Title: Advanced General Relativity - 240320

Speakers: Eric Poisson

Collection: Advanced General Relativity (PHYS7840)

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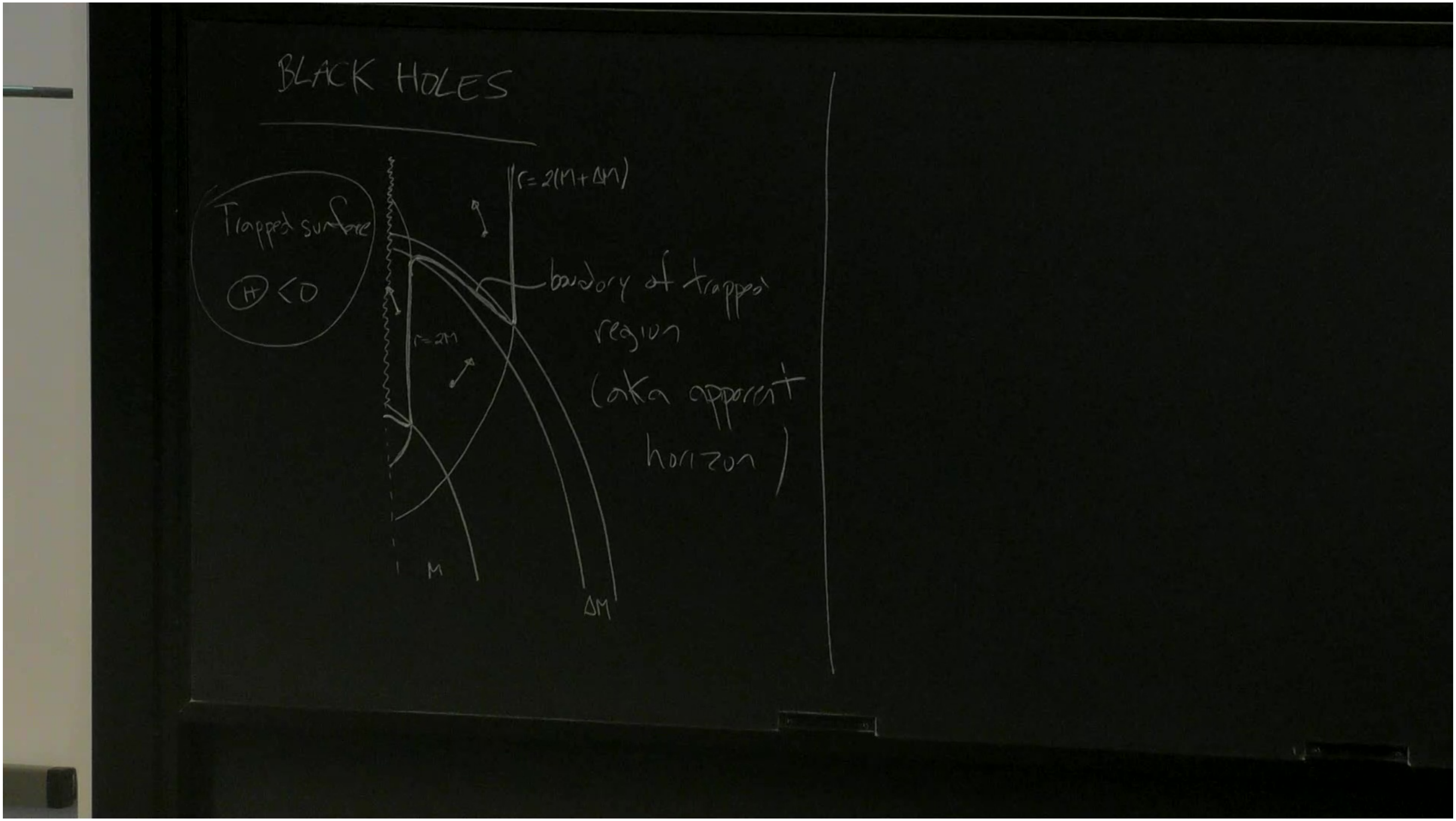
URL: https://pirsa.org/24030002


