a = D_b K^b{}_a - D_a K$$

$$K_{ab} = e^\alpha_a e^\beta_b \nabla_\alpha n_\beta$$

$$= e^\alpha_a e^\beta_b \nabla_\beta n_\alpha$$

$$= \frac{1}{2} e^\alpha_a e^\beta_b (\nabla_\alpha n_\beta + \nabla_\beta n_\alpha)$$

$$= \frac{1}{2} e^\alpha_a e^\beta_b \mathcal{L}_n\, g_{\alpha\beta}$$

Painlevé–Gullstrand slice of Schwarzschild

$$= -f\,dt^2 + f^{-1}\,dr^2 + r^2\,d\Omega^2$$

$$= 1 - 2M/r$$

surface : $\Phi = t + \int \frac{\sqrt{2M/r}}{f}\,dr = t + 4M\left\{\sqrt{r/2M} + \frac{1}{2}\ln\frac{\sqrt{r/2M}-1}{\sqrt{r/2M}+1}\right\}$

$- \nabla_\alpha \Phi = -\left(1, \frac{\sqrt{2M/r}}{f}, 0, 0\right)$ $\qquad g^{\alpha\beta} n_\alpha n_\beta = -1$

oordinates : $y^a = (r, \theta, \phi)$ $\qquad e^\alpha_a = \partial x^\alpha / \partial y^a$

relations : $t = \text{const} - \int \frac{\sqrt{2M/r}}{f}\,dr$ $\qquad e^\alpha_r =$

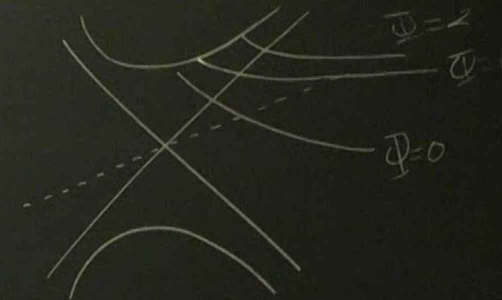
$r = r$

$\theta = \theta$

$\phi = \phi$

warzschild

$$\frac{\sqrt{2M/r}}{f}\,dr = t + 4M\left\{\sqrt{r/2M} + \frac{1}{2}\ln\frac{\sqrt{r/2M}-1}{\sqrt{r/2M}+1}\right\}$$



