Title: Levin-Wen is a gauge theory: entanglement from topology

Speakers: Kyle Kawagoe

Series: Quantum Matter

Date: February 20, 2024 - 2:30 PM

URL: https://pirsa.org/24020087

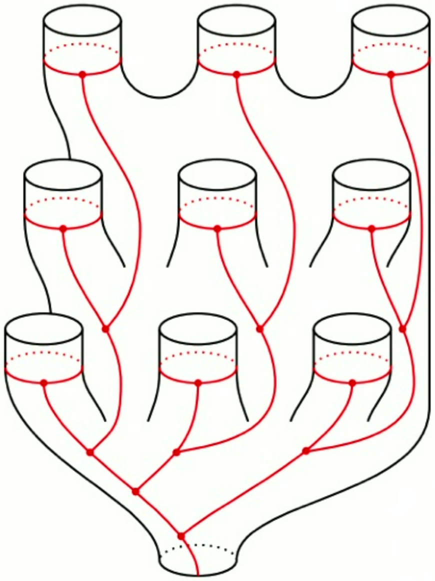
Abstract: The Levin-Wen model is known to produce a vast array of topological phases of matter. Among these theories are gauge theories such as the twisted quantum double. In this talk, we will show that the Levin-Wen model is itself a gauge theory. In particular, given a unitary fusion category C, we construct a globally tube algebra (Tube(C)) symmetric lattice model and gauge this symmetry to produce the Levin-Wen model with anyons described by the Drinfeld center Z(C). This construction endows the terms of the Levin-Wen Hamiltonian with the interpretation of flux and charge operators for the Tube(C) gauge symmetry. Furthermore, this construction gives a gauge theoretic interpretation to the mathematical fact that the category of representations of Tube(C) is equivalent to Z(C). To demonstrate this new class of Tube(C) symmetric theories, we will explicitly explore the case where C is the Fibonacci category Fib. We will write down the ungauged Tube(Fib) symmetric theory, compute the symmetry action, and show how to gauge the Tube(Fib) global symmetry to produce the double Fibonacci Levin-Wen model.

Zoom link

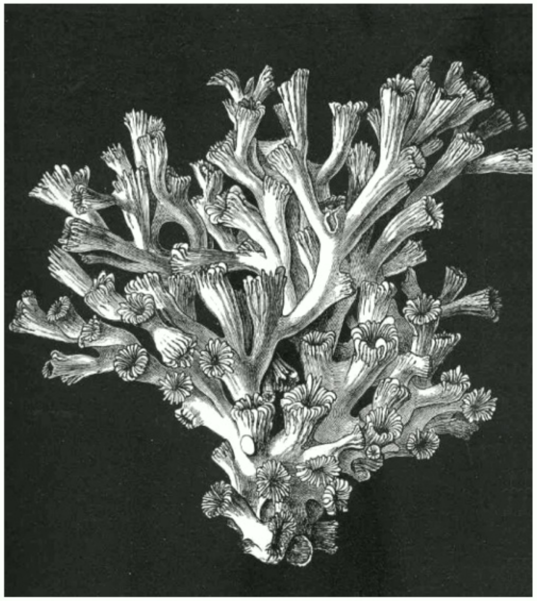
Levin-Wen is a Gauge Theory: Entanglement from Topology

Kyle Kawagoe

Department of Physics
Department of Mathematics
The Ohio State University



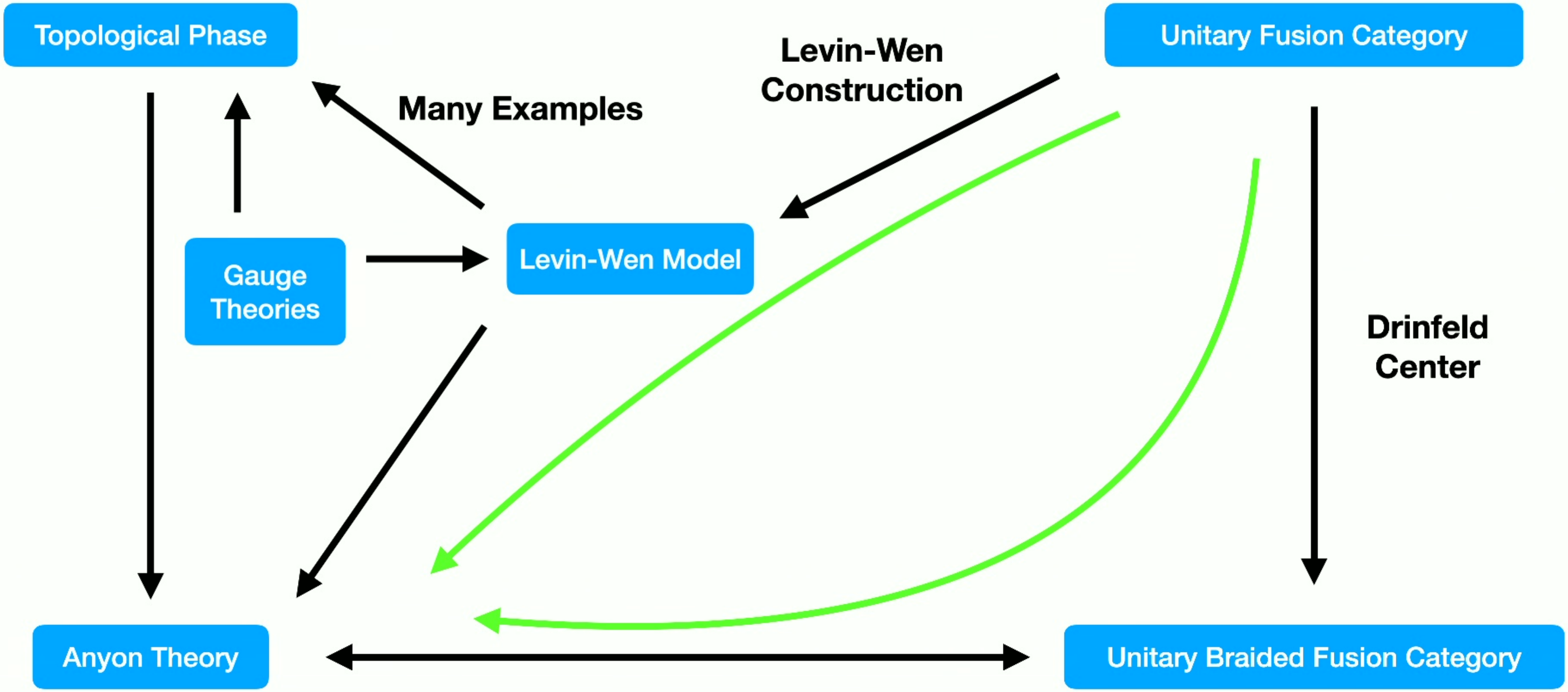
C. Wyville Thomson. New York, Macmillan and co, 1873



ArXiv: 2401.13838
(2024)

In collaboration with Corey Jones, Sean Sanford, David Green, and David Penneys

Background: Synoptic Chart



Topological Phase

Unitary Fusion Category

Many Examples

Levin-Wen Construction

Gauge Theories

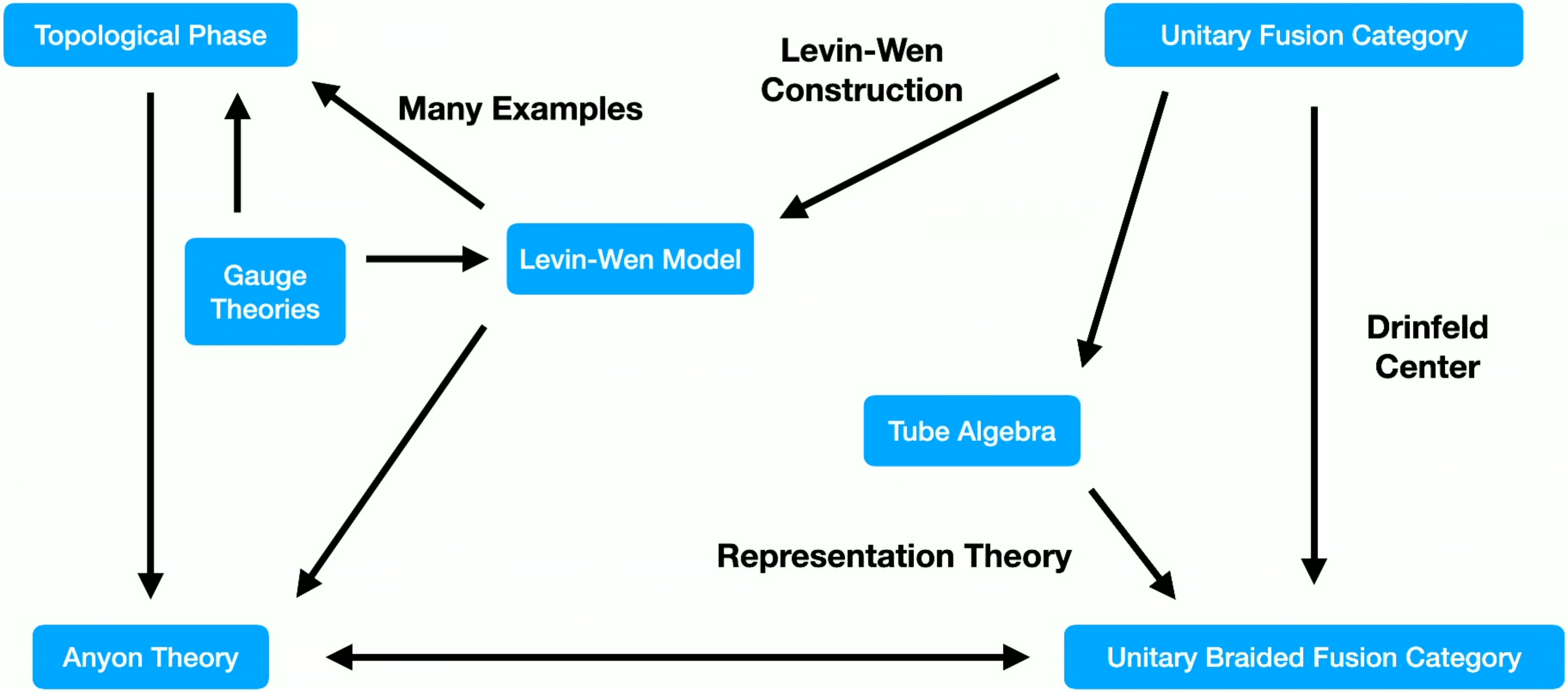
Levin-Wen Model

Drinfeld Center

Anyon Theory

Unitary Braided Fusion Category

Background: Synoptic Chart



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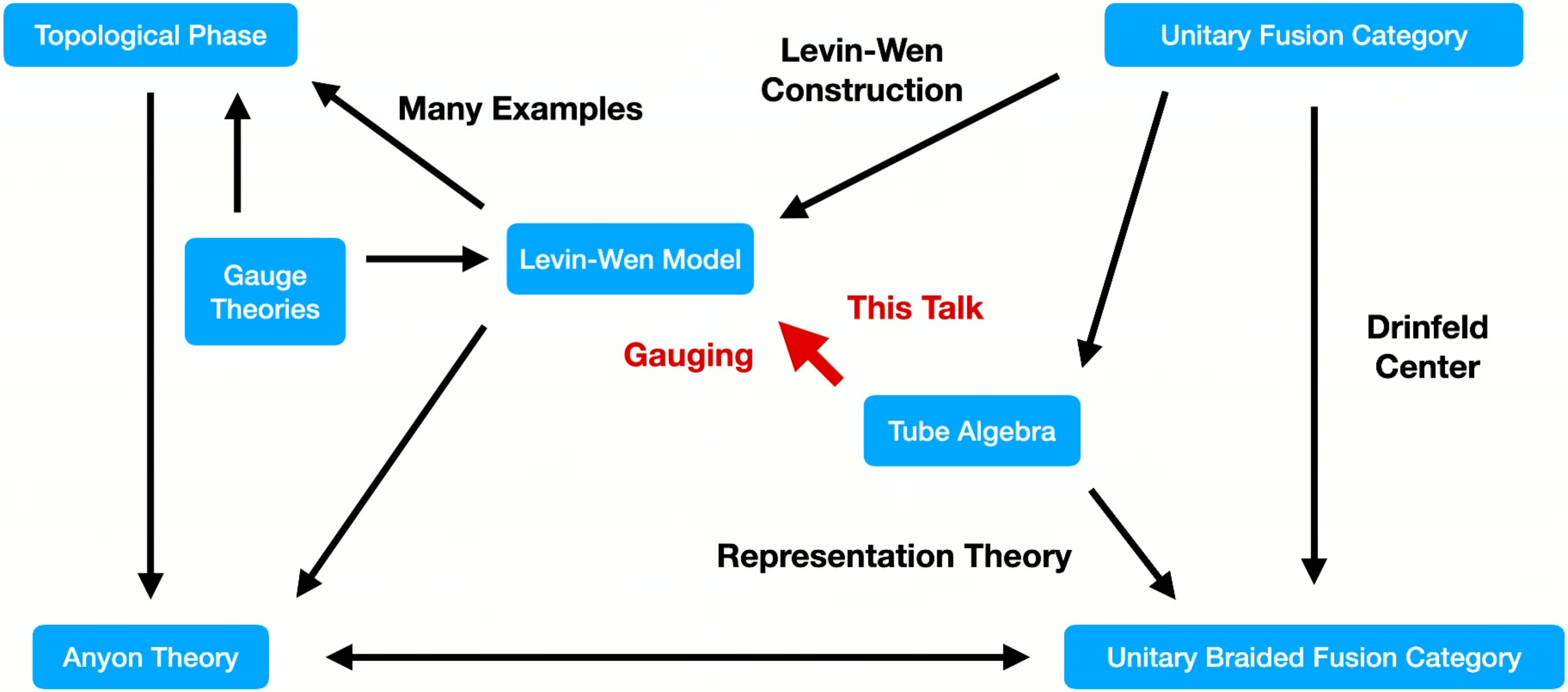

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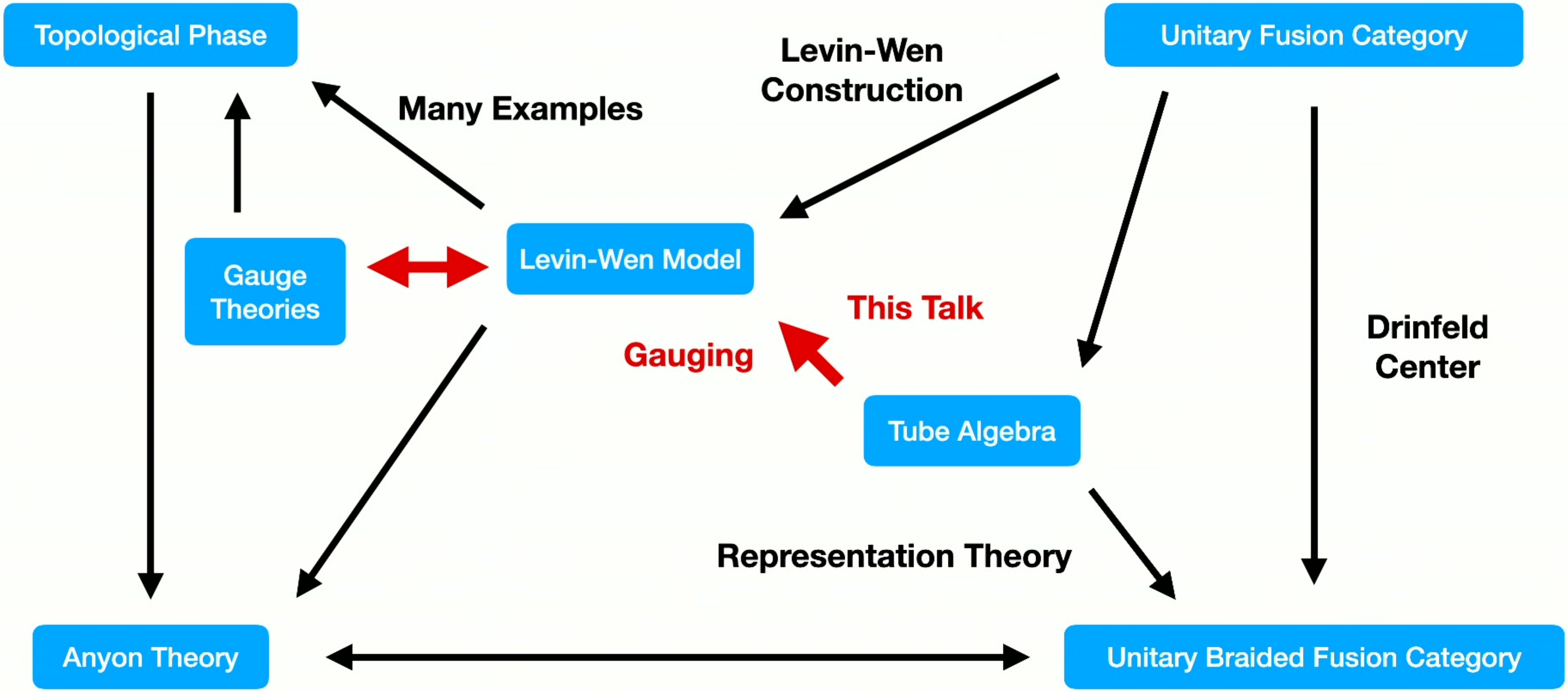

Topological Phase

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This Talk

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Drinfeld Center

Representation Theory

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Motivation

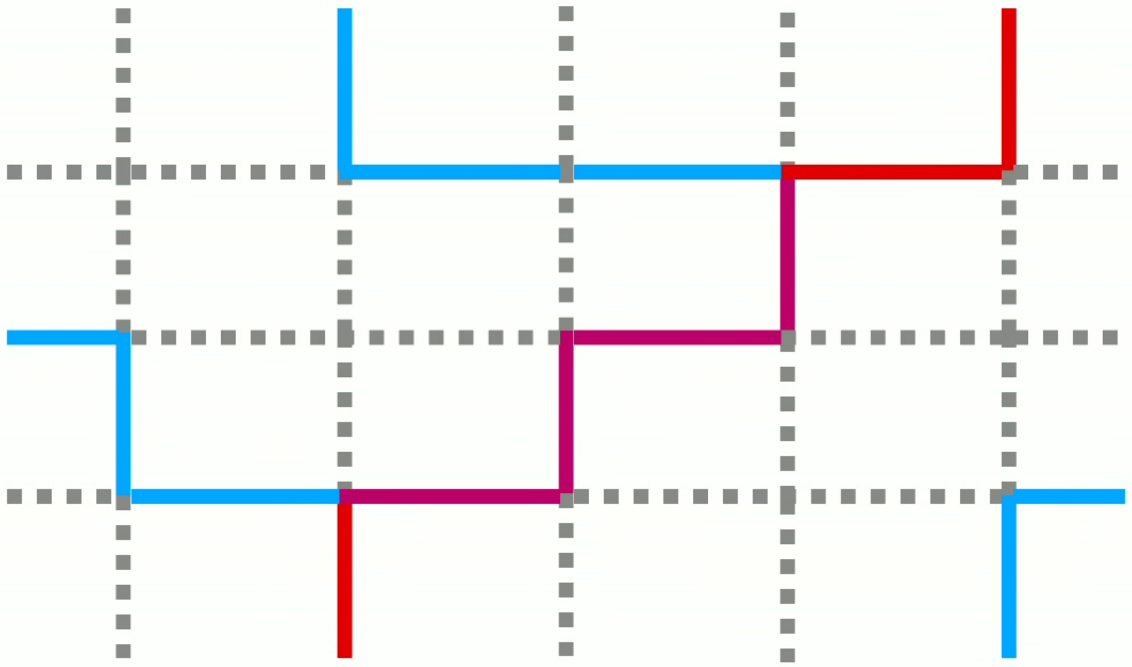
- The Levin-Wen model describes a vast array of gapped topological phases
- Many of these models are gauge theories.
- Levin-Wen Hamiltonian *looks* like a gauge theory

$$H = -\sum_v A_v - \sum_p B_p$$

- $Rep(Tube(\mathscr{C})) \cong Z(\mathscr{C}) =$ Levin-Wen anyons

M. Levin and X.-G. Wen Phys. Rev. B **71**, 045110, (2005)

What is the Levin-Wen model?



