Title: Sifting quantum black holes through the principle of least action

Speakers: Alessia Platania

Series: Quantum Gravity

Date: February 22, 2024 - 2:30 PM

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Abstract:

We tackle the question of whether regular black holes or other alternatives to the Schwarzschild solution can arise from an action principle in quantum gravity. Focusing on an asymptotic expansion of such solutions and inspecting the corresponding field equations, we demonstrate that their realization within a principle of stationary action would require either fine-tuning, or strong infrared non-localities in the gravitational effective action. We will also show that the black hole entropy of theories displaying such infrared non-localities diverge. The principle of least action and the consistency of Wald entropy thus yield non-trivial asymptotic constraints on the metric of quantum black holes.

Zoom link

Sifting quantum black holes through the principle of least action

Alessia Platania

Based on:

B. Knorr, A. Platania - arXiv:2202.01216

A. Platania, J. Redondo-Yuste - arXiv:2303.17621

Quantum Gravity Seminar

Perimeter Institute for Theoretical Physics

22.02.2024

PERIMETER INSTITUTE

Driving philosophy

cross-fertilization of ideas and approaches may be key to make progress in quantum gravity

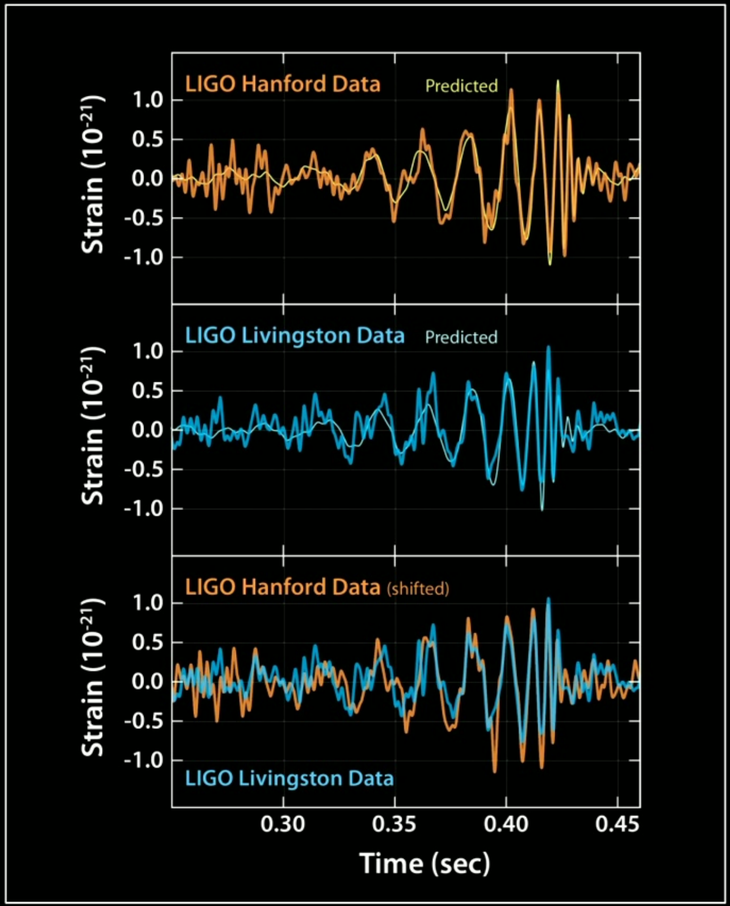
Different directions:

- **Across different theories (e.g., string theory and asymptotic safety)**
  [Basile, AP '21+'21+'21 + work in progress with I. Basile, ...]

- **Across different fields (e.g., quantum gravity and effective field theory)**
  [work in progress with L. Heisenberg, B. Knorr, ...]

- **Combining top-down and bottom-up approaches**
  [this talk: application to black holes]

How do quantum black holes look like?



LIGO & Virgo data
Gravitational waves from binary black hole mergers

Event Horizon Telescope (EHT) Images
supermassive black hole at the core of M87*

Consistent to date with black holes in General Relativity…

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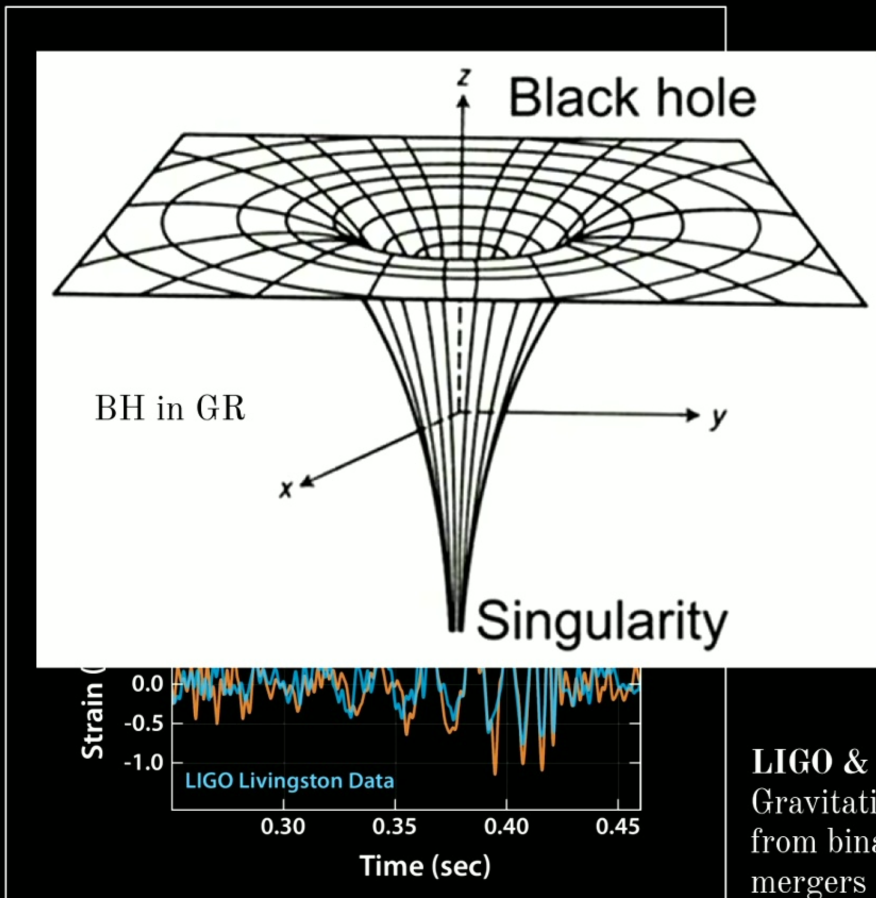

