Title: Tidal Deformations of Black Holes

Speakers: Maria Rodriguez

Series: Strong Gravity

Date: February 08, 2024 - 1:00 PM

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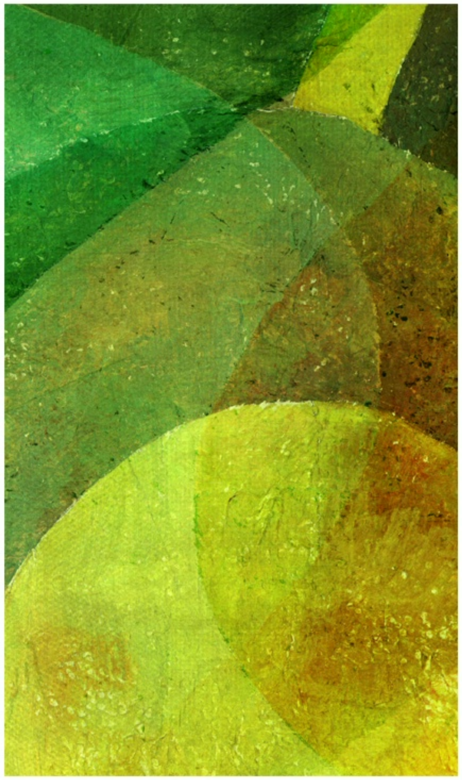
Abstract: Recent developments point to a remarkable resistance of black holes to tidal deformations under the influence of external gravitational fields. Relying on hidden symmetries, compelling progress has been achieved to explain that the Love numbers, characterizing tidal deformations for Kerr black holes, vanish. How does the phenomenon of tidal squeezing manifest in broader dynamical contexts? An examination of the dynamical tidal deformations in rotating black holes will be presented.

Zoom link



Dynamical Love Numbers for Kerr Black Holes

Maria J Rodriguez

UtahStateUniversity ift

On sabbatical

BLACK HOLE INITIATIVE

Perimeter Institute - Strong Gravity Seminar- Feb, 8th, 2024

Perimeter Institute

James Bardeen

Maria J Rodriguez

Tidal Deformations of Black Holes

<u>Outline</u>

1) Motivation

2) Review: **Static** Love Numbers for Black Holes (BHs)

"Love Symmetry" by Charalambous, Dubovsky and Ivanov arXiv:2209.02091 [hep-th]
"Near-Zone Symmetries of Kerr Black Holes" by Hui, Joyce, Penco, Santoni and Solomon. arXiv:2203.08832 [hep-th]

3) Dynamical Tidal Coefficients for BHs

"Dynamical Tidal Love Numbers for Kerr Black Holes" by Malcolm Perry and M.J.R. arXiv: 2310.03660 [gr-qc]

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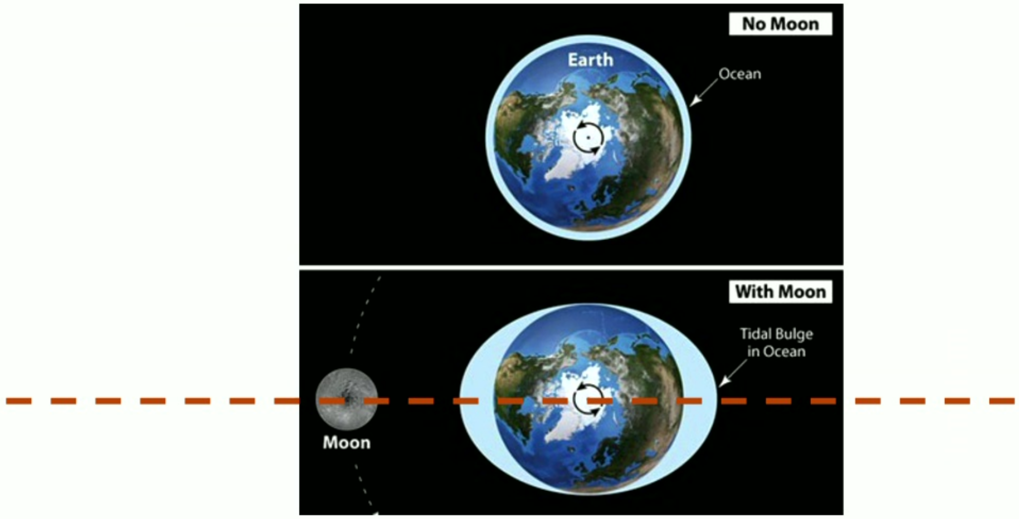
Tidal deformations are a gravitational phenomenon that causes a body to stretch along the line pointing towards and away from the center of mass of another compact object.



This is a result of spatial variations in the gravitational field exerted on one body by another, that is not constant across its parts.

