Title: Weak measurement in conformal field theory and holography - VIRTUAL

Speakers: Shaokai Jian

Series: Quantum Matter

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Abstract: Weak measurements can be viewed as a soft projection that interpolates between an identity operator and a projection operator, and can induce an effective central charge distinct from the unmeasured CFT. In the first part, I will discuss the effect of measurement and postselection on the critical ground state of a Luttinger liquid theory. Depending on the Luttinger parameter K, the effect of measurement is irrelevant, marginal, or relevant, respectively. When the measurement is marginal, and we find a critical state whose entanglement entropy exhibits a logarithmic behavior with a continuous effective central charge as a function of measurement strength. Inspired by this result, in the second part, I will discuss a holographic description of the weak measurement. The weak measurement is modeled by an interface brane, separating different geometries dual to the post-measurement state and the unmeasured CFT. In an infinite system, the weak measurement is related to ICFT via a spacetime rotation. We find that the holographic entanglement entropy with twist operators located on the defect is consistent in both calculations for ICFT and weak measurements. In a finite system, the weak measurement can lead to a rich phase diagram, in which the post-measurement geometry can realize a Python's lunch.

Zoom link TBA

Weak measurement in conformal field theory and holography

Shao-Kai Jian

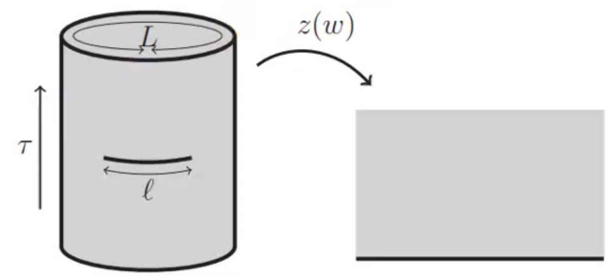
Tulane University

Xinyu Sun, Hong Yao, SKJ *New critical states induced by measurement*, 2301.11337
Xinyu Sun, SKJ *Holographic weak measurement*, JHEP12(2023)157

Perimeter Institute
Feb 27, 2024

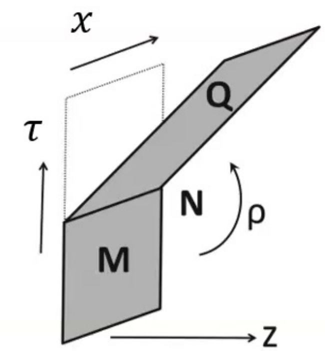
Local Projection Measurements

Projection measurement often induces a radical change of the wavefunction. For instance, local projection causes the wavefunction unentangled.



- In a conformal field theory, the local projection measurement onto a boundary state is described by a slit at $\tau = 0$ in the imaginary time path integral. It can be mapped to a BCFT, e.g.

  J-M Stephan, PRB 90, 045424 (2014)
  M A Rajabpour, PRB 92, 075108 (2015); J. Stat. Mech. 063109 (2016)



- Accordingly, in a holographic system, local projection measurements create end-of-the-world brane anchors at the boundary of measurement region.

  T Takayanagi, PRL 107 101602 (2011)
  M Fujita, T Takayanagi, & E Tonni, J. High Energ. Phys. 2011, 43
  T Numasawa, N Shiba, T Takayanagi, et al. J. High Energ. Phys. 2016, 77

- The boundary state has trivial spacetime dual

  M Miyaji, S Ryu, T Takayanagi and X Wen, JHEP 1505 (2015)

S Antonini, G Bentsen, CJ Cao, B Grado-White, J Harper, SKJ, B Swingle: 2209.12903, 2211.07658, 2304.06743

Weak Measurements

Boundary state resulted from local projection measurement has no entanglement and thus has a trivial spacetime dual, can a more general measurement support finite entanglement and nontrivial spacetime?

Consider weak measurement operator:

$$M = e^{W\sigma^z}$$

- For $W = 0$, $M = 1$
- For $W \to \infty$, $M \propto |\uparrow\rangle\langle\uparrow|$
- When $\bigotimes_i M_i$ acts on a critical state, the parameter W interpolates between the original state and a boundary state

For a generic W, what is the resulted state? How to describe it holographically?

X Sun, H Yao, SKJ, New critical states induced by measurement, 2301.11337
X Sun, SKJ, Holographic weak measurement, JHEP12(2023)157

Note that there are related studies

SJ Garratt, Z Weinstein, E Altman. PRX 021026 (2023); Z Yang, D Mao, CM Jian, 2301.08255; Z Weinstein, R Sajith, E Altman, SJ Garratt, 2301.08268

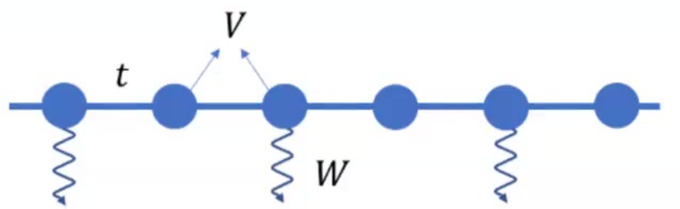
Outline

- Weak measurement in a Luttinger liquid (compactified boson CFT)
- Weak measurement in holography

Luttinger Liquid Theory

Consider a spinless fermion chain with hopping t and interaction V

$$H = -t\sum_i \left(c_i^\dagger c_{i+1} + h.c.\right) + V\sum_i \left(n_i - \frac{1}{2}\right)\left(n_{i+1} - \frac{1}{2}\right)$$



The ground state is described by Luttinger liquid (compactified free boson $c = 1$), with left and right movers and

$$K = \frac{\pi}{2(\pi - \arccos V/t)}$$

- $K = 1$: noninteracting fermion; $K > 1$ ($K < 1$): attractive (repulsive) interaction

Implement weak measurements on the ground state

$$\rho_m = \frac{M\rho M^\dagger}{Tr[M\rho M^\dagger]} \text{ with the measurement operator: } M = e^{-W\sum_i (-1)^i n_i}$$

- Measures the occupation number in a staggered way. Inhomogeneity creates potential that scatters left and right movers (in boson language, measurement = vortex operators)
- Measurement can be implemented by coupling to ancilla qubits, and then projecting them out
- When $W \to \infty$ it becomes local projection measurements. For finite W, it is weak measurements

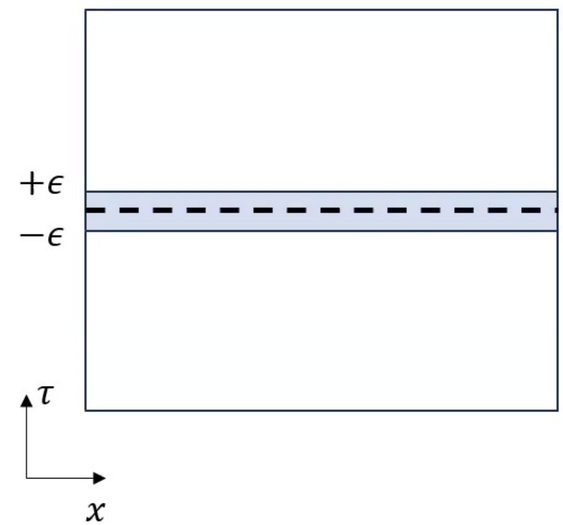
Path Integral Representation

To construct the path integral representation of the weak measurements, we introduce a UV cutoff ϵ for the measurement $M = \lim_{\epsilon \to 0} M_\epsilon$

$$\langle \psi_2 | M_\epsilon | \psi_1 \rangle = \int_{\psi(-\epsilon)=\psi_1}^{\psi(0)=\psi_2} D\psi \exp[-S_M] , \qquad S_M = \int_{-\epsilon}^{0} d\tau \sum_i \left(\psi_i^\dagger \partial_\tau \psi_i + \frac{W}{\epsilon} (-1)^i n_i \right)$$

