Title: Renormalized Volume/Area from Conformal Gravity

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Abstract: We explore the physical consequences of embedding Einstein gravity with negative cosmological constant in Conformal Gravity in four dimensions. In the bulk, the procedure is equivalent to Holographic Renormalization, as the Einstein-AdS action appears augmented by the correct boundary counterterms. In codimension-2, 4D Conformal Gravity induces a 2D conformal invariant, which leads to a renormalized area for minimal surfaces attached to the conformal boundary. For arbitrary surfaces, this proposal reproduces other energy functionals as Willmore Energy and Reduced Hawking Mass. In particular, this procedure provides a more geometric approach to the computation of Holographic Entanglement Entropy for CFTs dual to 4D Einstein gravity.

Zoom link TBA

Renormalized Volume/Area from Conformal Gravity

Perimeter Institute, Feb 29th, 2024

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HOLOGRAPHYCL

Outline

Energy Functionals as Codimension-2 Structures

Membranes, bubbles and Holographic Entanglement Entropy

Renormalized Action/Volume in AdS gravity

Holographic Renormalization

Renormalized Area

From a Renormalized Action

Renormalized Area

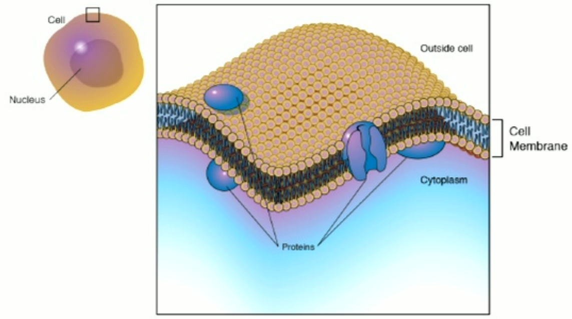
From Einstein gravity + Topological Term

(Generalized) Renormalized Area

From Conformal Gravity

Energy functionals

Membranes in biological systems

- 

Soap bubbles

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Willmore Energy

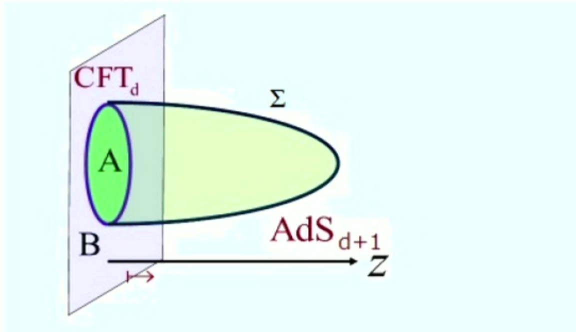
Functional defined on compact and orientable 2D surfaces immersed in $\mathbb{R}^3$

In terms of the (spatial) mean curvature

$$\mathcal{W}\left[\Sigma\right] = \int\limits_{\Sigma} d^2y\sqrt{\gamma}H^2$$

Energy functionals

Holographic Entanglement Entropy from Minimal Surface [Ryu-Takayanagi, 2006]



HEE from a Cosmic Brane [Lewkowycz-Maldacena, 2013]
Black Hole Entropy [Solodukhin, 1993] [Banados, Teitelboim, Zanelli, 1994]

$$S = -\lim_{\alpha \to 1} \partial_\alpha I_{\text{grav}}^{(\alpha)}$$

Renormalized Volume: Black Hole Thermo in AdS gravity

Euclidean static black hole metric

$$ds^2 = f^2(r)d\tau^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega^2_{D-2}\,, \quad f^2(r) = 1 - \frac{2\omega_D G_N M}{r^{D-3}} + \frac{r^2}{\ell^2}$$

Einstein-AdS gravity

$$I_{EH} = \frac{1}{16\pi G_N}\int\limits_M d^D x\,\sqrt{-g}\,(R - 2\Lambda)\,, \qquad \Lambda = -\frac{(D-1)(D-2)}{2\ell^2}$$

Gibbs free energy $G = TI^E$

$$TI^E_{bulk} = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty}\frac{V(S^{D-2})}{8\pi G_N}\frac{r^{D-1}}{\ell^2}$$

