Title: Mixed-state quantum anomaly and multipartite entanglement

Speakers: Leonardo Almeida Lessa
Date: February 26, 2024 - 2:00 PM

URL: https://pirsa.org/24020093

Abstract: Quantum entanglement measures of many-body states have been increasingly useful to characterize phases of matter. Here we explore the surprising connection between symmetry-protected topology (SPT) and separability of their boundary mixed states. More specifically, we consider lattice systems in d space dimensions with anomalous symmetry G, where the anomaly is characterized by a bulk SPT invariant in the group cohomology Hd+2(G,U(1)). We show that any mixed state ? that is strongly symmetric under G, in the sense that G ? ? ?, is necessarily (d+2)-nonseparable, i.e. is not the mixture of tensor products of d+2 states in the Hilbert space. Furthermore, such states cannot be prepared from any (d+2)-separable states using finite-depth local quantum channels, so the non-separability is long-ranged in nature. The anomaly-nonseparability connection also allows us to generate simple examples of mixed states with nontrivial multi-partite entanglement.

Zoom link TBA

Mixed-state quantum anomaly and multipartite separability

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PI Grad student seminars - Feb. 26, 2024

arXiv:2401.17357

Mixed-state quantum anomaly and multipartite entanglement

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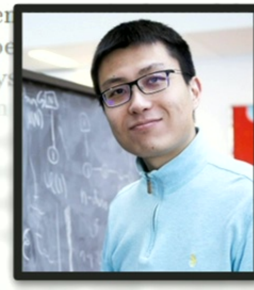


Quantum entanglement measures of mar ... been ... characterize phases of matter. Here we explore a s ... be ... entanglement and 't Hooft anomaly. More specifically, ... sys ... ensions with anomalous symmetry G where the anomal ... an ... cohomology

Contents

1. Quantum anomaly
2. Mixed-state quantum anomaly
3. Main result in 1d
4. Example of anomalous mixed state in 1d
5. Summary