BLACK HOLES
Trapped surface
Θ < 0
r = 2(M + ΔM)
boundary of trapped region
(aka apparent horizon)
r = 2M
M
ΔM

rapped

pparent

zon

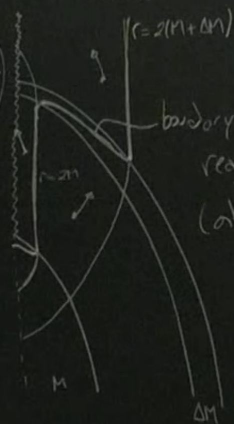
Schwarzschild spacetime

$$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2 \qquad f = 1 - 2M/r$$

Birkhoff: S metric is the outside metric of any spherically symmetric body.

Trapped surface

$\Theta < 0$



boundary of trapped region (aka apparent horizon)

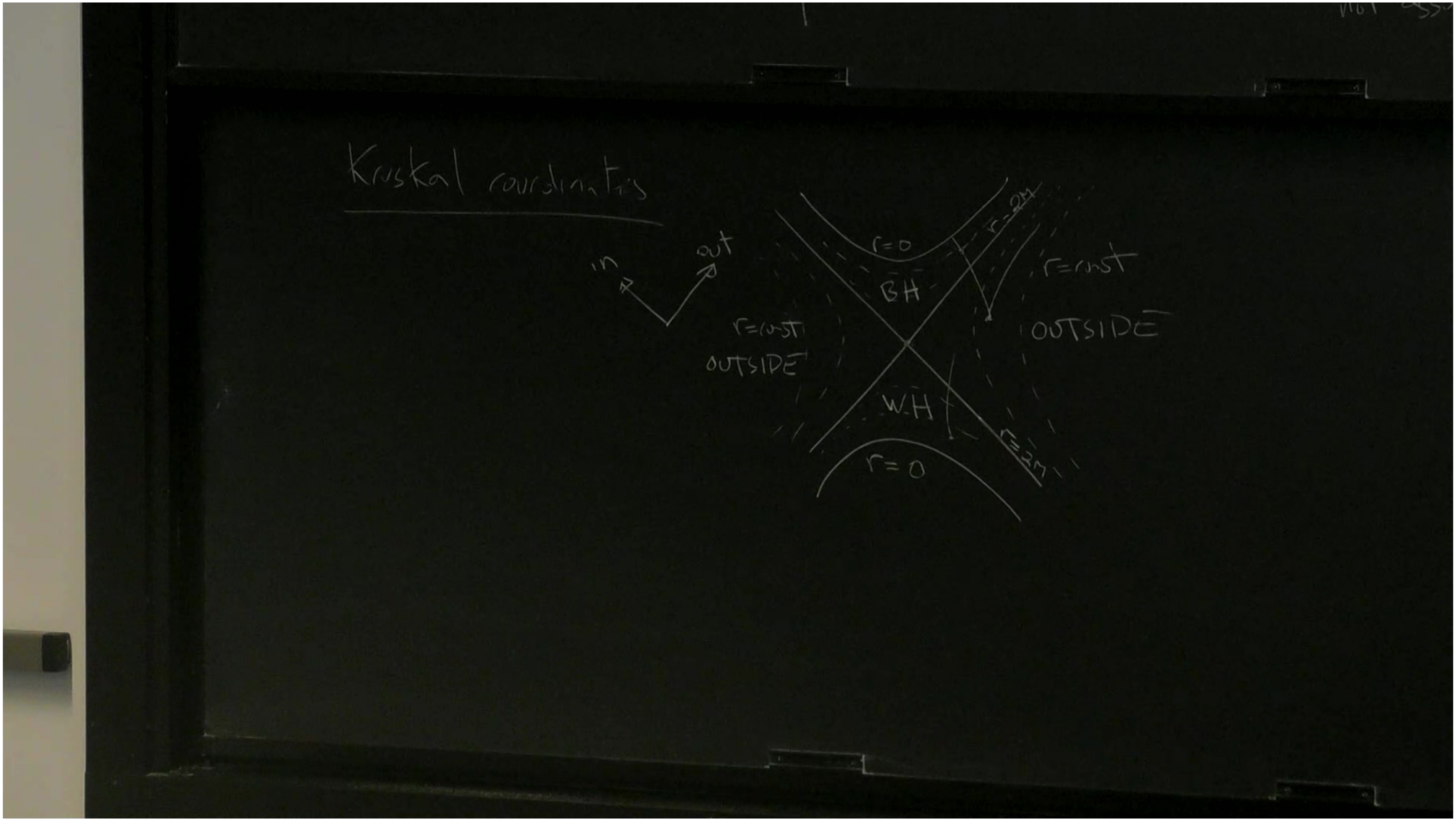
Schwarzschild spacetime

$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2\,d\Omega^2 \qquad f = 1 - 2M/r$$

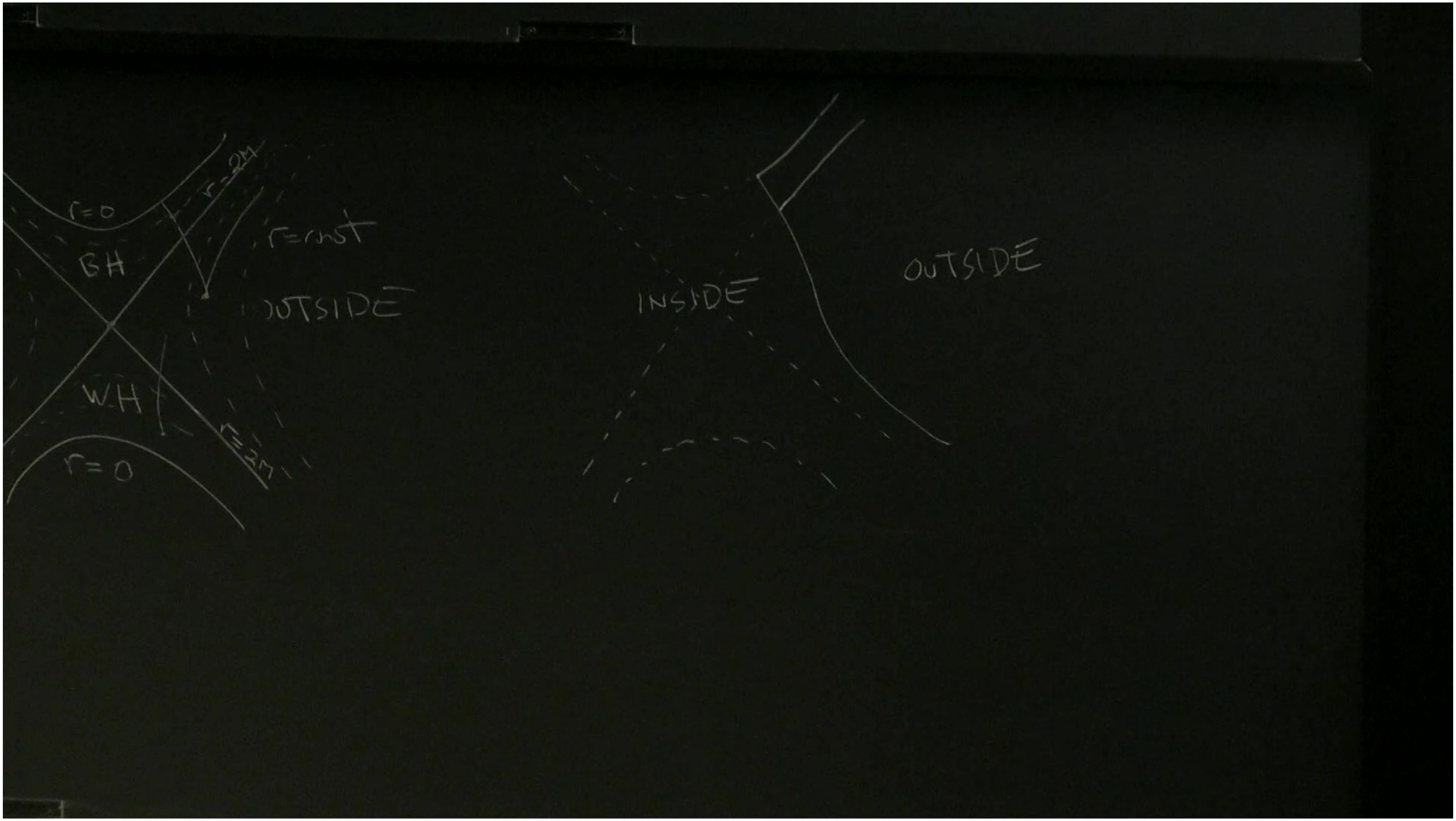
Birkhoff: S metric is the outside metric of any spherically symmetric body.