The full path integral is

$$S = \int_{-\infty}^{\infty} d\tau \sum_i \left(\psi_i^\dagger \partial_\tau \psi_i + H + f(\tau) W (-1)^i n_i \right), \quad f(\tau) = \begin{cases} \frac{1}{2\epsilon}, & |\tau| < \epsilon \\ 0, & |\tau| \geq \epsilon \end{cases}$$



such that $\int d\tau \, f(\tau) = 1$. At $\epsilon \to 0$ limit, we have

$$S = \int d\tau \sum_i \left(\psi_i^\dagger \partial_\tau \psi_i + H + \delta(\tau) W (-1)^i n_i \right)$$

- The full path integral lives in the entire complex plane
- The weak measurement creates an interface at $\tau = 0$

Bosonization

Abelian bosonization: $M \sim \psi_L^\dagger \psi_R + h.c. \sim e^{i\sqrt{2}\phi_L} e^{i\sqrt{2}\phi_R} + h.c. \sim \cos 2\phi$

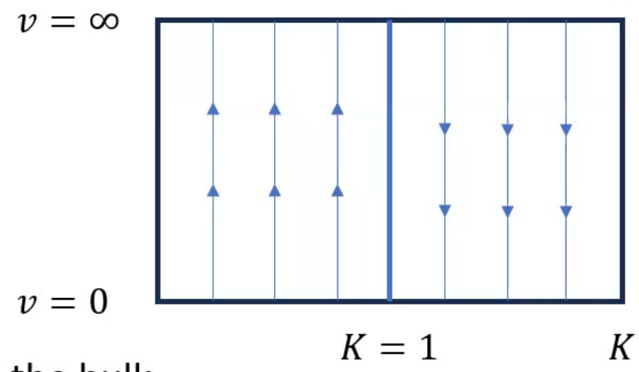
We have a free compactified boson with an interface (the measurement strength is $v \propto W$)

$$S = \int d\tau dx \left(\frac{(\partial\phi)^2}{2\pi K} + \delta(\tau) v \cos 2\phi \right)$$

Renormalization group equation

$$\frac{dv}{dl} = (1-K)v$$
$$\frac{dK}{dl} = 0$$

SJ Garratt, Z Weinstein, E Altman. PRX 021026 (2023)



- The vortex operator strength is controlled by the Luttinger parameter in the bulk
- The measurement at interface is not able to renormalize the coupling in the bulk
- $K > 1$ (attractive interaction) irrelevant, $K < 1$ (repulsive interaction) relevant.
- At $K = 1$, measurement is marginal, it is consistent because the theory is effectively a free fermion theory

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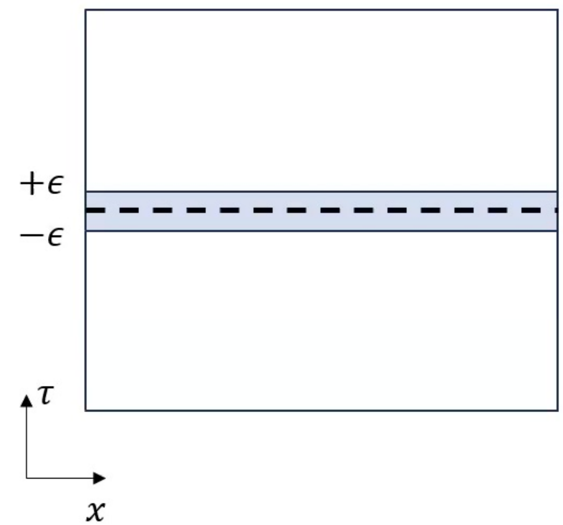

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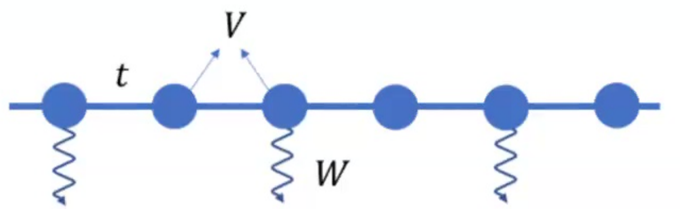

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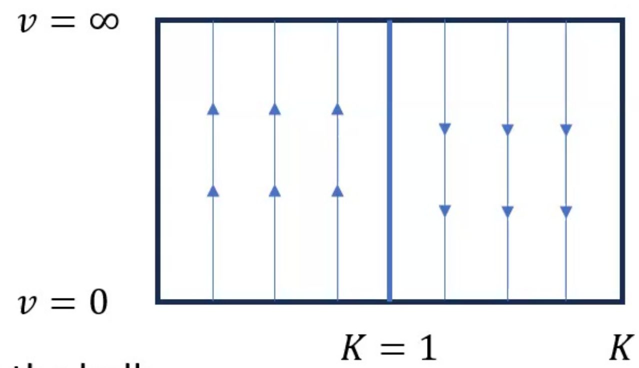

SJ Garratt, Z Weinstein, E Altman. PRX 021026 (2023)

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Entanglement Transition

Consider entanglement entropy of subsystem A with length x_A:

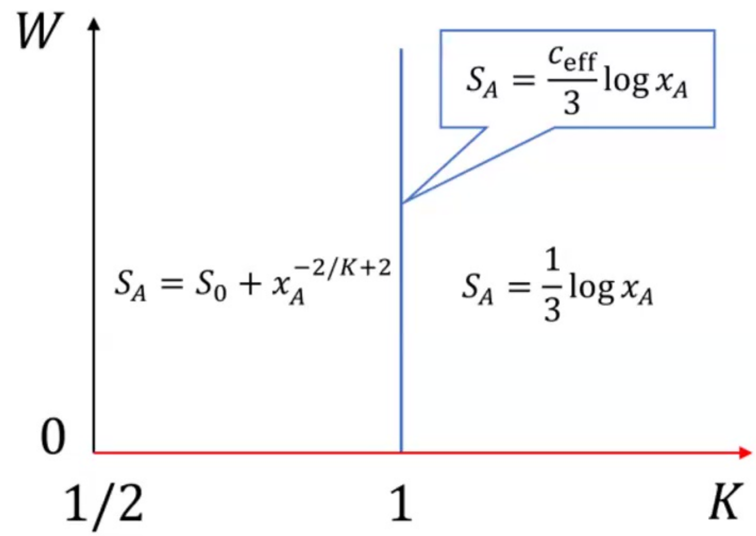
When the measurement is irrelevant, we recover the entanglement entropy of Luttinger liquid

$$S_A = \frac{1}{3}\log x_A$$

When the measurement is relevant, the entanglement entropy is

$$S_A = S_0 + x_A^{-\frac{2}{K}+2}$$

- The leading component is an area law, and the subleading correction exhibits a power law decay