Counterterms in AdS gravity

Holographic Renormalization [Henningson and Skenderis, 1998]

$$I_{\text{ren}} = I_{EH} - \frac{1}{8\pi G_N} \int\limits_{\partial M} d^d x \sqrt{-h}\, K + \int\limits_{\partial M} d^d x\, L_{ct}(h, \mathcal{R}, \nabla\mathcal{R})$$

Counterterms [Balasubramanian-Kraus, 1999], [Emparan, Johnson, Myers, 1999]

$$\begin{aligned} 8\pi G_N\, L_{ct} = {} & \tfrac{d-1}{\ell}\sqrt{-h} + \tfrac{\ell\sqrt{-h}}{2(d-2)}\mathcal{R} + \tfrac{\ell^3\sqrt{-h}}{2(d-2)^2(d-4)}\left(\mathcal{R}^{ab}\mathcal{R}_{ab} - \tfrac{d}{4(d-1)}\mathcal{R}^2\right) \\ & + \tfrac{\ell^5\sqrt{-h}}{(d-2)^3(d-4)(d-6)}\left(\tfrac{3d-2}{4(d-1)}\mathcal{R}\mathcal{R}^{ab}\mathcal{R}_{ab} - \tfrac{d(d+2)}{16(d-1)^2}\mathcal{R}^3 \right. \\ & \left. -2\mathcal{R}^{ac}\mathcal{R}^{bd}\mathcal{R}_{abcd} - \tfrac{d}{4(d-1)}\nabla_c\mathcal{R}\nabla^c\mathcal{R} + \nabla^c\mathcal{R}^{ab}\nabla_c\mathcal{R}_{ab}\right) + \ldots \end{aligned}$$

Energy functionals

Bulk gravity action evaluated in conical defects

$$I_{\mathsf{EH}}^{(\alpha)}=\frac{1}{16\pi G_N}\int\limits_{M^{(\alpha)}}d^4x\sqrt{g}R^{(\alpha)}=\frac{1}{16\pi G_N}\int\limits_{M}d^4x\sqrt{g}R+\frac{(1-\alpha)}{4G_N}\mathcal{A}\left[\Sigma\right]$$

Area Functional (codimension-2)

$$\mathcal{A}\left[\Sigma\right]=\int\limits_{\Sigma}d^2y\sqrt{\gamma}$$

HEE dual to Einstein gravity (Minimal surface $\mathcal{K}=0$)

$$S=\frac{1}{4G_N}\mathcal{A}\left[\Sigma_{\mathsf{min}}\right]$$

Renormalized Area/HEE

Area Functional is divergent in AAdS spaces

$\mathcal{A}_{\text{ren}}$ from Renormalized Bulk Action [Taylor-Woodhead, 2016]

$$S_{\text{ren}} = -\lim_{\alpha\to 1}\partial_\alpha I_{\text{ren}}^{(\alpha)}$$

Renormalized AdS Action $I_{\text{ren}}^{(\alpha)}$ (Holographic Renormalization)
[de Haro-Skenderis-Solodukhin, 2000]

Topological Terms in 4D AdS gravity

Renormalization with Kounterterms [RO, 2005]

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G_N} \int\limits_{M} d^4x \sqrt{g}\,(R - 2\Lambda) + \frac{\ell^2}{64\pi G_N} \int\limits_{\partial M} d^3x\, B_3$$

$$B_3 = 4\sqrt{h}\,\delta^{a_1 a_2 a_3}_{b_1 b_2 b_3} K^{b_1}_{a_1} \left(\tfrac{1}{2}\mathcal{R}^{b_2 b_3}_{a_2 a_3}(h) - \tfrac{1}{3} K^{b_2}_{a_2} K^{b_3}_{a_3}\right)$$

Euler Theorem

$$\int\limits_{M} d^4x\, GB = 32\pi^2 \chi\,[M] + \int\limits_{\partial M} d^3x\, B_3$$

$$GB = \sqrt{g}\,(Ric^2 - 4Ric^2 + 4R^2)$$

Topological Terms in 4D AdS gravity

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Renormalized Action = Renormalized Volume

Black Hole Thermodynamics

$$T I^E_{bulk} = \frac{M}{2} - TS + \lim_{r\to\infty} \frac{V(S^2)}{8\pi G}\frac{r^3}{\ell^2}$$

Euclidean Kounterterms

$$T\, c_3 \int_{\partial M} B_3 = \frac{M}{2} - \lim_{r\to\infty} \frac{V(S^2)}{8\pi G}\frac{r^3}{\ell^2}$$