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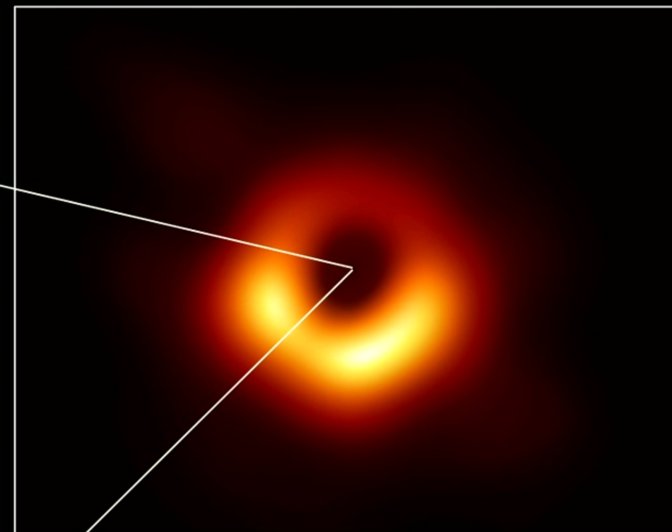

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Consistent to date with black holes in General Relativity…

…but black holes in General Relativity are inconsistent…

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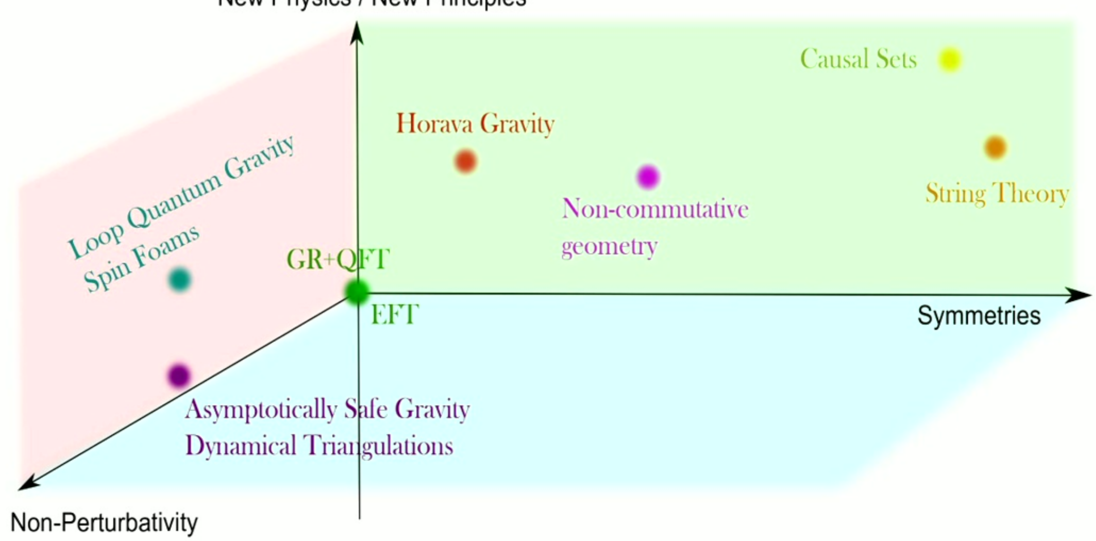
- **Singularity resolution** requires some sort of new physics
  - **Quantum gravity**
  - Exotic matter

- **Alternatives** to classical black holes:
  - **Regular black holes, wormholes, compact objects** (fuzzballs, gravastars, boson stars, etc)
  - Depend on the **specific theory or model**



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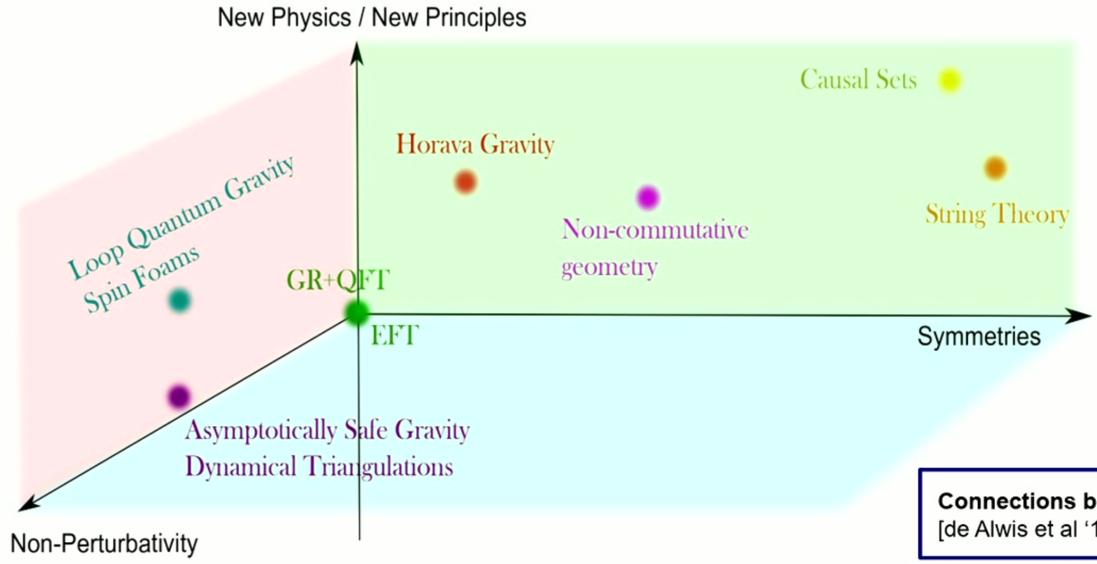

Connections between approaches?
[de Alwis et al ‘19; Basile, AP ‘21+’21+’21]

