Title: The quantum mechanics of a perfect fluid

Speakers: Riccardo Rattazzi

Series: Particle Physics

Date: February 29, 2024 - 10:00 AM

URL: https://pirsa.org/24020104

Abstract: Finite density systems can be described by effective field theories with non-linearly realized space-time symmetries, whose construction resembles that of the QCD chiral lagrangian. Based on that similarity, one would expect the construction to work equally well classically and quantum mechanically. While that is true for superfluids and solids, one instead finds that for genuine fluids things are made more complicated by the unusual dynamics of their transverse modes, which are not described by a Fock space. Focussing on the incompressible limit for a fluid in 2+1 dimensions, I illustrate its analogy with the ordinary rigid body. Indeed both systems describe motion on a group manifold. Proceeding from this analogy I develop a consistent quantum mechanical description of a perfect fluid using the known equivalence between the area preserving diffeomorfism group in 2D and SU(N) for N->\infty

The Quantum Mechanics of a Perfect Fluid

Riccardo Rattazzi - EPFL

- S. Dubovsky, T. Gregoire, A. Nicolis, RR 2005

- S. Endlich, A. Nicolis, RR, J. Wang 2010

- A. Nicolis, R. Penco, F. Piazza, RR 2015

- S. Endlich, W. Goldberger, RR, I. Rothstein 2015 unpublished

- A. Dersy, A. Khmelnitsky, RR 2021

- G. Cuomo, F. Eustachon, E. Firat, B. Henning, RR

- S. Dubovsky, E. Firat, A. Gomes, A. Nicolis, RR, ... <u>Now</u>

What is cosmology?

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{gravity}} = \underbrace{T_{\mu\nu}}_{\text{condensed matter}}$$

$$T_{\mu\nu} = T^{0}_{\mu\nu} + \delta T_{\mu\nu}$$

$T^{0}_{\mu\nu} \rightarrow$ finite density QFT

Poincaré $\longrightarrow$ ISO(3)

$\delta T_{\mu\nu} \rightarrow$ hydrodynamic modes

$\equiv$ "Goldstones"

- Which options for dynamics behind $T_{\mu\nu}$ are even possible?

$$\sim$$

What is the "space" of sensible EFTs realizing $ISO(3,1) \xrightarrow{\text{spontaneously}} ISO(3)$?

Ex: relativistic superfluid

|Son 2002|

$$\begin{cases} U(1): \phi \to \phi + c \\ \\ \text{Poincaré} \times U(1) \to ISO(3) \times \overline{P_0} \end{cases}$$

$\phi = \mu t + \pi$

$\phi \qquad t$

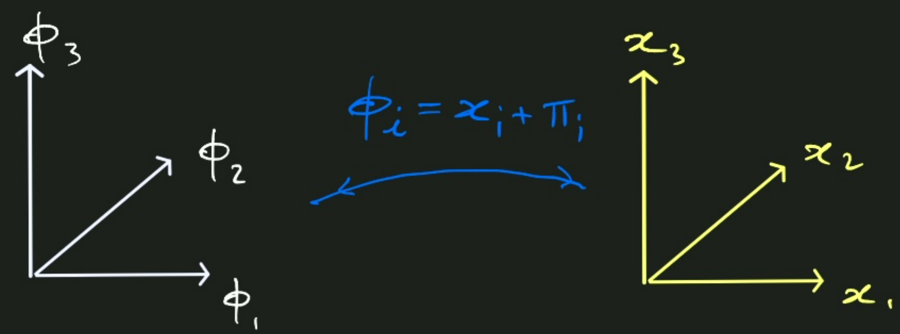
$$X \equiv \partial_\mu \phi \, \partial^\mu \phi$$

$$\mathcal{L} = \mathcal{L}(X) \iff \text{eq. of state} \implies$$

soft phonon scattering

Ex Fluid

Carter 1973



$$\begin{cases} \phi^I \longrightarrow \xi^I(\phi) \in \mathrm{Sdiff} \\ \left|\dfrac{\partial \xi}{\partial \phi}\right| = 1 \end{cases}$$

$$\left.\begin{aligned} X^{IJ} &\equiv \partial_\mu \phi^I \partial^\mu \phi^J \\ \mathcal{L} &= F(\det X) \end{aligned}\right\} \Longrightarrow$$