Black Hole Thermo OK

Topological Terms in 4D AdS gravity

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Boundary conditions in AdS gravity

Fefferman-Graham expansion for AAdS Einstein spaces

$$ds^2 = \frac{\ell^2}{z^2}\,dz^2 + \frac{1}{z^2}\,g_{ab}(x,z)\,dx^a dx^b\,, \quad g_{ab}(x,z) = g_{(0)ab}(x) + z^2 g_{(2)ab}(x) + \cdots$$

Dirichlet b.c. $\delta h_{ab} = 0$ does not make sense in AAdS spaces `[Papadimitriou and Skenderis, 2004]`

$$h_{ab} = \frac{g_{(0)ab}}{z^2} + \ldots$$

Renormalization = variational problem in $g_{(0)ab}$

$$\delta I_{ren} = \frac{1}{2}\int\limits_{\partial M} \sqrt{-g_{(0)}}\,T^{ab}[g_{(0)}]\,\delta g_{(0)ab}$$

Kounterterms and Holography

Asymptotic form of the extrinsic curvature

$$K_{ab} = \frac{1}{\ell}\frac{g_{(0)ab}}{z^2} + \ldots$$

Counterterms of a different sort...

$$\tilde{I}_{ren} = I_{EH} + c_d \int\limits_{\partial M} d^d x\, B(f(h), K)$$

...as long as the theory is *holographic*

$$\delta\tilde{I}_{ren} = \frac{1}{2}\int\limits_{\partial M} \sqrt{-g_{(0)}}\tau^{ab}\,\delta g_{(0)ab}$$

From extrinsic to intrinsic renormalization in 4D

Adding zero...

$$\tilde{I}_{\text{ren}} = I_{EH} - \frac{1}{8\pi G_N} \int\limits_{\partial M} d^3x \sqrt{-h}\, K + \int\limits_{\partial M} d^3x \, L_{ct} .$$

Fefferman-Graham expansion

$$K^a_b = \frac{1}{\ell}\delta^a_b - \ell S^a_b(h) + \mathcal{O}(\mathcal{R}^2)\,, \qquad S^a_b(h) = \frac{1}{d-2}\left(\mathcal{R}^a_b(h) - \frac{1}{2(d-1)}\delta^a_b \mathcal{R}(h)\right)$$

Kounterterms turn into *counterterms*

$$L_{ct} = \frac{1}{8\pi G}\sqrt{-h}\left(\frac{2}{\ell} + \frac{\ell}{2}\mathcal{R}(h)\right) .$$

Renormalized AdS Action

For a fixed GB coupling

$$I_{\text{ren}} = \frac{1}{16\pi G_N}\int\limits_M d^4x\sqrt{g}\left[(R-2\Lambda)+\frac{\ell^2}{4}(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-4R_{\mu\nu}R^{\mu\nu}+R^2)\right]-\frac{\pi\ell^2}{2G_N}\chi[M]$$

GB coupling is also singled out by SUSY [Andrianopoli, D'Auria, 2014]

Addition of Topological Terms = Holographic Renormalization in 4D [Miskovic, RO, 2009]

Addition of Topological Terms = Holographic Renormalization in $D=2n$ (for ACF spaces) [Anastasiou, Miskovic, RO, Papadimitriou, 2020]

Curvature-squared terms and conical defects

Riemann squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Rie^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Rie^2 + 8\pi\,(1-\alpha)\int_\Sigma d^2y\sqrt{\gamma}\left(R^{AB}_{AB} - \mathcal{K}^{(A)}_{ij}\mathcal{K}^{ij}_{(A)}\right) + \dots$$

Ricci squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Ric^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Ric^2 + 4\pi\,(1-\alpha)\int_\Sigma d^2y\sqrt{\gamma}\left(R^{A}_{A} - \frac{1}{2}\mathcal{K}^{(A)}\mathcal{K}_{(A)}\right) + \dots$$

Ricci scalar squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(R^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,R^2 + 8\pi\,(1-\alpha)\int_\Sigma \sqrt{\gamma}\left(R^{AB}_{AB} + \mathscr{R} - \mathcal{K}^{(A)}\mathcal{K}_{(A)} + \mathcal{K}^{(A)}_{ij}\mathcal{K}^{ij}_{(A)}\right) + \dots$$