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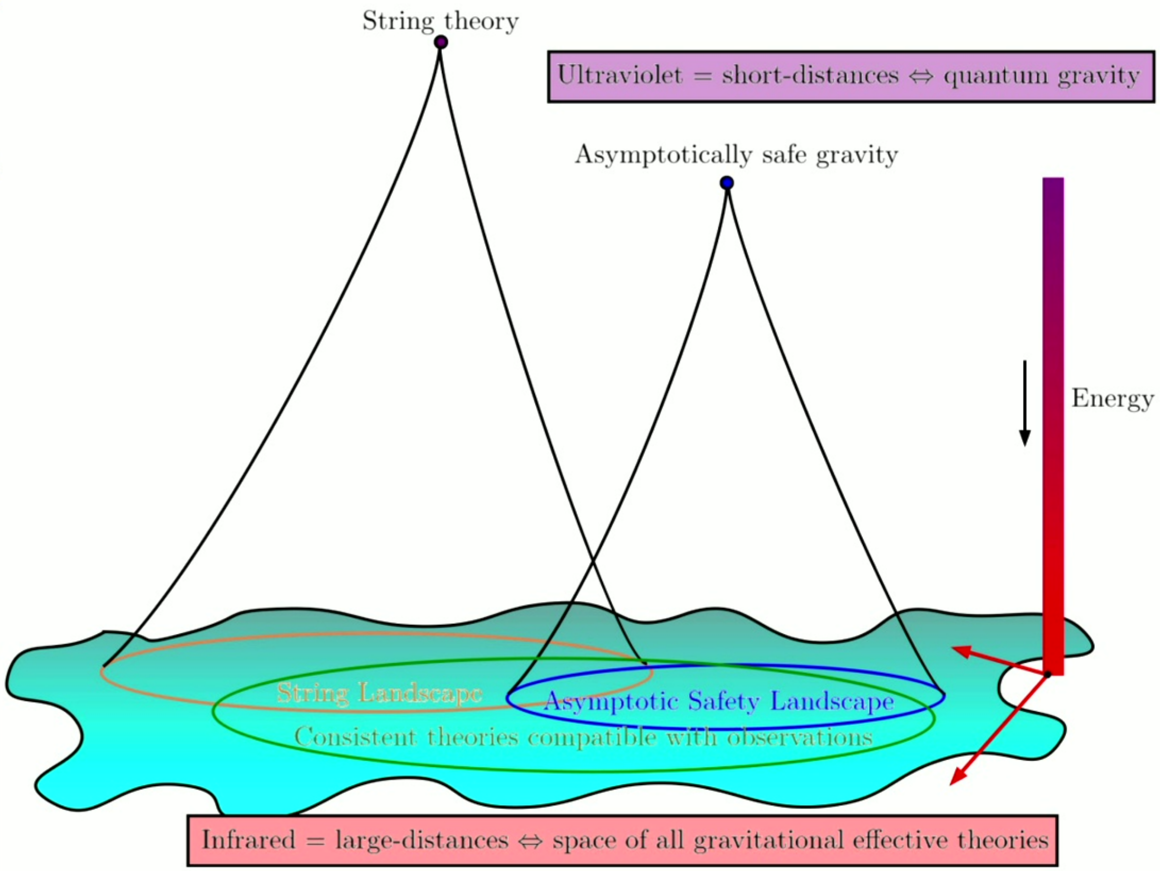
A step back: general strategies in quantum gravity

Top-down approach
(the "ideal path", but a long one!):
Predictions from scratch

Bottom-up approach:
(advantage: model-independent)
Model building, parameterizations
Theoretical constraints
Observational constraints



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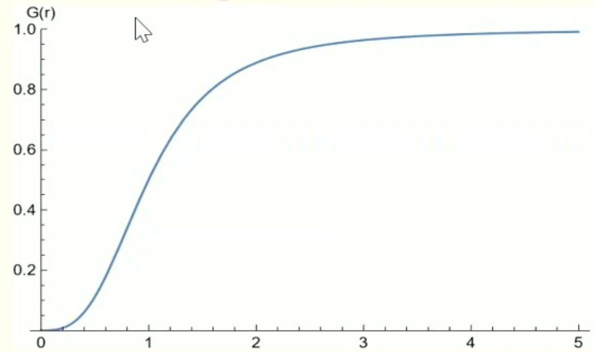
Quantum Black Holes: model building



Model building based on consistency or quantum gravity hallmarks:

$$g_{\mu\nu} = \mathrm{diag}\left(-f_{tt}(r), f_{rr}^{-1}(r), r^2, r^2 \sin\theta\right)$$

$$f_{tt}(r) \sim f_{rr}(r) \simeq 1 - \frac{2mG(r)}{r}$$

1) Violate hypothesis of Penrose's singularity theorems [Bardeen; Dymnikova; Hayward; ...]:

$$f_{tt}(r) = f_{rr}(r) = 1 - \frac{2M\,G_N}{r}\frac{r^3}{r^3 + 2MG_N^2} \sim 1 - \frac{2MG_N}{r} + \frac{4G_N^3 M^2}{r^4} \qquad \text{Hayward BH}$$

$$f_{tt}(r) = f_{rr}(r) = 1 - \frac{2M\,G_N}{\sqrt{r^2+a^2}} \sim 1 - \frac{2MG_N}{r} + \frac{4G_N M a^2}{r^3} \qquad \text{Simpson-Visser Spacetime}$$

2) Loop Quantum Gravity inspired: Bounce inside black holes and Planck stars [Rovelli, Vidotto '14; ...]

3) Asymptotic safety inspired: gravitational anti-screening and renormalization group improvement [Bonanno, Reuter '00; ...]

4) The fuzzball paradigm: the region around the horizon is not in the vacuum state [Lunin, Mathur '01; ...]

More substantial relation to fundamental Quantum Gravity Theories?