Theory of a dynamical net of labeled strings with branching structure

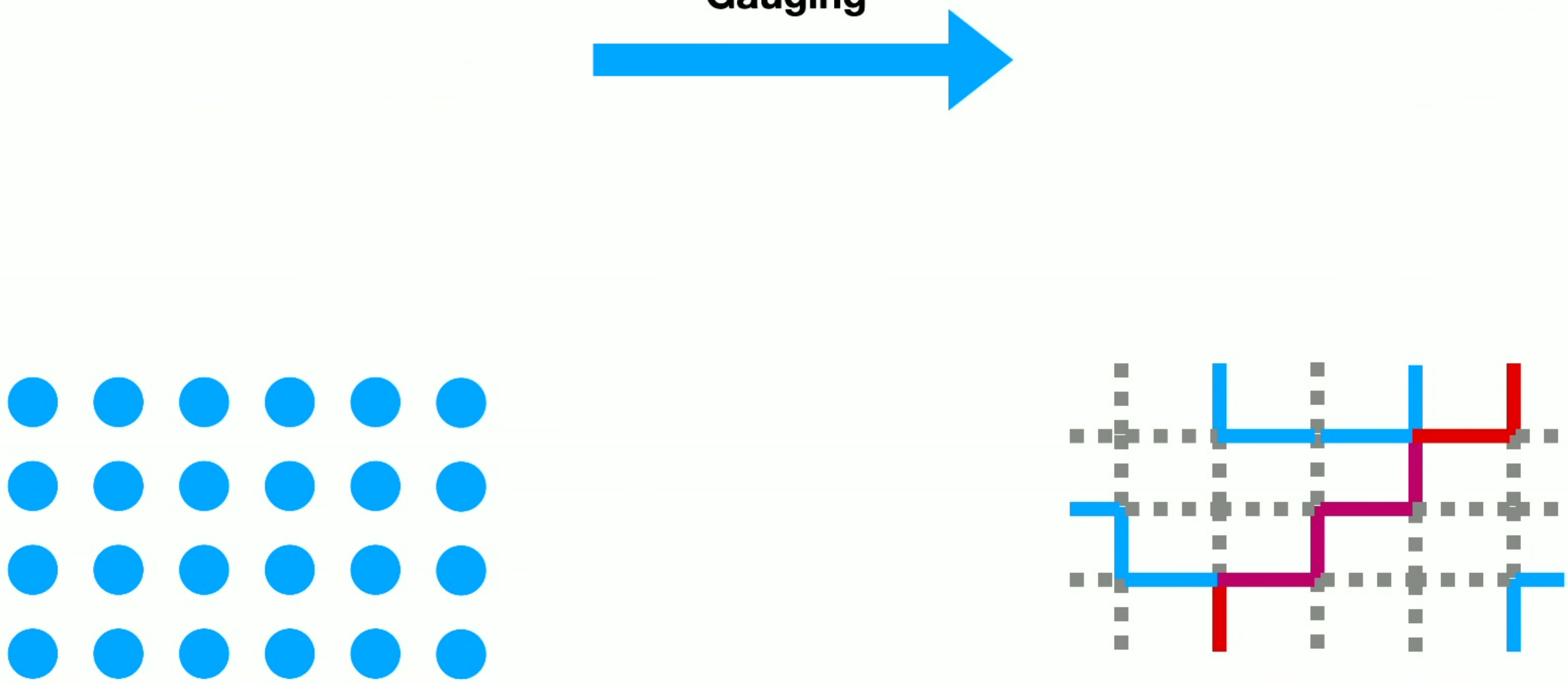
M. Levin and X.-G. Wen Phys. Rev. B **71**, 045110, (2005)

Is it a theory of domain walls?

Theory of domains

Krammers
Wannier
Gauging

Theory of domain walls



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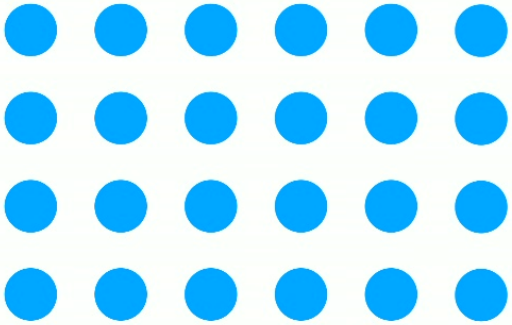
Theory of domains

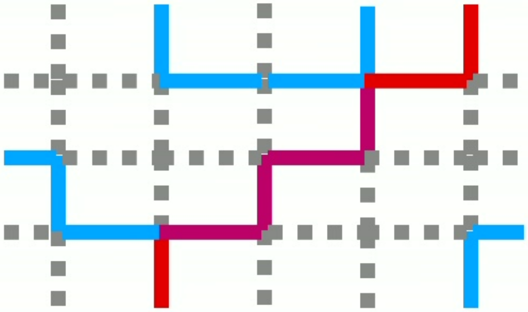
Trivial $\mathbb{Z}_2$ Paramagnet

Krammers Wannier Gauging

Theory of domain walls

Toric Code





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Non-trivial $\mathbb{Z}_2$ Paramagnet

Symmetry Protected Topological Order

Krammers Wannier Gauging

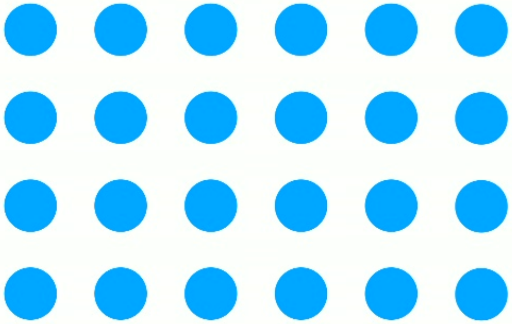
Theory of domain walls

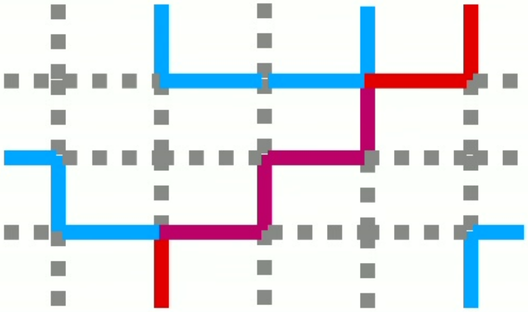
Toric Code

M. Levin and Z.-C. Gu
Phys. Rev. B **86**, 115109 (2012)

Double Semion

Quantum Double





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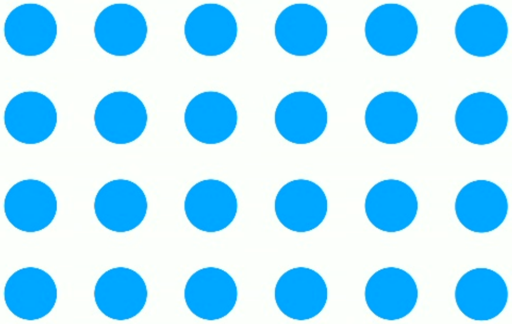
Symmetry Protected Topological Order

????



Krammers Wannier Gauging

Theory of domain walls

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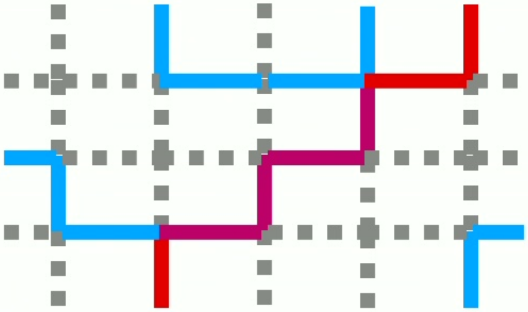
Levin-Wen Model

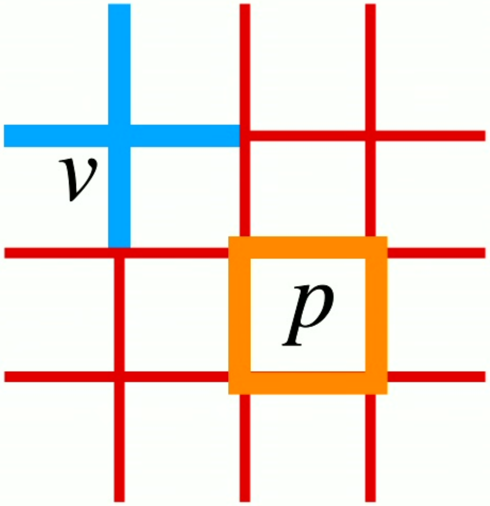


Outline

1. Toric Code as gauge theory
2. Levin-Wen through diagrammatics
3. Levin-Wen as gauge theory
4. Example: Fibonacci symmetric model

Toric Code

1/2 - Spin on each edge

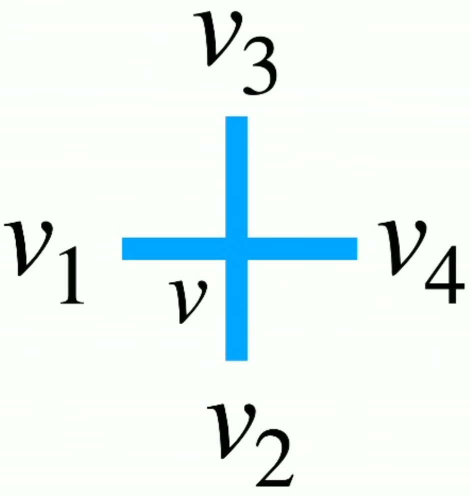


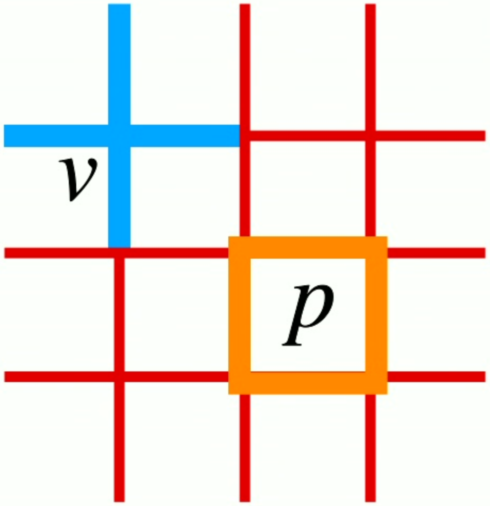
$$H = -\sum_{v} A_v - \sum_{p} B_p$$

Toric Code

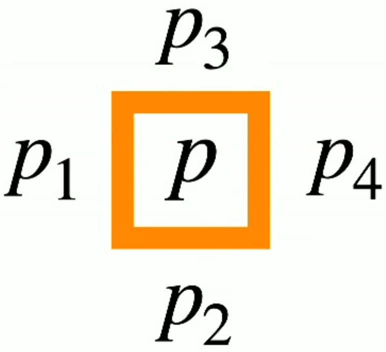
1/2 - Spin on each edge

$$A_v = \sigma^x_{v_1}\sigma^x_{v_2}\sigma^x_{v_3}\sigma^x_{v_4}$$





$$B_p = \sigma^z_{p_1}\sigma^z_{p_2}\sigma^z_{p_3}\sigma^z_{p_4}$$



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Toric Code is a Z_2 gauge theory

$$H = -\sum_v \sigma_v^z$$

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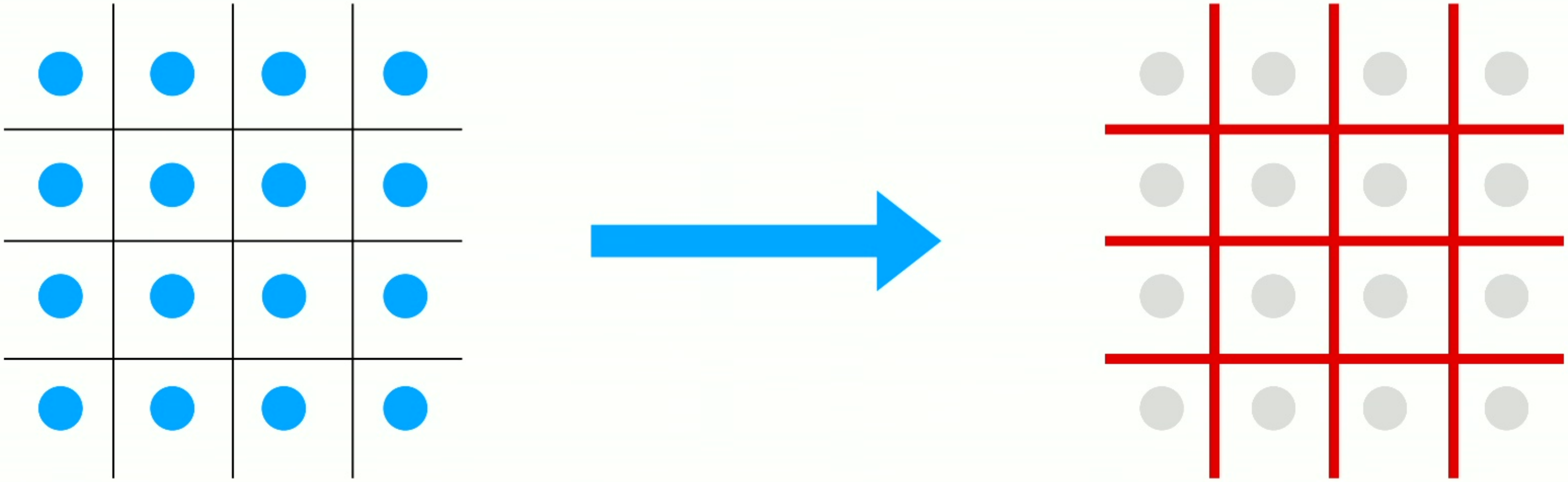

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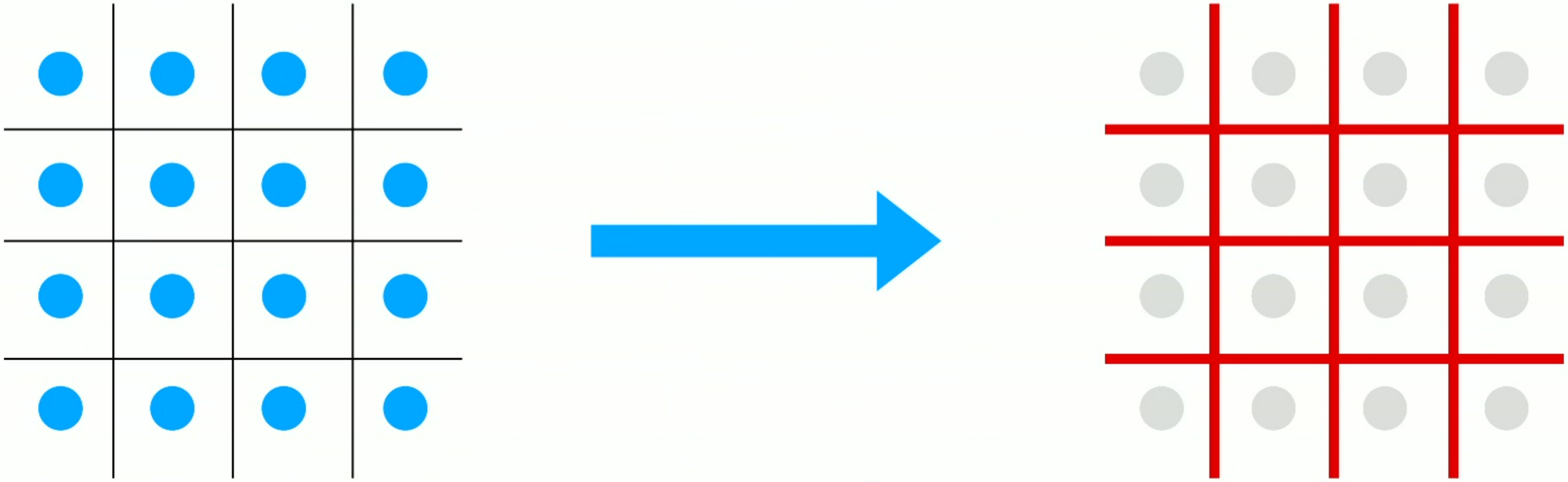


2^{16} **dim**

2^{24} **dim**

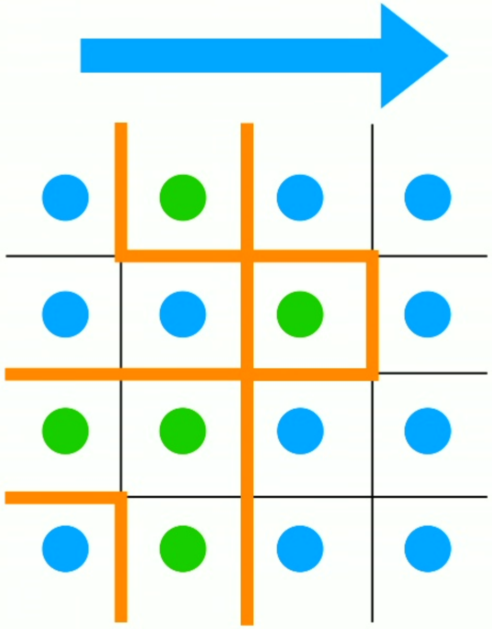
Gauging $\mathbb{Z}_2$ trivial paramagnet

1/2 - Spin on each site

$\uparrow_x$ **spin** ●

$\downarrow_x$ **spin** ●

Replace with:



1/2 - Spin on each edge

$\uparrow_x$ **spin** —

$\downarrow_x$ **spin** ▬

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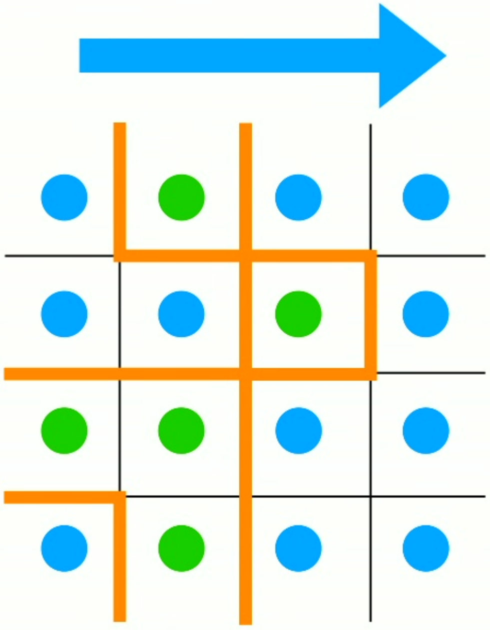

1/2 - Spin on each edge

$\uparrow_x$ **spin** 🔵

$\downarrow_x$ **spin** 🟢

$$H = -\sum_v \sigma_v^z$$

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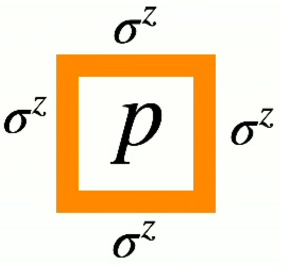
$\downarrow_x$ **spin** ━

$$H_0 = -\sum_p B_p$$



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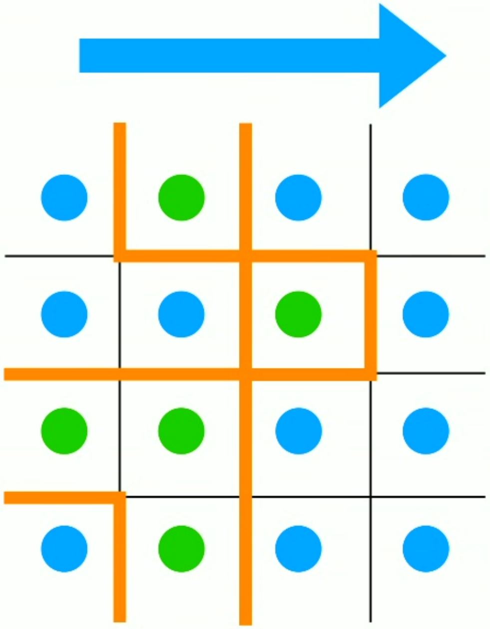

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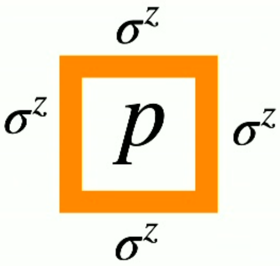

2^{24} **dim**

2^{15} **dim. symmetric space**

Gauging $\mathbb{Z}_2$ trivial paramagnet

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$\uparrow_x$ **spin** 🔵

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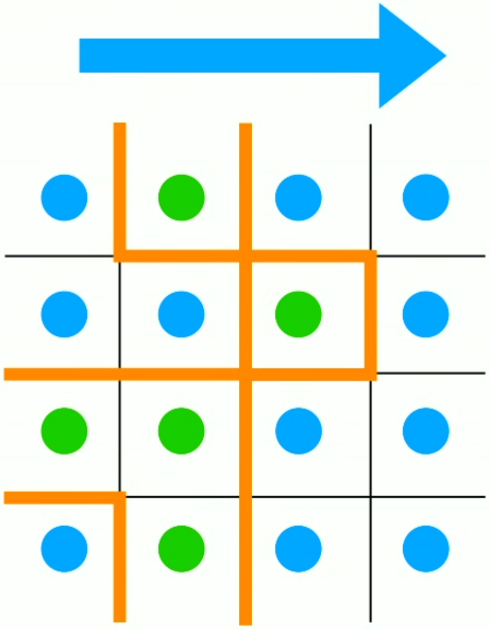
$$H = -\sum_v \sigma_v^z$$