[Fursaev-Patrushev-Solodukhin, 2013]

Renormalized Area from Topological Terms

$$S_{\text{ren}} = -\partial_{\alpha} I_{\text{ren}}^{tot} = \frac{\mathcal{A}_{\text{ren}}[\Sigma]}{4G_N}$$

Renormalized AdS action and conical defects

Gauss-Bonnet term

$$\int_{M^{(\alpha)}} d^4x\, GB^{(\alpha)} = \int_M d^4x\, GB + 8\pi\,(1-\alpha)\int_\Sigma d^2y\,\sqrt{\gamma}\mathscr{R}$$

Euler characteristic

$$\chi\left[M^{(\alpha)}\right] = \chi\,[M] + (1-\alpha)\,\chi\,[\Sigma]$$

Renormalized AdS action

$$I_{\mathsf{ren}}^{(\alpha)} = I_{\mathsf{ren}} + \frac{(1-\alpha)}{4G_N}\left(\mathcal{A}\,[\Sigma] + \frac{\ell^2}{2}\int_\Sigma d^2y\,\sqrt{\gamma}\mathscr{R} - 2\pi\ell^2\chi\,[\Sigma]\right)$$

Renormalized Area from Topological Terms

Renormalized HEE [Anastasiou-Araya-RO, 2018]

$$S_{\text{ren}} = -\partial_\alpha I^{(\alpha)}_{\text{ren}} = \frac{\mathcal{A}_{\text{ren}}[\Sigma]}{4G_N}$$

Renormalized Area [Alexakis-Mazzeo, 2010]

$$\mathcal{A}_{\text{ren}}[\Sigma] = \frac{\ell^2}{4}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\delta^{km}_{ij}\left(\mathcal{R}^{ij}_{km} + \frac{1}{\ell^2}\delta^{ij}_{km}\right) - 2\pi\ell^2\chi[\Sigma]$$

It removes divergences from the anchoring points to the conformal boundary (for surfaces anchored orthogonally)

Renormalized Area

Volume Renormalization $\Longrightarrow$ Area Renormalization

•

It measures the deviation respect to a constant-curvature surface

• 

For surfaces of constant curvature, HEE is purely topological

$$S_{\text{ren}} = -\frac{\pi \ell^2}{2G_N}\chi\left[\Sigma\right]$$

•

Motivation

$\mathcal{A}_{\mathtt{ren}}$ is not finite for surfaces anchored to the boundary at an arbitrary angle.

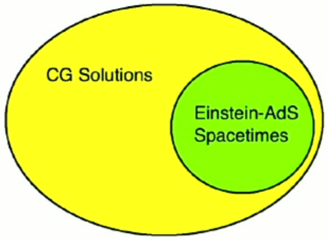
$\mathcal{A}_{\mathtt{ren}}$ is not conformally invariant.

Renormalization and Conformal Invariance in codimension-2

Conformal Renormalization

Conformal Renormalization

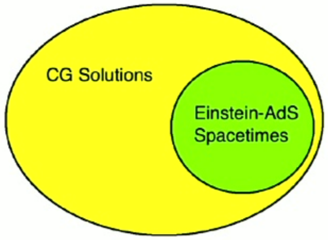
Embedding Einstein theory in Conformal Gravity



- Why?: Conformal Gravity is finite for AAdS conditions. [Grumiller et al., 2013]
- What for?: Renormalization should be inherited by the Einstein sector.
- How?: a *holographic* mechanism to turn CG into Einstein

Conformal Renormalization

Embedding Einstein theory in Conformal Gravity



- Why?: Conformal Gravity is finite for AAdS conditions. [Grumiller et al., 2013]
- What for?: Renormalization should be inherited by the Einstein sector.
- How?: a *holographic* mechanism to turn CG into Einstein

Renormalized AdS Action

Renormalized AdS action = MacDowell-Mansouri action (1977)

$$I_{\text{ren}} = \frac{\ell^2}{256\pi G_N} \int\limits_M d^4x\sqrt{-g}\,\delta^{[\nu_1\cdots\nu_4]}_{[\mu_1\cdots\mu_4]} \left[R^{\mu_1\mu_2}_{\nu_1\nu_2} + \frac{\delta^{\mu_1\mu_2}_{\nu_1\nu_2}}{\ell^2}\right] \left[R^{\mu_3\mu_4}_{\nu_3\nu_4} + \frac{\delta^{\mu_3\mu_4}_{\nu_3\nu_4}}{\ell^2}\right] - \frac{\pi\ell^2}{2G_N}\chi\left[M\right]$$