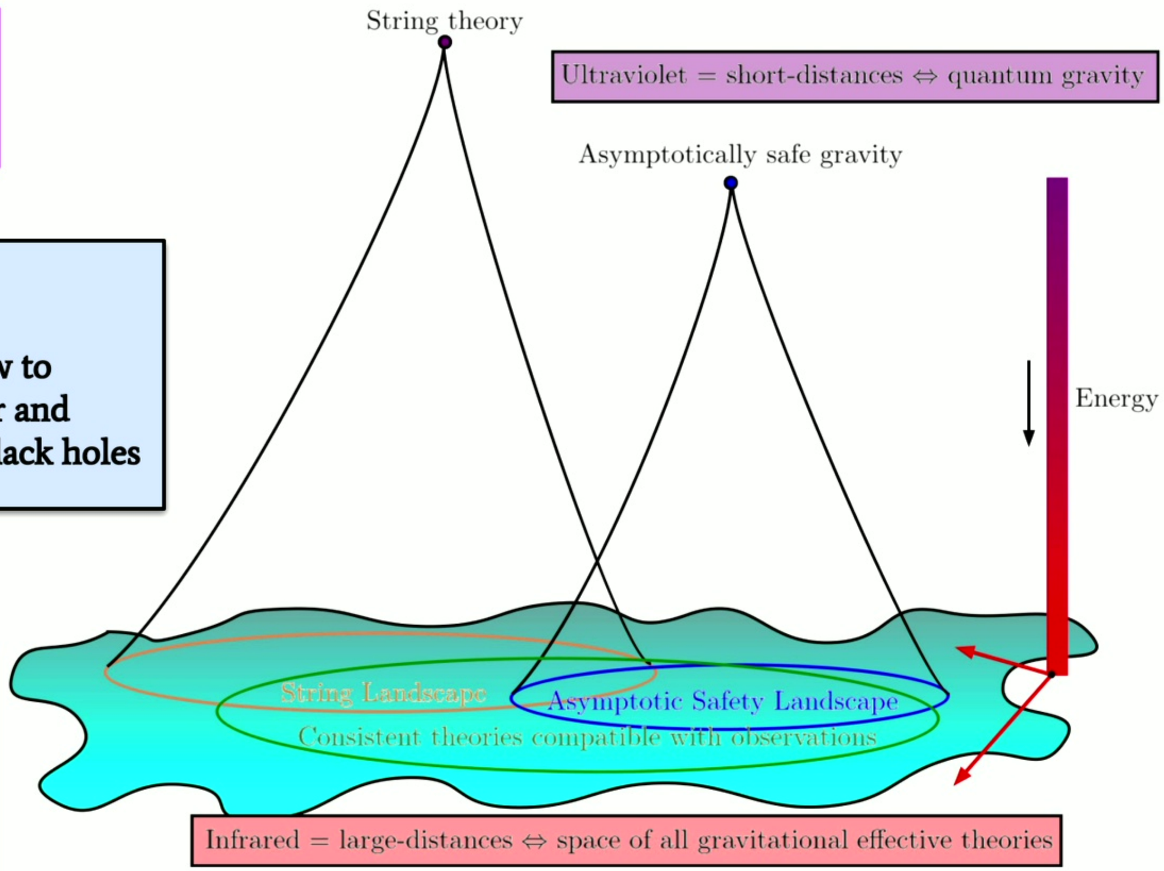
A step back: general strategies in quantum gravity

Top-down approach
(the "ideal path", but a long one!):
Predictions from scratch

This talk:

Two examples on how to exploit them together and constrain quantum black holes

Bottom-up approach:
(advantage: model-independent)
Model building, parameterizations
Theoretical constraints
Observational constraints



Quantum Black Holes: fundamental aspects

- **Goal: predictions, contrast different theories, put constraints.**
  Interim goal (regardless of the method):
  Re-sum all quantum-gravity fluctuations (non-perturbative!)

- **Effective action from path integral over metrics**
  [= sum over all 1PI vacuum diagrams]

$$\Gamma_{\text{eff}} \quad \Leftrightarrow \quad \int \mathcal{D}g_{\mu\nu} e^{iS[g_{\mu\nu}]}$$

Note:

- Can use different fundamental dof & symmetries;
- Expected to have/recover description in terms of effective actions (AS gravity, CDT, non-local gravity, string theory, etc).
- In discrete theories more difficult but same concepts applicable

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Different theories (or approx.) are characterized by different form factors:

$$\Gamma_{\text{eff}} = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} + \mathcal{L}_{\text{HD}} \right)$$

$$\mathcal{L}_{\text{HD}} = \frac{1}{16\pi G} \left(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\mathcal{F}_3(\Box)R^{\mu\nu\rho\sigma} + \mathcal{O}(R^3) \right)$$

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$$\frac{\delta \Gamma_{\text{eff}}}{\delta g_{\mu\nu}} = 0 \quad \Rightarrow \quad \text{quantum dynamics}$$

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If QG is to be the key to singularity resolution

⇒ regular black holes or compact objects ought to arise as effective solutions to the fully-quantum dynamics dictated by a quantum effective action

(at least in approaches to quantum gravity are supposed to reconcile with a QFT/EFT of gravity in the infrared)

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Top-down+bottom-up 1: models under the test of action principle

Reversing the logic:

- Given a modified solution to GR, can I find an effective action which produces it?
- Idea: **use the principle of least action as a selection principle**:
  which proposed alternatives to classical black holes can *potentially* come from Quantum Gravity?

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⇒ regular black holes or compact objects ought to arise as effective solutions to the fully-quantum dynamics dictated by a quantum effective action

Focus on the asymptotic limit:

- Results/constraints apply to many proposed alternatives to classical black holes
- Universal QFT language, for any theory of quantum gravity (also discrete ones).

State-of-the-art: effective action not known yet,

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Modified black holes from gravitational effective actions?

[Knorr, AP '22]

Consider **modified (static, spherically symmetric) black-hole spacetime**:

$$g_{\mu\nu} = \mathrm{diag}\left(-f_{tt}(r), f_{rr}^{-1}(r), r^2, r^2 \sin\theta\right)$$

with **lapse functions admitting an asymptotic expansion**:

$$f_{tt}(r) \sim 1 - \frac{2G_N M}{r} + \frac{c_t}{r^{n_t}}, \qquad f_{rr}(r) \sim 1 - \frac{2G_N M}{r} + \frac{c_r}{r^{n_r}}$$

Task: **understand whether it can be generated by a quantum effective action**

$$\Gamma_{\text{eff}} = \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g} \left[-R - \frac{1}{6} R f_R(\Box) R + R^{\mu}{}_{\nu} f_{Ric}(\Box) R^{\nu}{}_{\mu} + \mathcal{O}(\mathcal{R}^3) \right]$$

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Are quantum effective corrections to the classical action compatible with modifications to Schwarzschild?