2^{16} **dim**

2^{15} **dim. symmetric space**

Replace with:



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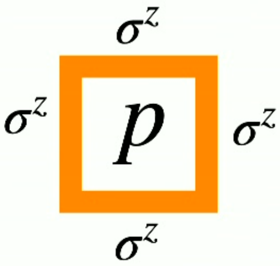

2^{24} **dim**

2^{15} **dim.** $A_v = 1$ **space**

(15=24-9)

Lessons from Toric Code

- This gauge theory is theory of domain walls.
- Symmetric ungauged space $\cong$ Gauged space with no “flux”
- Not all anyons are irreps of group symmetry ($\mathbb{Z}_2$ has two irreps)
- (Not shown) Anyons are irreps of $D(\mathbb{Z}_2) \cong Tube(Vec\mathbb{Z}_2)$

(Drinfeld Double)

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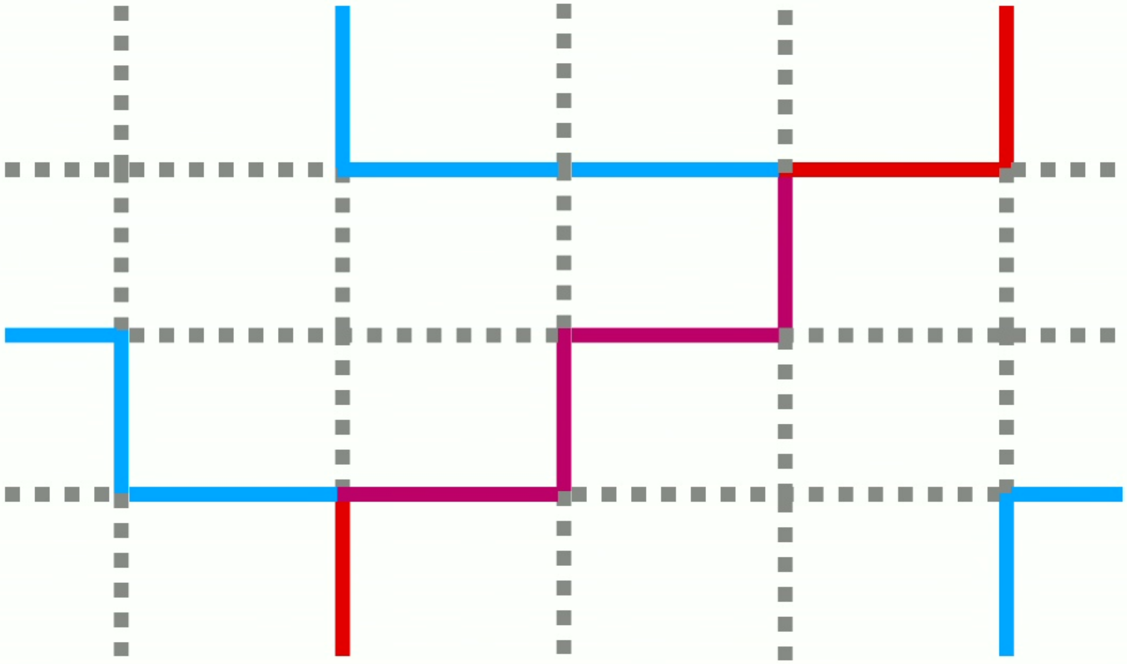
(Drinfeld Double)

All symmetry groups generate a symmetry algebra
(But we will see that not all symmetry algebras have a symmetry group)

What is the Levin-Wen model?



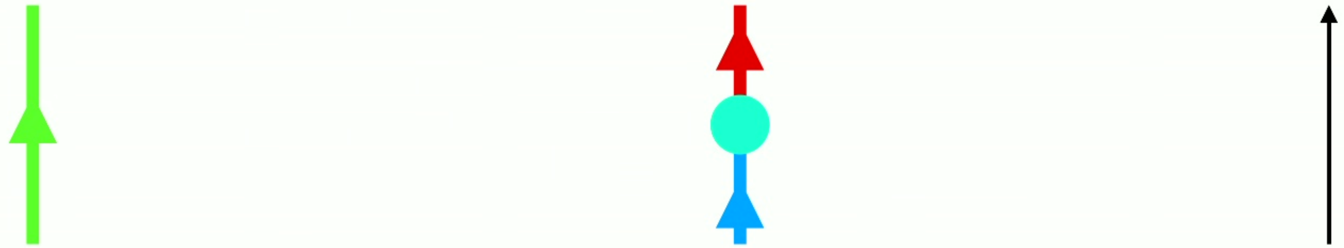
To introduce Hilbert space, need diagrammatics

D. Green, P. Huston, **KK**, D. Penneys, A. Poudel, S. Sanford arXiv:2305.14068 (2023)
(To appear in Quantum)

M. Levin and X.-G. Wen Phys. Rev. B **71**, 045110, (2005)

Fusion Categories and You

Categories: Objects and morphisms
Diagrammatic calculus



Objects

Morphisms

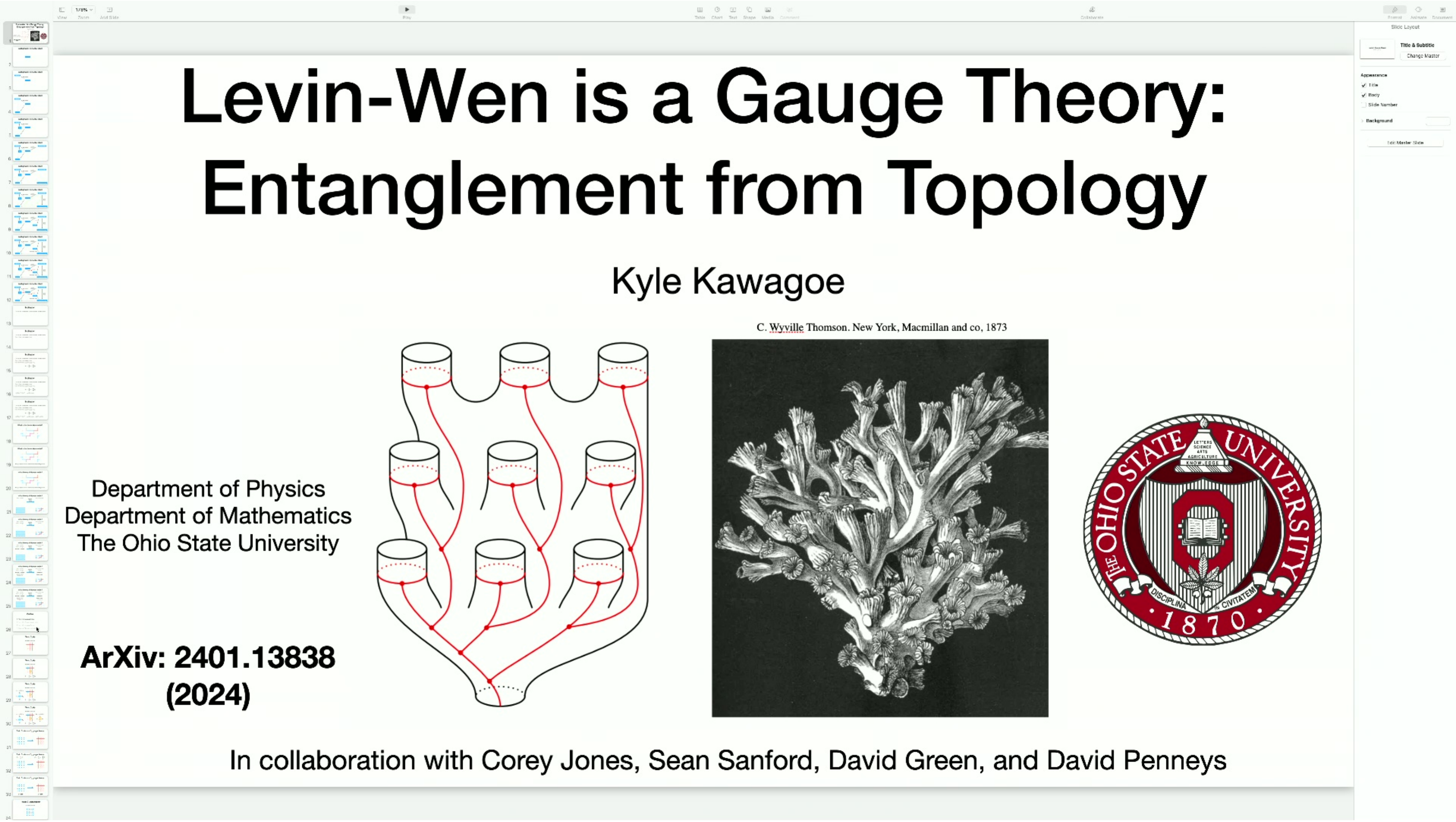
Quantum Circuits

Qubits

Gates

Space of gates on a qubit is a 4D Hilbert space.

$$\langle A \mid B \rangle = Tr(A^{\dagger}B)/2$$



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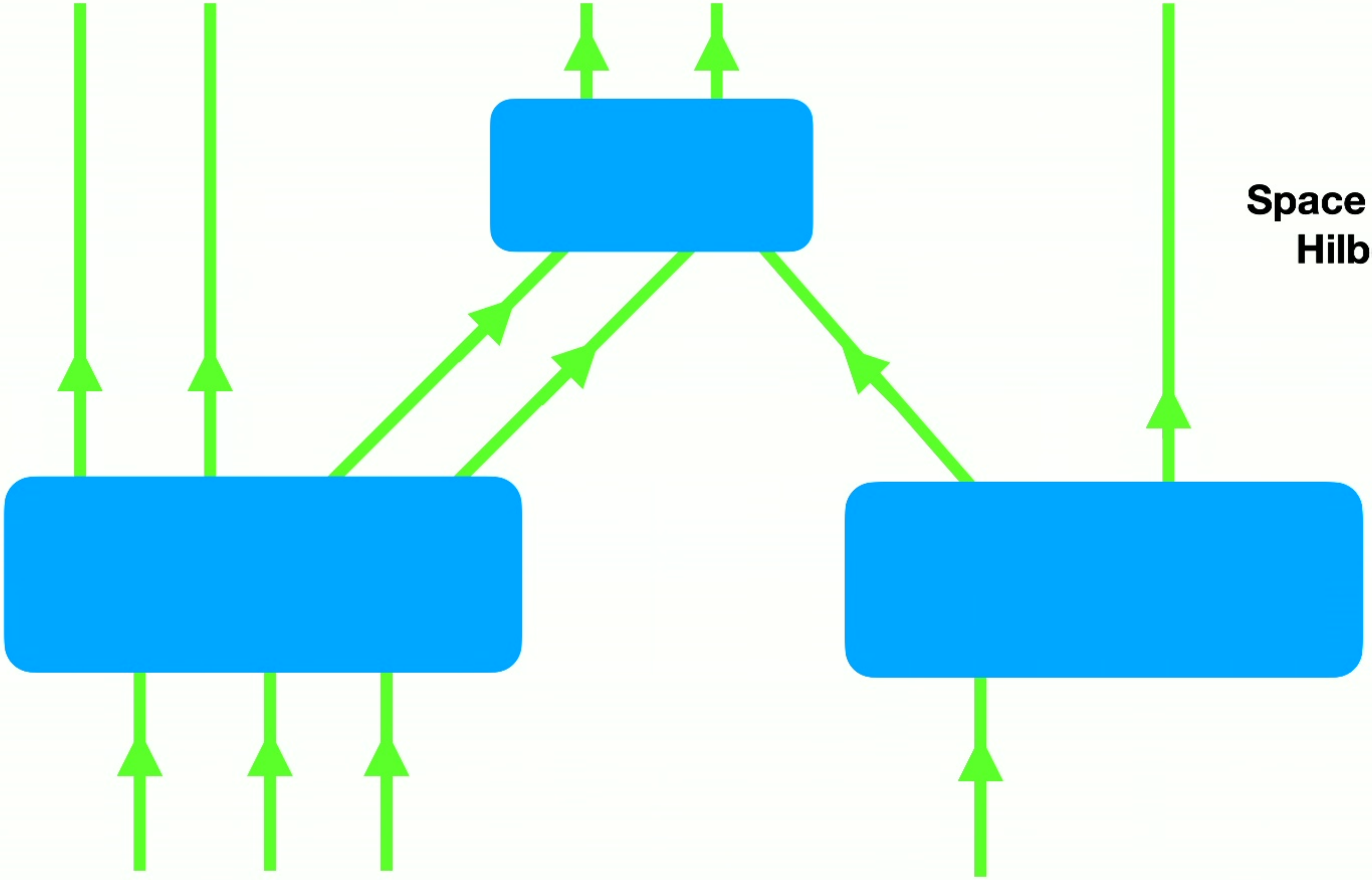
Quantum Circuits



Space of gates is a Hilbert space.

Fusion Categories and <u>You</u>

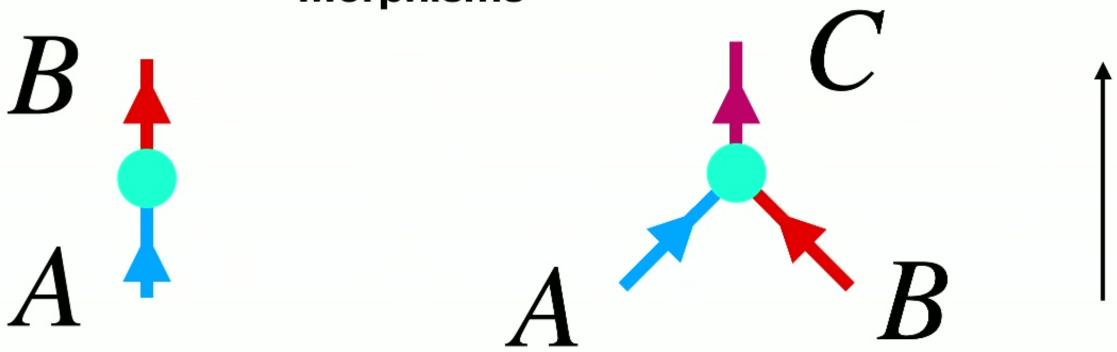
Categories: Objects and morphisms
Diagrammatic calculus

Objects

Finite list of “simple” objects

$$Irr(\mathscr{C})$$

Morphisms



$$\mathscr{C}(A \to B)$$

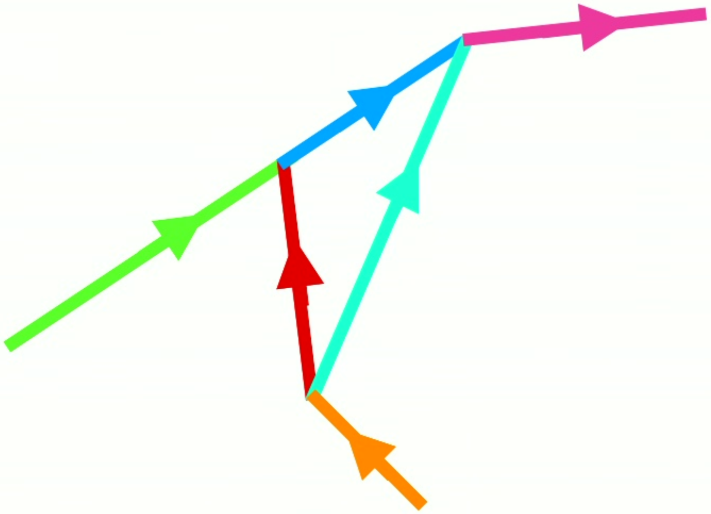
$$\mathscr{C}(A \otimes B \to C)$$

Space of morphisms from A to B
is a Hilbert space.

Diagrammatic Calculus on Plane

Assign diagram to vector space

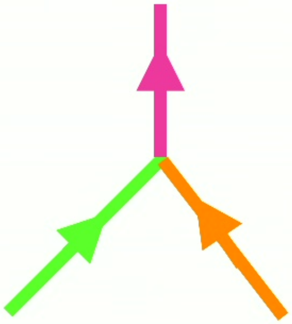
Lines are simple objects



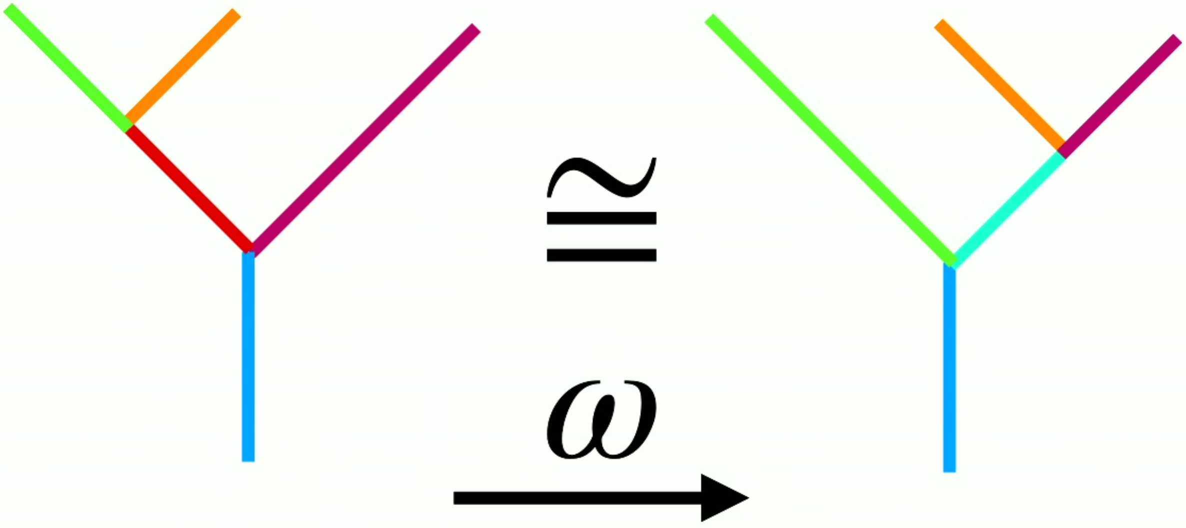
Vector space

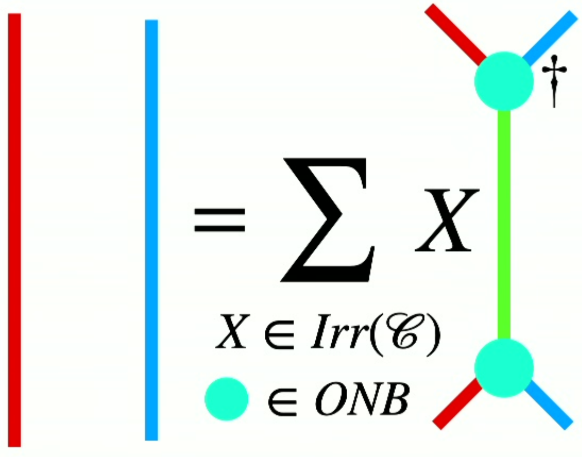
$$\frac{\mathscr{C}(\bullet \otimes \bullet \to \bullet) \otimes \mathscr{C}(\bullet \to \bullet \otimes \bullet) \otimes \mathscr{C}(\bullet \otimes \bullet \to \bullet)}{\text{Relations}}$$