- Euler eqs. of relativistic perfect fluid
- $S_{\mathrm{diff}} \xrightarrow{\text{Noether}}$ Kelvin's theorem

- fluctuations $\vec{\Phi} = \vec{x} + \vec{\pi}$

$$\mathcal{L} \propto \frac{1}{2} \dot{\vec{\pi}}^2 - \frac{1}{2} c_s^2 \left(\vec{\nabla} \cdot \vec{\pi}\right)^2 + O(\pi^3)$$

- $\vec{\pi} \propto \vec{\nabla} \pi_L \longrightarrow \omega = c_s k$

- $\vec{\pi} \propto \vec{\nabla} \wedge \vec{\pi}_\perp \longrightarrow \omega = 0$!!

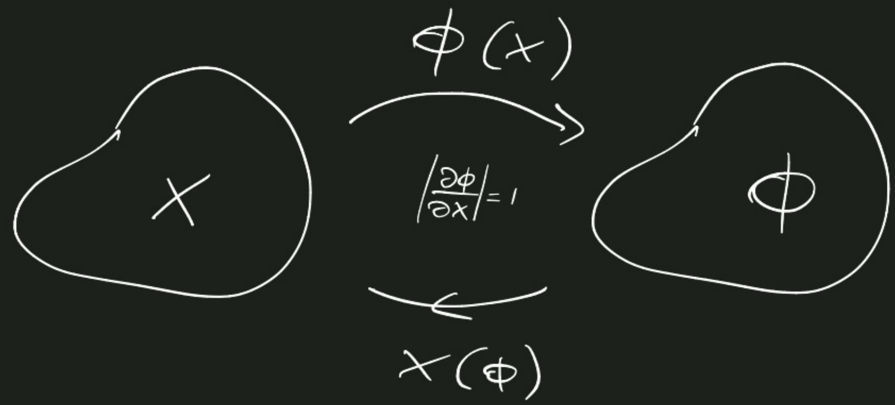
$\Rightarrow$ transverse phonons $\neq$ Fock space

vacuum <u>not-localized</u> in field space

$\boxed{\text{non-perturbative}}$

☆ Zoom on transverse modes $\Longrightarrow$ incompressible limit

$$v_{flow} \ll c_s$$



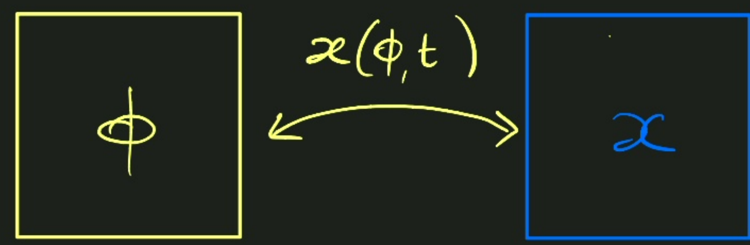
$$v = \frac{d}{dt} x(\phi, t)$$

$$S = \int dt\, d^n x\, \frac{1}{2} \rho v^2 + \dots$$

$$\Longrightarrow H \sim \frac{1}{\rho} (\dots) \longrightarrow \frac{\Lambda^{2+n}}{\rho} \quad \text{Landau 1941}$$

$$\longrightarrow \frac{K^{2+n}}{\rho} \quad \text{some of us 2010}$$

Fluid on T^2

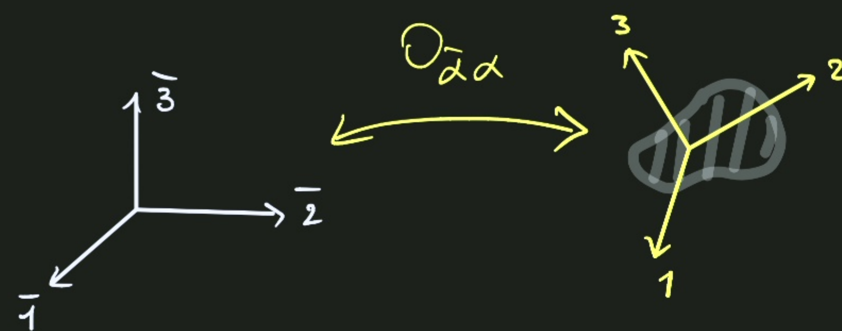


- $x(\phi, t) \in \mathrm{Sdiff}(T^2) \implies$ motion on group manifold

---

Rigid Body

- $O_{\bar{\alpha}\alpha}(t) \in SO(3)$



see V.I. Arnol'd "Mechanics"