$)$

$$g^{\alpha\beta} n_\alpha n_\beta = -1$$

$$e^\alpha_a = \partial x^\alpha / \partial y^a$$

$$\int \frac{\sqrt{2M/r}}{f}\,dr$$

$$e^\alpha_r = \left(-\frac{\sqrt{2M/r}}{f}, 1, 0, 0\right)$$

$$e^\alpha_\theta = (0, 0, 1, 0)$$

$$e^\alpha_\phi = (0, 0, 0, 1)$$

$r = r$

$\theta = \theta$

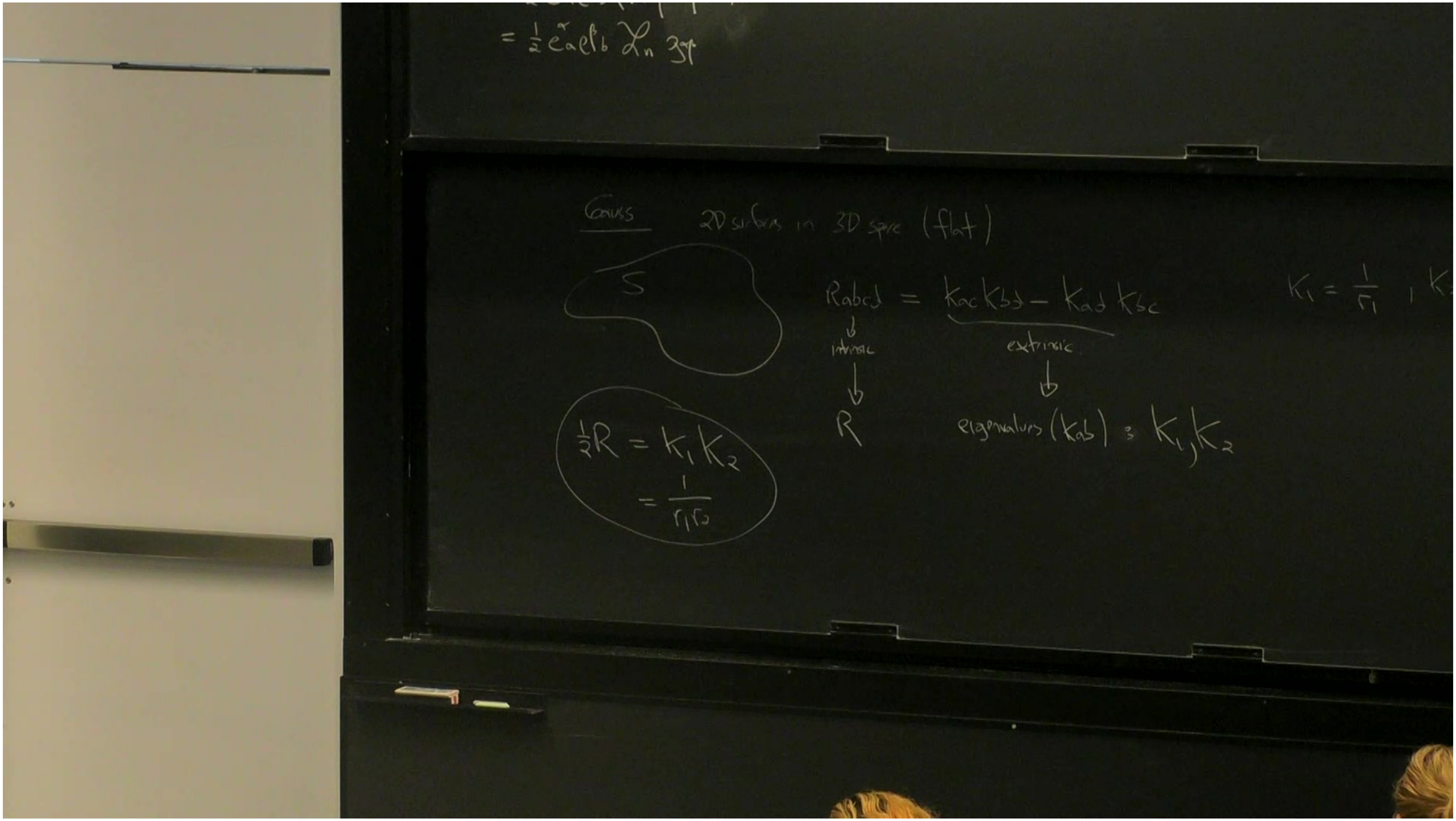
$\varphi = \varphi$

$e^\alpha_\theta = (0, 0, 1, 0)$

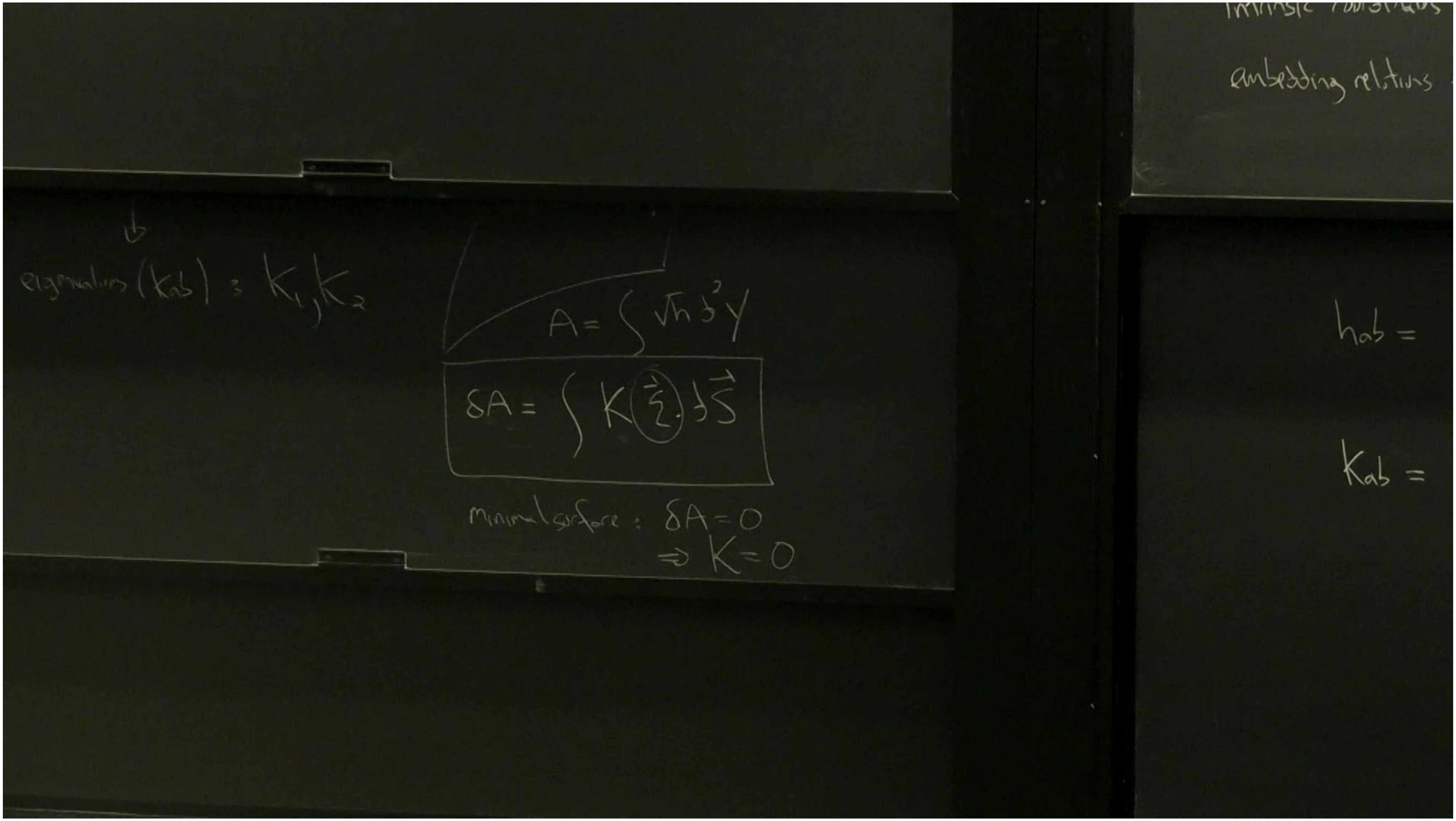
$e^\alpha_\varphi = (0, 0, 0, 1)$

$$h_{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2\theta \end{pmatrix} \qquad h_{ab}\, dy^a dy^b = dr^2 + r^2 d\Omega^2 = \text{flat!}$$