Israel: S metric is unique sol to EFE in vacuum for a static, asymptotically spacetime with a nonsingular event horizon.

(spherical symmetry is consequence, not assumption)



Kruskal coordinates
in
out
r=0
r=2M
r=const
BH
r=const
OUTSIDE
OUTSIDE
WH
r=2M
r=0

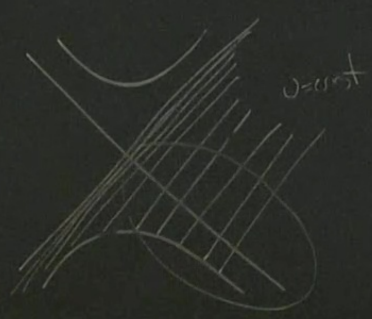

r=0
BH
WH
r=0
r=2M
r=2M
r=const
OUTSIDE
INSIDE
OUTSIDE

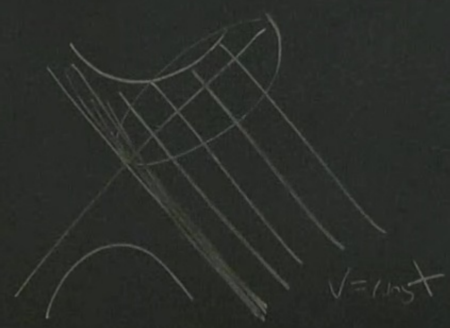
Eddington–Finkelstein coordinates

$$ds^2 = -f\,\underbrace{(dt - f^{-1}dr)}_{dU}\,\underbrace{(dt + f^{-1}dr)}_{dV} + r^2 d\Omega^2$$

$U = t - \int \frac{dr}{f}$ – constant on outgoing light rays

$V = t + \int \frac{dr}{f}$ – constant on ingoing light rays





$) + r^2 d\Omega^2$

$$dt = dv - f^{-1} dr$$

$$ds^2 = -f\left(dv - f^{-1} dr\right)^2 + f^{-1} dr^2 + r^2 d\Omega^2$$

$$= -f\left(dv^2 - 2f^{-1} dv\, dr + \cancel{f^{-2} dr^2}\right) + \cancel{f^{-1} dr^2} + r^2 d\Omega^2$$

ht rays

ht rays

$$\boxed{ds^2 = -f\, dv^2 + 2\, dv\, dr + r^2 d\Omega^2}$$

$v = \text{const}$

$$0 = \partial u = \partial t - f^{-1}\partial r = \partial v - 2f^{-1}\partial r$$

$u = \text{const}$,
$$\bar{k}_\alpha = -\nabla_\alpha u = \left(-1, +\frac{2}{f}, 0, 0\right)$$

$$\bar{k}^\alpha = g^{\alpha\beta}\bar{k}_\beta = \left(\frac{2}{f}, 1, 0, 0\right)$$

$$\bar{k}_\alpha \bar{k}^\alpha = 0$$

$$\bar{k}^\beta \nabla_\beta \bar{k}^\alpha = 0$$

$$0 = \partial U = \partial t - f^{-1}\partial r = \partial v - 2f^{-1}\partial r$$