Group Action

- rigid body $\longrightarrow$ $O_{\bar{\alpha}\alpha} \longrightarrow U_L\, O\, U_R^{-1}$

$$SO(3)_L \times SO(3)_R$$

$SO(3)_L$: symm

$SO(3)_R$: not symm

- fluid $\longrightarrow$ $x(\Phi) \longrightarrow f_R(x(f_L^{-1}(\Phi)))$

$$\equiv f_R \circ x \circ f_L^{-1}$$

$$\Longrightarrow (\mathrm{Sdiff})_L \times (\mathrm{Sdiff})_R$$

$(\mathrm{Sdiff})_L$: symm

$(\mathrm{Sdiff})_R$: not symm

Hamiltonian

$$R_i = -I_{ij}\,\Omega_j$$

$$H = \frac{1}{2} I^{-1}_{ij} R_i R_j$$

$$[R_i, R_j] = i\,\varepsilon_{ijk} R_k$$

$$\Big\Downarrow \text{ Peter-Weyl}$$

$$\mathcal{H} = \bigoplus (\underbrace{e}_{\text{left}}, \underbrace{e}_{\text{right}})$$

$$R(x) = \rho\, \nabla \wedge v(x)$$

$$H = \frac{1}{2\rho}\int d^2x \; R\, \nabla^{-2} R$$

$$\boxed{(\partial_t + \vec{v}\cdot\vec{\nabla})\, R = 0}$$

$$[R_{\underline{n}}, R_{\underline{m}}] = i\,(\underline{n} \wedge \underline{m})\, R_{\underline{n}+\underline{m}}$$

$$\Downarrow$$

$$\mathcal{H} = \bigoplus (\underbrace{g}_{\phi_a(x)}, \underbrace{g}_{v(x)}) = ?$$

How to deal with "hidden" degeneracy?

A// gauge $(\mathrm{Sdiff})_L$ $\sim$ project on ground state $(0,0)$

$\Longrightarrow$ fluid $\longrightarrow$ superfluid Feynman ☺

B// postpone interpretation to concrete realization

Ex drop (r,r) structure: just $(\mathrm{Sdiff})_R$ on some $\mathcal{H}$

Hamiltonian

$$R_i = -I_{ij}\,\Omega_j$$

$$H = \frac{1}{2} I^{-1}_{ij}\, R_i R_j$$

$$[R_i, R_j] = i\,\varepsilon_{ijk} R_k$$

$$\Downarrow \quad \text{Peter-Weyl}$$

$$\mathcal{H} = \bigoplus (\ell, \ell)$$

ℓ → left, ℓ → right

$$R(x) = \rho\, \nabla \wedge v(x)$$

$$H = \frac{1}{2\rho}\int d^2x\; R\, \nabla^{-2} R$$

$$\boxed{(\partial_t + \vec{v}\cdot\vec{\nabla})\, R = 0}$$

$$[R_{\underline{n}}, R_{\underline{m}}] = i\,(\underline{n} \wedge \underline{m})\, R_{\underline{n}+\underline{m}}$$

$$\Downarrow$$

$$\mathcal{H} = \bigoplus (q, q) = ?$$

q → $\phi_a(x)$, q → $v(x)$

How to deal with "hidden" degeneracy?

A// gauge $(\mathrm{Sdiff})_L$ $\sim$ project on ground state $(0,0)$

$\Longrightarrow$ fluid $\longrightarrow$ superfluid   Feynman ☺

B// postpone interpretation to concrete realization

Ex drop (r,r) structure: just $(\mathrm{Sdiff})_R$ on some $\mathcal{H}$

A remarkable fact

learned from George Savvidy

$$\mathrm{Sdiff}(T^2) \sim \lim_{N\to\infty} SU(N)$$

- J. Hoppe, PhD thesis 1982
- Fairlie, Fletcher, Zachos 1988
- Pope, Stelle 1989

- Girvin, MacDonald, Platzman 1985
-
-
- Wiegmann 2013

3) A clever basis of SU(N) algebra ('t Hooft 1978)