$$K_{ab} = \begin{pmatrix} \frac{1}{2}\sqrt{2m/r^3} & 0 & 0 \\ 0 & -\sqrt{2mr} & 0 \\ 0 & 0 & -\sqrt{2mr}\,\sin^2\theta \end{pmatrix} \qquad K = -\frac{3}{2}\sqrt{2m/r^3}$$



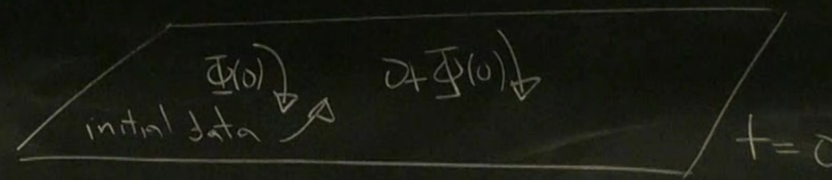
$= \frac{1}{2} e^{\alpha}_{a} e^{\beta}_{b} \mathcal{L}_n g_{\alpha\beta}$
Gauss
2D surfaces in 3D space (flat)
S
$R_{abcd} = \underbrace{K_{ac} K_{bd} - K_{ad} K_{bc}}_{\text{extrinsic}}$
intrinsic
R
eigenvalues (K_{ab}) : K_1, K_2
$\frac{1}{2} R = K_1 K_2 = \frac{1}{r_1 r_2}$
$K_1 = \frac{1}{r_1}$, K


embedding relations
eigenvalues (K_{ab}) : K_1, K_2
$A = \int \sqrt{h}\, d^2y$
$\delta A = \int K \vec{\xi} \cdot d\vec{S}$
minimal surface : $\delta A = 0$
$\Rightarrow K = 0$
$h_{ab} =$
$K_{ab} =$

Initial-value problem

flat 4D spacetime,

$$\Box \Phi = 0$$



curved spacetime:

Σ : spacelike hypersurface

Initial-value problem

curved spacetime:

Φ

$n^\alpha \nabla_\alpha \Phi$

any Σ: spacelike hypersurface

initial data: $\Phi(\Sigma)$

$n^\alpha \nabla_\alpha \Phi(\Sigma)$

flat 4D spacetime,

$$\Box \Phi = 0$$

$\Phi(0)$ $\partial_t \Phi(0)$

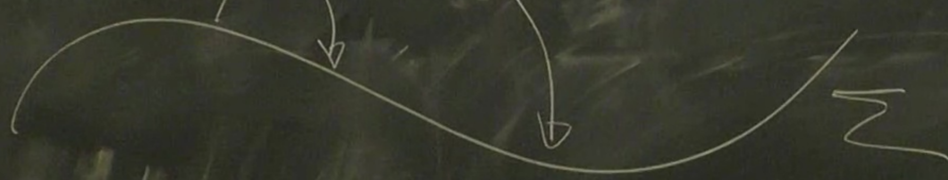
initial data

$t = 0$

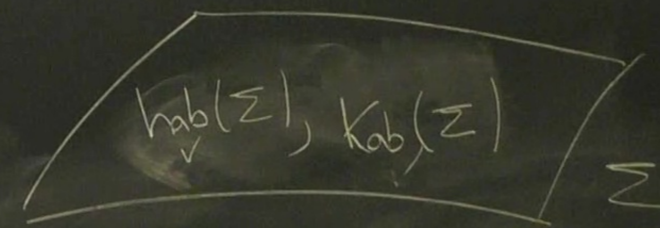
GR

initial moment $\equiv$ any spacelike hypersurface