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Field equations from the action principle:

$$\frac{\delta \Gamma_{\text{eff}}}{\delta g_{\mu\nu}} = \frac{\delta \Gamma_{\text{GR}}}{\delta g_{\mu\nu}} + \frac{\delta \Gamma_{R^2}}{\delta g_{\mu\nu}} + \frac{\delta \Gamma_{R^3}}{\delta g_{\mu\nu}} + \cdots = 0$$

(procedure: compute each contribution to field equations, replace the modified metric above, and check conditions for it to be a solution)

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$$\frac{\delta\Gamma_{GR}}{\delta g_{\mu\nu}} \sim \frac{c_r}{16\pi G_N}(n-1)\,r^{-2-n}$$

$$\frac{\delta\Gamma_{Stelle}}{\delta g_{\mu\nu}} \sim a\,r^{-n-4}$$

It cannot cancel EH terms

$$\frac{\delta\Gamma_{\mathcal{R}^3}}{\delta g_{\mu\nu}} \sim b\,r^{-8}$$

It can possibly work if:

- n=6, specific coefficients;
- Higher-order contributions cancel Stelle terms;
- Also sub-leading cancel out

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Local terms only $\Rightarrow$ fine tuning of operators, works for specific powers n only

Constraint on *n*:
n≥6 & n≠7

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Example Hayward black holes — DO NOT WORK

$$f_{tt}(r) = f_{rr}(r) = 1 - \frac{2M\,G_N}{r}\frac{r^3}{r^3+2MG_N^2} \sim 1 - \frac{2MG_N}{r} + \frac{4G_N^3M^2}{r^4}$$

Example Simpson-Visser wormhole

$$f_{tt}(r) = f_{rr}(r) = 1 - \frac{2M\,G_N}{\sqrt{r^2+a^2}} \sim 1 - \frac{2MG_N}{r} + \frac{4G_N M a^2}{r^3}$$

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[Knorr, AP '22]

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$$\frac{\delta \Gamma_{GR}}{\delta g_{\mu\nu}} \sim \frac{c_r}{16\pi G_N} (n-1)\, r^{-2-n}$$

Non-local ansatz that can cancel GR terms:

$$f_R(\Box) = \frac{\alpha}{\Box}, \qquad f_{Ric}(\Box) = \frac{\beta}{\Box}$$

$$\frac{\delta \Gamma_{f_R}}{\delta g_{\mu\nu}} \sim c\, r^{-2-n}$$

$$\frac{\delta \Gamma_{f_{Ric}}}{\delta g_{\mu\nu}} \sim d\, r^{-2-n}$$

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$$\frac{\delta\Gamma_{f_{Ric}}}{\delta g_{\mu\nu}} \sim d\, r^{-2-n}$$

Important implication:

Aside from <u>specific values of n</u>, **modified black holes** with metrics admitting the asymptotic expansion above (case asymptotically flat spacetime) require **large distance non-localities**.

<u>Examples</u>: Hayward and Bardeen;
<u>Exception</u>: Dymnikova;

Modified black holes from gravitational effective actions?

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$$\frac{\delta\Gamma_{GR}}{\delta g_{\mu\nu}} \sim \frac{c_r}{16\pi G_N}(n-1)\,r^{-2-n}$$

$$\frac{\delta\Gamma_{f_R}}{\delta g_{\mu\nu}} \sim c\,r^{-2-n}$$

$$\frac{\delta\Gamma_{f_{Ric}}}{\delta g_{\mu\nu}} \sim d\,r^{-2-n}$$

Non-local ansatz that can cancel GR terms:

$$f_R(\Box) = \frac{\alpha}{\Box}, \qquad f_{Ric}(\Box) = \frac{\beta}{\Box}$$

Are such infrared non-localities consistent?

Important implication:

Aside from <u>specific values of n</u>, **modified black holes** with metrics admitting the asymptotic expansion above (case asymptotically flat spacetime) require **large distance non-localities**.

<u>Examples</u>: Hayward and Bardeen;
<u>Exception</u>: Dymnikova;

Top-down+bottom-up 2: non-localities in the QG effective actions

Remember: higher-derivative part of effective action

$$\mathcal{L}_{\rm HD} \propto R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\mathcal{F}_3(\Box)R^{\mu\nu\rho\sigma} + \mathcal{O}(R^3)$$

Different QG theories will yield different form factors. Examples:

Non-local gravity [Buoninfante, Koshelev, Mazumdar, Modesto, Tokareva, …]: $\mathcal{F}_i \propto \frac{e^{H(-\Box)}-1}{\Box}$

Spin foams [Borissova, Dittrich]: $\mathcal{F}_i \propto \frac{1}{a+b\Box}$

Asymptotic-safety models [Draper, Knorr, Ripken, Saueressig]: $\mathcal{F}_i \propto \tanh(a\Box)$

Causal dynamical triangulations [Knorr, Saueressig]: $\mathcal{F}_i \propto \Box^{-2}$

Polyakov-type operators in some string models [Polyakov]: $\mathcal{F}_i \propto \Box^{-1}$

$$\mathcal{F}_i(\Box) = \sum_{n=-\infty}^{\infty} c_{i,n}\Box^{-n}$$

Pheno models: focus on IR non-localities

- **IR deviation from GR and dynamical dark energy, seemingly compatible with observations**
  [Amendola, Akrami, et al; Capozziello e al; Deser, Woodard; Maggiore et al; Wetterich; …]
- **Can make popular regular black-hole models compatible with a principle of least action**
  [Knorr, AP '22]

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Are infrared non-localities consistent?