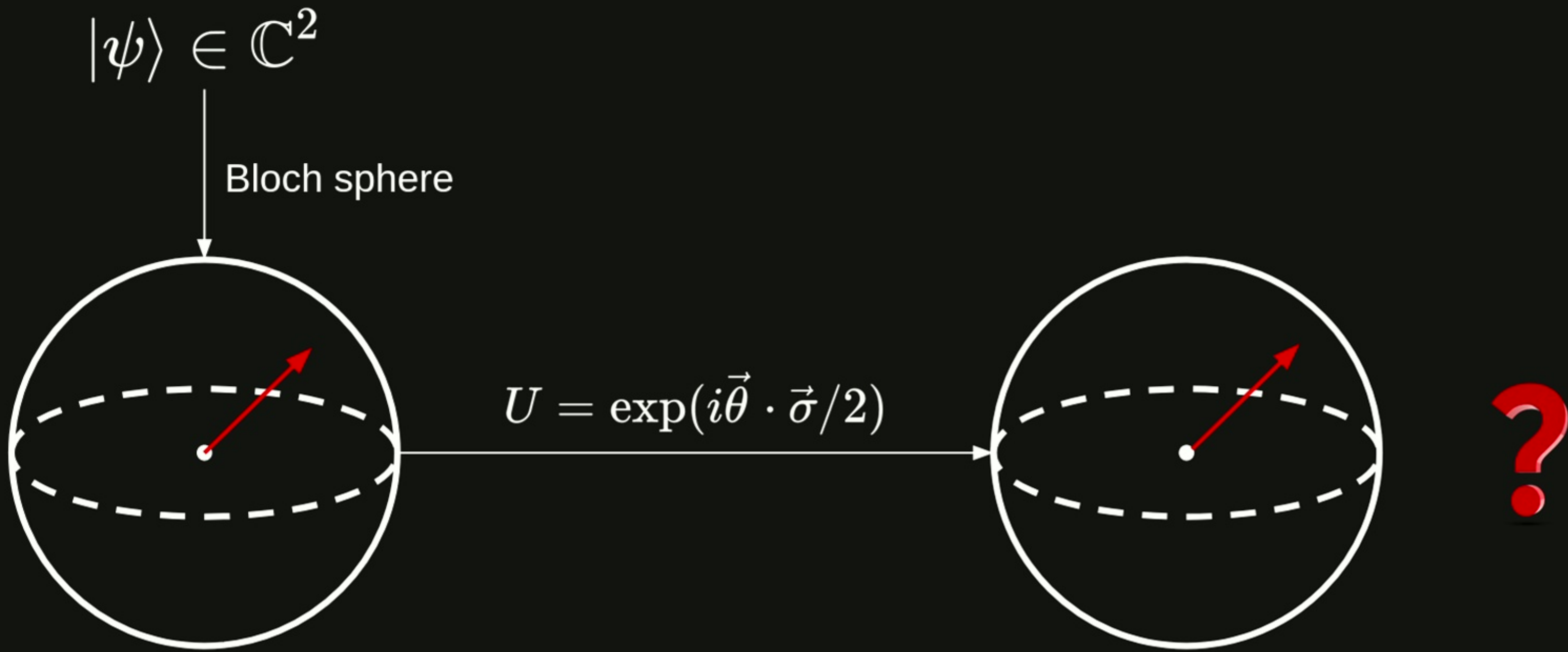
Quantum anomalous symmetry

$\approx$

Symmetric states are *nontrivial**

Example 1

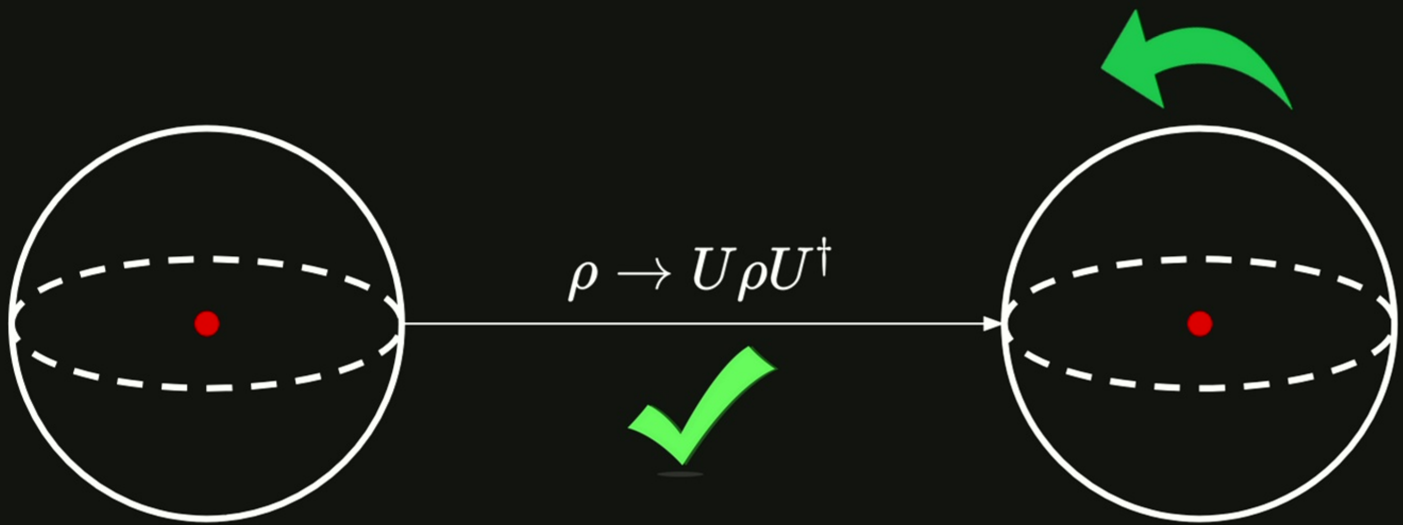
Is there a SO(3) symmetric **pure** qubit state?



Example 1

Is there a SO(3) symmetric **mixed** qubit state?

Answer: Only the maximally mixed state $I/2$.



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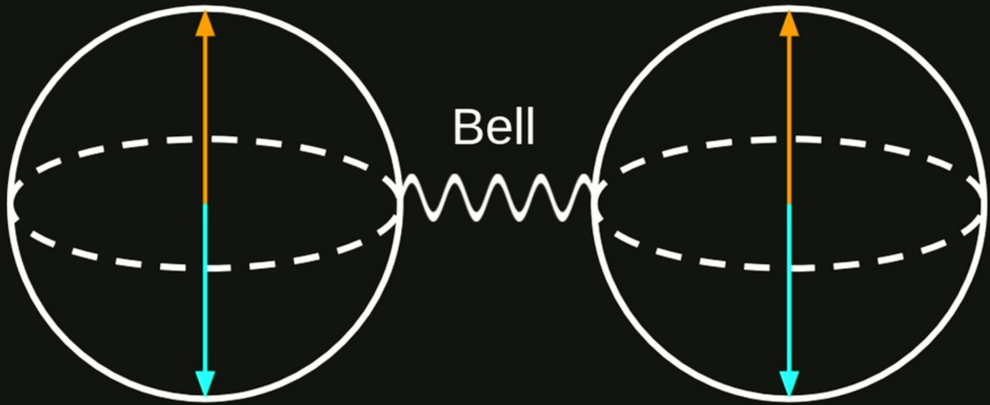
Is there a SO(3) symmetric **two-qubit** state?