General Weyl tensor

$$W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} - \left(S^{\alpha}_{\mu}\delta^{\beta}_{\nu} - S^{\beta}_{\mu}\delta^{\alpha}_{\nu} - S^{\alpha}_{\nu}\delta^{\beta}_{\mu} + S^{\beta}_{\nu}\delta^{\alpha}_{\mu}\right)$$

Weyl tensor for Einstein spaces

$$W^{\alpha\beta}_{(\mathsf{E})\mu\nu} = R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\delta^{\alpha\beta}_{\mu\nu}$$

Renormalized AdS Action

Renormalized action for Einstein spaces

$$I_{\mathrm{ren}} = \frac{\ell^2}{64\pi G_N} \int\limits_M d^4x \sqrt{g}\, W_{(E)\mu\nu\alpha\beta} W_{(E)}^{\mu\nu\alpha\beta} - \frac{\pi \ell^2}{2G_N} \chi\left[M\right]$$

Renormalized action is a sector of Conformal Gravity

Einstein Gravity from Conformal Gravity in 4D

Einstein gravity from CG with Neumann bc's [Maldacena, 2011]

$$I_{CG} = \alpha \int_M d^4x \sqrt{-g}\, W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta} - \frac{\pi \ell^2}{2G_N} \chi[M]$$

EOM for Conformal Gravity

$$B_{\mu\nu} = \nabla^\lambda C_{\mu\nu\lambda} + S^{\lambda\sigma} W_{\lambda\mu\sigma\nu} = 0\,, \qquad C^\mu{}_{\nu\lambda} = \nabla_\nu S^\mu_\lambda - \nabla_\lambda S^\mu_\nu$$

Fefferman-Graham expansion for AAdS spaces in CG

$$ds^2 = \frac{\ell^2}{z^2} dz^2 + \frac{1}{z^2} g_{ab}(x,z)\, dx^a dx^b\,, \quad g_{ab}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ab}(x) + \cdots + z g_{(1)ab}(x) + \cdots$$

Einstein spaces: holographic prescription

Einstein-AdS spaces

$$S^\mu_\nu = -\frac{1}{2\ell^2}\delta^\mu_\nu, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

Traceless Ricci tensor

$$H^\mu_\nu = R^\mu_\nu - \frac{1}{D}R\delta^\mu_\nu = 0$$

$$H_{\mu\nu} = 0 \Longleftrightarrow \partial_z g_{ab} = g_{(1)ab} = 0 \text{ (Maldacena)}$$

CG action for Einstein spaces = Renormalized Einstein-AdS action

$$I_{\mathsf{CG}}\left[E\right] = I_{\mathsf{HR}}$$

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Conformal Gravity and Conical Defects

Conformal Gravity [Anastasiou, Araya, RO, 2022]

$$I_{\mathsf{CG}} = \frac{\ell^2}{64\pi G_N} \int\limits_M d^4x\sqrt{g}\left(Rie^2 - 2Ric^2 + \frac{1}{3}R^2\right) - \frac{\pi\ell^2}{2G_N}\chi\left[M\right]$$

In a manifold with a conical singularity

$$\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}\left|W^{(\alpha)}\right|^2 = \int\limits_M d^4x\sqrt{g}\left|W\right|^2 + 8\pi\left(1-\alpha\right)\int\limits_\Sigma d^2y\sqrt{\gamma}K_\Sigma + \mathcal{O}\left(\left(1-\alpha\right)^2\right)$$

Conformal Gravity and Conical Defects

Conformal Invariant in co-dimension 2

$$\sqrt{\gamma}K_{\Sigma} = \sqrt{\gamma}\left(W_{ij}^{ij} - P_{ij}^{(A)}P_{(A)}^{ij}\right), \qquad P_{ij}^{(A)} = \mathcal{K}_{ij}^{(A)} - \frac{1}{2}\mathcal{K}^{(A)}\gamma_{ij}$$

General Energy Functional

$$L_{\Sigma}[M] = \frac{\ell^2}{2}\int_{\Sigma} d^2y\sqrt{\gamma}K_{\Sigma} - 2\pi\ell^2\chi\left[\Sigma\right]$$

Conformal Invariance in Codimension-2 is inherited from the Bulk!