$$\cong \mathscr{C}(\bullet \otimes \bullet \to \bullet)$$



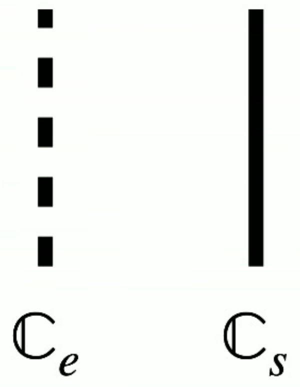
Main two relations





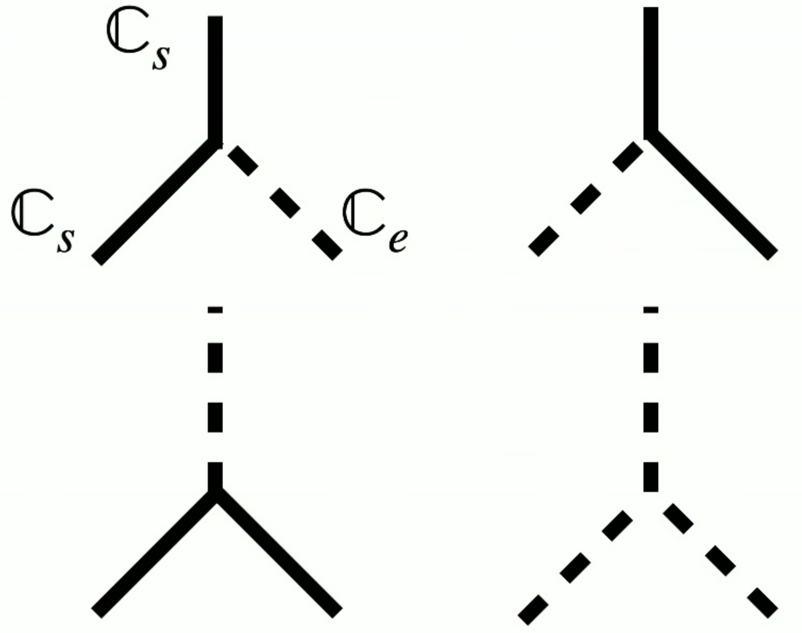
$Vec\mathbb{Z}_2$

Simple Objects



$$\mathbb{C}_s \otimes \mathbb{C}_s = \mathbb{C}_e$$

Non-zero trivalent vertex spaces



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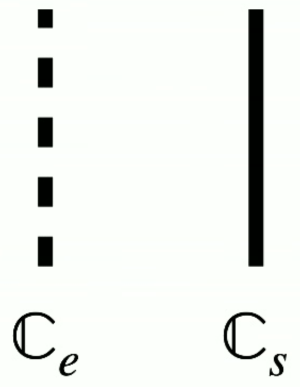
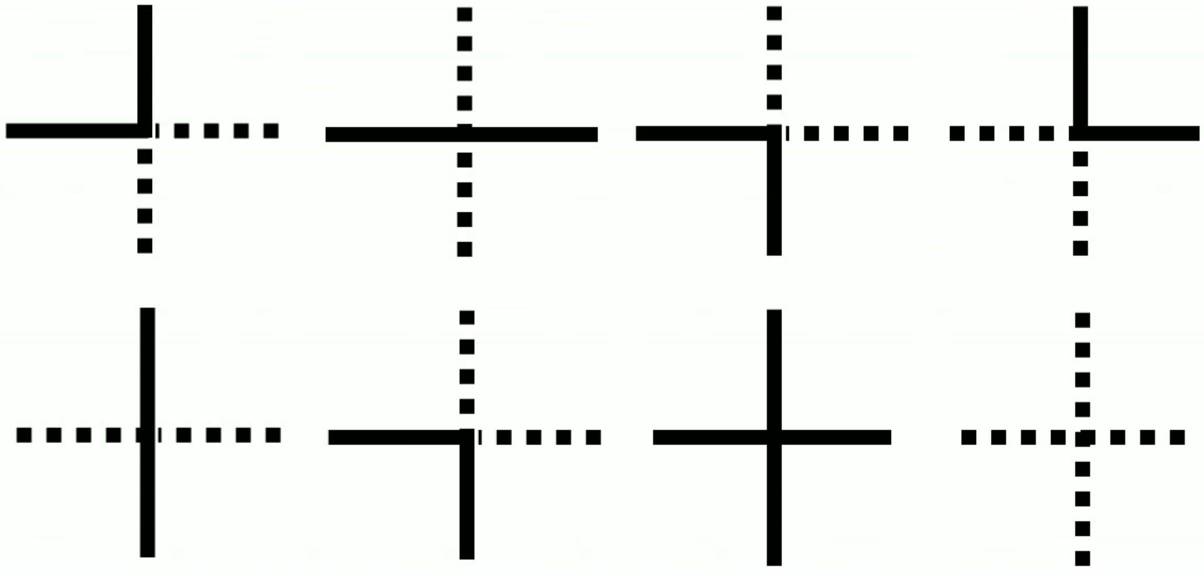
Simple Objects



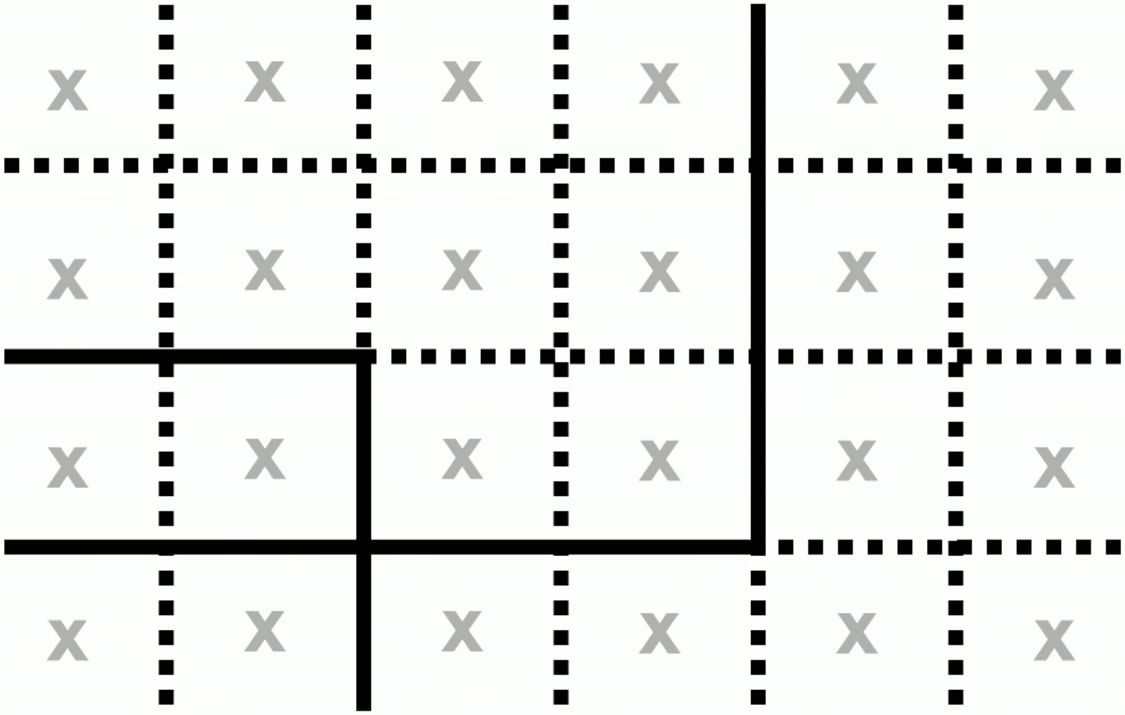
$$\mathbb{C}_s \otimes \mathbb{C}_s = \mathbb{C}_e$$

Non-zero 4-valent vertex spaces



Flux free Toric Code ($Vec\mathbb{Z}_2$)

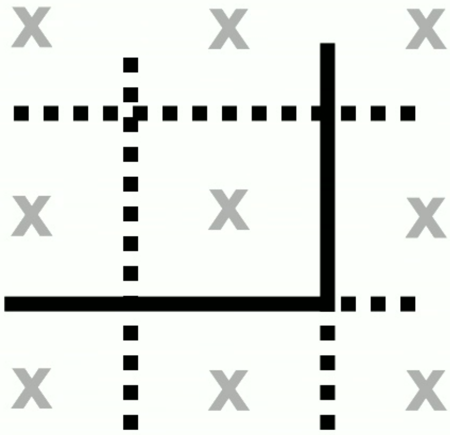
Need punctures to distinguish states



Ground States of A_v

Diagram on punctured plane

Flux free Toric Code ($Vec\mathbb{Z}_2$)



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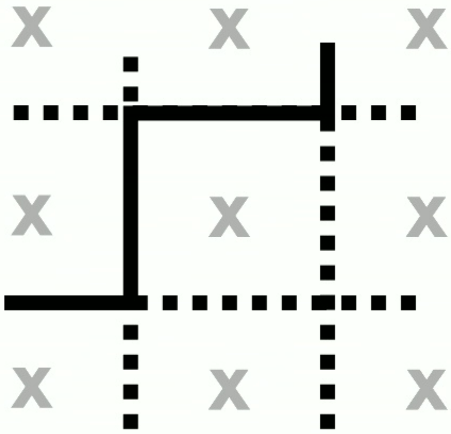
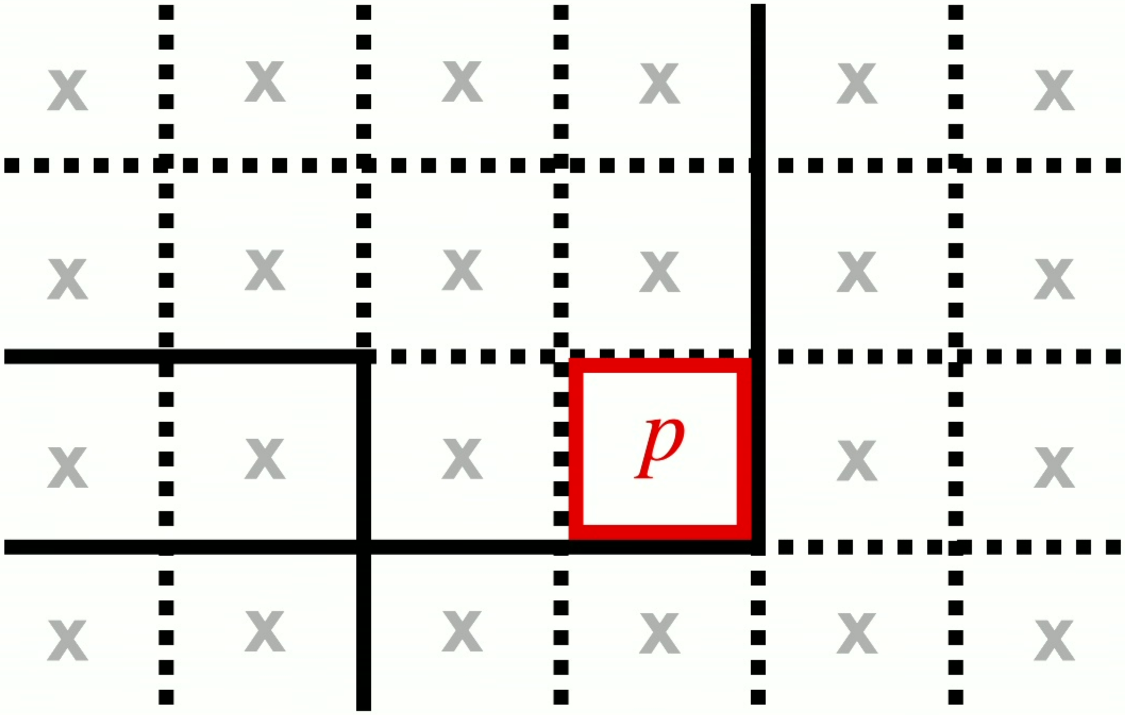


Flux free Toric Code ($Vec\mathbb{Z}_2$)

Need punctures to distinguish states



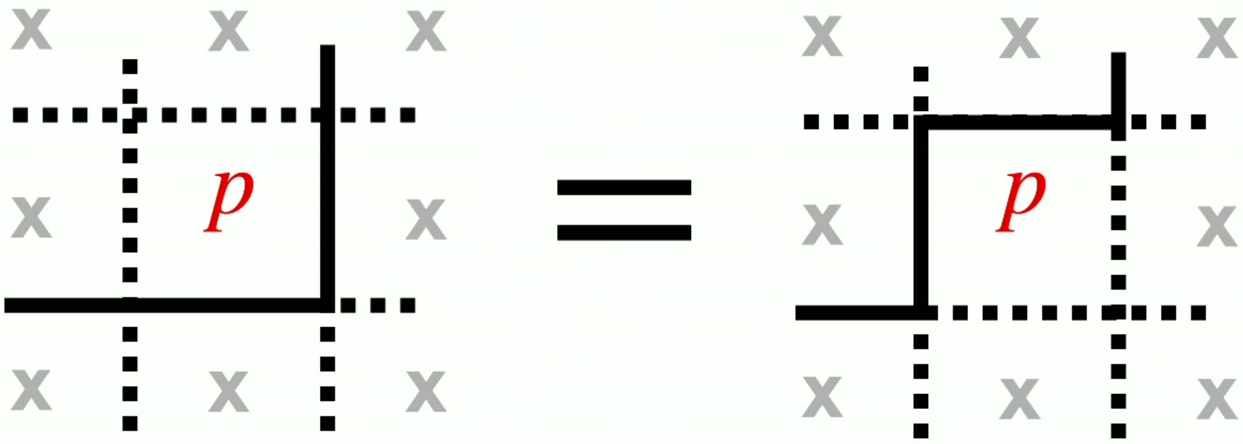
Ground States of A_v

And this B_p

Diagram on punctured plane

Flux free Toric Code ($Vec\mathbb{Z}_2$)

Image of $B_p = 1$ projection has no puncture

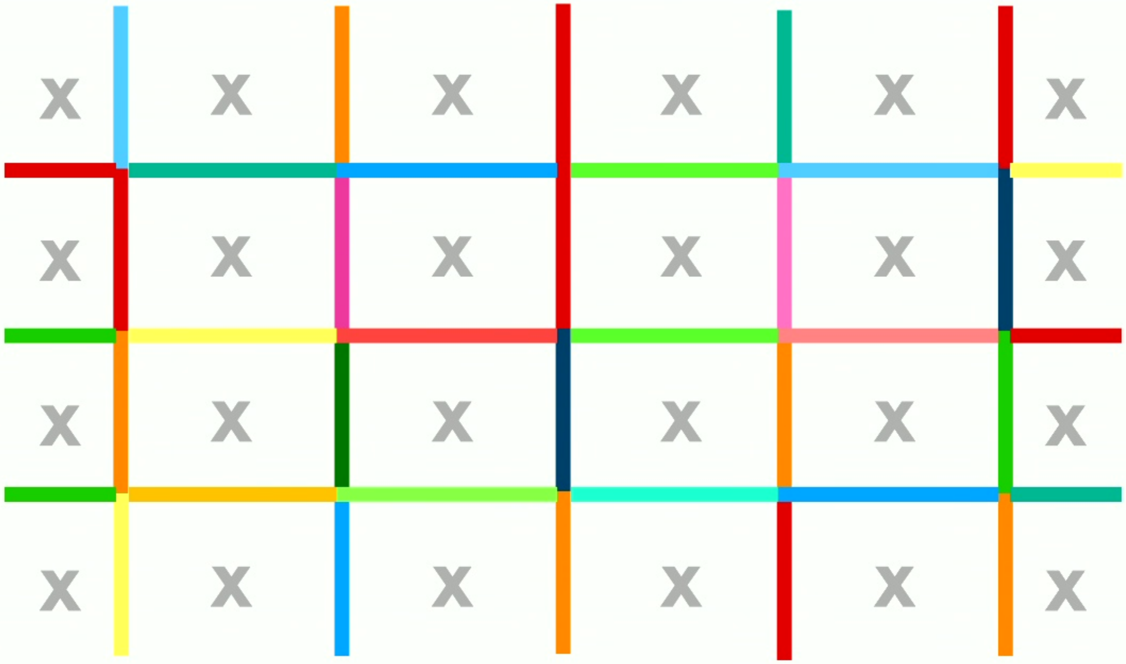


Ground States of A_v

And this B_p

Flux free Levin-Wen

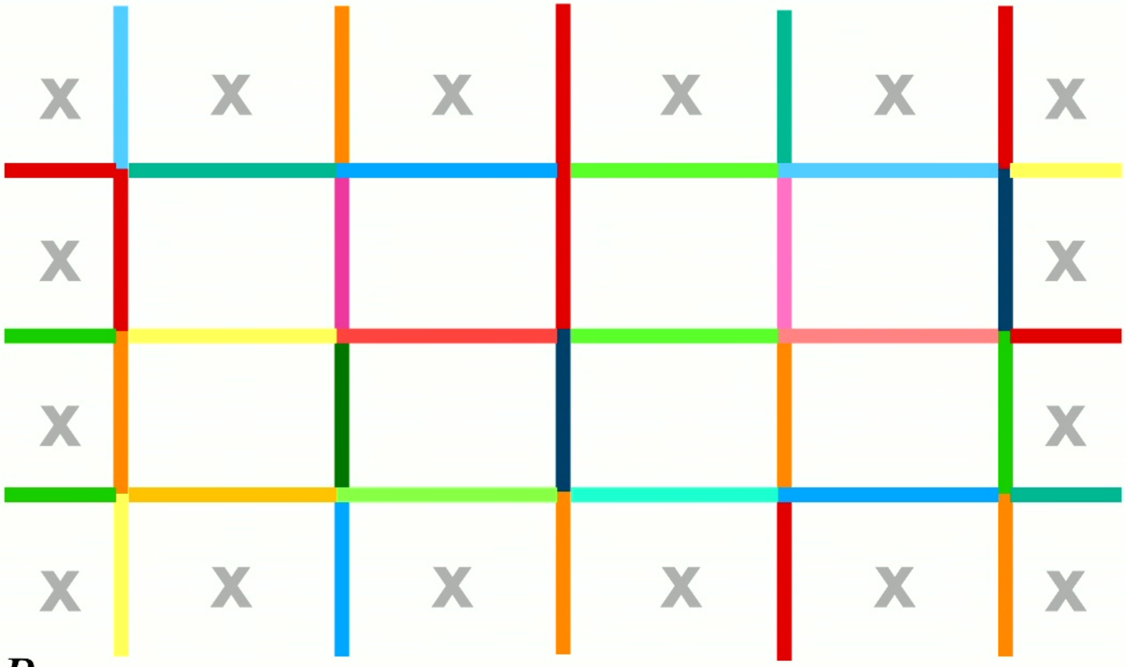
Ground states of A_v are diagrams on punctured plane



$$H_0 = -\sum_v A_v$$

Flux free Levin-Wen

Ground states of A_v are diagrams on punctured plane



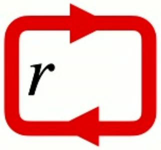
$$H = -\sum_v A_v - \sum_p B_p$$

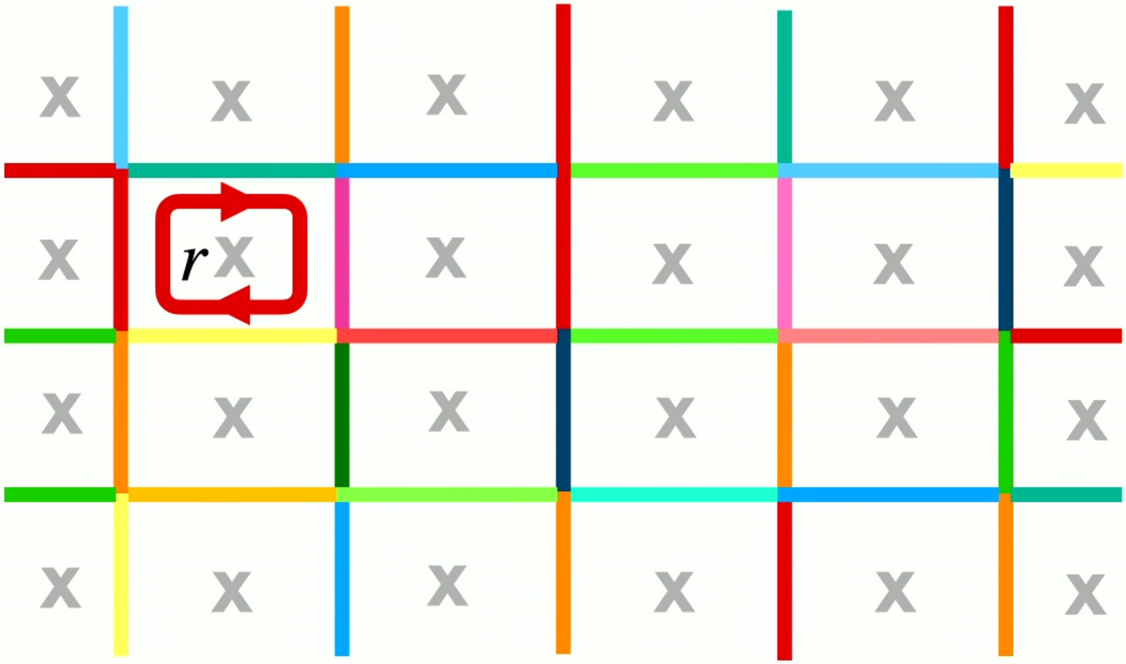
Ground states are diagrams on disk (can fix boundary)

Flux free Levin-Wen

Ground states of A_v are diagrams on punctured plane

$$B_p = \frac{1}{D_{\mathscr{C}}} \sum_{r \in Irr(\mathscr{C})} d_r$$





$$H = -\sum_{v} A_v - \sum_{p} B_p$$

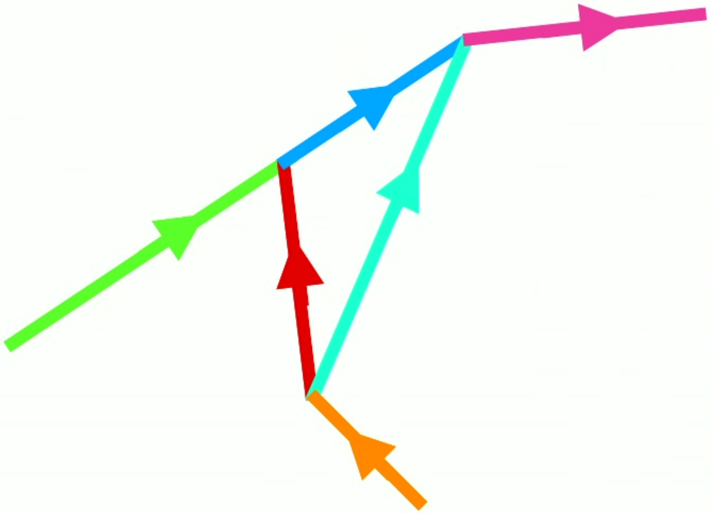
Ground states are diagrams on disk (can fix boundary)