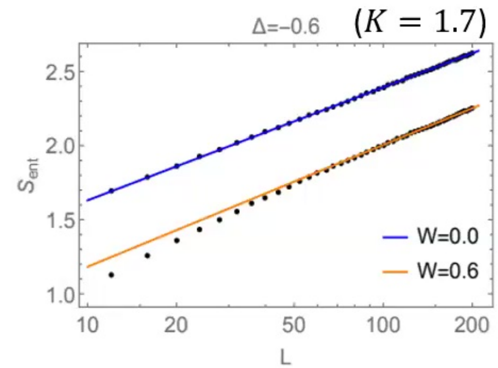
Density Marix Renormalization Group

We implement DMRG simulation to confirm the entanglement transition. We simulate the XXZ model, which is equivalent to the spinless fermion after Jordan-Wigner transformation

$$H = \sum_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right)$$

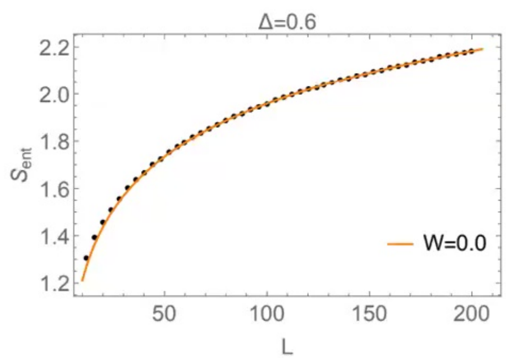
Consider half chain entanglement entropy

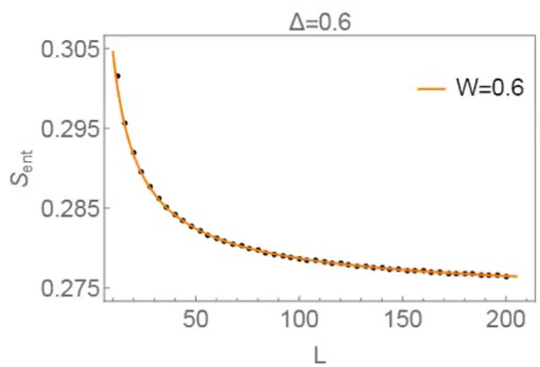
- Irrelevant phase ($K = 1.7 > 1$):

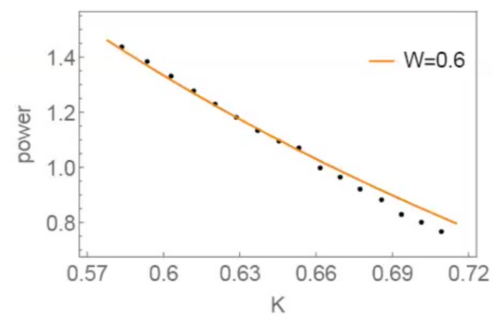


- Relevant phase ($K = 0.7 < 1$):

subleading power law $\mathrm{Abs}\left(-\frac{2}{K} + 2\right)$







Critical Point: Free Fermion

At the critical point $K = 1$, which means $V = 0$. The theory is a free fermion theory with left and right movers

$$H = \int dx\, \psi^\dagger(-i\sigma^z\partial_x)\psi \text{ with } \psi = (\psi_L, \psi_R)^T$$

Recall that the measurement operator creates a potential that scatters between left and right movers

$$h = \int dx\, \psi^\dagger\sigma^x\psi$$

This is effectively a mass term as σ^x anticommutes with σ^z. Let's ask what other similar measurements (as an effective mass term) can be implemented. Include the possibility of superconductivity $\Psi = \left(\psi_L, \psi_R, \psi_L^\dagger, \psi_R^\dagger\right)$, there are two additional mass terms:

- SSH mass: $$h_1 = \int dx\, \Psi^\dagger\sigma^y\Psi$$
- p-wave superconductivity (μ stands for the Nambu space)

$$h_2 = \int dx\, \Psi^\dagger\sigma^y\mu^x\Psi$$

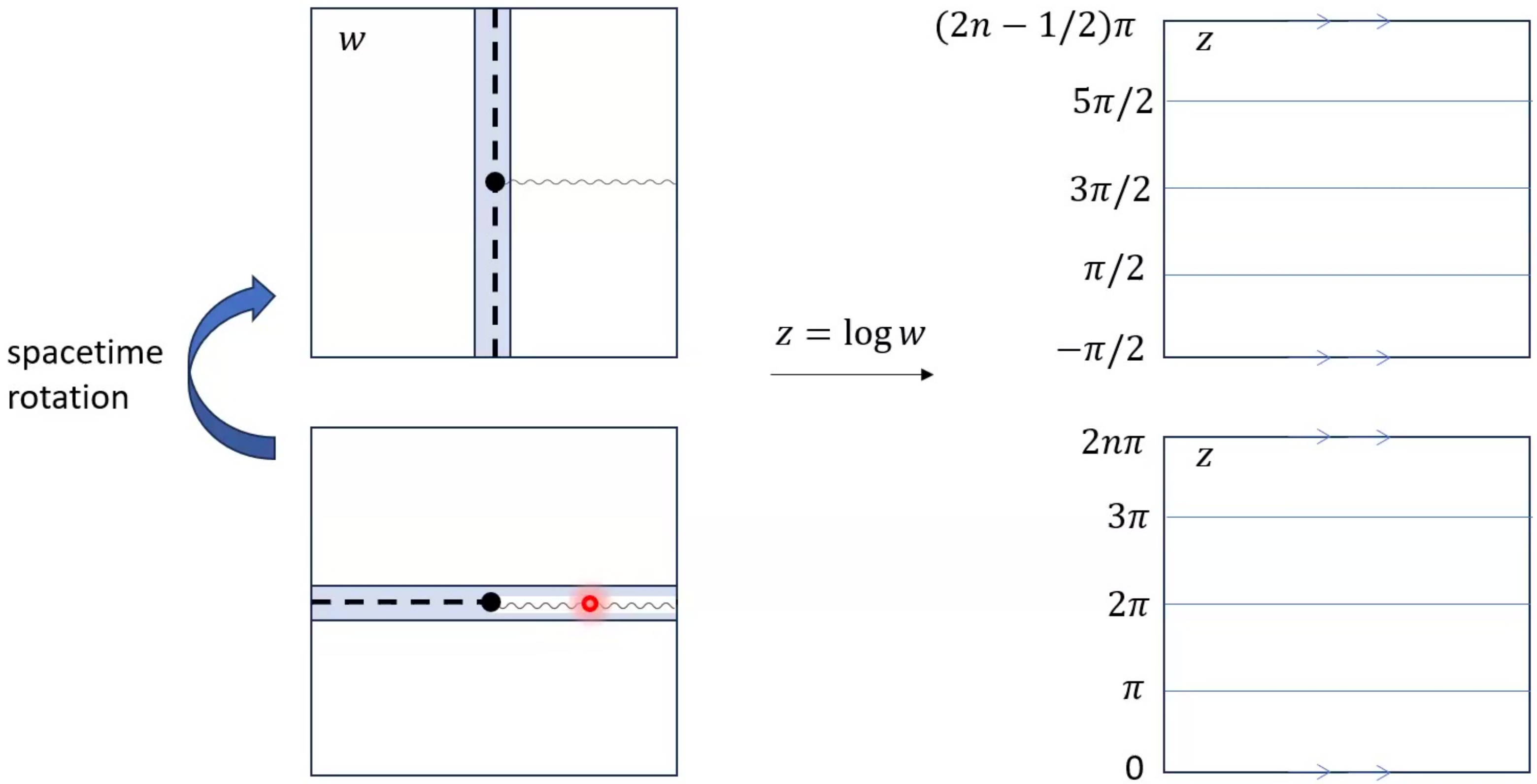
In the low energy theory, the effect of these mass terms is the same, so we expect to have a universal behavior for all three measurements protocols