- $\square = Z_\alpha \quad \alpha = -\frac{N-1}{2}, \ldots, 0, \ldots \frac{N-1}{2} \qquad \alpha \in \left[-\frac{N-1}{2}, \frac{N-1}{2}\right]$

- $\underline{n} \equiv (n_1, n_2) \in \left[-\frac{N-1}{2}, \frac{N-1}{2}\right] \times \left[-\frac{N-1}{2}, \frac{N-1}{2}\right] \longrightarrow N^2$

- $J_{\underline{n}} \qquad \underline{n} \neq (0,0)$

$$\boxed{J_{-\underline{n}} = J_{\underline{n}}^{\dagger} = J_{\underline{n}}^{-1}}$$

$$\boxed{\left[J_{\underline{n}}, J_{\underline{m}}\right] = -2i \sin\left(\frac{\pi\, \underline{n} \wedge \underline{m}}{N}\right) J_{\underline{n}+\underline{m}}}$$

$$R_{\underline{n}} \equiv -\frac{N}{2\pi} \mathcal{J}_{\underline{n}} \implies \left[R_{\underline{n}}, R_{\underline{m}}\right] = i\frac{N}{\pi}\sin\left(\frac{\pi\, \underline{n}\wedge\underline{m}}{N}\right) R_{\underline{n}+\underline{m}}$$

$$\boxed{|\underline{n}|, |\underline{m}| \ll \sqrt{N} \implies \left[R_{\underline{n}}, R_{\underline{m}}\right] \simeq i(\underline{n}\wedge\underline{m}) R_{\underline{n}+\underline{m}}}$$

$\boxed{\text{SU(N) rigid body}}$ $\sim$ $\boxed{\text{UV completion of perfect fluid}}$

$\underline{n} \in \left[-\frac{N-1}{2}, \frac{N-1}{2}\right]^2 \longrightarrow$ space latticized

$\underline{n} \xleftrightarrow{\text{Fourier}} \underline{x}$ $\quad \underline{x} \in \left[-\frac{N-1}{2}, \frac{N-1}{2}\right]^2$

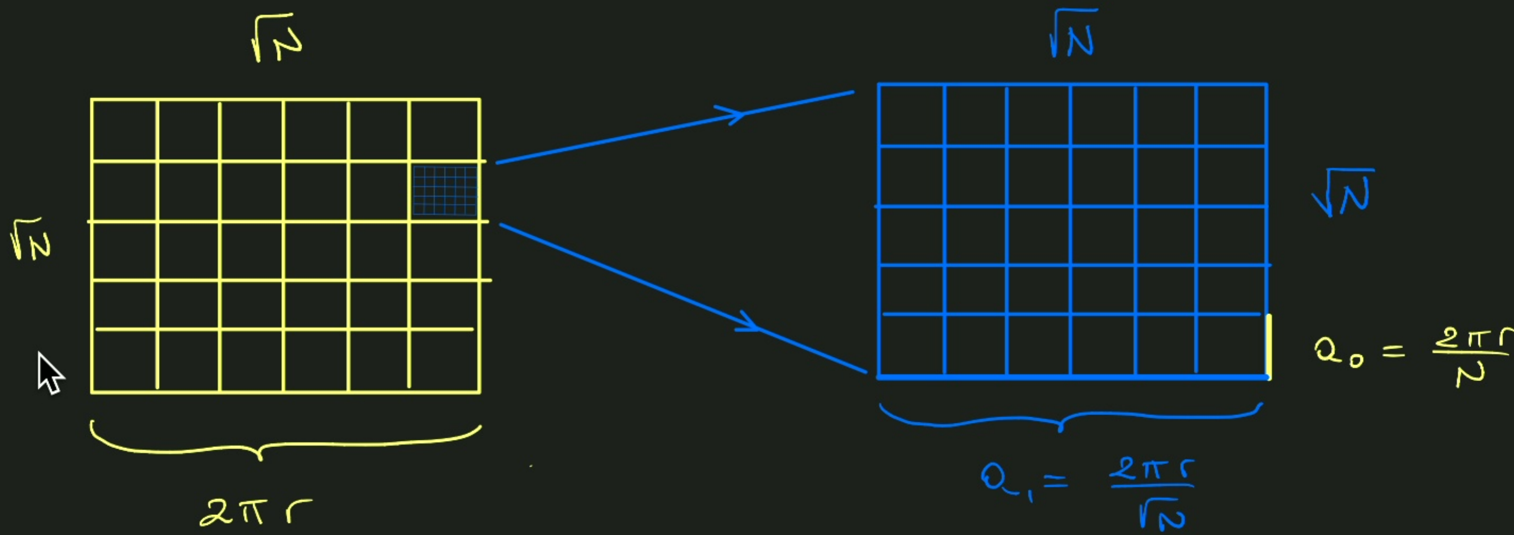
$$R(\underline{x}) \equiv \frac{1}{N} \sum_{\underline{n}} \omega^{-\underline{n}\cdot\underline{x}} R_{\underline{n}} \qquad \omega = e^{\frac{2\pi i}{N}}$$