Levin-Wen is a gauge theory

Diagrammatic Calculus on Plane

Assign diagram to vector space

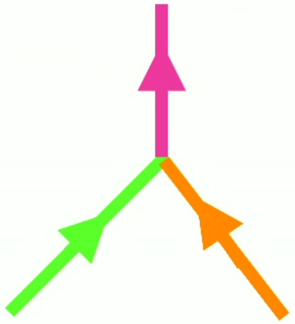
Lines are simple objects



Vector space

$$\frac{\mathscr{C}({\color{lime}\bullet}\otimes{\color{red}\bullet}\to{\color{deepskyblue}\bullet})\otimes\mathscr{C}({\color{orange}\bullet}\to{\color{red}\bullet}\otimes{\color{aqua}\bullet})\otimes\mathscr{C}({\color{deepskyblue}\bullet}\otimes{\color{aqua}\bullet}\to{\color{magenta}\bullet})}{\textbf{Relations}}$$

$$\cong \mathscr{C}({\color{lime}\bullet}\otimes{\color{orange}\bullet}\to{\color{magenta}\bullet})$$



Diagrammatic Calculus on Plane

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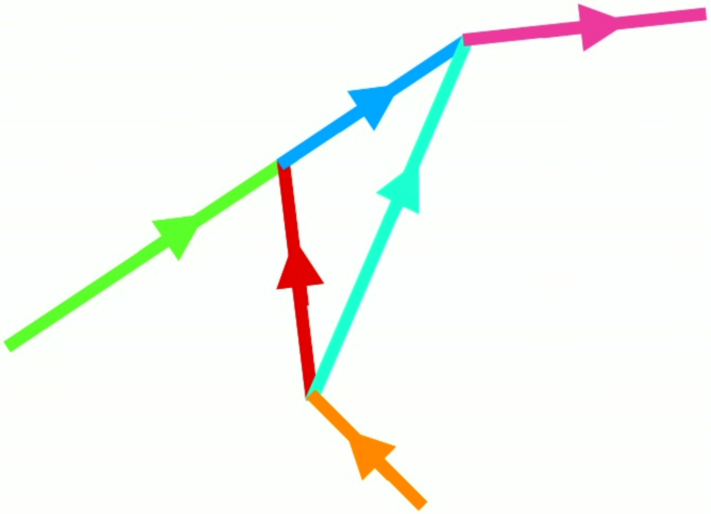
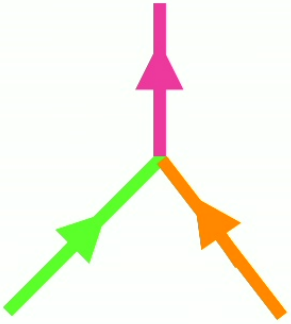


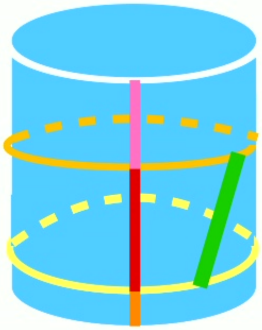
Reduction



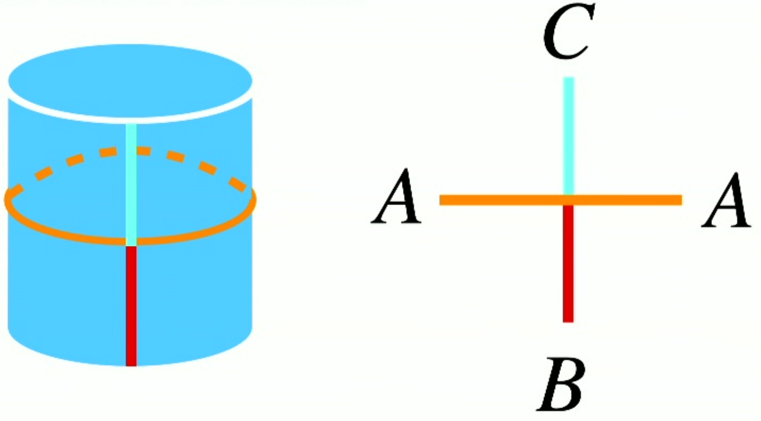
Diagrams on plane all reduce* to points with incoming and outgoing lines
***(Really a subspace of single point)**

Diagrammatic Calculus on a tube

Assign diagram on tube to vector space



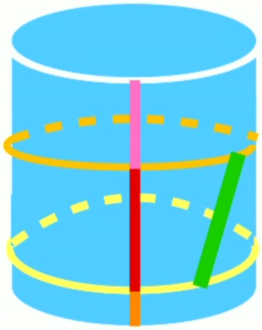
Diagrams on tube all reduce to diagram with below structure



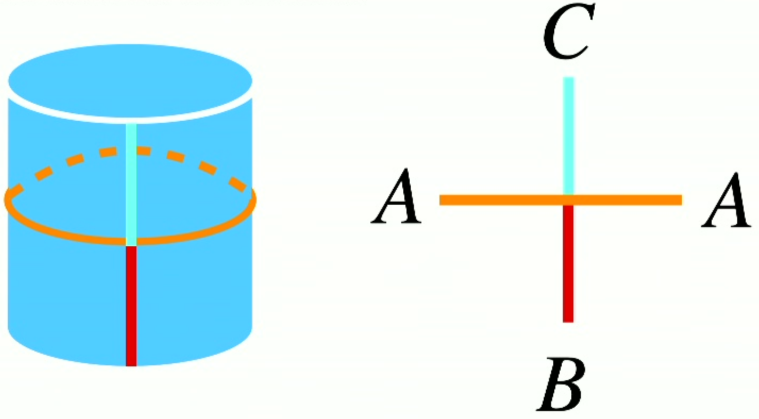
$$\mathscr{C}(AB \to CA)$$

Diagrammatic Calculus on a tube

Assign diagram on tube to vector space



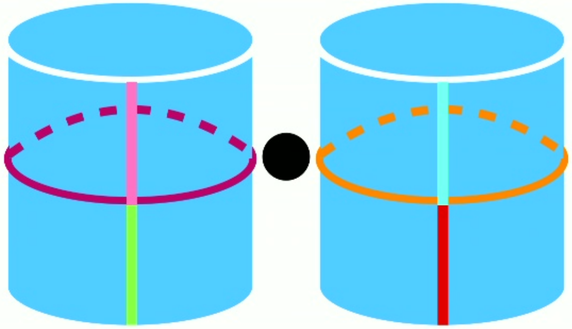
Diagrams on tube all reduce to diagram with below structure



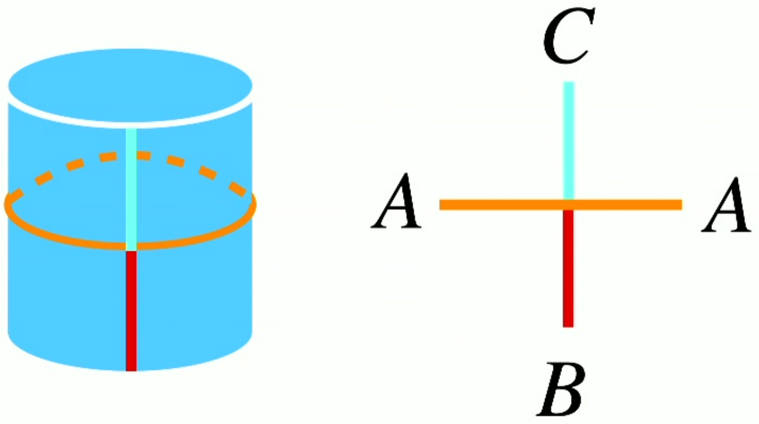
As a vector space $Tube(\mathscr{C}) \cong \bigoplus_{A,B,C \in Irr(\mathscr{C})} \boxed{\mathscr{C}(AB \to CA)}$

Tube Algebra

Algebras have multiplication



Diagrams on tube all reduce to diagram with below structure



$$Tube(\mathscr{C}) \cong \bigoplus \mathscr{C}(AB \to CA)$$

Tube Algebra

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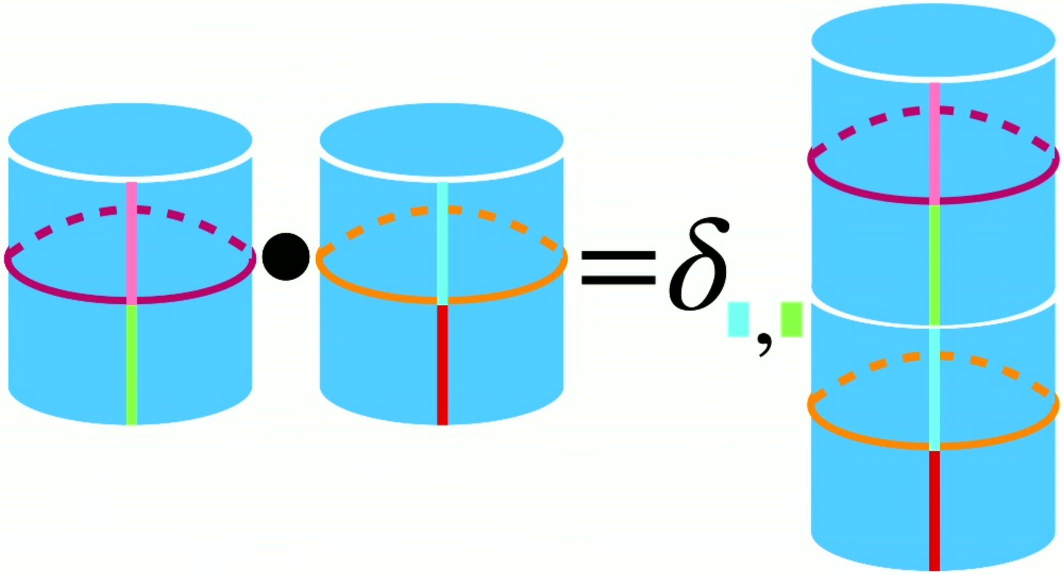
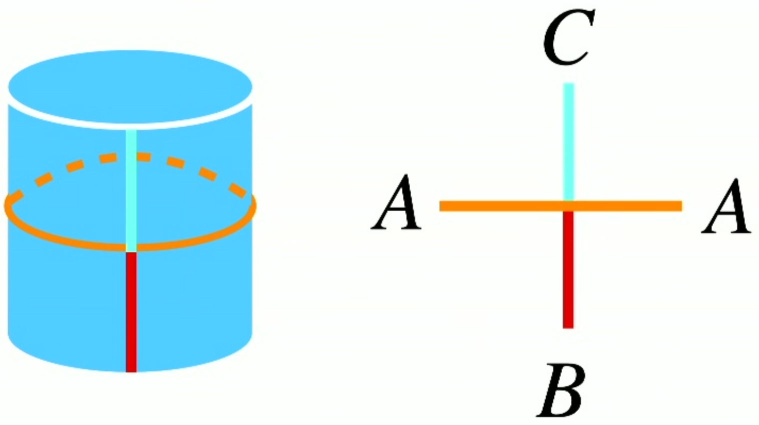


Diagrams on tube all reduce to diagram with below structure



$$Tube(\mathscr{C}) \cong \bigoplus \mathscr{C}(AB \to CA)$$

Tube Algebra Facts

$Rep(Tube(\mathscr{C})) \cong Z(\mathscr{C})$ **= Levin-Wen anyon theory**

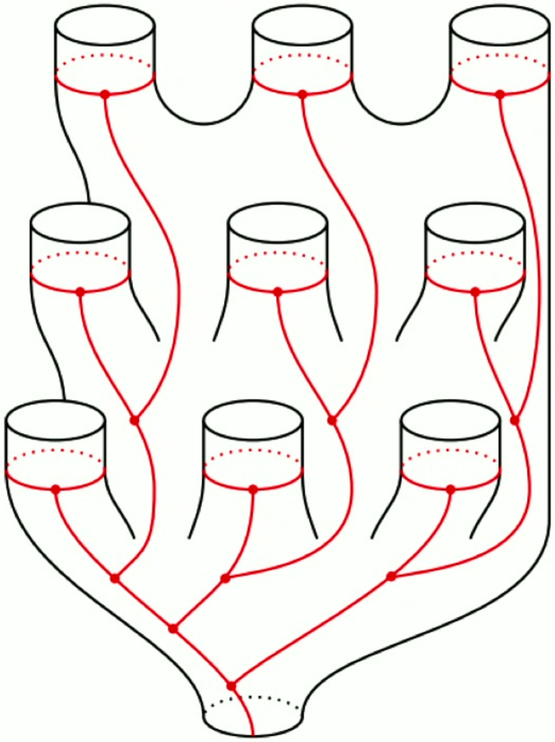
- **No standard tensor product of representations**
- **Must define symmetric lattice model without tensor product**

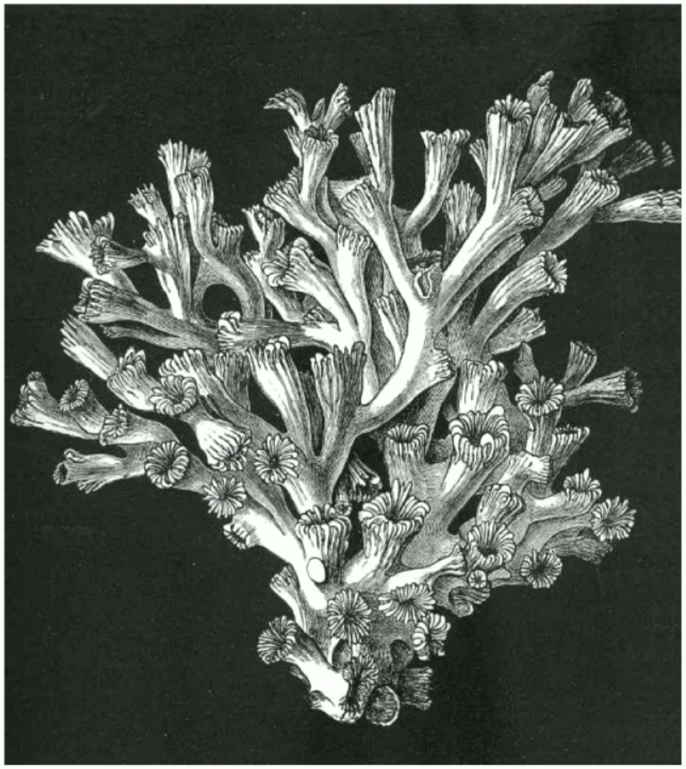
Tube Algebra Facts

$Rep(Tube(\mathscr{C})) \cong Z(\mathscr{C})$ **= Levin-Wen anyon theory**

- **No standard tensor product of representations**
- **Must define symmetric lattice model without tensor product**
- **Our model is local in that the algebra of operators has a local structure**

Spine Coral Skein Module

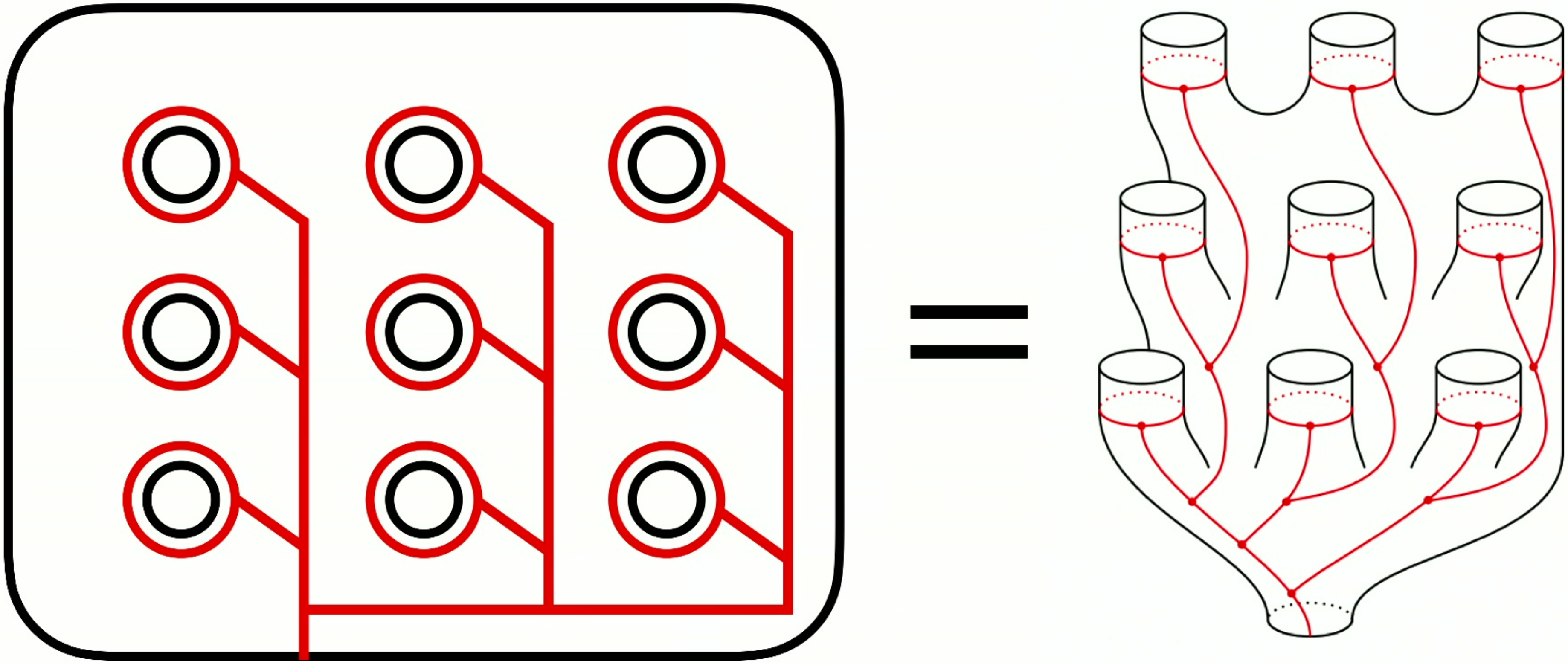




C. Wyville Thomson. New York, Macmillan and co, 1873

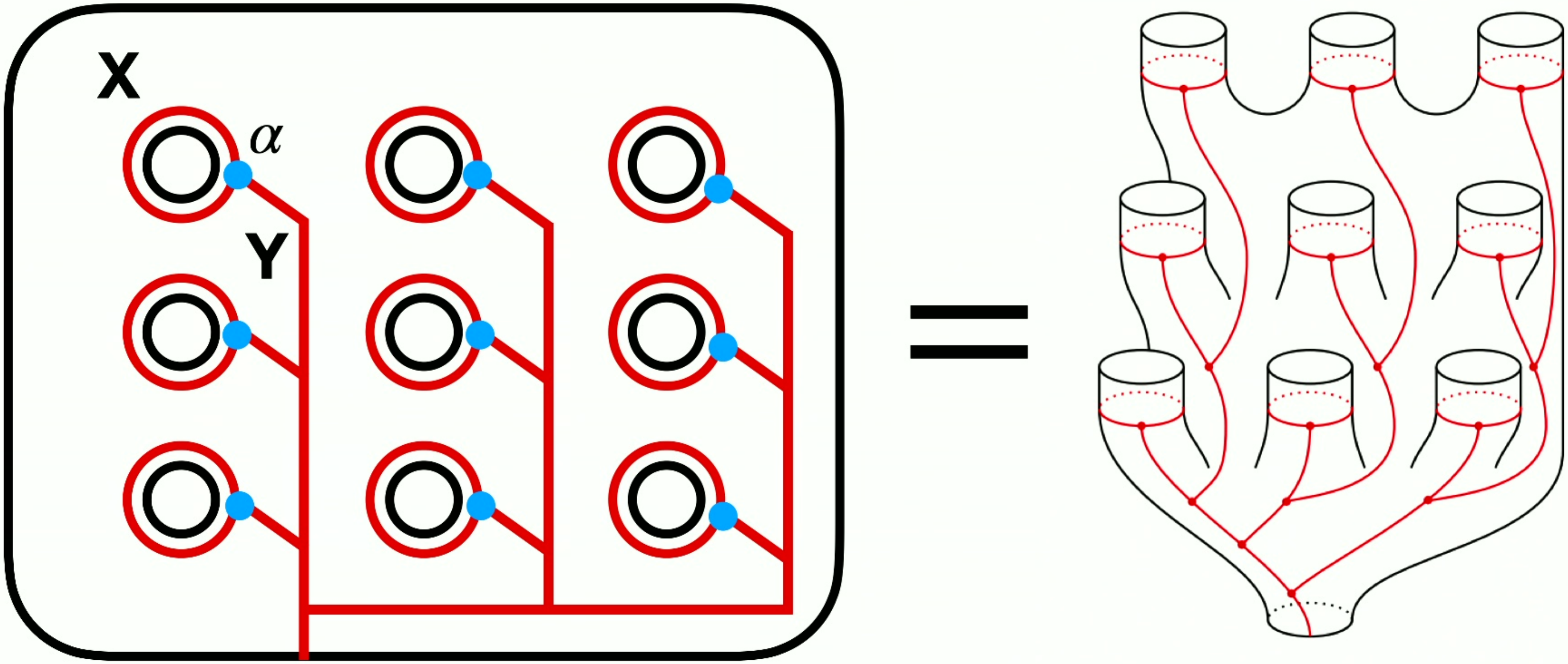
KK, C. Jones, S. Sanford, D. Green, D. Penneys, arXiv: 2401.13838 (2024)