$\cancel{g_{\alpha\beta}(\Sigma)}$, $\mathcal{L}_n g_{\alpha\beta}(\Sigma)$



$h_{ab}(\Sigma)$, $K_{ab}(\Sigma)$



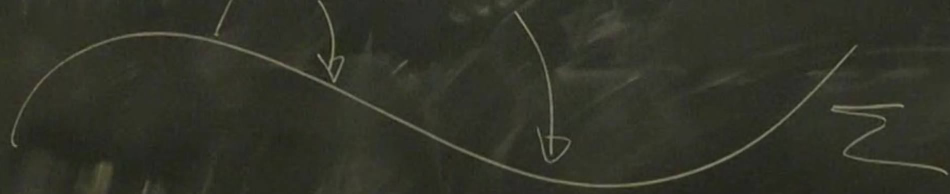
EFE : $\partial_t h_{ab} = \cdots$

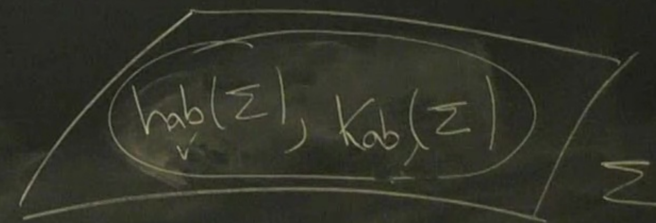
$\partial_t K_{ab} = \cdots$

GR

initial moment ≡ any spacelike hypersurface

$\cancel{g_{\alpha\beta}(\Sigma)}$, $\mathcal{L}_n g_{\alpha\beta}(\Sigma)$