$\underline{n} \in \left[-\frac{N-1}{2}, \frac{N-1}{2}\right]^2 \longrightarrow$ space latticized

$\underline{n} \xleftrightarrow{\text{Fourier}} \underline{x}$ $\quad\quad$ $\underline{x} \in \left[-\frac{N-1}{2}, \frac{N-1}{2}\right]^2$

$$R(\underline{x}) \equiv \frac{1}{N} \sum_{\underline{n}} \omega^{-\underline{n}\cdot\underline{x}} R_{\underline{n}} \qquad \omega = e^{\frac{2\pi}{N} i}$$

$$|\underline{n}| \lesssim \sqrt{N} \longleftrightarrow |\Delta x| \gtrsim \sqrt{N}$$



fluid regime $\quad \Delta x \gtrsim a_1$

$$p \lesssim \frac{1}{a_1} \equiv \Lambda \qquad \vec{p} = \frac{\vec{n}}{r}$$

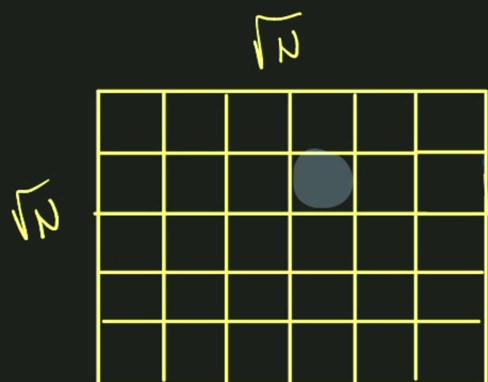
Hamiltonian

$$H = \frac{1}{2\rho}\left(\frac{1}{2\pi r^2}\right)^2 \sum_{\underline{n}}^{|\underline{n}|<\sqrt{N}} \frac{R_{\underline{n}}\ R_{-n}}{\underline{n}^2}$$

$$= \frac{\Lambda^4}{32\pi^2\rho} \cdot \sum_{\underline{n}}^{|n|<\sqrt{N}} \frac{\mathcal{J}_{\underline{n}}\ \mathcal{J}_{-\underline{n}}}{\underline{n}^2}$$

Spectrum

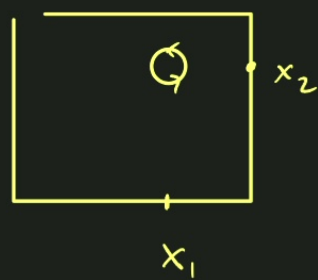
- trivial irrep $\bullet$ $\Rightarrow$ $E = 0$ $\equiv$ ground state
- fundamental $\Box$ $\Rightarrow$ $E_\alpha \equiv E_\Box = \frac{\Lambda^4}{16\pi^2 \rho} \ln \Lambda r$ Landau!