Bell state is invariant under *joint* SO(3) rotations.



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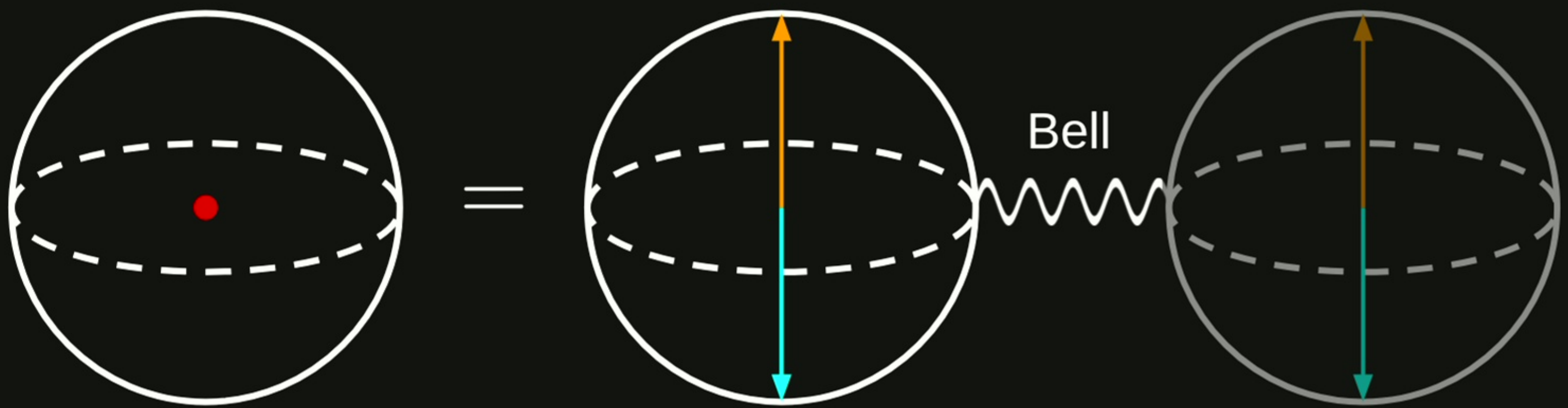


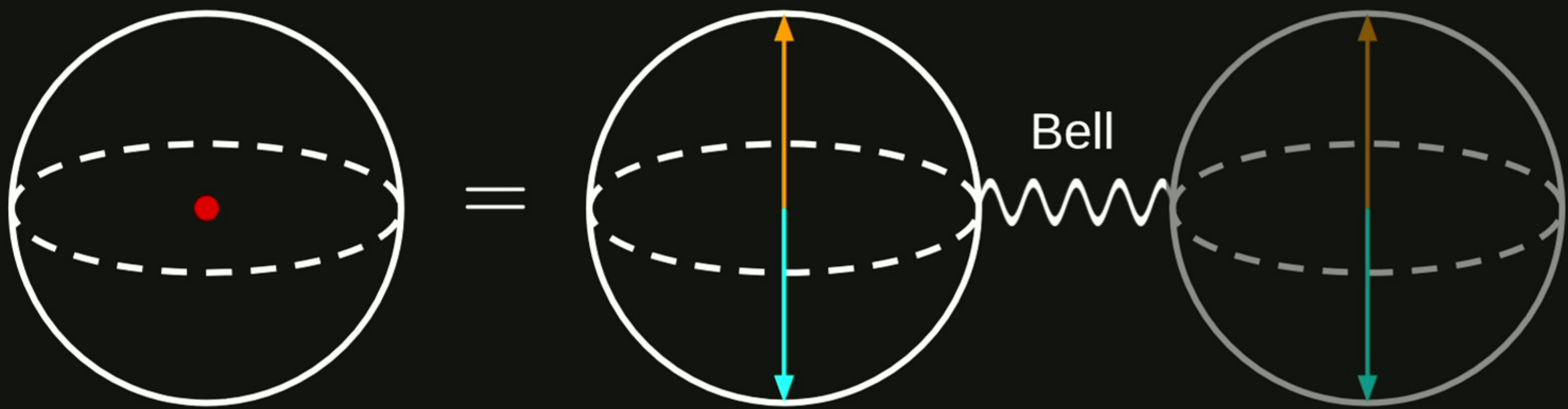
Example 1

Is there a SO(3) symmetric **two-qubit** state?