A non-trivial test: BH entropy

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Black hole entropy from IR non-localities [AP, Redondo-Yuste, '23]

- **Black hole entropy depends on higher-derivative/curvature corrections via Wald formula**

$$S_W = -2\pi \int_\Sigma \left(\frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \right) \epsilon_{\mu\nu} \epsilon_{\rho\sigma} dV_2^2$$

$$\mathcal{L}_{\rm HD} \propto R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\mathcal{F}_3(\Box)R^{\mu\nu\rho\sigma} + \mathcal{O}(R^3)$$

- **One-loop corrections** [Xavier, Kuipers] **⇒ Log corrections to area law**
- **Analytic part of form factor within the annulus of convergence:**

$$\mathcal{F}_i(\Box) = \sum_{n=-\infty}^{\infty} c_{i,n} \Box^{-n}$$

- **Positive-degree operators ⇒ sub-leading corrections to Bekenstein-Hawking area law** [Mazumdar et al; Myung]**:**

$$S_W = \frac{\mathcal{A}}{4G}(1 + \sum_{n>0} a_n \, r_H^{-2n})$$

- **Negative-degree terms, aka, IR non-localities?**
  - **Expectation**: dominant contribution over Bekenstein-Hawking term; <u>how dominant?</u>
  - **Computational caveat**: localization procedure, recursion formulas, large black holes

Black hole entropy from IR non-localities [AP, Redondo-Yuste, '23]

Computational setup: large, static, spherically-symmetric asymptotically flat black holes.

Modified field equations will admit asymptotic solutions with [Knorr, Platania, '22]:

$$f_{tt}(r) \simeq 1 - \frac{r_s}{r} + \frac{A}{r^\alpha}, \quad f_{rr}(r) \simeq 1 - \frac{r_s}{r} + \frac{B}{r^\beta}$$

Focus on **IR non-localities**:

$$\mathcal{F}_i(\Box) = \sum_{n=1}^{N} c_{i,n} \Box^{-n}$$

$$\mathcal{L}_{\rm HD} \propto R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\mathcal{F}_3(\Box)R^{\mu\nu\rho\sigma} + \mathcal{O}(R^3)$$

⇒ Corresponding **dimensionless Wald entropy contributions**:

$$\tilde{S}_W^{(n)} \equiv \lim_{r\to r_H} \left(2c_{1,n} \frac{1}{\Box^n} R_1 + c_{2,n} \frac{1}{\Box^n} R_2 - 4c_{3,n} \frac{f_{rr}}{f_{tt}} \frac{1}{\Box^n} R_3 \right) \qquad R_i = R_i(f_{tt}, f_{rr})$$

To be computed **recursively**. For n=1 [Knorr, AP, '22]:

$$\Box^{-1}\phi(r) = \int_r^{R_x} \int_x^{R_y} dx\, dy \frac{-y^2 \phi(y)}{x^2 \sqrt{f_{rr} f_{tt}(x)}} \sqrt{\frac{f_{tt}(y)}{f_{rr}(y)}}$$

with R_x, R_y boundary conditions

[AP, Redondo-Yuste, '23]

Lapse function, simple case:

$$f_{tt} = f_{rr} \simeq 1 - \frac{r_s}{r} + \frac{A}{r^\alpha}$$

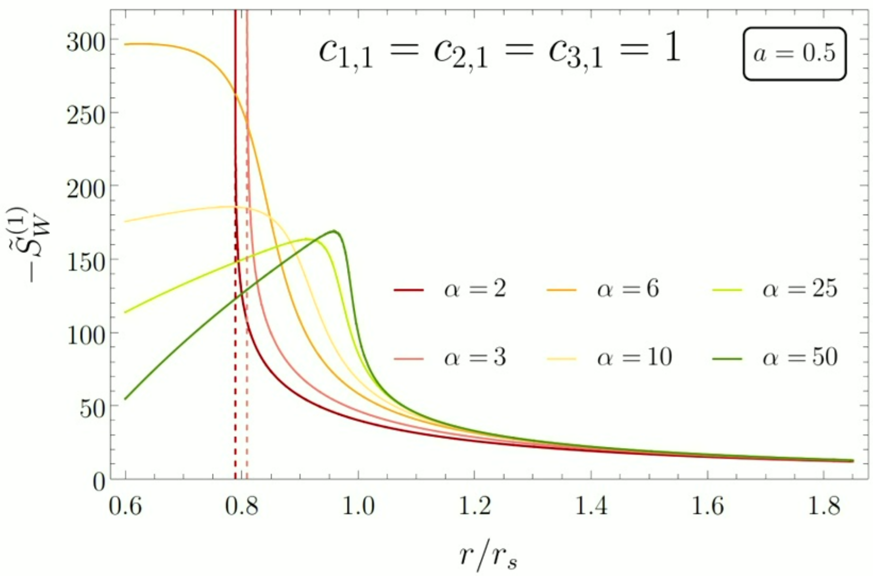
Remember:

$$\mathcal{F}_i(\Box) = \sum_{n=1}^{N} c_{i,n} \Box^{-n}$$

The event horizon, when it exists, is located at:

$$r_H \simeq \left(1 + \frac{a}{(a-1)\,\alpha - 1}\right) r_s \qquad a \equiv (\alpha+1) r_s^{-\alpha} A$$

⇒ study entropy dependence on a, α, form-factor coefficients



[AP, Redondo-Yuste, '23]