Ungauged Levin-Wen Model

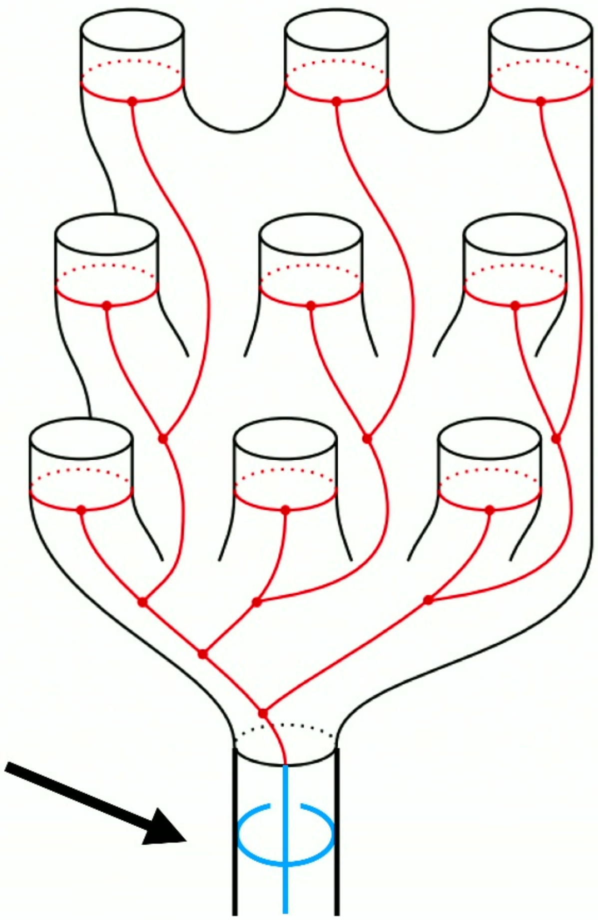


Ungauged Levin-Wen Model

Label basis states by line and vertex labels

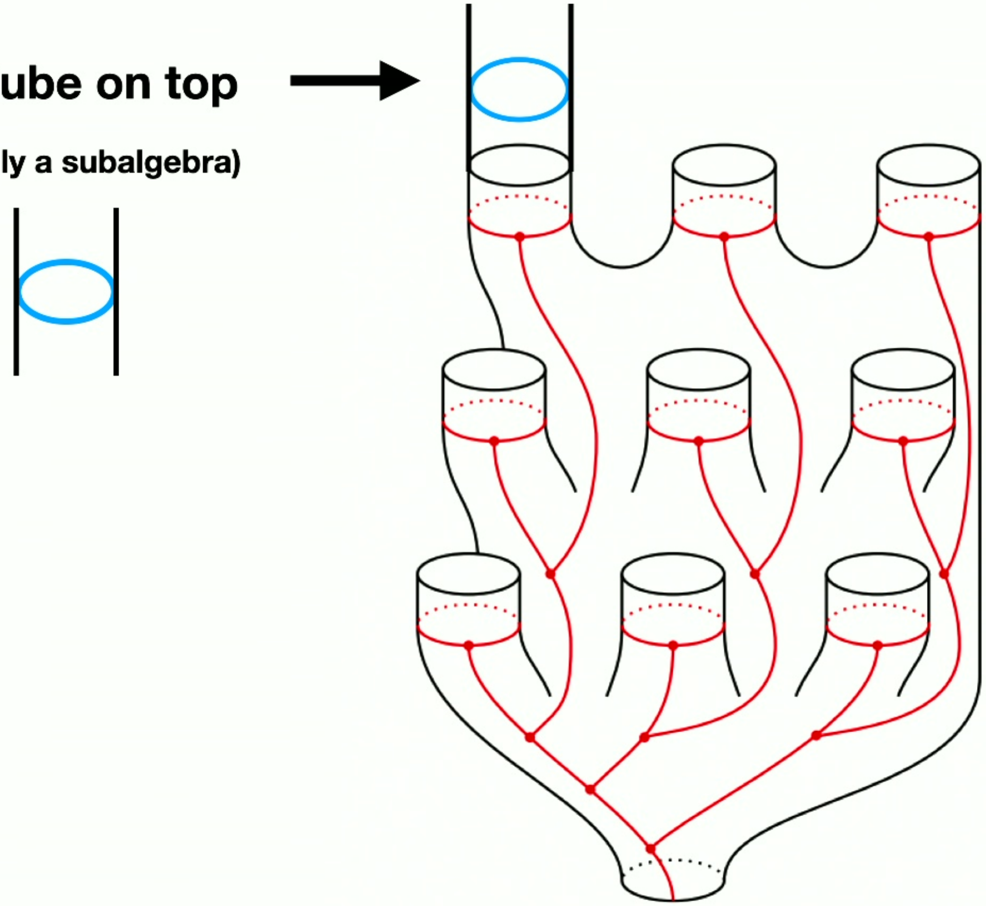


Tube algebra symmetry action



Act tube on bottom

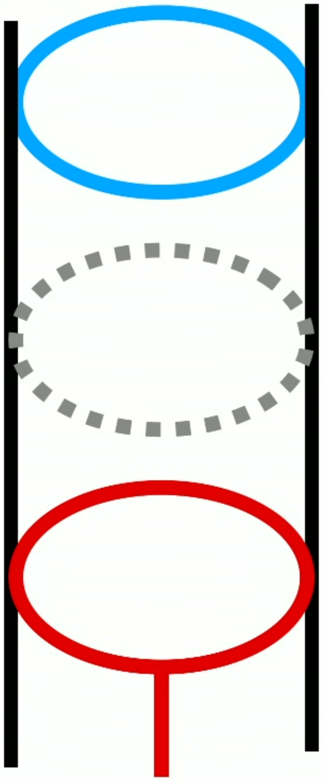
Tube algebra as local operators



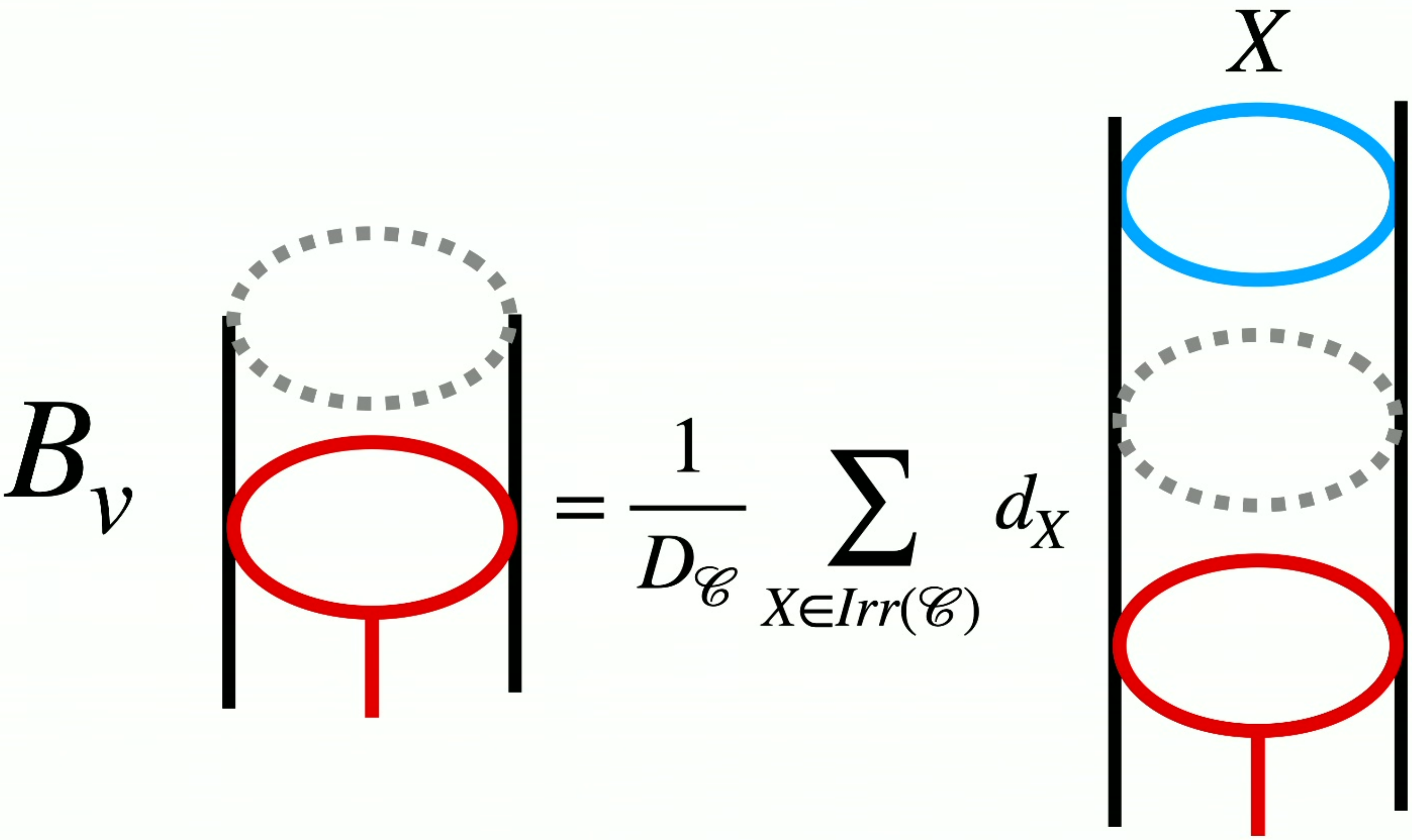
Act tube on top

(But only a subalgebra)

Tube algebra as local operators

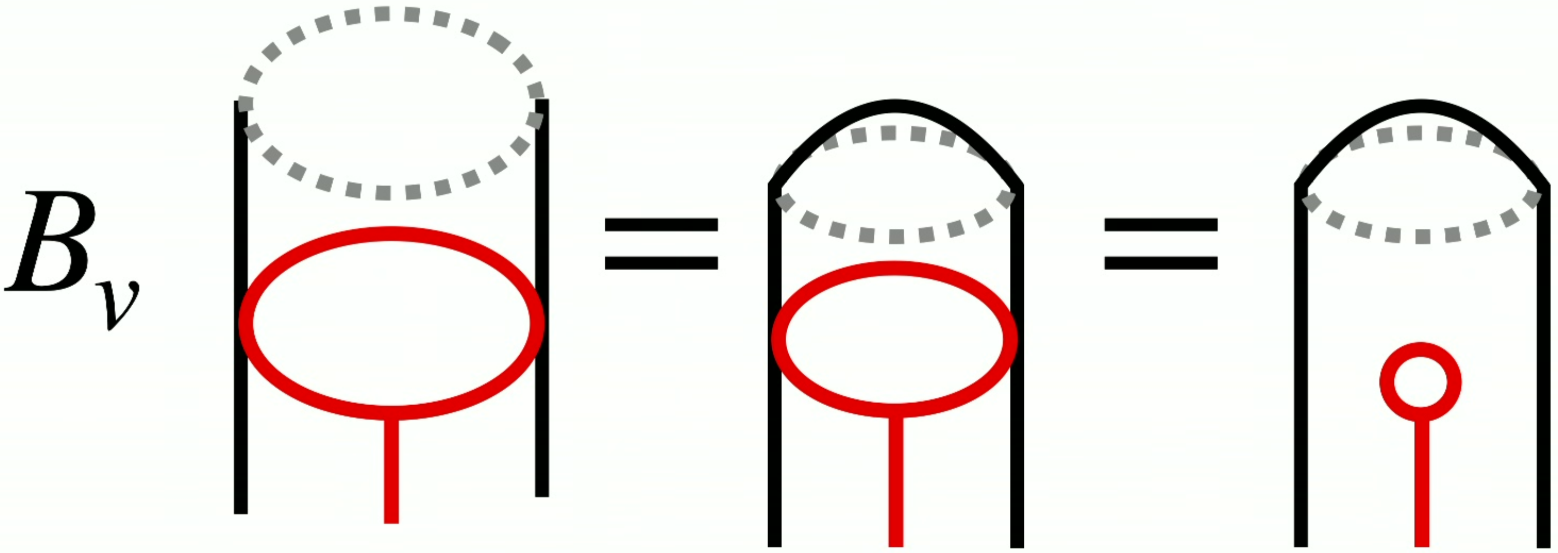


Hamiltonian



$$B_v \quad = \frac{1}{D_{\mathscr{C}}} \sum_{X\in Irr(\mathscr{C})} d_X$$

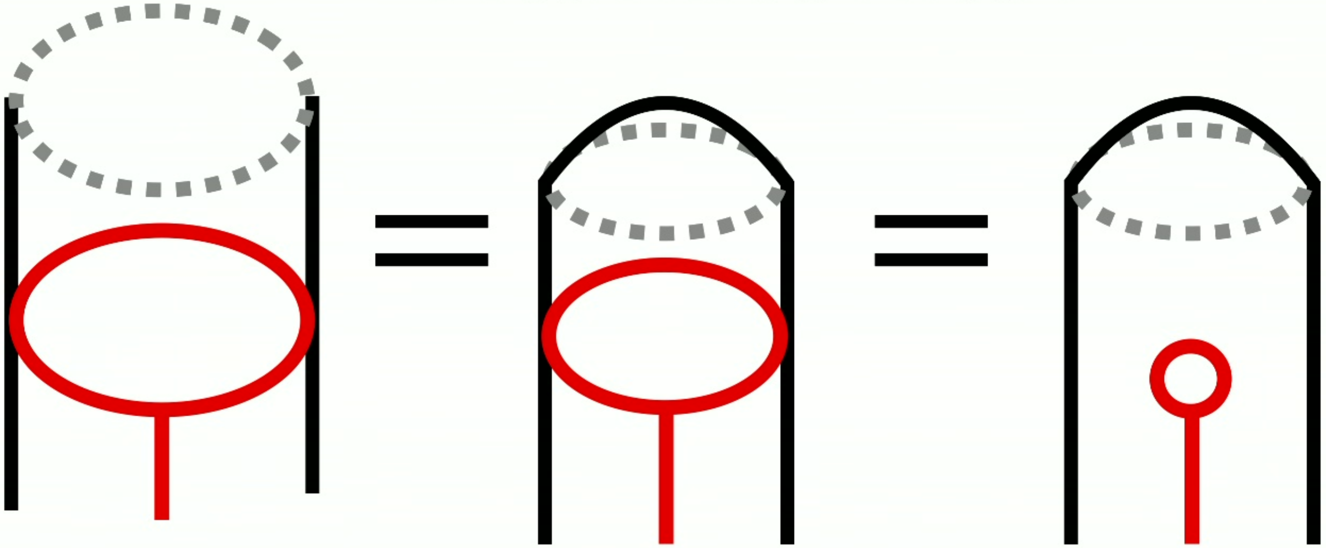
Hamiltonian



Only non-zero if line is 1

Hamiltonian

B_v

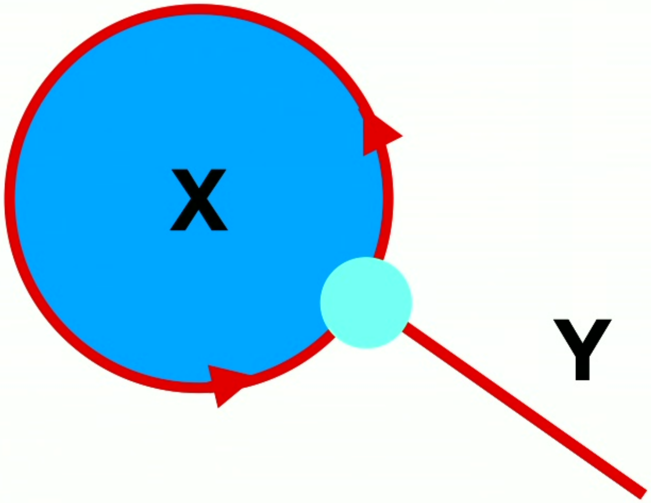


Only non-zero if line is 1

- Commuting projector model
- Ground state is empty, capped off coral
- Gapped ground state on finite lattice with symmetric Hamiltonian implies* trivial order

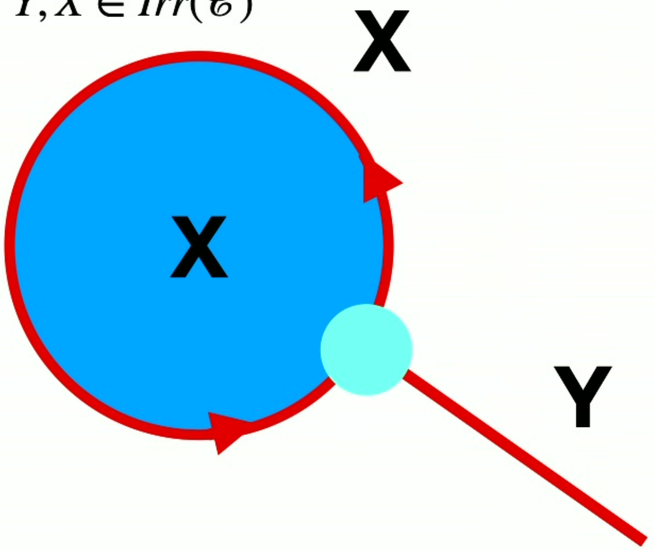
*in standard story

Domain interpretation



Domain interpretation

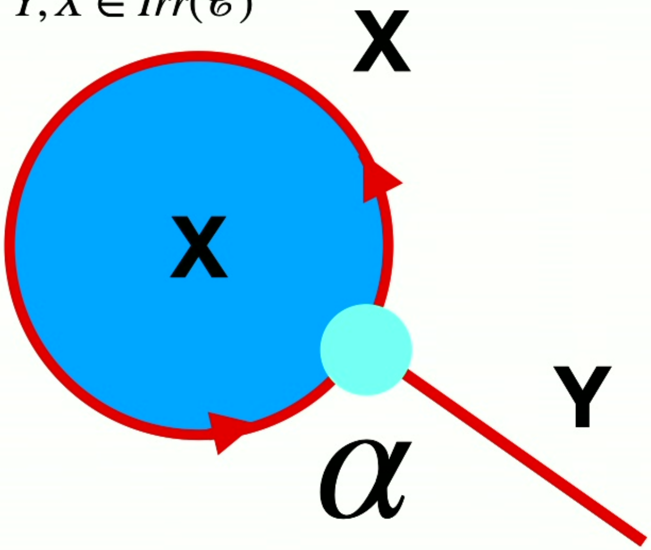
$$\mathscr{H}_v = \bigoplus_{Y,X \in Irr(\mathscr{C})} \mathscr{C}(Y \to X^\vee X)$$



$X^\vee$ **is something like** X^{-1}

Domain interpretation

$$\alpha \in \mathscr{H}_v = \bigoplus_{Y,X \in Irr(\mathscr{C})} \mathscr{C}(Y \to X^\vee X)$$

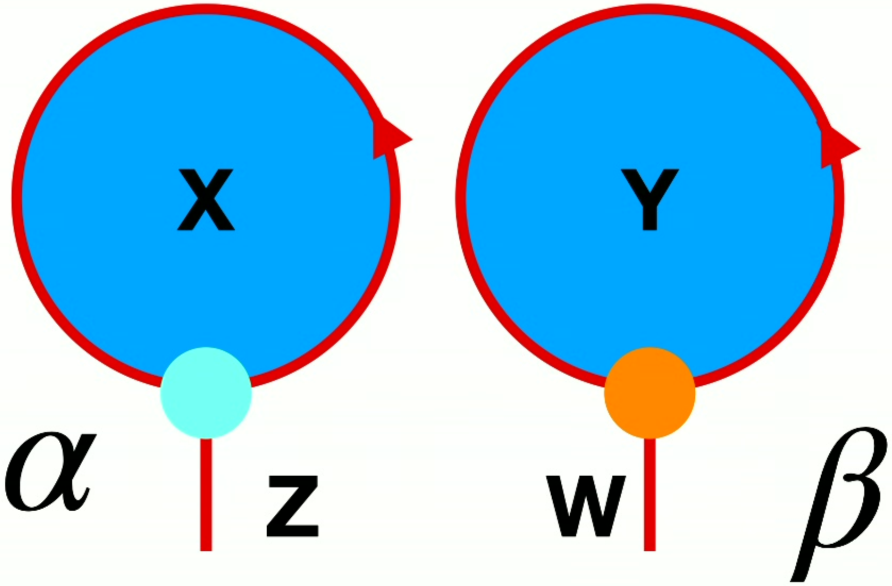


X **domain twisted by** (α, Y)