Bell state is invariant under *joint* SO(3) rotations.

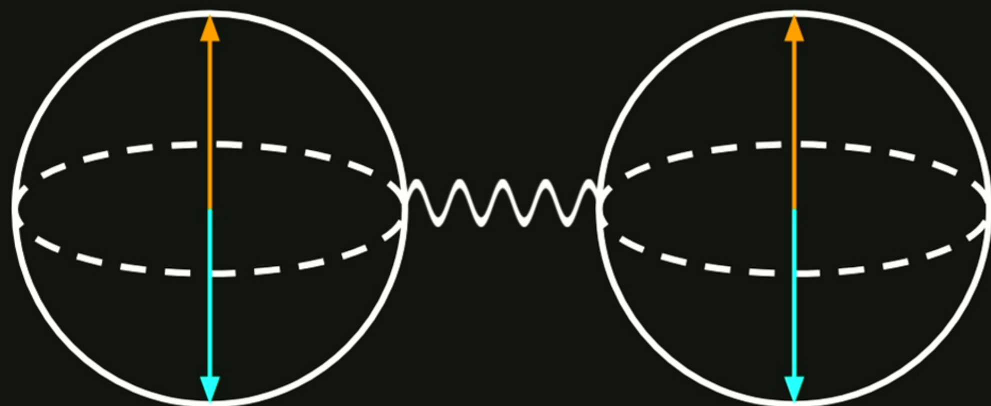
Reason: spin-½ is projective rep., but ½+½ = 0,1 is not!

$$U(g)U(h) = \omega(g,h)U(g \cdot h)$$



Example 1: Projective representations

Anomaly ⇒ nontrivial symmetric states



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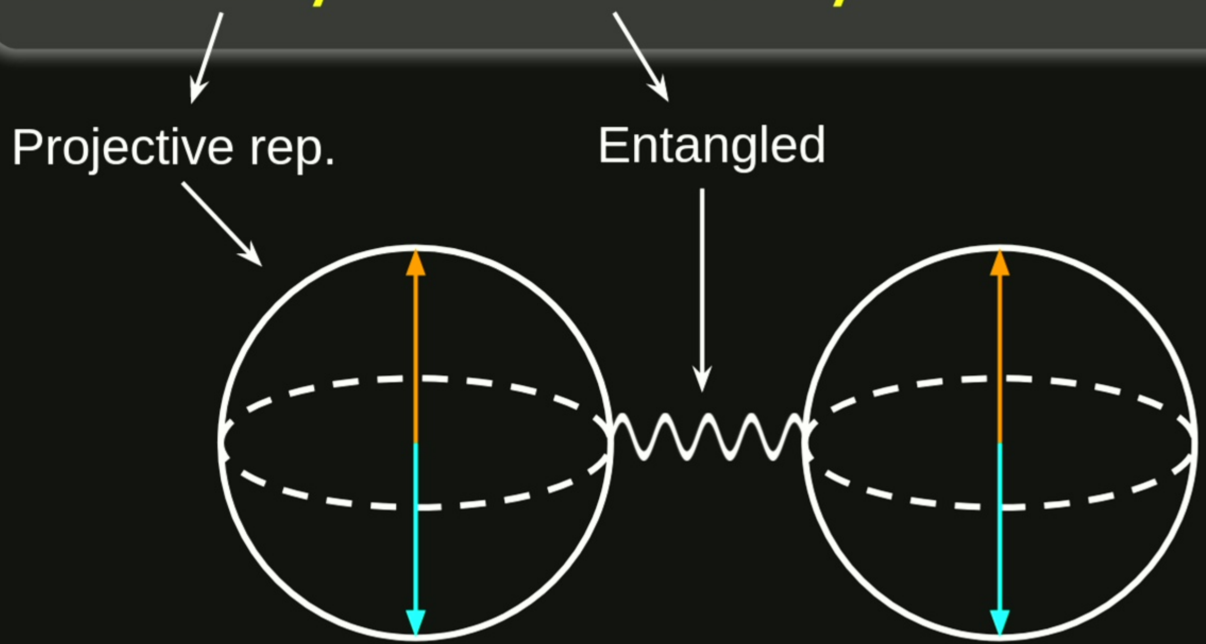
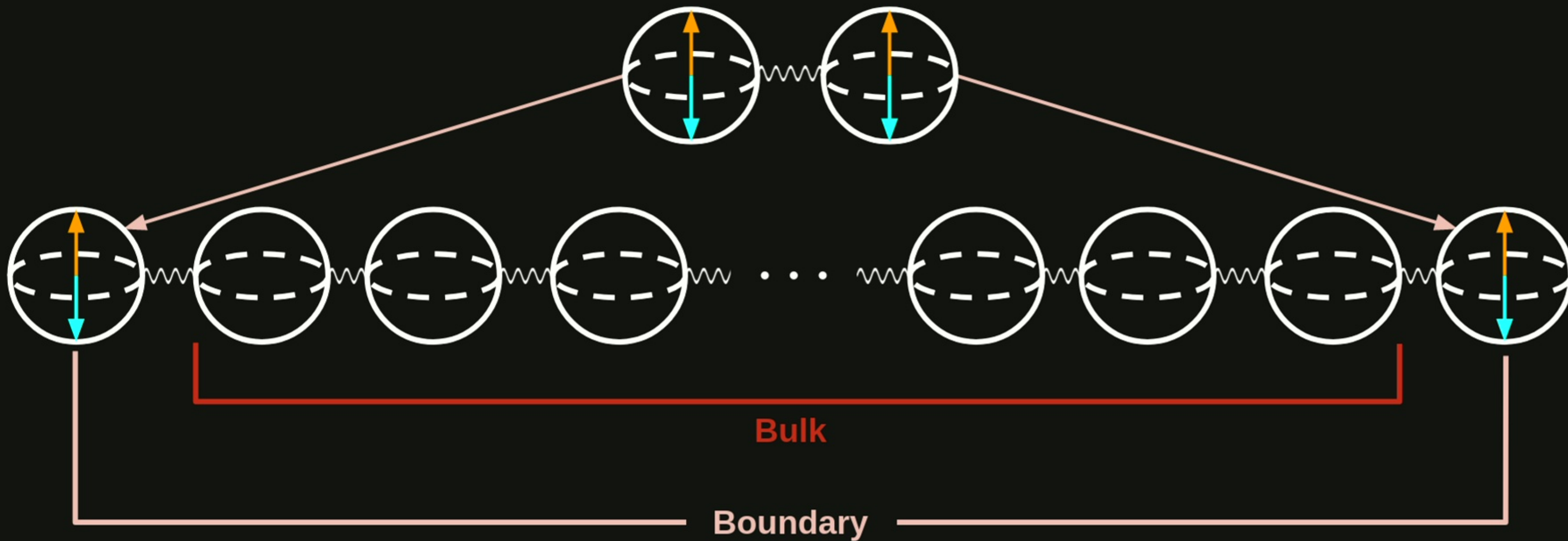