$U = \text{const}$, $\quad \bar{K}_\alpha = -\nabla_\alpha U = \left(-1, +\frac{2}{f}, 0, 0\right)$

$$\bar{K}^\alpha = g^{\alpha\beta}\bar{K}_\beta = \left(\frac{2}{f}, 1, 0, 0\right)$$

$$\bar{K}_\alpha \bar{K}^\alpha = 0$$

$$\bar{K}^\beta \nabla_\beta \bar{K}^\alpha = 0$$

$$\frac{\partial v}{\partial \lambda} = \bar{K}^v = \frac{2}{f}$$

$$\frac{\partial r}{\partial \lambda} = \bar{K}^r = 1$$

$$\frac{\partial \theta}{\partial \lambda} = \bar{K}^\theta = 0$$

$$\frac{\partial \varphi}{\partial \lambda} = \bar{K}^\varphi = 0$$

= const

increasing

New parametrization (v)

$$\bar{K}^\alpha = \underbrace{\frac{2}{f}}_{p} K^\alpha \, , \quad \boxed{K^\alpha = \left(1, \tfrac{1}{2} f, 0, 0\right)}$$

$$\boxed{\begin{aligned} K^\alpha &= p\, \bar{K}^\alpha \\ p &= \tfrac{1}{2} f = \frac{\partial r}{\partial v} \end{aligned}}$$

geodesic eqn : $K^\beta \nabla_\beta K^\alpha = \rho \bar{K}^\beta \nabla_\beta (\rho \bar{K}^\alpha) = \rho^2 \underbrace{\bar{K}^\beta \nabla_\beta \bar{K}^\alpha}_{0} + (\rho \bar{K}^\beta \nabla_\beta \rho) \bar{K}^\alpha$

$$= \left(\frac{1}{\rho} K^\beta \nabla_\beta \rho\right) K^\alpha$$

$$\rightarrow \boxed{\begin{aligned} K^\beta \nabla_\beta K^\alpha &= \kappa K^\alpha \\ \kappa &\equiv \frac{1}{\rho} K^\alpha \nabla_\alpha \rho \end{aligned}} = \frac{2M}{r^2}$$

$$\kappa = \cancel{\frac{2}{f} \left(\frac{1}{2} f\right)} \frac{\partial}{\partial r} \left(\frac{1}{2} f\right) = \frac{M}{r^2}$$

new parametrization:

$$\Theta = \frac{1}{\delta A}\frac{\partial}{\partial V}\delta A =$$

$$= \frac{\partial r}{\partial V}\,\overline{\Theta}$$

$$= \mu \nabla_\alpha \left(\mu^{-1} K^\alpha\right)$$

$$= \nabla_\alpha K^\alpha - \frac{1}{\mu} K^\alpha \nabla_\alpha \mu$$

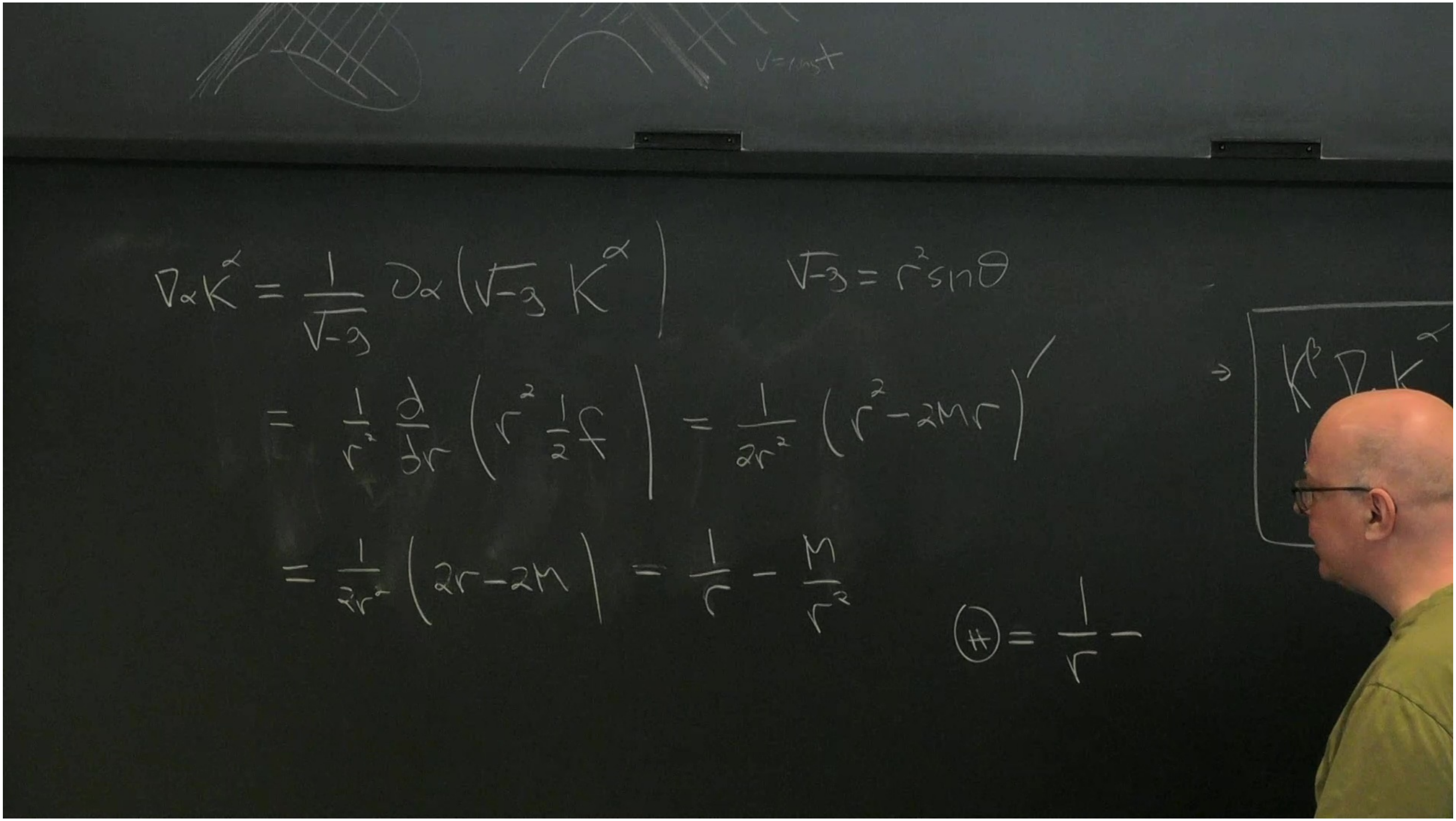
$$\sqrt{-g} = r^2 \sin\theta$$

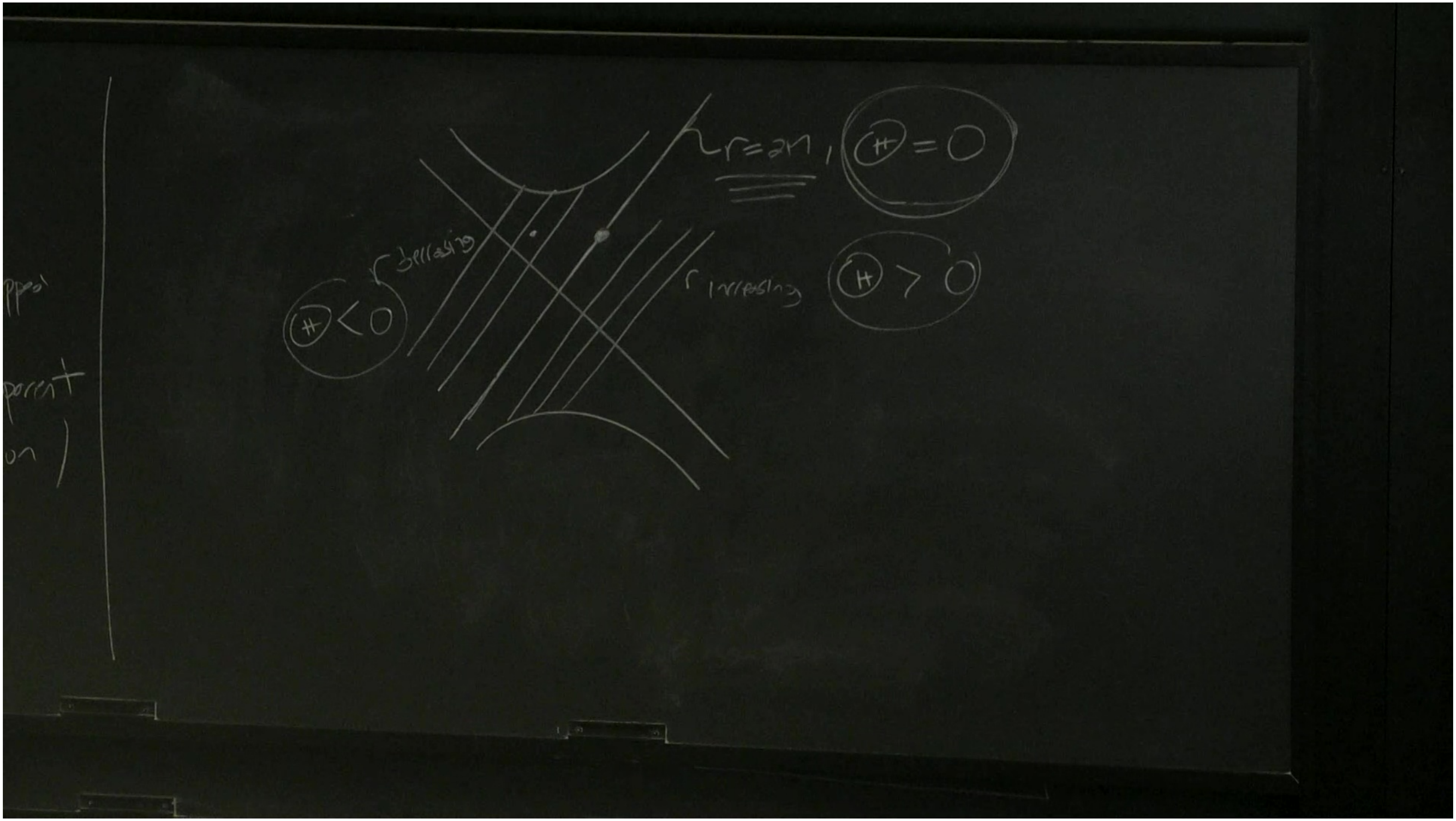
$$= \frac{1}{2r^2}\left(r^2 - 2Mr\right)'$$

$$\frac{1}{r} - \frac{M}{r^2}$$

$$\Rightarrow \boxed{\begin{aligned} K^\beta D_\beta K^\alpha &= \kappa K^\alpha \\ \kappa &\equiv \frac{1}{\rho} K^\alpha D_\alpha \rho \end{aligned}} = \frac{M}{r^2}$$

$$H = \frac{1}{r} - \frac{2M}{r^2} \quad \Rightarrow \quad \boxed{H = \frac{r-2M}{r^2}}$$


v = const
$\nabla_\alpha K^\alpha = \frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}\,K^\alpha\right)$
$\sqrt{-g} = r^2\sin\theta$
$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{1}{2}f\right) = \frac{1}{2r^2}\left(r^2-2Mr\right)'$
$= \frac{1}{2r^2}\left(2r-2M\right) = \frac{1}{r} - \frac{M}{r^2}$
$(H) = \frac{1}{r} -$



r=2m,
r decreasing
r increasing