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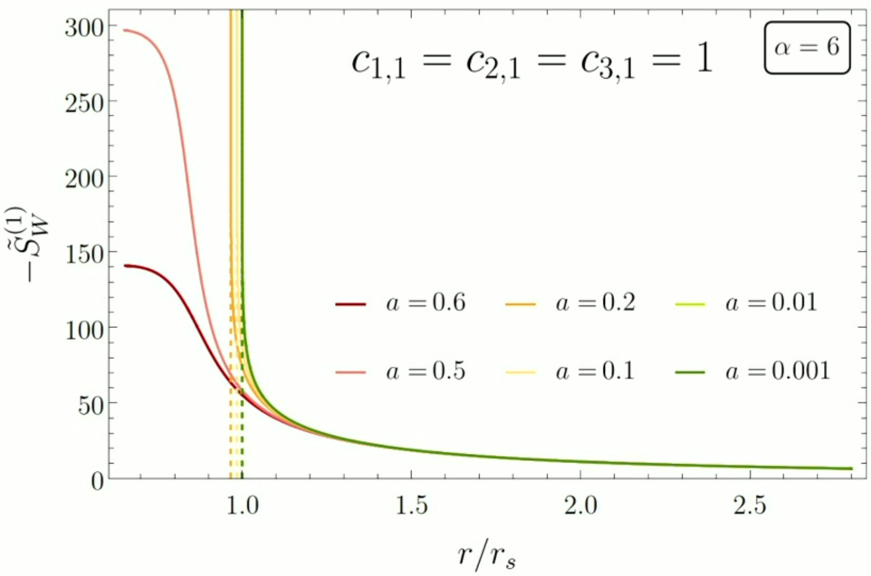
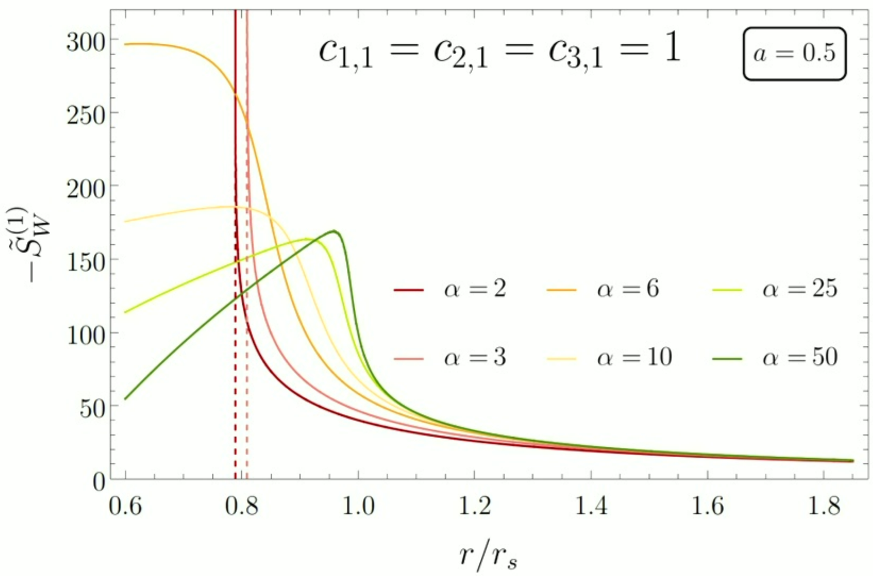


[AP, Redondo-Yuste, '23]



The first contribution to the entropy diverges anytime an event horizon exists.

Lapse function, simple case:

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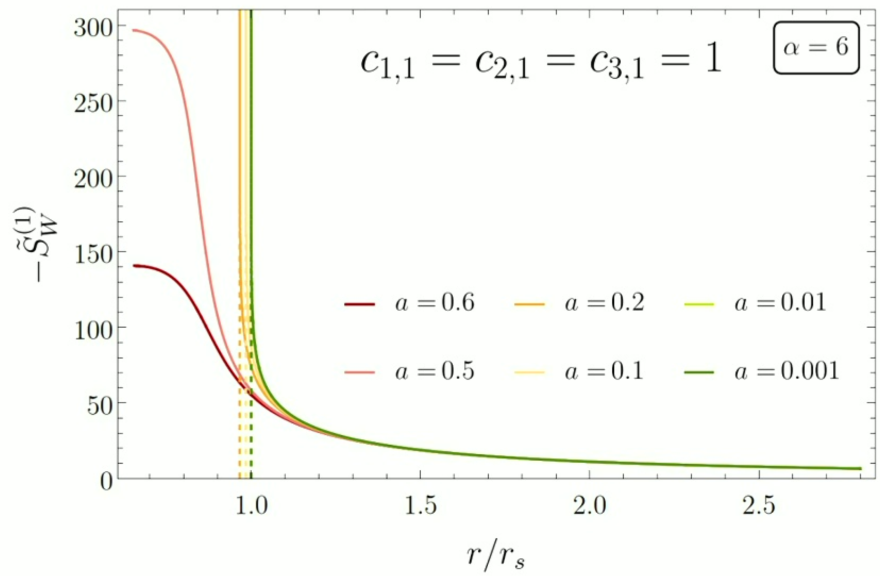


Note:

- Divergence of the first contribution ⇒ divergence of all of the others

The result is independent of:

- The parameters one puts in
- Whether the lapse functions coincide
- Initial conditions

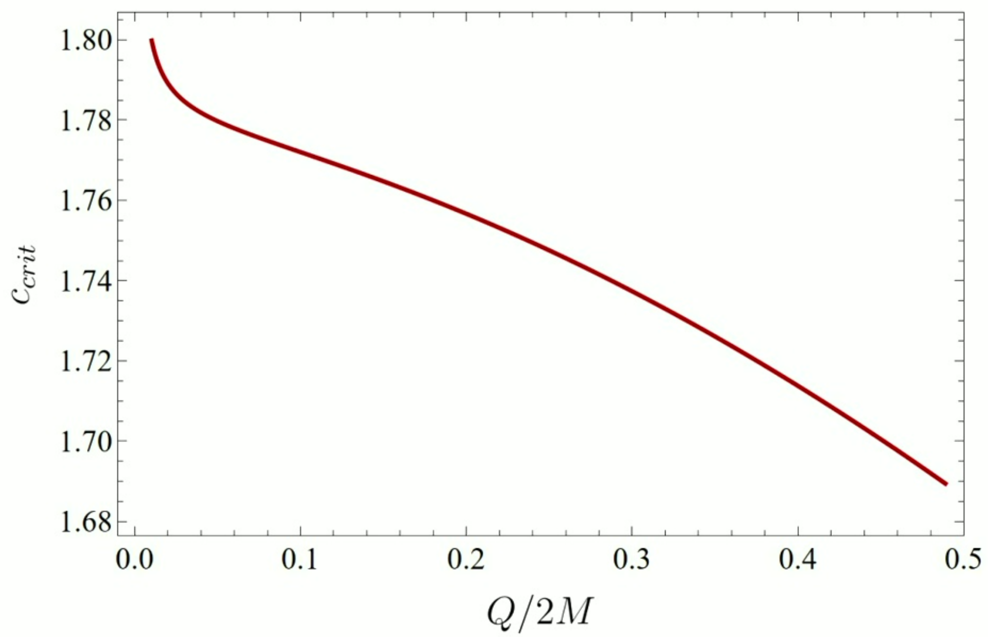
[AP, Redondo-Yuste, '23]

Possible cancellations of divergences is non-trivial.