Spacetime Rotation

To calculate the entanglement entropy after measurements, we insert twist operators in the time reversal invariant $\tau = 0$ slice. It creates a branch cut.

E Brehm, I Brunner, JHEP 2015, 80

This problem is closely related to defect CFT via a spacetime rotation that has been extensively studied



Effective Central Charge for Marginal Measurement

- The defect CFT has been studied in [e.g. V Eisler, I Peschel, 1005.2144], there is an effective central charge

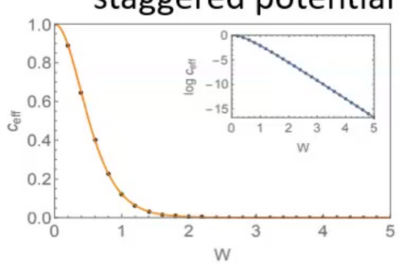
$$c_{\text{eff}} = -\frac{6}{\pi^2}\left\{ [(1+s)\log(1+s) + (1-s)\log(1-s)]\log(s) + (1+s)\text{Li}_2(-s) + (1-s)\text{Li}_2(s) \right\}$$

- So, with spacetime rotation the entanglement entropy of a subsystem A with length x_A is $S_A = \frac{c_{eff}}{3}\log\frac{x_A}{a}$

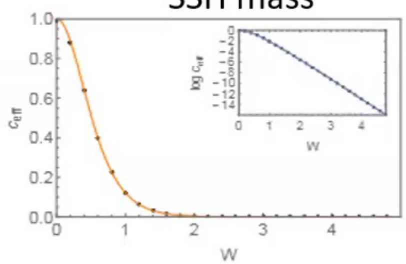
also in Z Yang, D Mao, CM Jian, 2301.08255

- We calculate the half chain entanglement entropy using fermionic Gaussian state simulation. For the three different measurement protocols, the results are identical

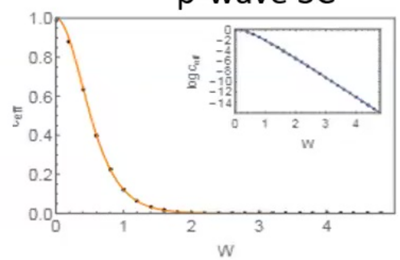
staggered potential



SSH mass

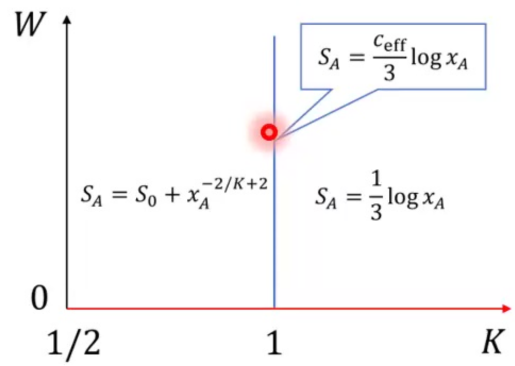


p-wave SC



Outline

- Weak measurement in a Luttinger liquid (compactified boson CFT)



- Weak measurement in holography

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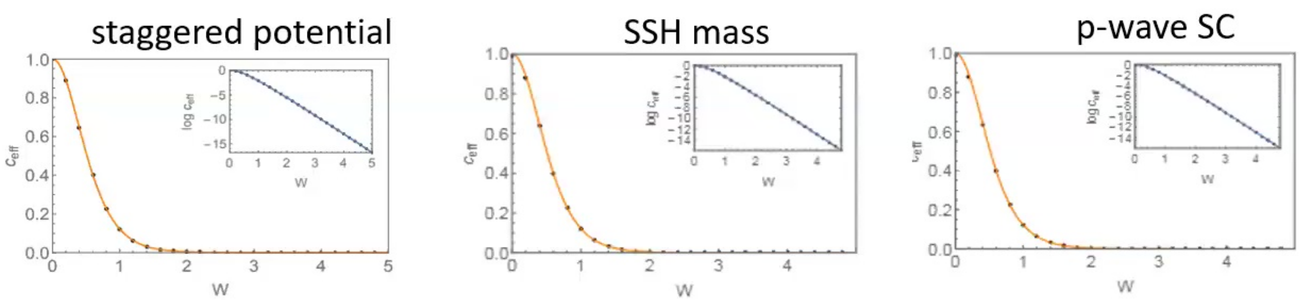
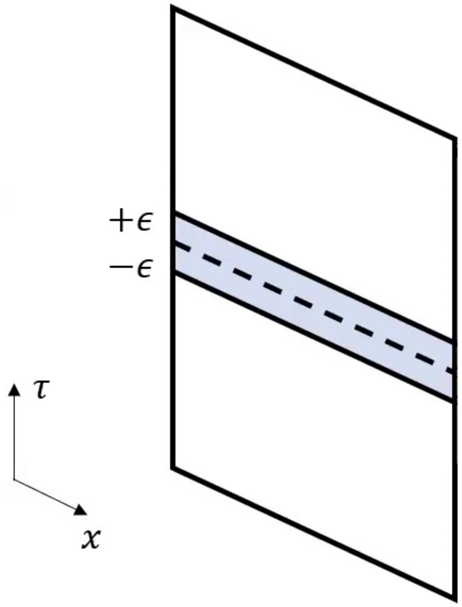


Holographic Weak Measurement



Recall the boundary theory is a CFT with measurement at $\tau \in (-\epsilon, \epsilon)$

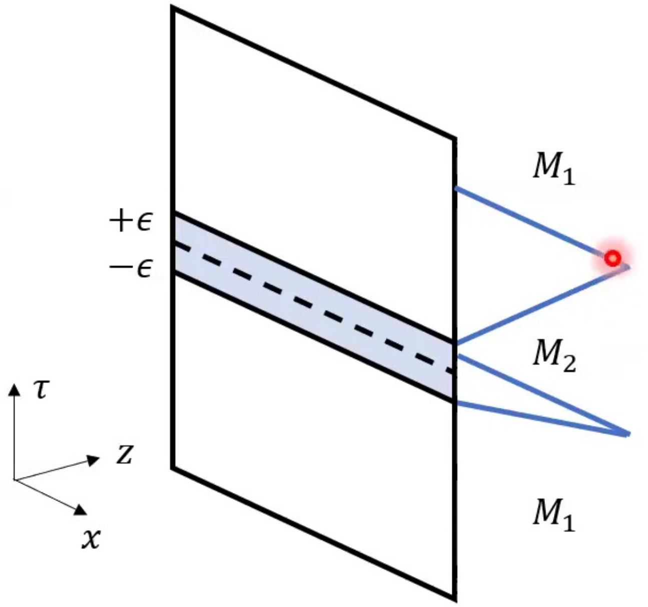
Euclidean path integral: partition function = $Z = \int D\phi\, e^{-S}$,

$$S = \int d\tau \sum_i \left(L_{CFT} + f(\tau) L_M\right), \qquad f(\tau) = \begin{cases} \frac{1}{2\epsilon}, & |\tau| < \epsilon \\ 0, & |\tau| \geq \epsilon \end{cases}$$