$X^\vee$ **is something like** X^{-1}

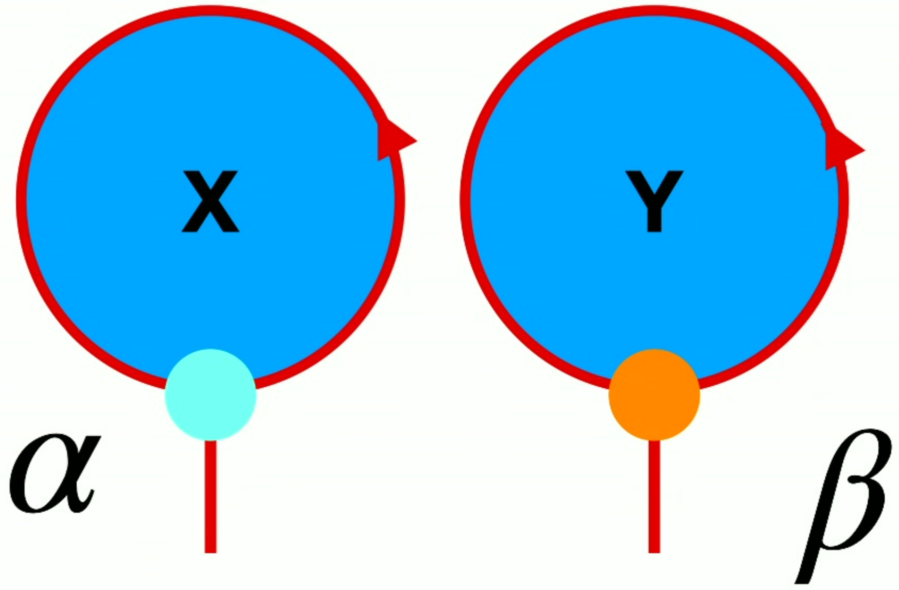
Mathematical formalism

Day convolution product builds spine coral

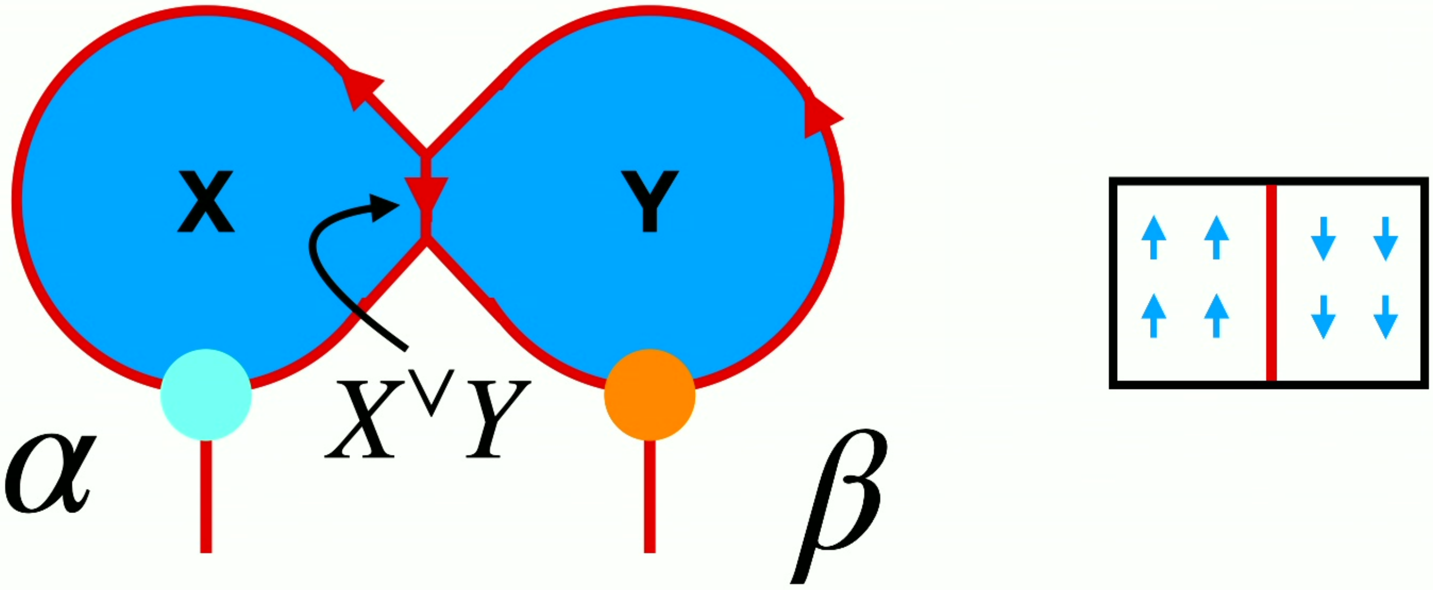


$$(\alpha \boxtimes \beta)_Q \in (H \boxtimes K)_Q = \sum_{Z,W,X,Y \in Irr(\mathscr{C})} \mathscr{C}(Q \to ZW) \otimes \mathscr{C}(Z \to X^\vee X) \otimes \mathscr{C}(W \to Y^\vee Y)$$

Kramers-Wannier transformation

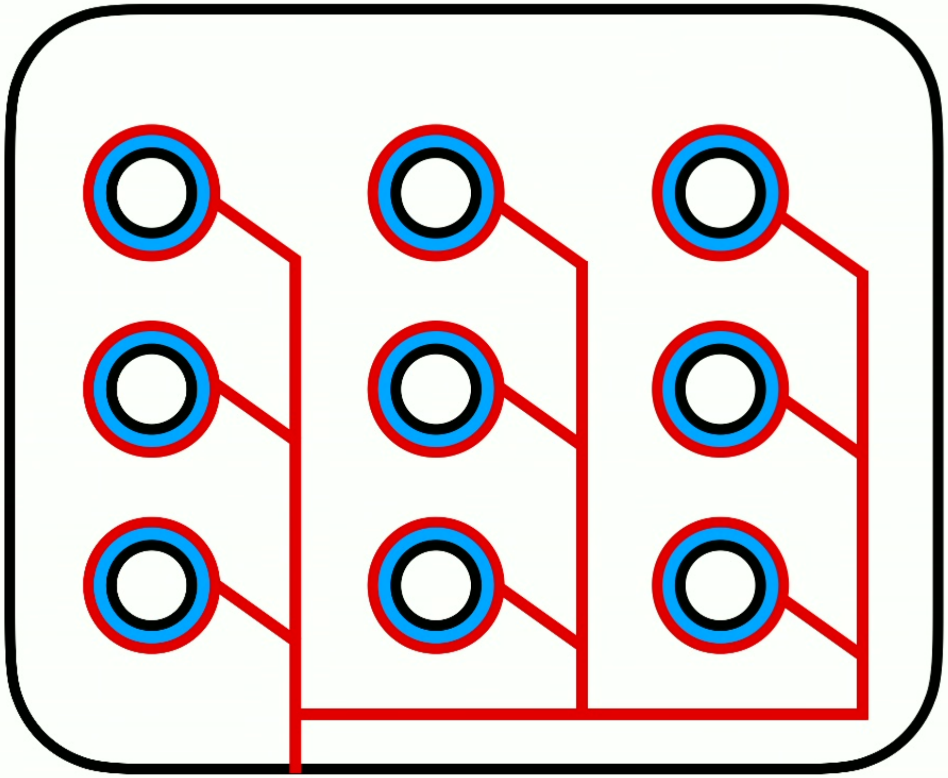


Kramers-Wannier transformation

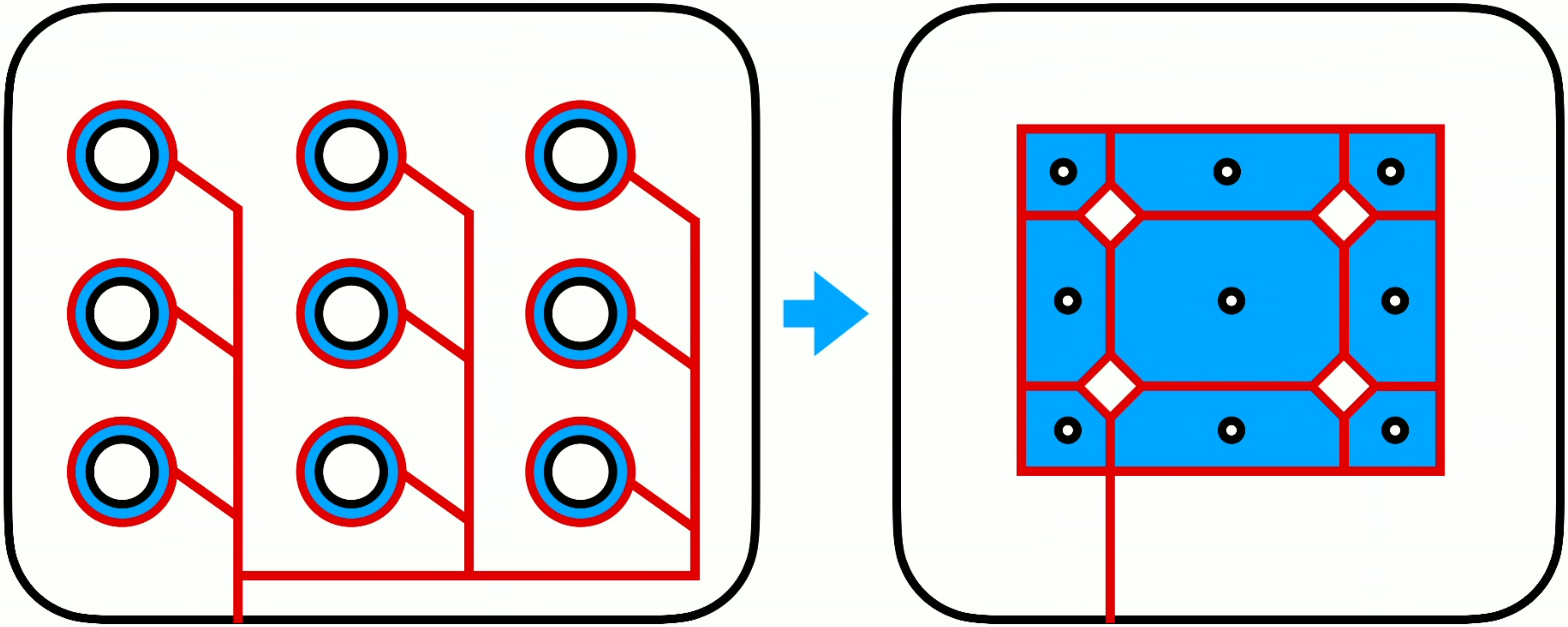


$X^\vee$ is something like X^{-1}

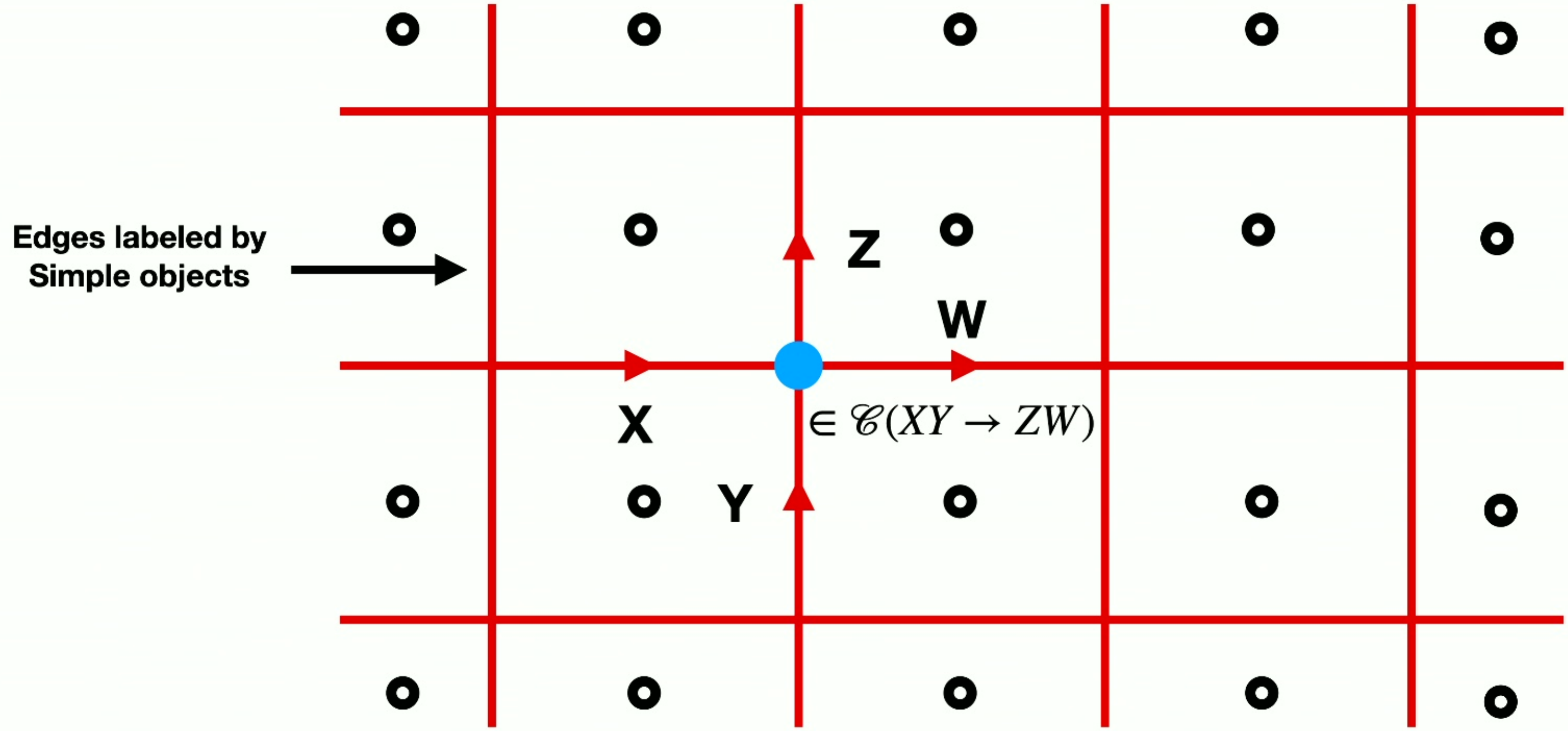
Krammers-Wannier Gauging



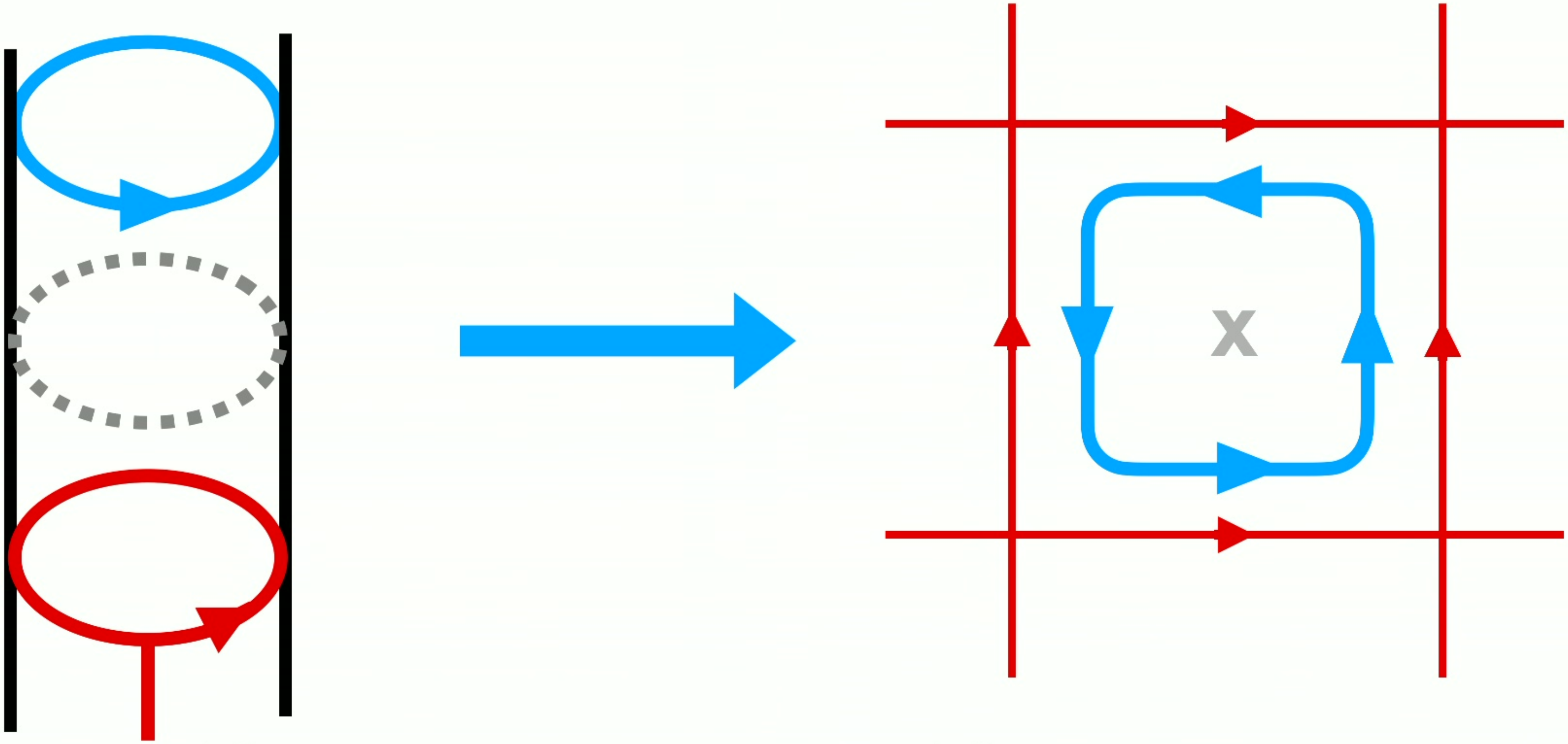
Krammers-Wannier Gauging



Flux-Free Levin-Wen Hilbert Space

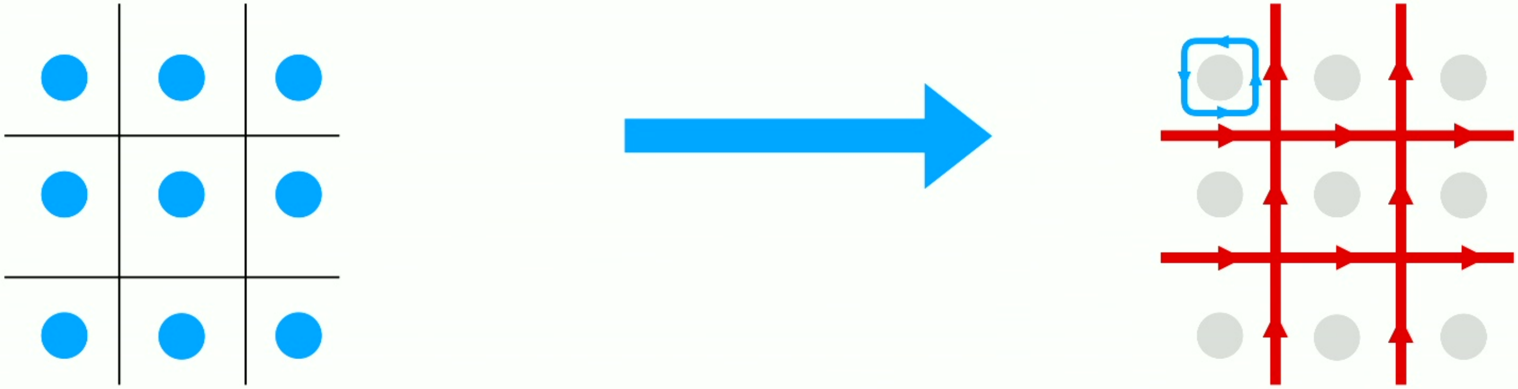


Induced Hamiltonian



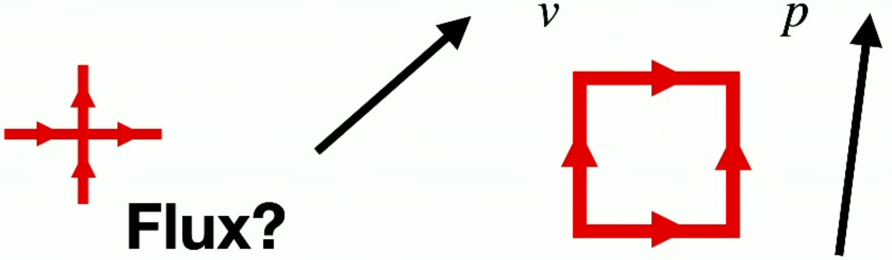
$$B_v \longrightarrow B_p$$

Levin-Wen Krammers-Wannier Gauging



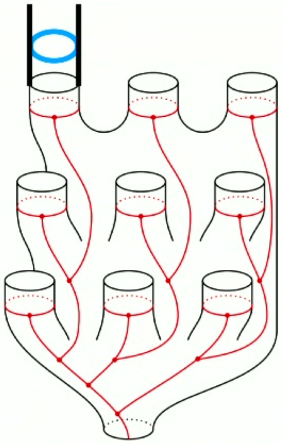
$$H = ??? \longrightarrow H = -\sum_v A_v - \sum_p B_p$$

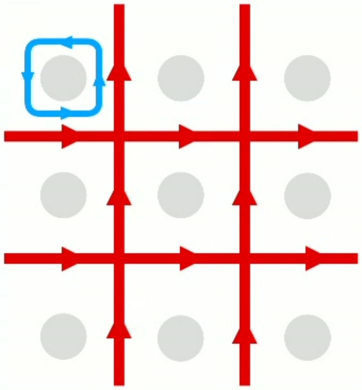
$$\mathscr{H}_T = ???$$



Charge?