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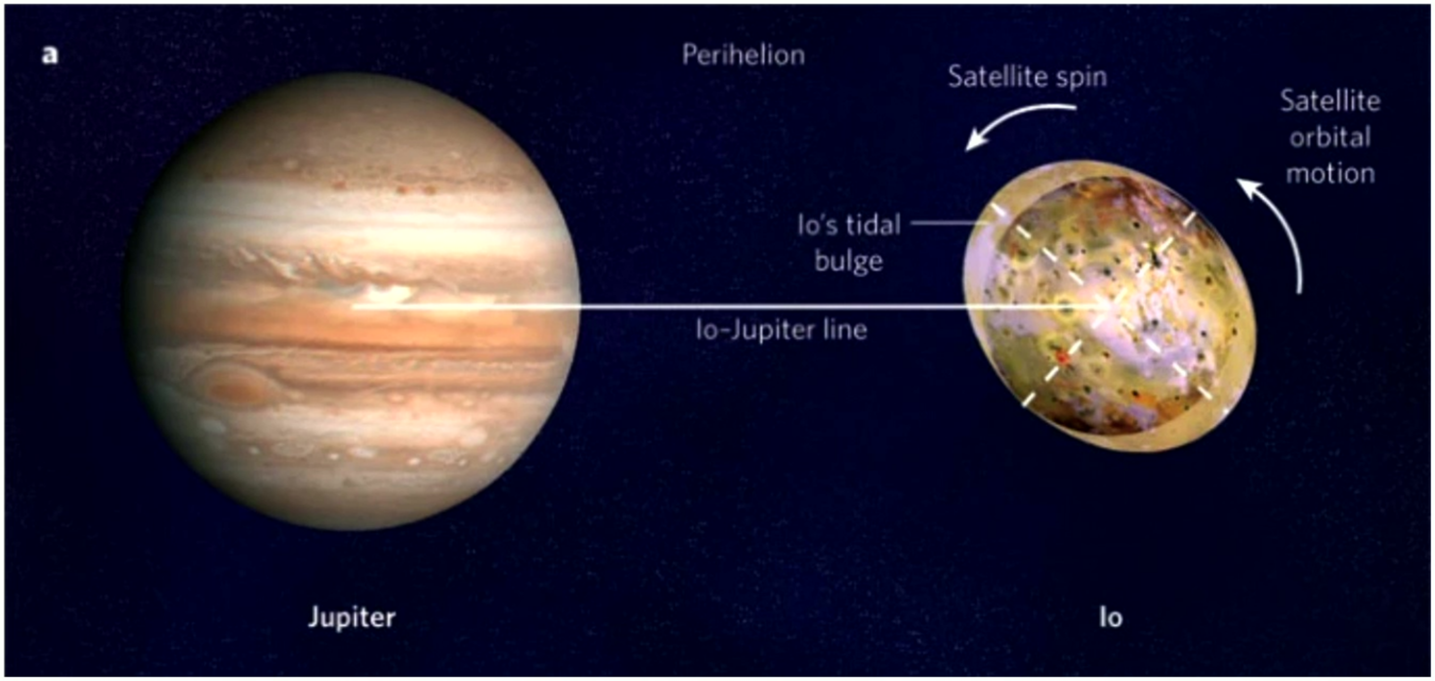
Tidal Squeezing in the Solar System

Figure 1: Jupiter–Io tidal interaction.



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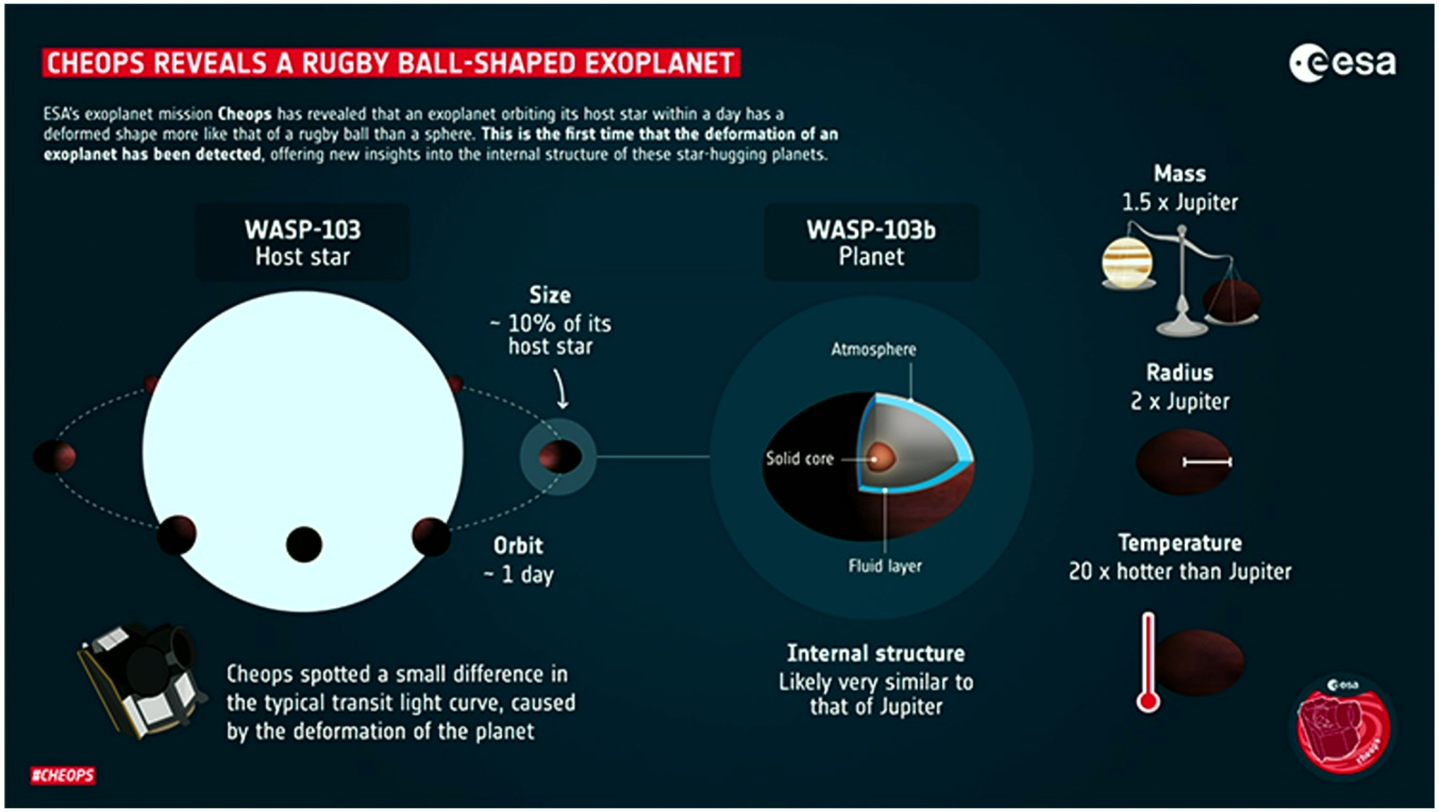
Tidal Squeezing in Exoplanets