For concreteness, the central charge of CFT is denoted as c_1. We denote effective central charge by c_{eff}

$$c_{eff} = \begin{cases} c_1 & irrelevant \\ c_{eff} \in (0, c_1) & marginal \\ 0 & relevant \end{cases}$$

Holographic Weak Measurement



We expect two bulk geometries separating by branes:

- M_1 denotes the region dual to CFT before measurement
- M_2 denotes the region dual to the measurement at boundary
- The branes end at $\tau = \pm\epsilon$

Consider Euclidean action with asymptotic infinite plane boundary

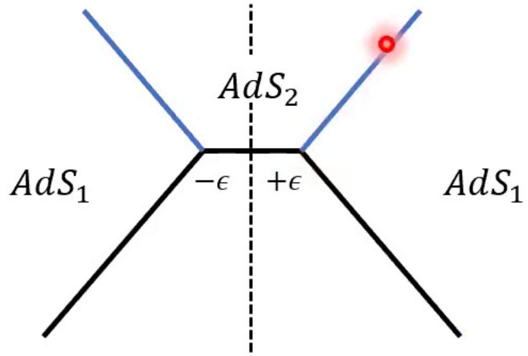
$$-16\pi G_N I = \int_{M_1} \sqrt{g_1}\left(R_1 + \frac{2}{L_1^2}\right) + \int_{M_2} \sqrt{g_2}\left(R_2 + \frac{2}{L_2^2}\right)$$
$$+2\int_{\partial M_{12}} \sqrt{h}\,(K_1 - K_2 - T)$$

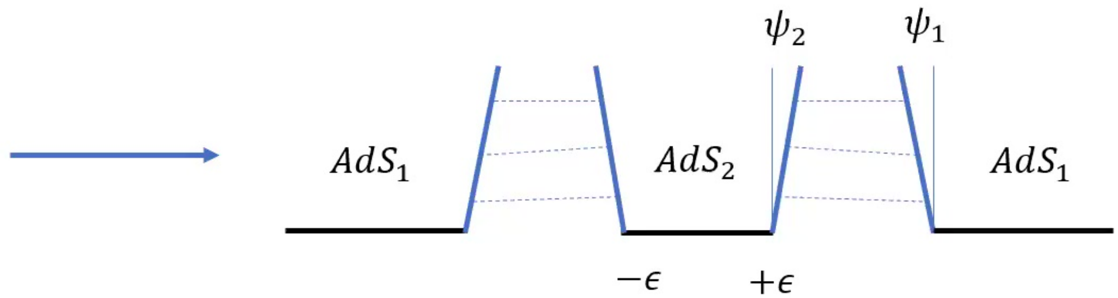
- Central charge: $c_1 = \frac{3L_1}{2G_N}, c_{eff} = \frac{3L_2}{2G_N};\ L_1 \geq L_2;$
- Tension = T and K_i = extrinsic curvature
- The bulk solution is $ds^2 = L_i \frac{dz^2 + d\tau^2 + dx^2}{z^2}$ with interface brane determines by junction conditions

Interface Brane

Brane solution

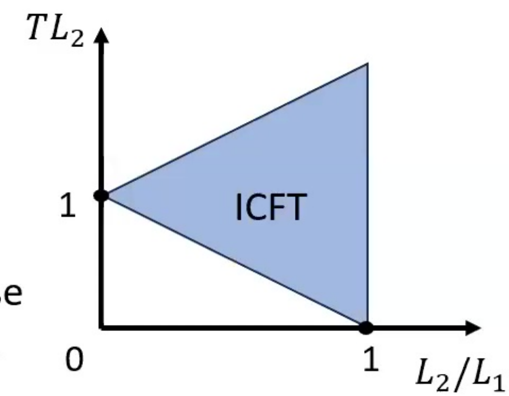
$$\sin\psi_1 = \frac{L_1}{2T}\left(T^2 + \frac{1}{L_1^2} - \frac{1}{L_2^2}\right), \ \sin\psi_2 = \frac{L_2}{2T}\left(T^2 + \frac{1}{L_2^2} - \frac{1}{L_1^2}\right)$$





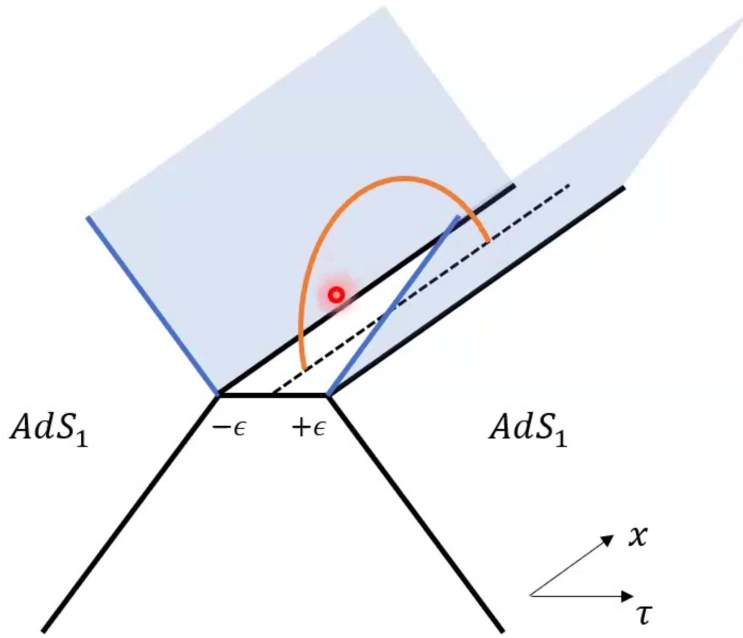
Tension locates within the range: $\frac{1}{L_2} - \frac{1}{L_1} \leq T \leq \frac{1}{L_1} + \frac{1}{L_2}$. The phase diagram is

- At $\frac{L_2}{L_1} = 0$, BCFT limit, the interface brane reduces to ETW brane. This corresponds the relevant case. State after measurement becomes area law and does not have a dual
- At $\frac{L_2}{L_1} = 1, T = 0$, two geometries merge into one. This corresponds to irrelevant case
- ICFT correspond to the marginal case with an effective central charge and boundary entropy



Entanglement Entropy

Consider the marginal case: consider entanglement entropy of a subsystem A after measurement at the time symmetric slice:

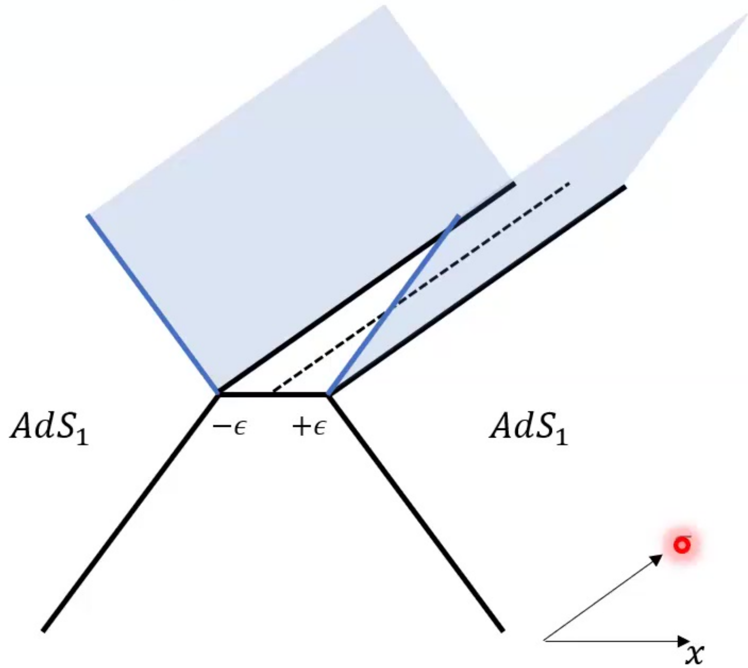


The geodesic length is $d = L_2 \cosh \frac{x_A^2}{2a^2} \approx L_2 \log \frac{x_A^2}{a^2}$, a=UV cut off, x_A= length of subregion A

According to RT formula $S_A = \frac{d}{4G_N} = \frac{1}{3}\left(\frac{3L_2}{2G_N}\right) \log \frac{x_A}{a} = \frac{c_{eff}}{3} \log \frac{x_A}{a}$

Spacetime Rotation

Because the bulk metric $ds^2 = L_i \frac{dz^2+d\tau^2+dx^2}{z^2}$ is symmetric in (x, τ), we can simply make a rotation and find the same solution of branes.



C Bachas, J de Boer, R Dijkgraaf, H Ooguri, JHEP 06 (2002) 027;
O DeWolfe, DZ Freedman, H. Ooguri, PRD 66 025009 (2002);
J Erdmenger, Z Guralnik, I Kirsch PRD 66, 025020 (2002);
J Erdmenger, M Flory, MN Newrzella, JHEP 01 (2015) 058;
T Anous, M Meineri, P Pelliconi, J Sonner, SciPost Physics, 13, 075 (2022)…

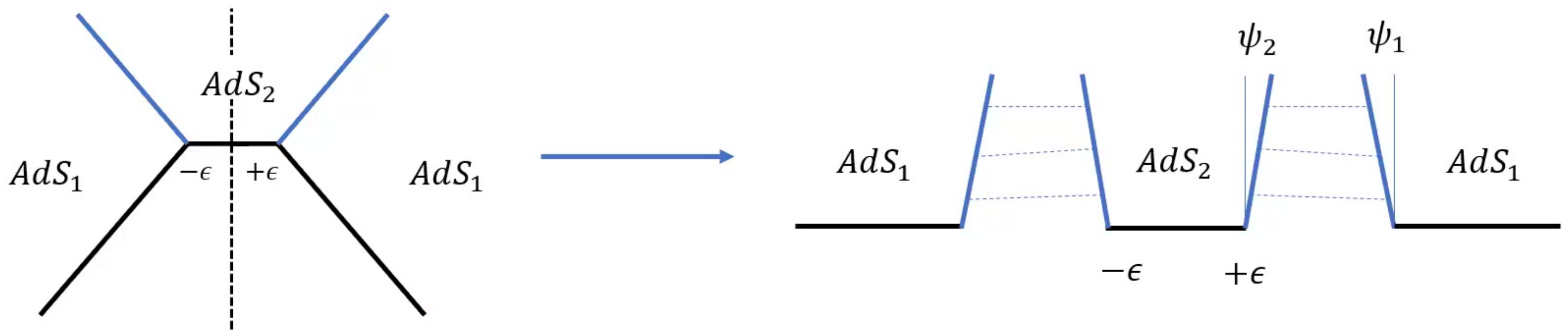
We are interested in the two cases

- One end of the RT surface anchors on AdS_2 $x = 0$, and the other on AdS_1
- RT surface is symmetric w.r.t $x = 0$

Interface Brane

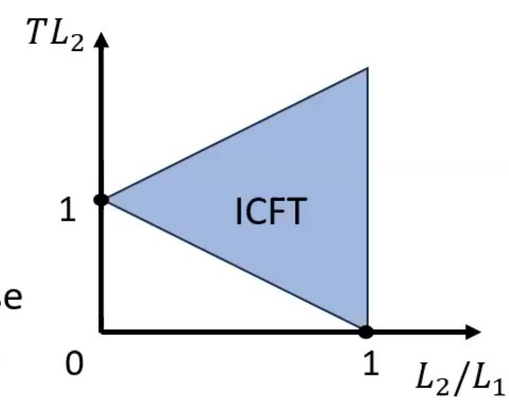
Brane solution

$$\sin\psi_1 = \frac{L_1}{2T}\left(T^2 + \frac{1}{L_1^2} - \frac{1}{L_2^2}\right), \ \sin\psi_2 = \frac{L_2}{2T}\left(T^2 + \frac{1}{L_2^2} - \frac{1}{L_1^2}\right)$$

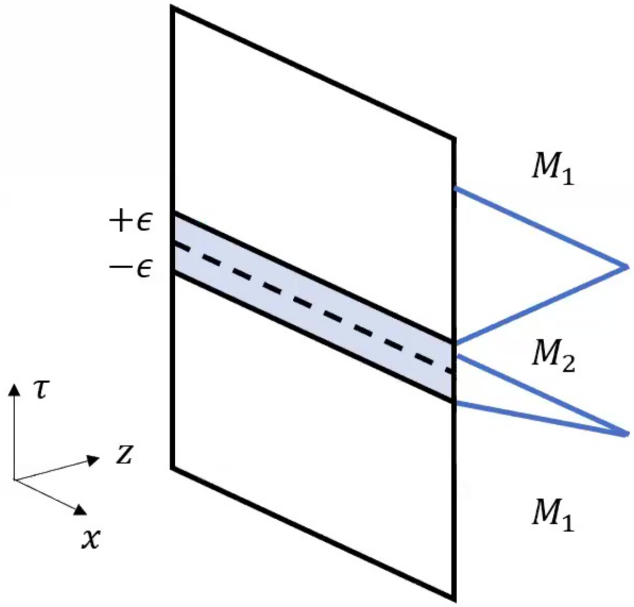


Tension locates within the range: $\frac{1}{L_2} - \frac{1}{L_1} \leq T \leq \frac{1}{L_1} + \frac{1}{L_2}$. The phase diagram is