Levin-Wen Krammers-Wannier Gauging



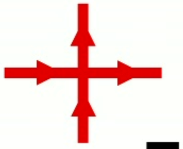


Maps to image of A_v projections

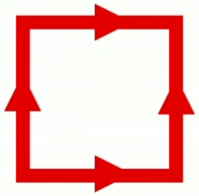
$$H = -\sum_v B_v \longrightarrow H = -\sum_v A_v - \sum_p B_p$$

$$\mathscr{H}_T = \boxtimes_v \mathscr{H}_v$$

Day

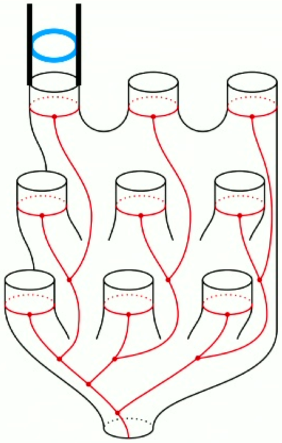


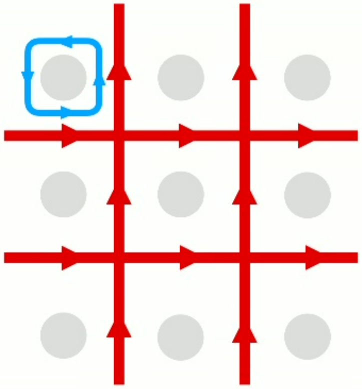
Flux



Charge

Levin-Wen is a gauge theory!



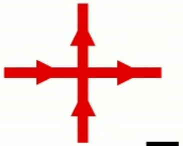


Maps to image of A_v projections

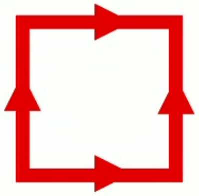
$$H = -\sum_v B_v$$

$$H = -\sum_v A_v - \sum_p B_p$$

$$\mathscr{H}_T = \underset{\text{Day}}{\boxtimes_v} \mathscr{H}_v$$



Flux



Charge

Outline

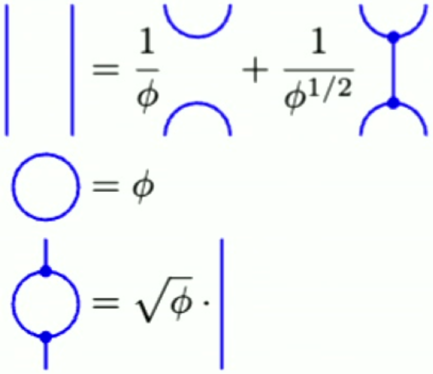
1. Toric Code as gauge theory
2. Levin-Wen through diagrammatics
3. Levin-Wen as gauge theory
4. Example: Fibonacci symmetric model

Fibonacci Category — Fib

- **Simple objects:** $1, \tau$
- **Fusion rules:** $\tau \otimes \tau = 1 \oplus \tau$ $\quad (\tau^{\otimes N} = F_{N-1} 1 \oplus F_N \tau)$

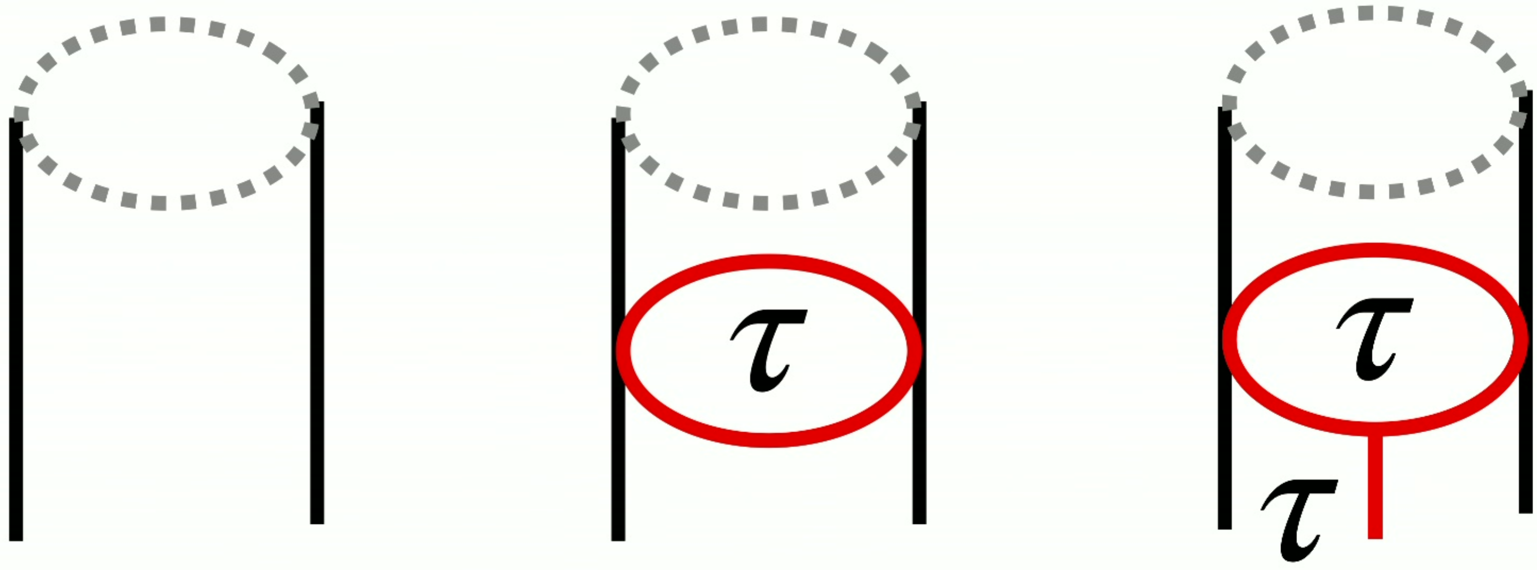
Fibonacci Category — Fib

- **Simple objects:** $1, \tau$
- **Fusion rules:** $\tau \otimes \tau = 1 \oplus \tau$ $\quad (\tau^{\otimes N} = F_{N-1} 1 \oplus F_N \tau)$



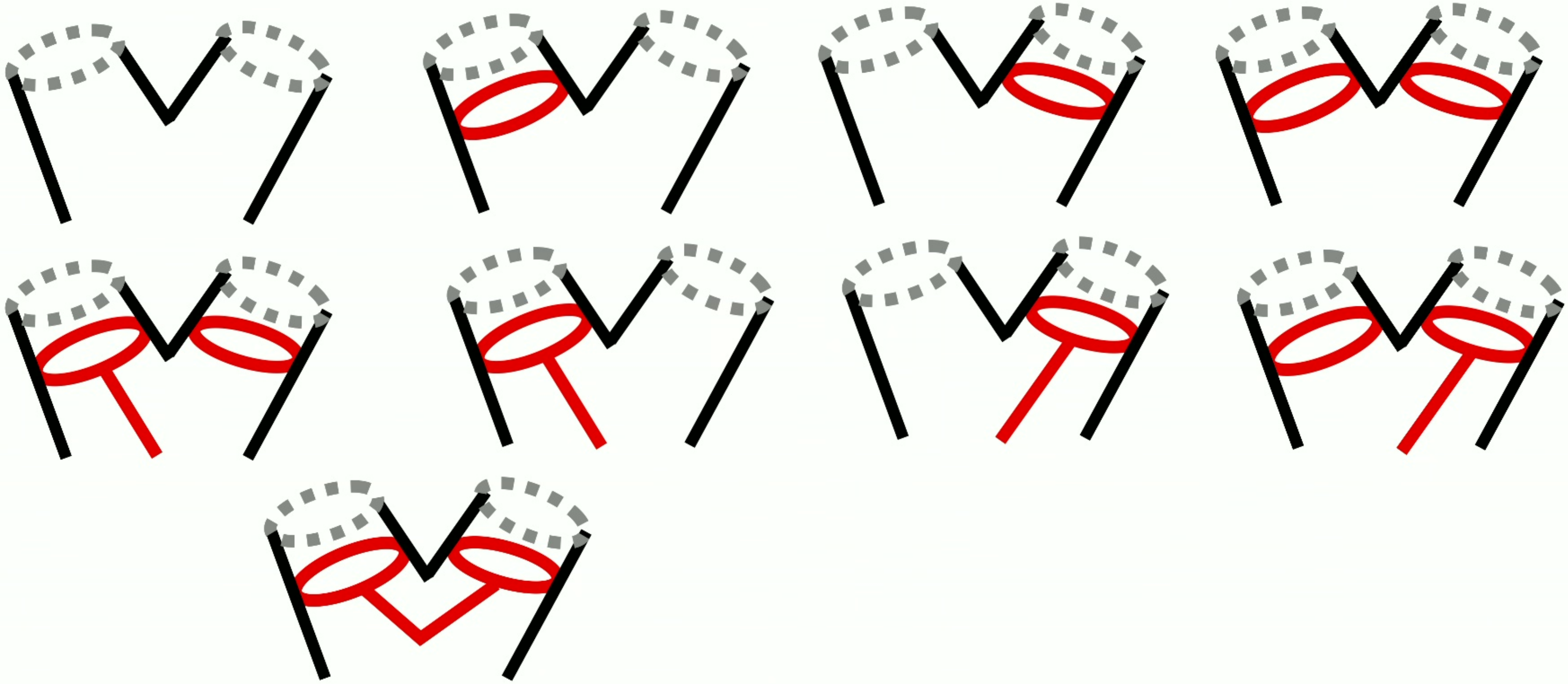
KK, C. Jones, S. Sanford, D. Green, D. Penneys, arXiv: 2401.13838 (2024)

Fibonacci Site

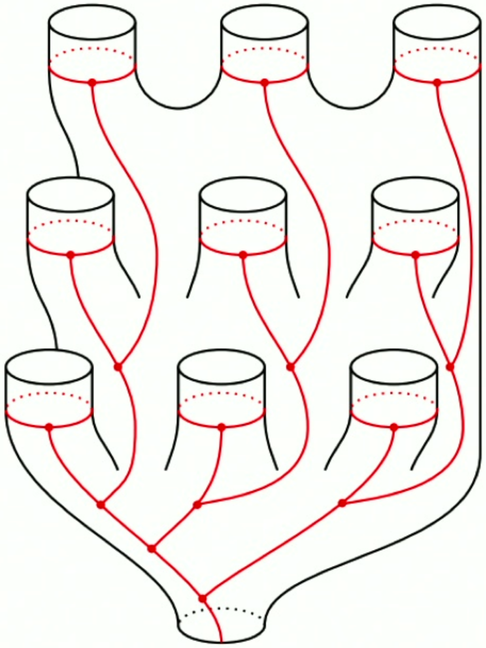


3D

Two Fibonacci Sites



N Fibonacci Sites

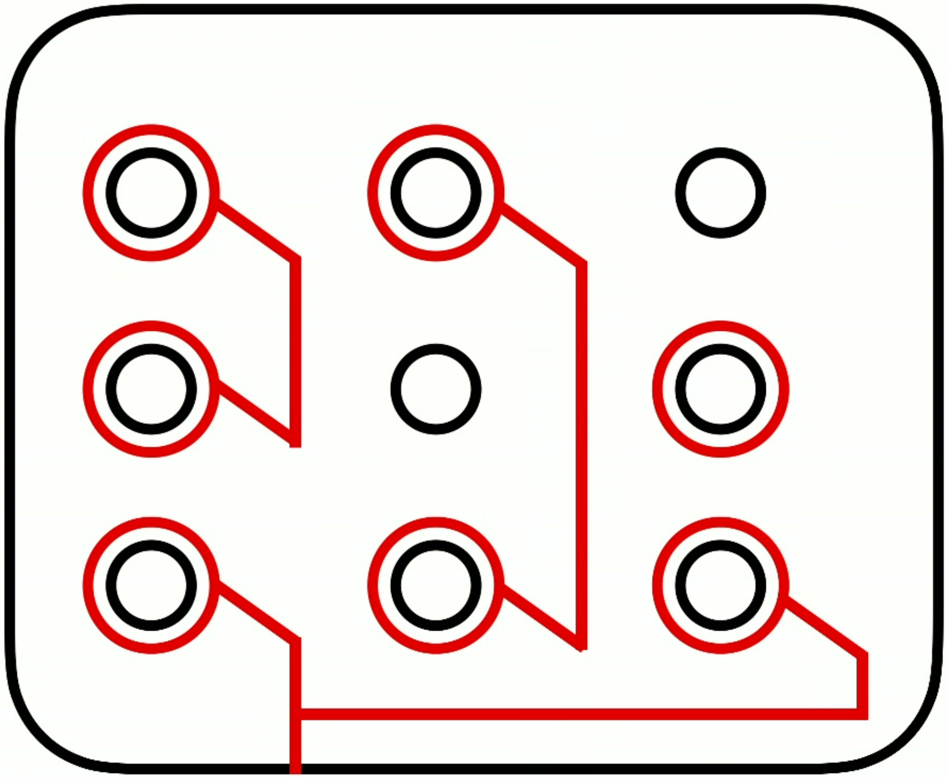


$$\left(\frac{1}{2}-\frac{\sqrt{5}}{10}\right)\left(\frac{5}{2}-\frac{\sqrt{5}}{2}\right)^{N}+\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{5}{2}\right)^{N}$$

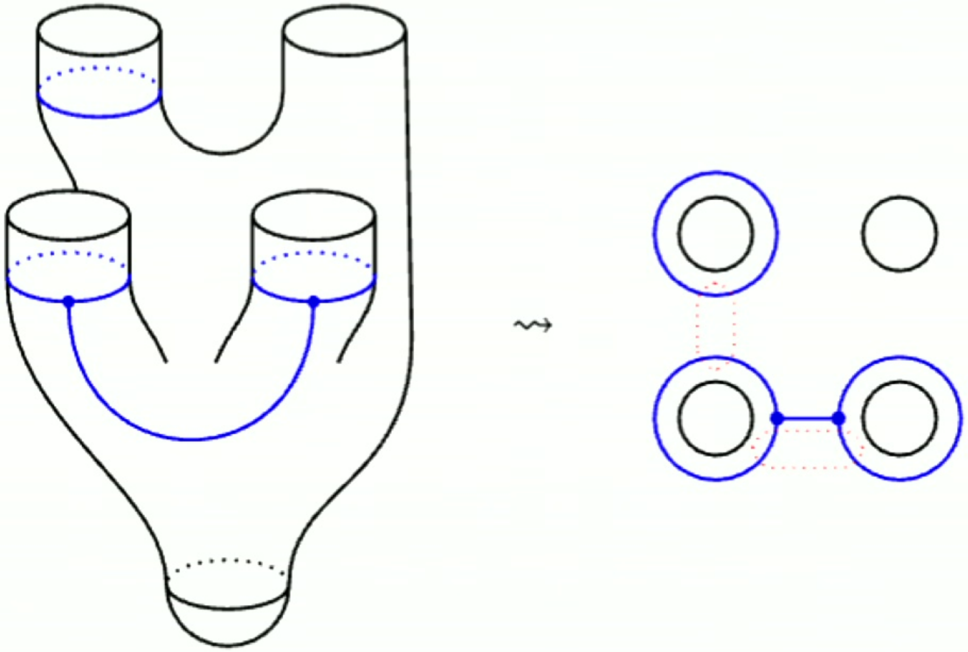
Dimensions

(OEIS A081567)

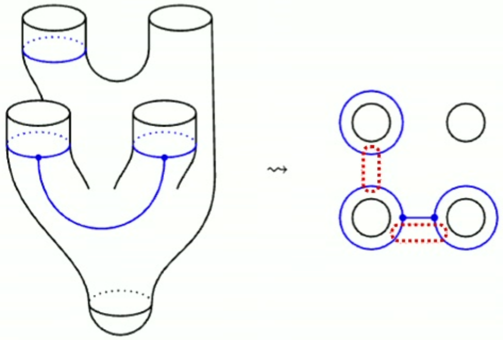
Example State



Example: Gauging of Fib



Example: Gauging of Fib



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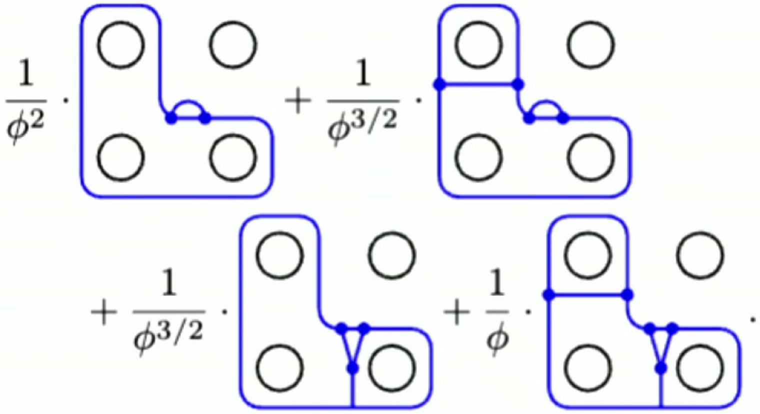

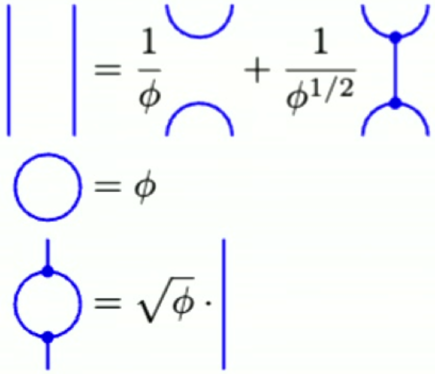

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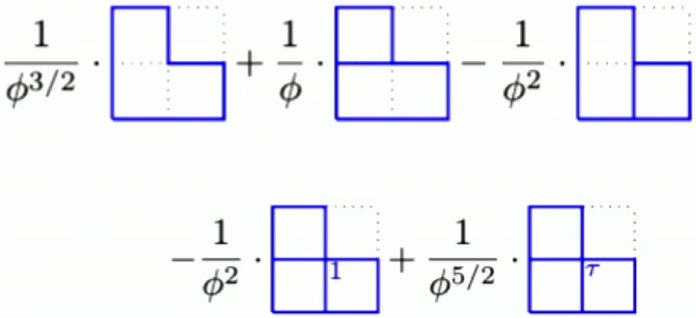

Conclusion

- The Levin-Wen model is a gauge theory
- Tube algebra as generalized gauge symmetry
- We showed the gauging procedure
- B_p measures charge and A_v measures flux
- Gave new physical meaning to $Rep(Tube(\mathscr{C})) \cong Z(\mathscr{C}) =$ Levin-Wen anyons

(Details in extra slides)

- New Tube algebra SPT models — Example: Fibonacci SPT

Main results alert!

Other research to talk about!

- Continuous symmetry enriched topological order
- 2-group crossed fusion categories
- Operator algebraic superselection theory
- Anomaly detection in N-dimensional symmetry protected phases
- Boundary algebras of topological order
- Boundary theories of Walker-Wang