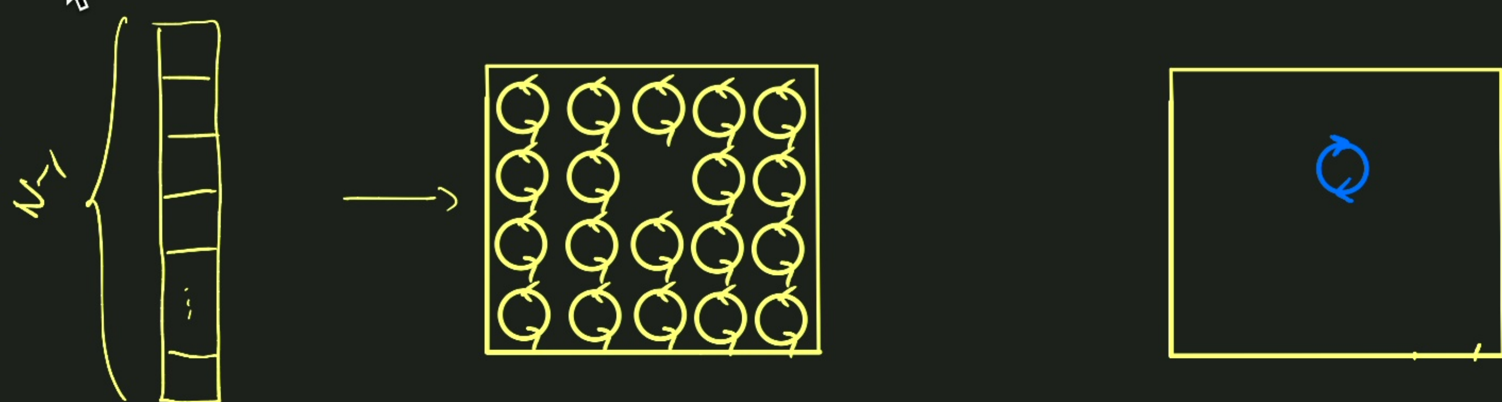


- $\sqrt{N} \times \sqrt{N} = N$ states
- $|\vec{x}\rangle$

- $\langle x | \partial \wedge v(y) | x \rangle \sim \frac{\Lambda^2}{2\pi\rho}\left[\delta^2(x-y) - \frac{1}{V}\right]$