Renormalized Area

Einstein ambient space

$$L_{\Sigma}\left[E\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left[W^{ij}_{(\mathsf{E})ij} - P^{(A)}_{ij}P^{ij}_{(A)}\right] - 2\pi\ell^2\chi\left[\Sigma\right]$$

Gauss-Codazzi relations

$$W^{ij}_{(E)ij} = \mathscr{R} - \mathcal{K}^{(A)}\mathcal{K}_{(A)} + \mathcal{K}^{(A)}_{ij}\mathcal{K}^{ij}_{(A)} + \frac{2}{\ell^2}\,,$$

Renormalized Area (Minimality)

$$L_{\Sigma_{\min}}[E] = \mathcal{A}_{\mathrm{ren}} = \frac{\ell^2}{4}\int\limits_{\Sigma_{\min}} d^2y\sqrt{\gamma}\,\delta^{km}_{ij}\left(\mathscr{R}^{ij}_{km} + \frac{1}{\ell^2}\delta^{ij}_{km}\right) - 2\pi\ell^2\chi\left[\Sigma_{\min}\right]\,. \qquad (1)$$

Willmore Energy

Einstein ambient space

$$L_\Sigma[E] = \frac{\ell^2}{2}\int_\Sigma d^2y\sqrt{\gamma}\left(W^{ij}_{(\mathsf{E})ij} - P^{(A)}_{ij}P^{ij}_{(A)}\right) - 2\pi\ell^2\chi[\Sigma]$$

For pure/global AdS_4 as ambient space, constant-time slice

$$W = 0 \qquad \mathcal{K}^{(t)} = 0$$

Gauss-Codazzi relations

$$L_\Sigma[AdS_4] = -\frac{\ell^2}{2}\int_\Sigma d^2y\sqrt{\gamma}\left(R^{ij}_{ij} - \mathscr{R} + 2H^2\right) - 2\pi\ell^2\chi[\Sigma]$$

Willmore Energy

Surface embedded in $\mathbb{R}^3$

$$L_{\Sigma}\left[AdS_4\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left(\mathscr{R} - 2H^2\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

For a compact surface

$$\int\limits_{\Sigma_{\mathsf{comp}}} d^2y\sqrt{\gamma}\mathscr{R} = 4\pi\chi\left[\Sigma_{\mathsf{comp}}\right]$$

Willmore Energy [Fonda, Seminara, Tonni, 2015]

$$L_{\Sigma_{\mathsf{comp}}}\left[AdS_4\right] = -\ell^2\mathcal{W}\left[\Sigma_{\mathsf{comp}}\right]$$

Reduced Hawking Mass

Arbitrary Σ, Einstein ambient space

$$L_{\Sigma}\left[E\right] = \frac{\ell^2}{4} I_H\left[\Sigma\right] - 2\pi\ell^2\chi\left[\Sigma\right]$$

Reduced Hawking Mass I_H [Fischetti, Wiseman, 2016]

$$I_H\left[\Sigma\right] = 2\int_{\Sigma} d^2y\sqrt{\gamma}\left[\mathscr{R} + \frac{2}{\ell^2} - \frac{1}{2}\left(\mathcal{K}^{(A)}\right)^2\right]$$

Generalized Renormalized Area

$$L_{\Sigma}\left[E\right] = \mathcal{A}_{\mathsf{ren}}\left[\Sigma\right] - \frac{\ell^2}{4}\int_{\Sigma} d^2y\sqrt{\gamma}\left(\mathcal{K}^{(A)}\right)^2$$

Main results

Energy functionals from Conformal Gravity

Σ \ M	Einstein	pure AdS
min	$\mathcal{A}_{\text{ren}}$	$\mathcal{W}$
non-min	I_H	

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Outlook

Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in Codimension-2

Renormalization in the Bulk $\Longrightarrow$ Renormalization in Codimension-2

Conformal Invariance $\Longrightarrow$ Renormalization (???) (as in 6D [Anastasiou, Araya, RO, 2021]

Willmore Energy in 4D from 6D Conformal Gravity (with Bueno, Vilar-Lopez)