Case of charged black holes (α=2, analytical formulas computed)

$$\tilde{S}_W^{(n)} \equiv \lim_{r\to r_H}\left(2c_{1,n}\frac{1}{\Box^n}R_1 + c_{2,n}\frac{1}{\Box^n}R_2 - 4c_{3,n}\frac{f_{rr}}{f_{tt}}\frac{1}{\Box^n}R_3\right)$$

$$\tilde{S}_W^{(1)} = \begin{cases}\text{sign}(c_{2,1})\times\infty\,, & c_{2,1} > c_{\text{crit}}(Q/2M)\,c_{3,1}\,,\\ -\text{sign}(c_{2,1})\times\infty\,, & c_{2,1} < c_{\text{crit}}(Q/2M)\,c_{3,1}\,,\end{cases}$$



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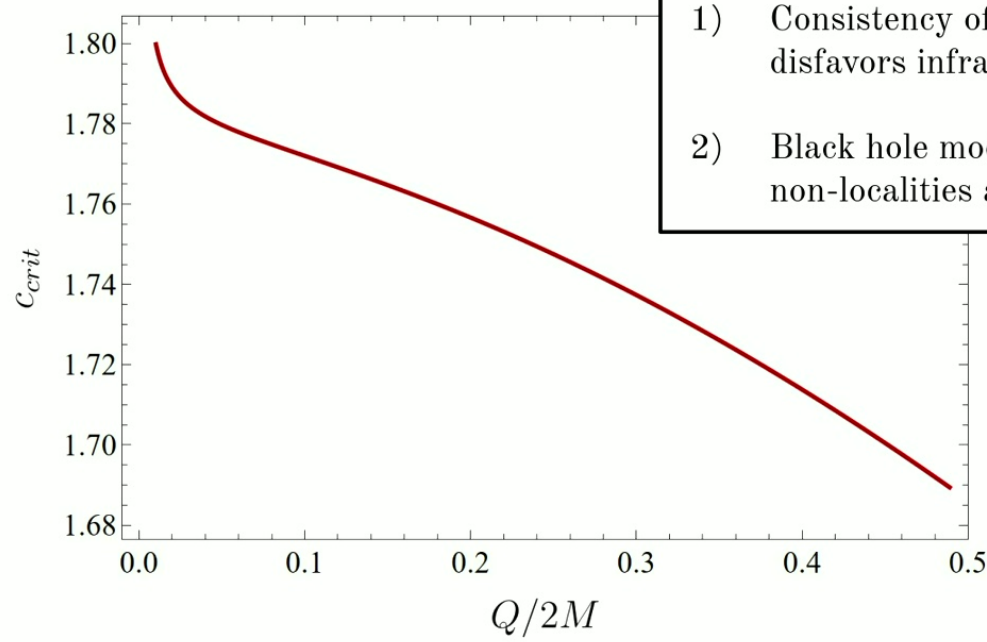

1) Consistency of black hole entropy disfavors infrared non-localities

2) Black hole models requiring infrared non-localities are likely ruled out

Summary and Conclusions

- **QG community (top-down):**

  Effective actions from different approaches
  ⇒ different effective non-localities

- **Pheno community (bottom-up):**

  Spacetime solutions: model building

- **Top-down + bottom-up:**
  - Fundamental tests for pheno models
  - Non-trivial constraints for fundamental theories

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  - Stationary-action principle ⇒ rule out models
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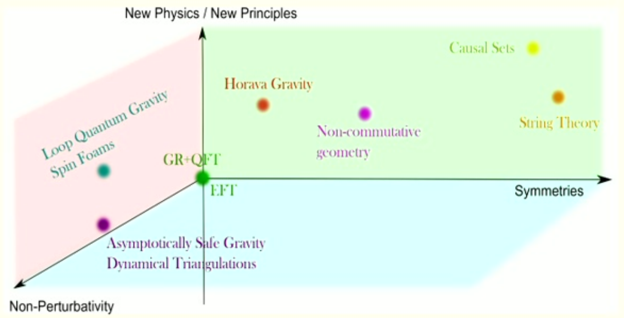

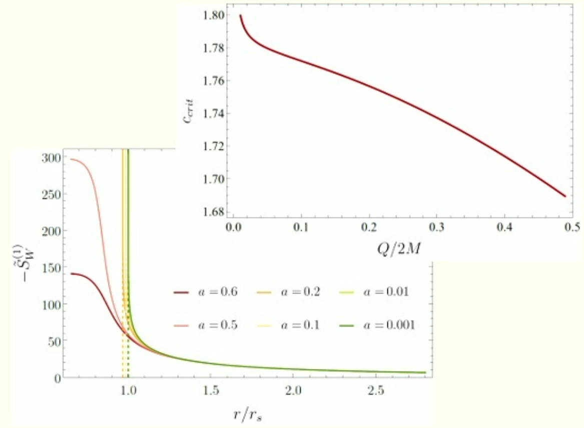

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Thank you!

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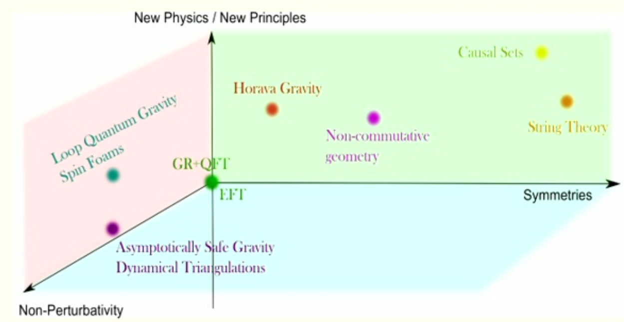

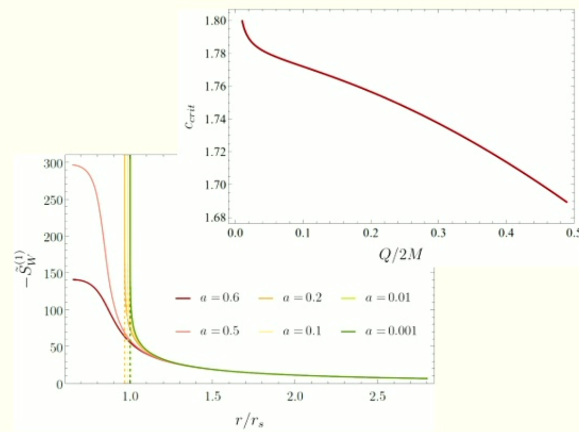

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