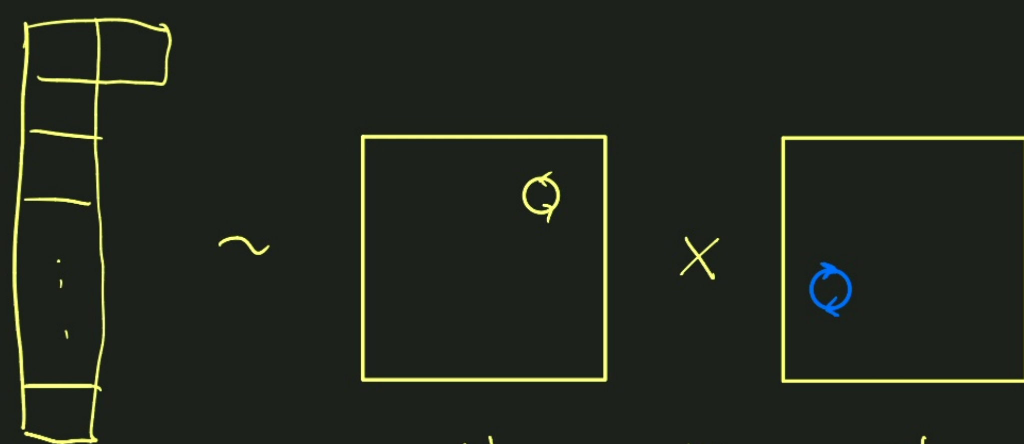
- $|x\rangle \longrightarrow$



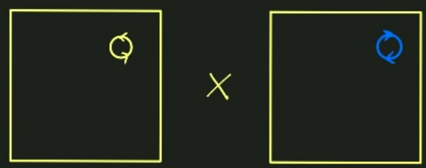
- 

- Lowest irreps 

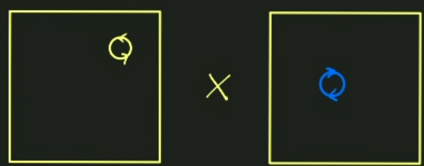
  all gapped

- Adjoint 

  $N \times N$



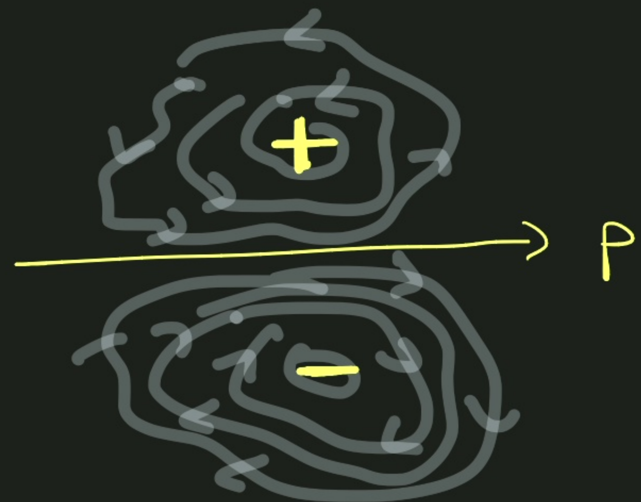
$\Longrightarrow$ ungapped $(\sim N)$



$\Longrightarrow$ gapped $\sim N^2$

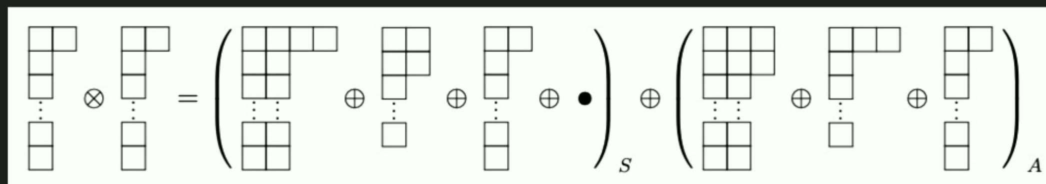
$$E_{adj}(P) = \begin{cases} \dfrac{P^2}{4\rho a_1^2} \equiv \dfrac{\Lambda^2 P^2}{16\pi^2 \rho} & P \ll \Lambda = \dfrac{1}{a_1} \\[2ex] \dfrac{\Lambda^4}{16\pi^3 \rho} \ln \dfrac{P^2}{\Lambda^2} & P \gtrsim \Lambda \end{cases}$$

- Vortons $\equiv$ quanta with vorticity dipole



$$d^I \equiv \int d^2x \, x^I \langle \vec{p} | \vec{\nabla} \wedge \vec{v} \, | \vec{p} \rangle = \frac{p^J \varepsilon^{JI}}{\rho}$$

Scattering



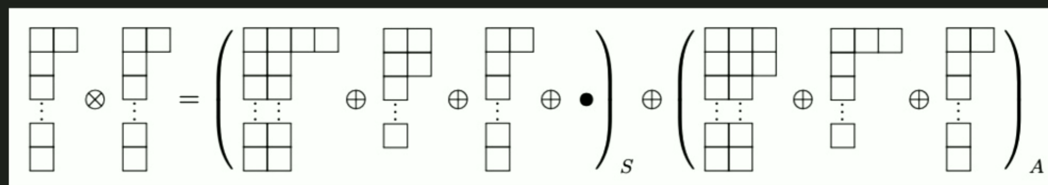
N^4 N^2 1 N^4 N^2

bosonic vorton EFT

Fermionic vorton EFT

$$\boxed{\mathcal{L} = i\phi^\dagger\dot{\phi} - \frac{1}{2\rho}\left(\partial\phi^\dagger \wedge \partial\phi\right)\frac{1}{-\nabla^2}\left(\partial\phi^\dagger \wedge \partial\phi\right)}$$

Scattering

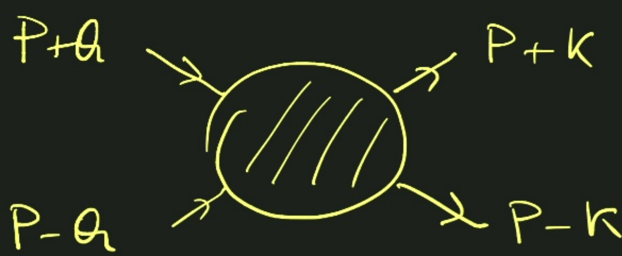


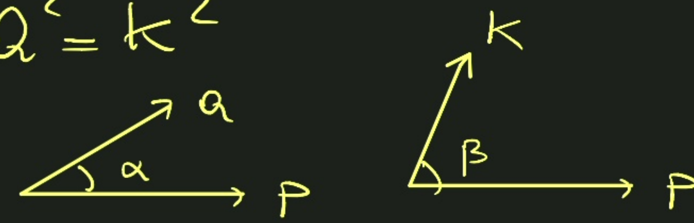
N^4 N^2 1 N^4 N^2

bosonic vorton EFT

Fermionic vorton EFT

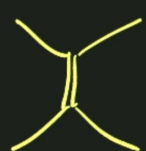
$$\mathcal{L} = i\phi^\dagger\dot{\phi} - \frac{1}{2\rho}\left(\partial\phi^\dagger\wedge\partial\phi\right)\frac{1}{-\nabla^2}\left(\partial\phi^\dagger\wedge\partial\phi\right)$$