Example 2: Boundary of SPT phases

Anomalous boundary of a 1d *symmetry-protected topological* (SPT) system



Beyond pure states?

Why? Interaction with environment is inevitable.

$$|\psi\rangle \to \rho$$

Does quantum anomaly makes sense for ρ ?

Does quantum anomaly imply ρ nontrivial?

Can we use quantum anomaly to find new *mixed-state phases of matter*?

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Yes!

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Can we use quantum anomaly to find new *mixed-state phases of matter*?

Yes!

Symmetry for mixed states?

Does quantum anomaly makes sense for ρ ?

Weak / average symmetry

$$g\rho g^{-1} = \rho$$

$$\Updownarrow$$

$$\rho = \sum_i p_i |\psi_i^{(\alpha_i)}\rangle\langle\psi_i^{(\alpha_i)}|$$

Mixture of symmetric states

$\supset$

Strong / exact symmetry

$$g\rho = e^{i\alpha}\rho$$

$$\Updownarrow$$

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Mixture of symmetric states
with same charge α.

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Mixture of symmetric states **with same charge α.**

Too weak!

Maximally-mixed state is symmetric for *any* symmetry

$$g \frac{I}{d} g^{-1} = \frac{I}{d}$$

Property X of symmetric **pure** states —Convexity→ Property X of *strongly symmetric* **mixed** states

Entanglement for mixed states?

Does quantum anomaly imply ρ nontrivial?