- At $\frac{L_2}{L_1} = 0$, BCFT limit, the interface brane reduces to ETW brane. This corresponds the relevant case. State after measurement becomes area law and does not have a dual
- At $\frac{L_2}{L_1} = 1, T = 0$, two geometries merge into one. This corresponds to irrelevant case
- ICFT correspond to the marginal case with an effective central charge and boundary entropy



Holographic Weak Measurement



We expect two bulk geometries separating by branes:

- M_1 denotes the region dual to CFT before measurement
- M_2 denotes the region dual to the measurement at boundary
- The branes end at $\tau = \pm\epsilon$

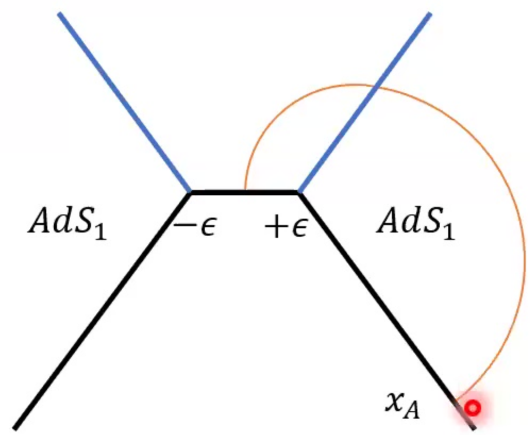
Consider Euclidean action with asymptotic infinite plane boundary

$$-16\pi G_N I = \int_{M_1} \sqrt{g_1}\left(R_1 + \frac{2}{L_1^2}\right) + \int_{M_2} \sqrt{g_2}\left(R_2 + \frac{2}{L_2^2}\right)$$
$$+2\int_{\partial M_{12}} \sqrt{h}\,(K_1 - K_2 - T)$$

- Central charge: $c_1 = \frac{3L_1}{2G_N}, c_{eff} = \frac{3L_2}{2G_N};\ L_1 \geq L_2;$
- Tension = T and K_i = extrinsic curvature
- The bulk solution is $ds^2 = L_i \frac{dz^2+d\tau^2+dx^2}{z^2}$ with interface brane determines by junction conditions

RT Surface Anchors on $x = 0$

RT surface (geodesic) anchors on $x = 0$



The geodesic length is $d = (L_1 + L_2) \log \frac{x_A}{a} + d_0$, a=UV cut off, x_A= length of subregion A

- We have taken $\epsilon \to 0$, and the result is well-behaved under such a limit
- d_0 is independent of x_A, d_0 is in general UV dependent because of the leading logarithmic function.

The entanglement entropy is

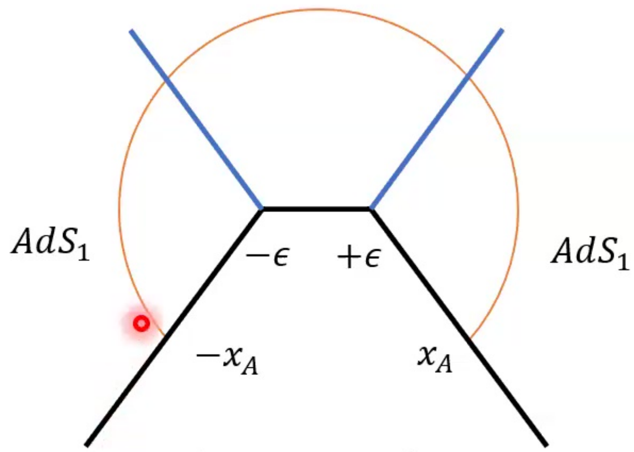
$$S_A = \frac{c_1 + c_{eff}}{6} \log \frac{x_A}{a}$$

- Each twist operators in two different regions contribute to the entanglement entropy
- Effective central charge is the same as the state under measurement

A Karch, ZX Luo, HY Sun, JHEP 09 (2021),
A Karch, M Wang, JHEP 06 (2023).

RT Surface Anchors on Symmetric Points

Geodesic anchors on symmetric points



The geodesic length is $d = d_1 + d_2$, $d_1 = 2L_1 \log\left(\frac{2x_A}{a}\frac{1+\sin\psi_1}{\cos\psi_1}\right)$, $d_2 = L_2 \cosh^{-1}\left(\frac{1+\sin^2\psi_2}{\cos^2\psi_2}\right)$, a=UV cut off,

- We have taken $\epsilon \to 0$, and the result is well-behaved under such a limit

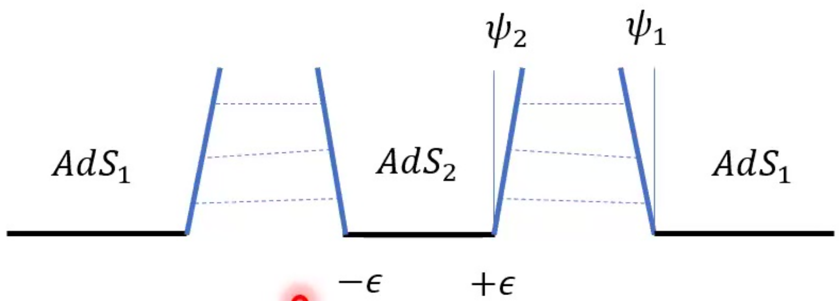
The entanglement entropy is $$S_A = \frac{c_1}{3}\log\frac{2x_A}{a} + S_{\text{bdy}}$$

- Boundary (interface) entropy $S_{\text{bdy}} = \frac{c_1}{3}\log\left(\tan\left(\frac{\psi_1}{2}+\frac{\pi}{4}\right)\right) + \frac{c_{eff}}{3}\log\left(\tan\left(\frac{\psi_2}{2}+\frac{\pi}{4}\right)\right)$ is well defined because we can subtract the case without defect, i.e., the leading term

Boundary Entropy from Partition Function

In the symmetric case, we obtain the boundary entropy associated with the interface. We can calculate the boundary entropy from the partition function:

$$Z = Tr\left[Me^{-\beta H}M^{\dagger}\right] = \lim_{\epsilon\to 0}\langle 0|M_\epsilon M_\epsilon^{\dagger}|0\rangle$$



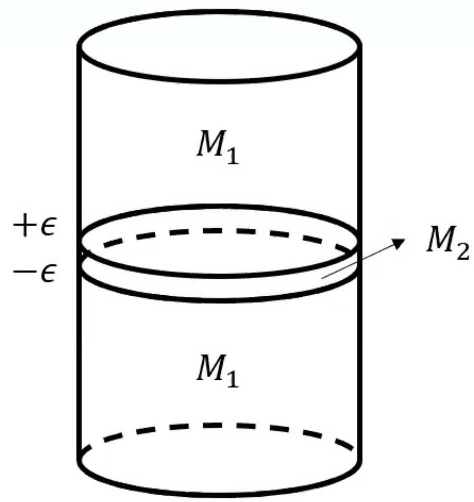
The boundary entropy is ($Z_{saddle} = e^{-I}$):

$$S_{\text{bdy}} = -(I - I_0) = 2(\rho_1 + \rho_2) \;\text{ with } \tanh\frac{\rho_i}{L_i} = \sin\psi_i$$