- $Q^2 = K^2$
- 

t $\qquad$ u

- $A_{\text{boson}} =$  $+$  $= \frac{1}{8\rho}\left(P^2 - Q^2\right)$

- $A_{\text{fermion}} =$  $-$  $= \frac{1}{8\rho}\left(P^2\cos^2(\alpha+\beta) - Q^2\cos^2(\alpha-\beta)\right)$

Summary

2D quantum perfect fluid $\sim$ $SU(N)$ rigid body
$N \to \infty$

- "quantum flows" $\sim$ $SU(N)$ irreps
  - vortices $\longrightarrow$ gapped
  - vortons $\longrightarrow$ ungapped

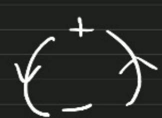
⚠ does this thing exist?

2+1 QED "realization

- $B^{ij} \equiv \varepsilon^{ij} B$
- $E^{i} \propto \varepsilon^{ij} v^{j}$ $\qquad \vec{\nabla}\cdot\vec{E} \propto \vec{\nabla}\wedge\vec{v}$

charge density $\sim$ vorticity

- Vortons $\equiv$ $e^{+}e^{-}$ bound states in external magnetic field



$\xrightarrow{P}$

$$E^{i} \sim \varepsilon^{ij}(P/m)B \Longrightarrow \vec{d}^{\,i} \propto \varepsilon^{ij} p^{j}$$

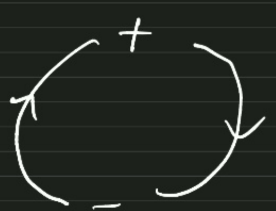
- parameters $\quad e,\ m,\ B$

$$E_{bind} \sim e^2$$

$$r_{Bohr}^2 \sim \frac{1}{e^2 m}$$

$$r_{Landau}^2 \sim \frac{1}{eB}$$

$\Rightarrow$ Choose $\quad r_B \sim r_L \equiv \frac{1}{\Lambda} \Rightarrow \rho \equiv \frac{m}{r_B^2} = e^2 m^2$



$v \sim P/2m \longrightarrow$

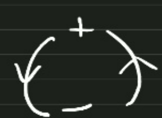
$$E \sim \frac{P}{2m} B \Rightarrow \frac{e^2}{r_B^2} \cdot d \sim e \cdot E$$

2+1 QED "realization"

- $B^{ij} \equiv \varepsilon^{ij} B$
- $E^{i} \propto \varepsilon^{ij} v^{j}$ $\vec{\nabla}\cdot\vec{E} \propto \vec{\nabla}\wedge\vec{v}$

charge density $\sim$ vorticity

- Vortons $\equiv$ $e^{+}e^{-}$ bound states in external magnetic field



$\xrightarrow{P}$

$E^{i} \sim \varepsilon^{ij}(P/m)B \Rightarrow \vec{d}^{i} \propto \varepsilon^{ij} p^{j}$

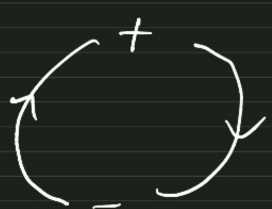
- parameters e, m, B

$$E_{bind} \sim e^2$$

$$r_{Bohr}^2 \sim \frac{1}{e^2 m}$$

$$r_{Landau}^2 \sim \frac{1}{eB}$$

$\Rightarrow$ Choose $r_B \sim r_L \equiv \frac{1}{\Lambda} \Rightarrow \rho \equiv \frac{m}{r_B^2} = e^2 m^2$



$v \sim P/2m$
$\longrightarrow$

$$E \sim \frac{P}{2m} B \Rightarrow \frac{e^2}{r_B^2} \cdot d \sim e \cdot E$$