- Pure state $|\psi\rangle$ is (bipartite) **entangled**, or **non-separable** if

Convexity

$$|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$$

Easy

- Mixed state ρ is (bipartite) **entangled**, or **non-separable** if

More parties

$$\rho \neq \sum_i p_i \rho_i^A \otimes \rho_i^B$$

(NP-)Hard

- Mixed state ρ is **k-partite entangled**, or **non-separable** if

$$\rho \neq \sum_i p_i \rho_i^{A_1} \otimes \rho_i^{A_2} \otimes \cdots \otimes \rho_i^{A_k}$$

Even harder

E.g., There are mixed states of three qubits that are separable under any bipartition, but still tripartite entangled! (Bennett *et al.*)

Main result in 1d

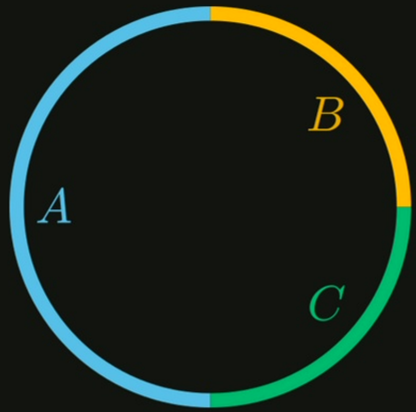
Anomalous symmetry G

&

$$G|\psi\rangle \propto |\psi\rangle$$

NEW

$$\Longrightarrow$$



$$|\psi\rangle \neq |A\rangle|B\rangle|C\rangle$$

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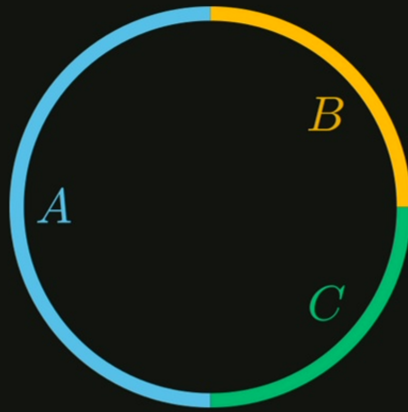
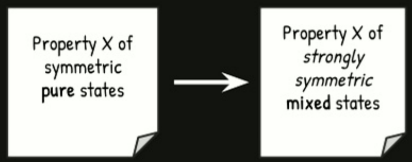


$$\rho \neq \sum_i p_i \rho_i^A \otimes \rho_i^B \otimes \rho_i^C$$



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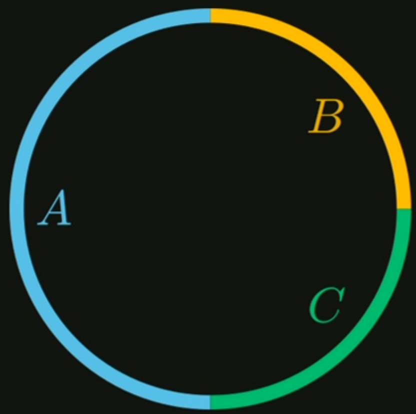

$$\rho \neq \mathcal{E}_{\text{Finite-depth}}\left[\sum_i p_i \rho_i^A \otimes \rho_i^B \otimes \rho_i^C\right]$$

Property X of symmetric **pure** states → Property X of *strongly symmetric* **mixed** states

Main result in 1d

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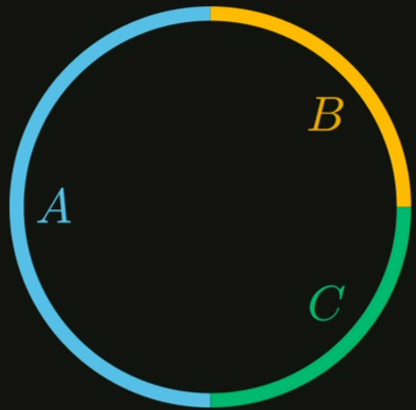

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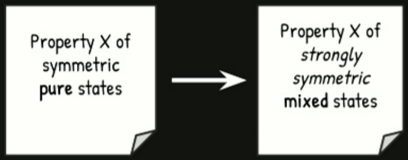
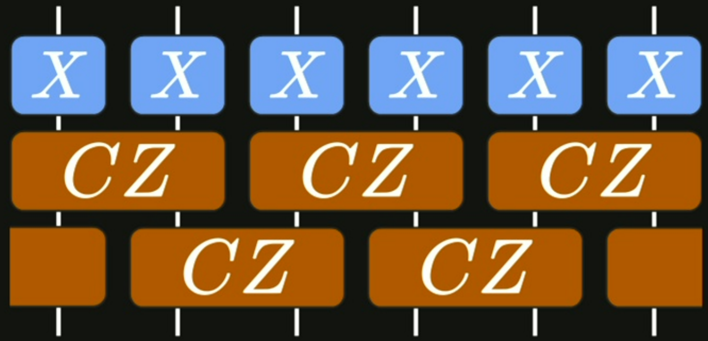