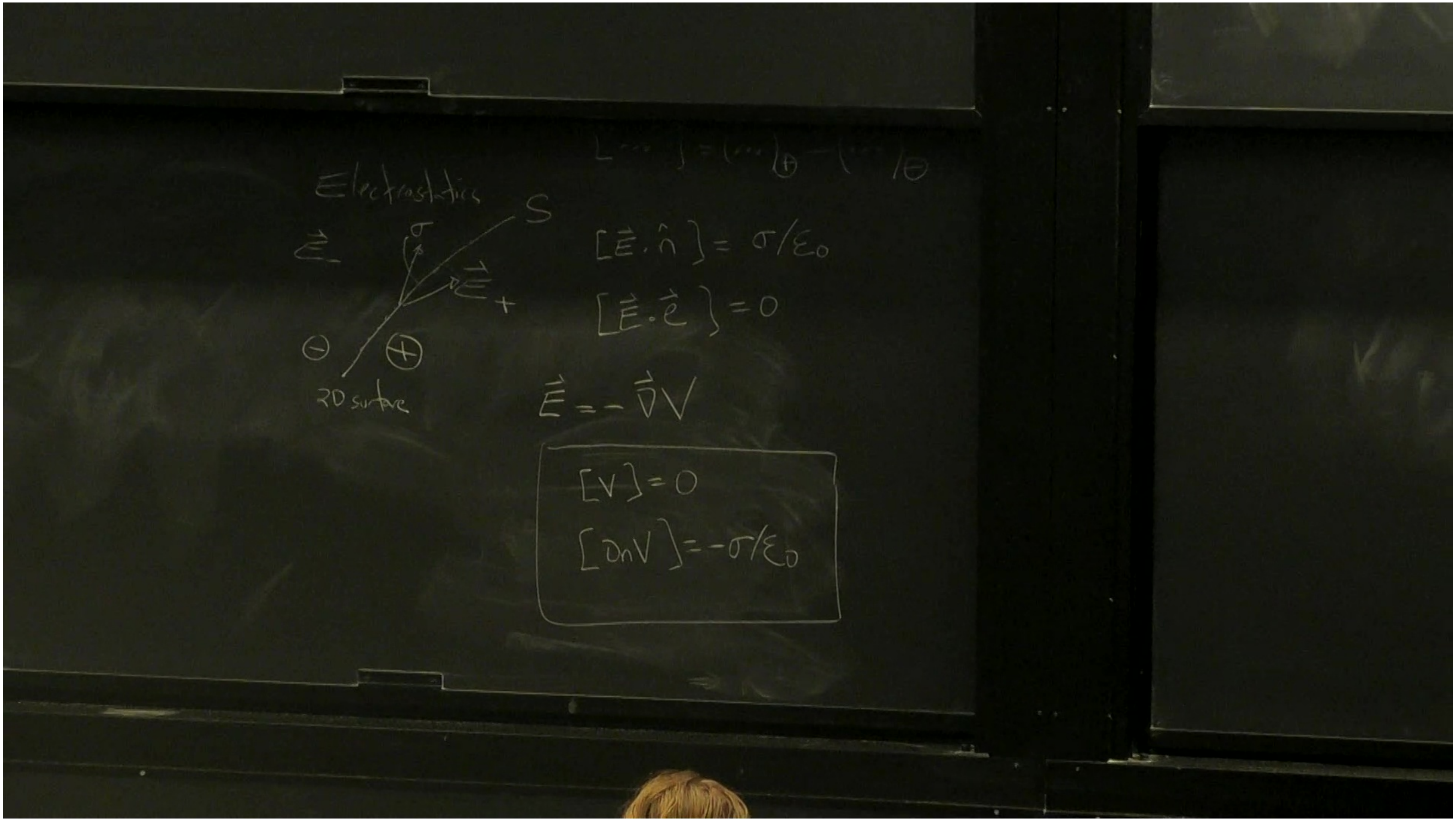


EFE : $\partial_t h_{ab} = \cdots$

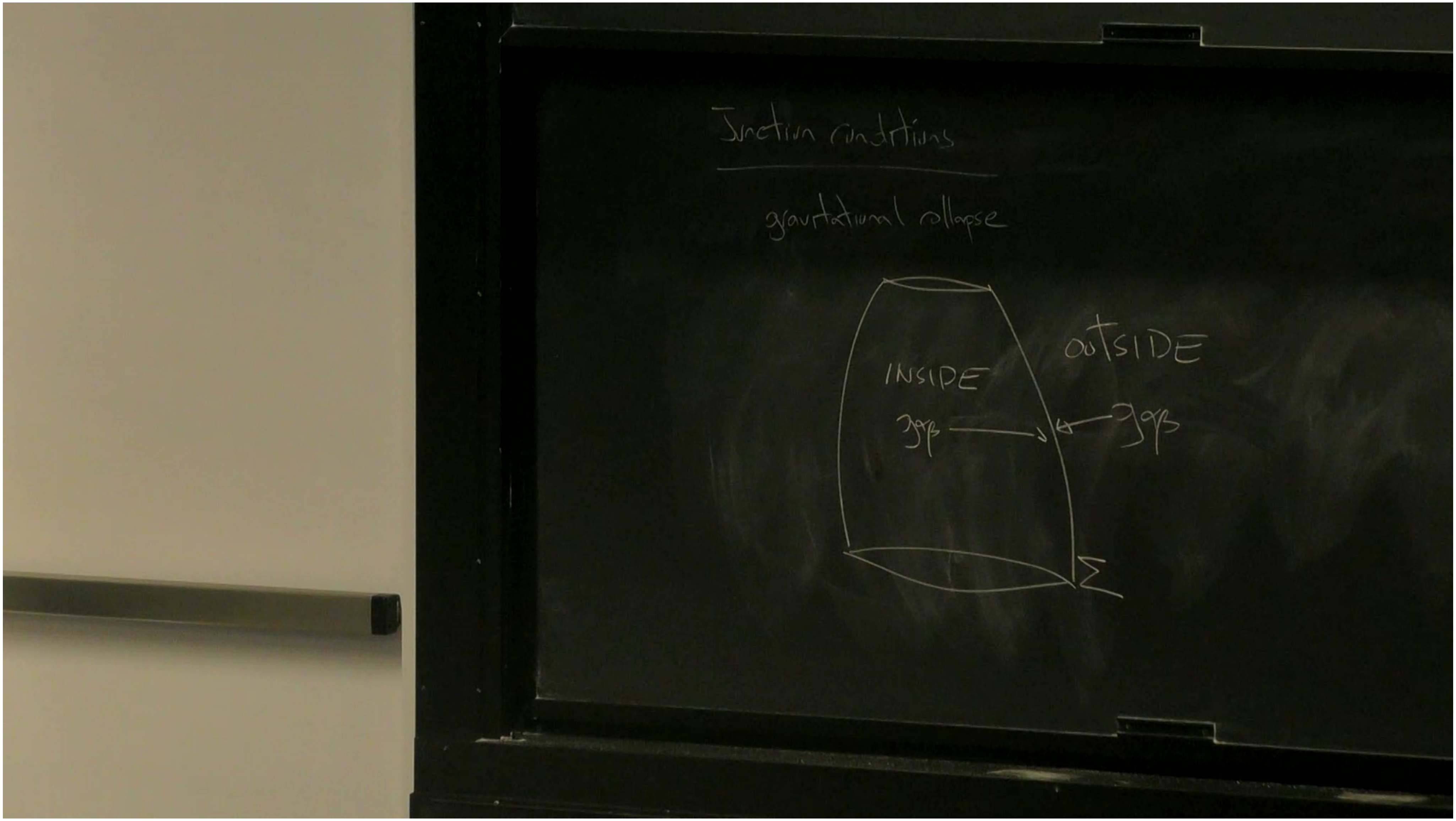
$\partial_t K_{ab} = \cdots$

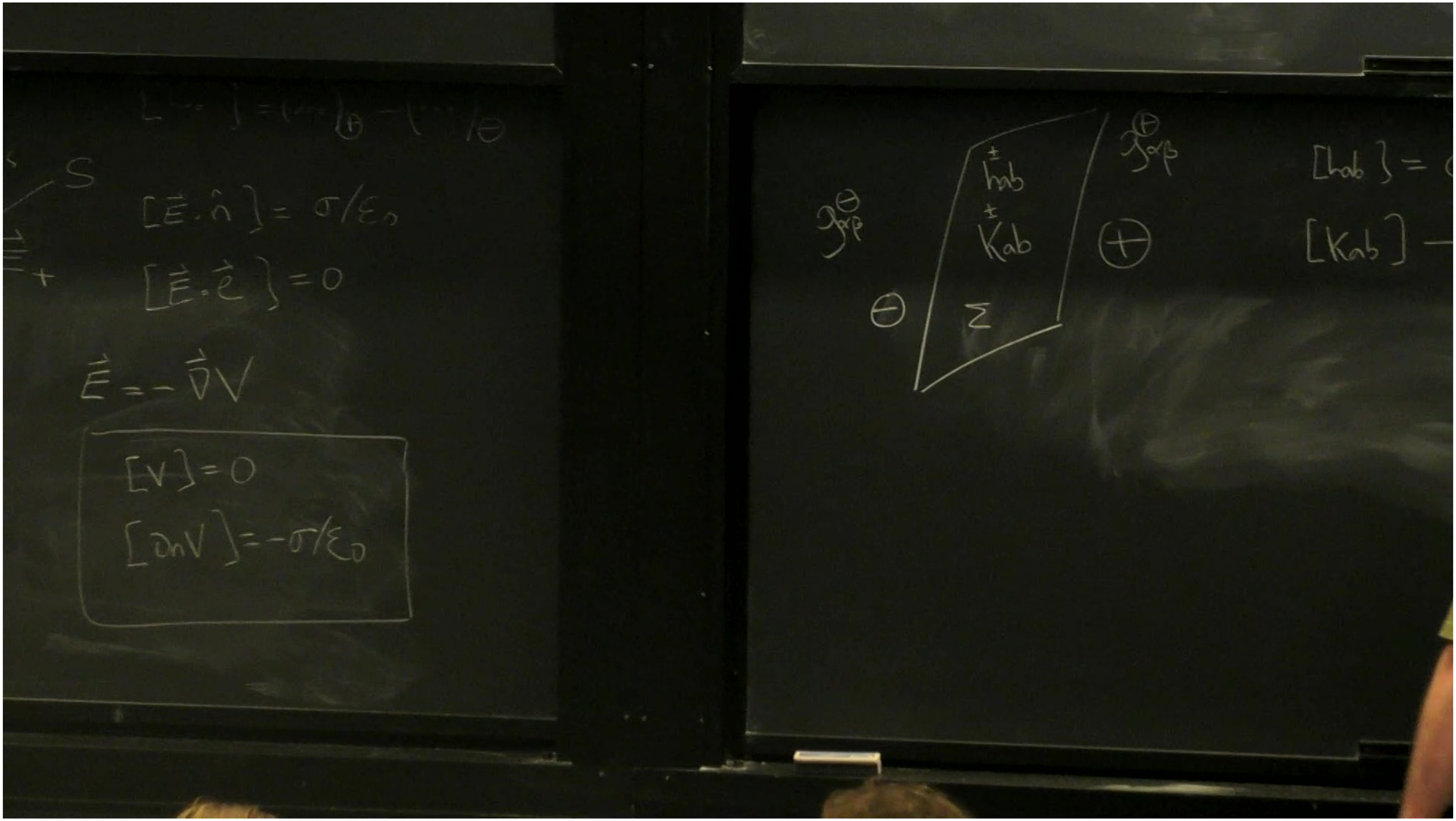
Constraints : $R - K_{ab}K^{ab} + K^2 = 8\pi T_{\mu\nu} n^\mu n^\nu$ ≡ energy density

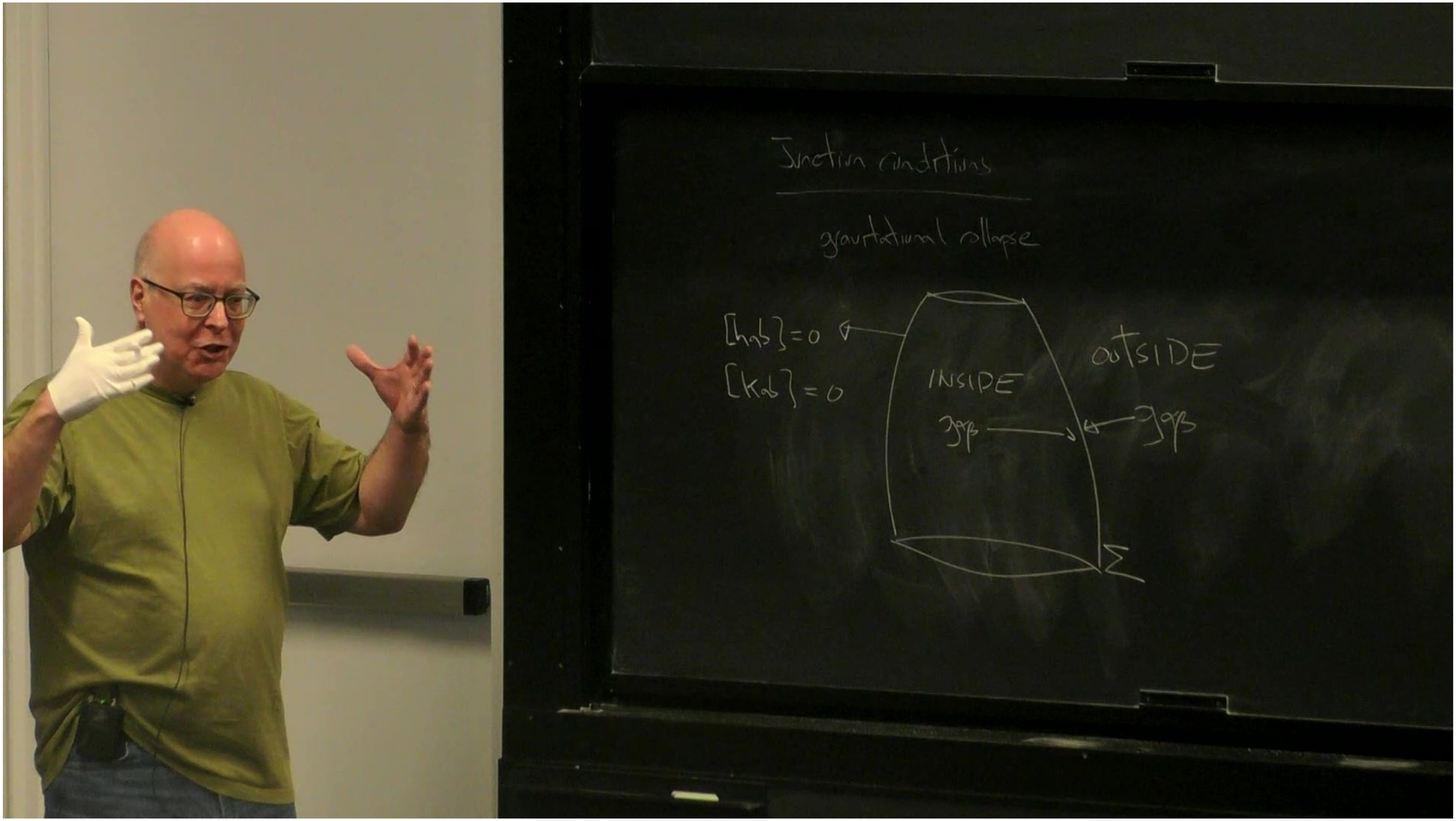
$D_b K^b{}_a - D_a K = 4\pi T_{\mu\nu} n^\mu e^\nu{}_a$ ≡ matter current