- I_0 is the action without the interface
- consistent with the symmetric geodesic $S_{\text{bdy}} = \frac{c_1}{3}\log\left(\tan\left(\frac{\psi_1}{2}+\frac{\pi}{4}\right)\right) + \frac{c_{eff}}{3}\log\left(\tan\left(\frac{\psi_2}{2}+\frac{\pi}{4}\right)\right)$
- Naively the result is two independent sum from two bulk dual, but it is not: ψ_i is a function of L_1, L_2 and T

Holographic Weak Measurement for Finite System

Consider the CFT in a circle with $R = 1$



Similarly, we expect two bulk geometries separating by branes:

- M_1 denotes the region dual to CFT before measurement
- M_2 denotes the region dual to the measurement at boundary
- The branes end at $\tau = \pm\epsilon$

We use the same convention

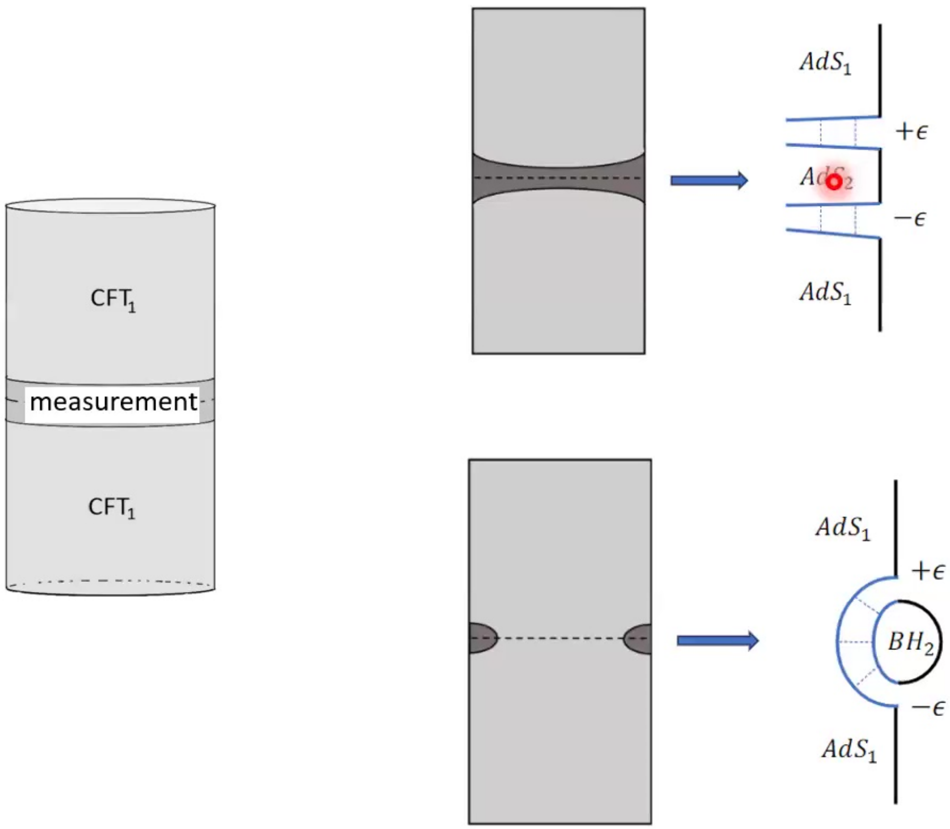
- Central charge: $c_1 = \frac{3L_1}{2G_N}, c_{eff} = \frac{3L_2}{2G_N};\ L_1 \geq L_2;$
- Tension = T

For region M_1, we have a global AdS metric, but now for region M_2 we can have either AdS or BTZ black hole metric

$$ds^2 = \left((1-\mu) + \frac{r^2}{L^2}\right)dt^2 + \left((1-\mu) + \frac{r^2}{L^2}\right)^{-1} dr^2 + r^2 dx^2$$

Phase Diagram

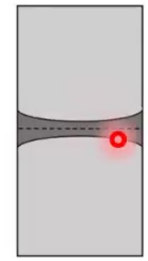
Fortunately, the phase diagram has been studied in [P Simidzija, and M Van Raamsdonk, "Holo-ween" JHEP (2020)]. There are two classes of phases:



- The no-bubble phase, it is similar to the infinite plane boundary case: two AdS with different L are separated by branes
- The bubble phase, it is given by the joint geometries of AdS (dual to the unmeasured CFT) and BH (induced by measurement)

Entanglement Entropy

We are interested in the entanglement properties of the state upon measurement. We focus on the time reflection symmetric slice.

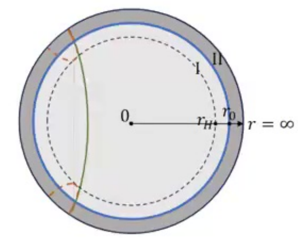


- In no-bubble phase, the time reversal invariant slice is located within AdS_2 phase

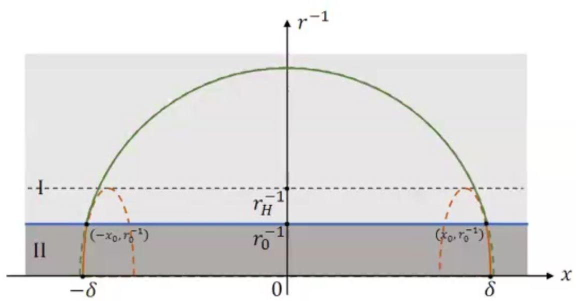
$$S_A = \frac{c_{eff}}{3} \log \frac{x_A}{a}$$

- In bubble phase, the time reversal invariant slice cuts through both geometries. The RT surface will cross the brane and enter AdS_1

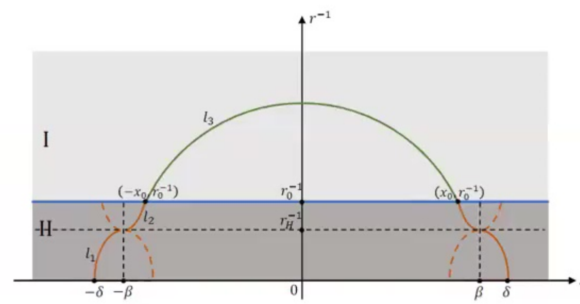
$$S_A = \frac{c_1}{3} \log \frac{x_A}{a}$$



bubble-outside-horizon phase



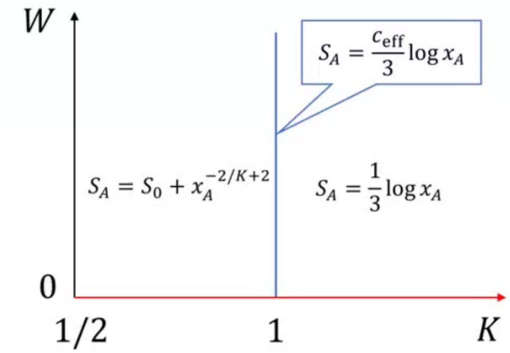
bubble-inside-horizon phase



Therefore, the no-bubble phase is the marginal case, and the bubble phase is the irrelevant case.

Conclusion

- Weak measurement in a Luttinger liquid (compactified boson CFT)



- Weak measurement in holography

$$\lim_{\epsilon \to 0} \langle 0 | M_\epsilon M_\epsilon^\dagger | 0 \rangle =$$