Anomaly ⇒ Long-range tripartite entanglement

Example: CZX model

Qubit chain with anomalous $\mathbb{Z}_2$ symmetry generated by (Chen, Liu, Wen)

$$U = \prod_i X_i \prod_i CZ_{i,i+1} =$$



Boundary symmetry of 2d bosonic SPT (Levin, Gu)

In 1d, *anomalous symmetry* ≈ *non-on-site symmetry*, like the one above

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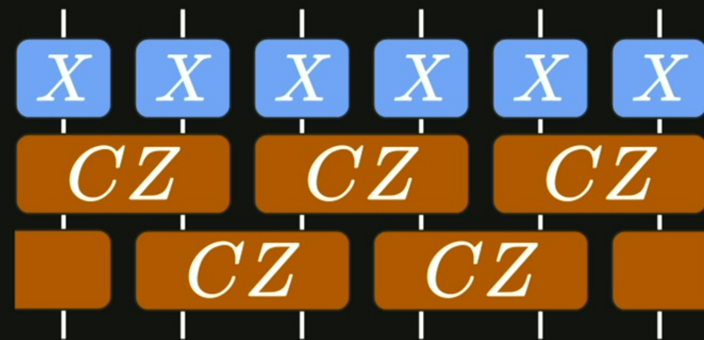

Boundary symmetry of 2d bosonic SPT (Levin, Gu)

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Claim: $U|\psi\rangle \propto |\psi\rangle \Rightarrow |\psi\rangle \neq |A\rangle|B\rangle|C\rangle$.

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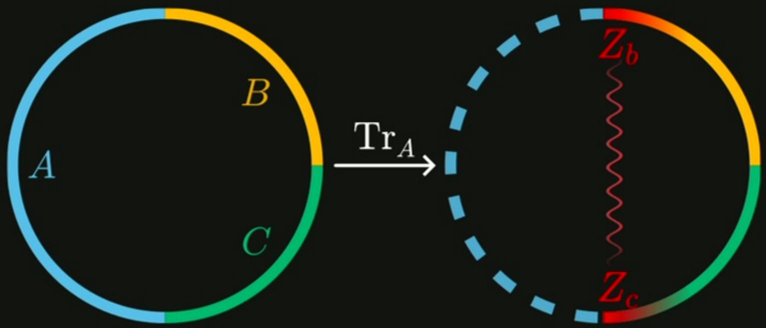
Proof by contradiction:

$U = \prod_i X_i \prod_i CZ_{i,i+1}$

$$U|A\rangle|B\rangle|C\rangle \propto |A\rangle|B\rangle|C\rangle$$

$$\downarrow \mathrm{Tr}_A$$

$$U_{BC}|B\rangle|C\rangle \propto |B\rangle|C\rangle$$



where U_{BC} acts like U in the bulk of BC

satisfying $U_{BC}^2 \propto Z_b Z_c$

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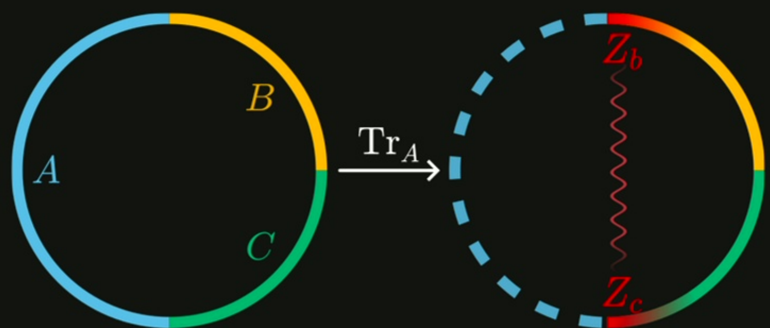

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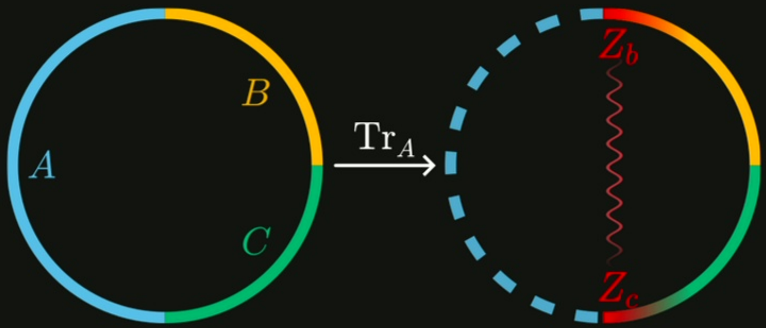