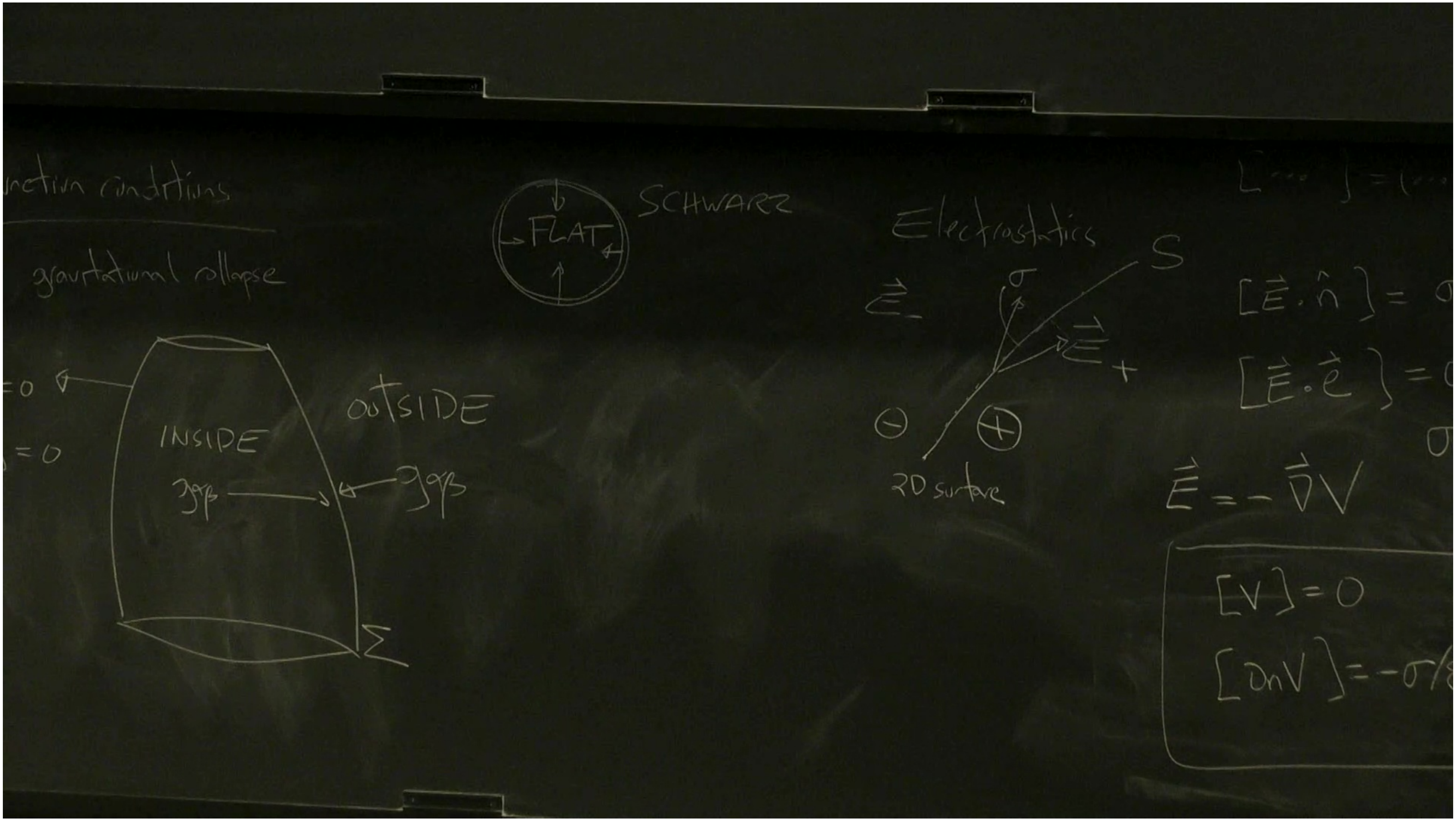


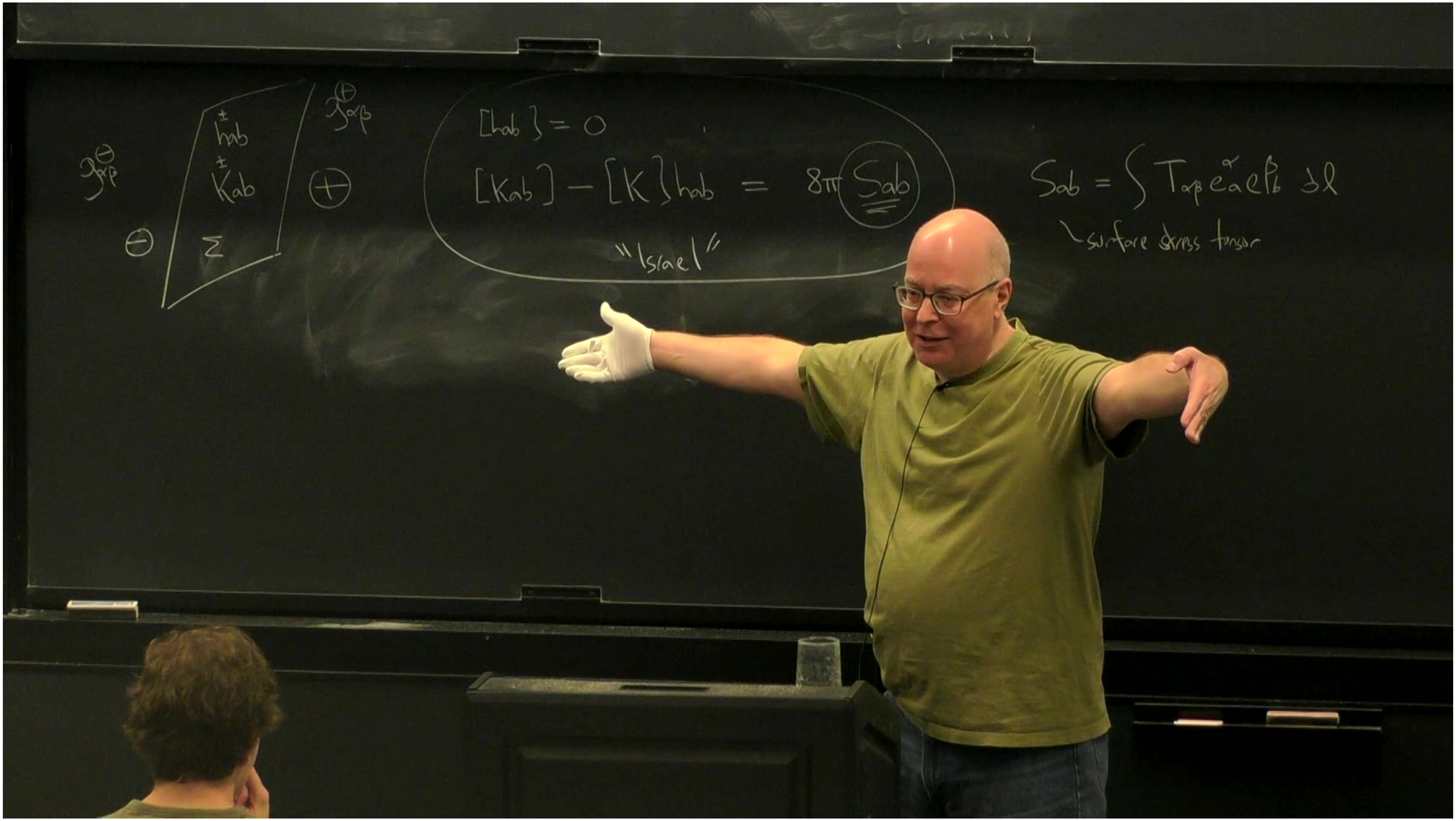
Electrostatics
S
σ
$\vec{E}_-$
$\vec{E}_+$
2D surface
$[\vec{E}\cdot\hat{n}] = \sigma/\varepsilon_0$
$[\vec{E}\cdot\vec{e}] = 0$
$\vec{E} = -\vec{\nabla}V$
$[V] = 0$
$[\partial_n V] = -\sigma/\varepsilon_0$


Junction conditions
gravitational collapse
INSIDE
OUTSIDE
$g_{\alpha\beta}$
$g_{\alpha\beta}$
Σ


S
[E · n̂] = σ/ε₀
[E · e] = 0
E = − ∇V
[V] = 0
[∂ₙV] = −σ/ε₀
g⊖αβ
h±ab
K±ab
⊖
Σ
⊕
g⊕αβ
[hab] =
[Kab] —


Junction conditions
gravitational collapse
[hab] = 0
[Kab] = 0
INSIDE
OUTSIDE
gαβ
gαβ
Σ


gravitational collapse
FLAT
SCHWARZ
OUTSIDE
INSIDE
Σ
Electrostatics
S
σ
2D surface
[V]=0


[hab] = 0
[Kab] − [K] hab = 8π Sab
"Israel"
Sab = ∫ Tαβ eαa eβb dl
surface stress tensor
hab
Kab
Σ
gαβ