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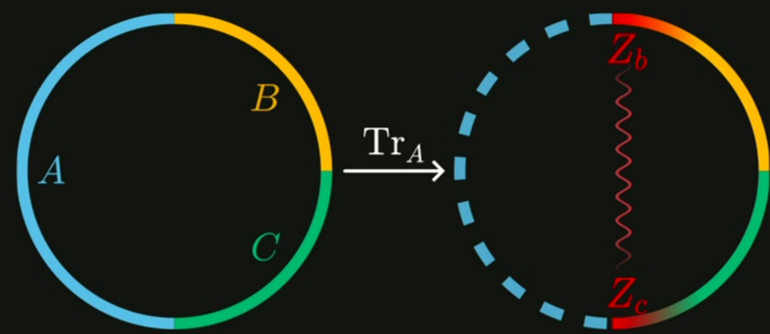

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→ **Anomaly** ←

∎

Exotic topological mixed state in 1d

Can we use quantum anomaly to find new *mixed-state phases of matter*?

Even highly mixed states are constrained by anomaly:

$$\rho_{\infty,\pm} = \frac{1}{2^L}(I \pm U_{\mathrm{CZX}})$$

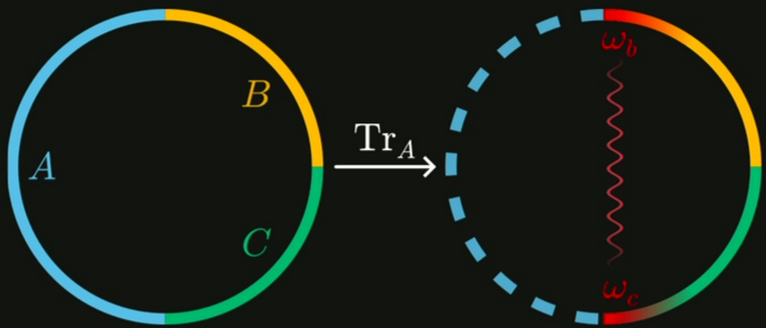
"Infinite-temperature" mixture of all states with same charge $U_{\mathrm{CZX}} = \pm 1$ ⇒ strongly symmetric

- Anomaly ⇒ long-range tripartite entangled
  - At the same time, it is bipartite separable!
- Only global correlations: any subsystem is trivial, $\mathrm{Tr}_A[\rho_{\infty,\pm}] \propto I$

Conclusion

- Quantum anomalous symmetries have *nontrivial* symmetric states
  - Pure states: projective reps., boundaries of SPTs, etc.
  - Strongly symmetric mixed states too:

Anomaly ⇒ Long-range tripartite entanglement



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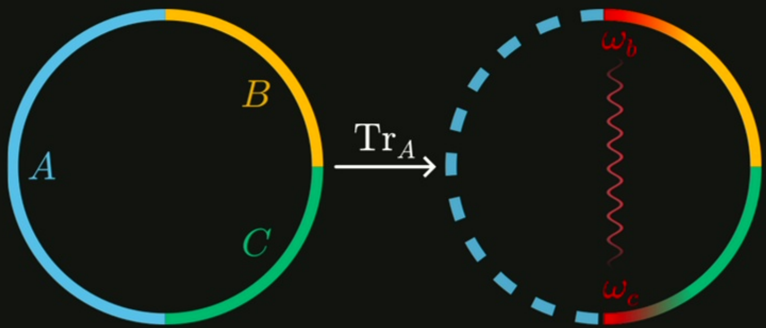

- Generalizations done in the paper:
    - Higher dimension (d-dimensional anomaly implies (d+2)-partite entangled symmetric states)
    - Mixed anomaly and Lieb-Schultz-Mattis

- Open questions:
    - Practical measurement of tripartite entanglement?
    - Higher-form symmetries? Fermions? Field theory?
    - …

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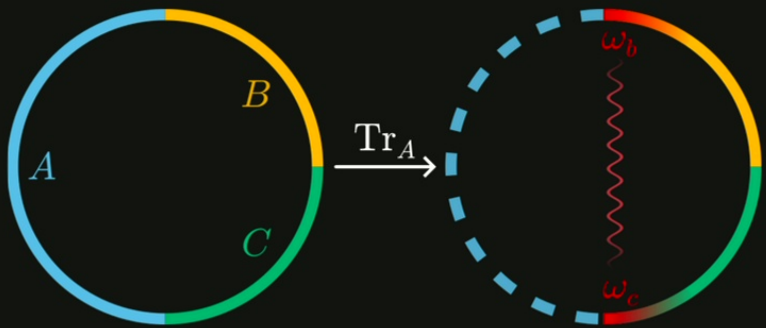

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