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Galactic Tidal Deformations

Hubble Space Telescope
Seyfert galaxy NGC 169 (bottom) and the galaxy IC 1559 (top)

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Significance: compact objects with distinct internal compositions undergo distinct deformations.

Tidal squeezing in the farm.

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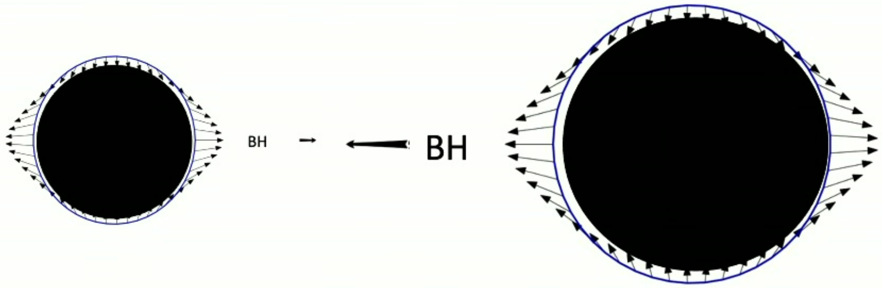
Tidal Deformation of Black Holes

Fundamental Idea:



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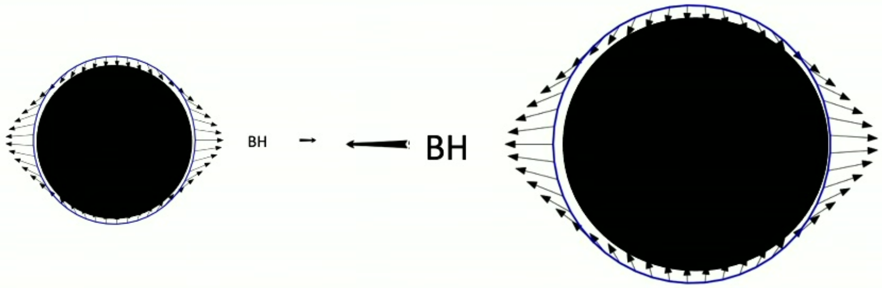
Tidal Deformation of Black Holes

Fundamental Idea:



Black Holes are nothing, simply boundaries of space-time.

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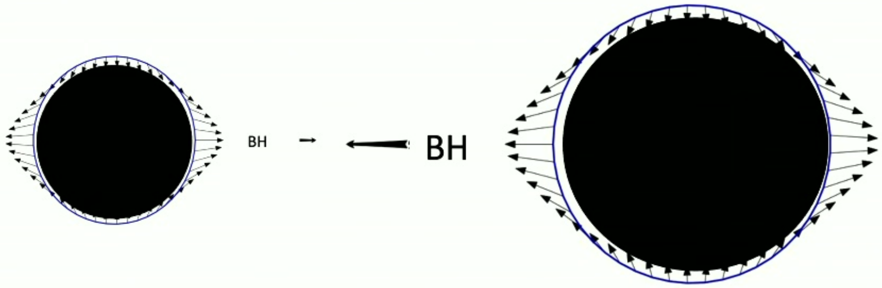
Tidal Deformation of Black Holes

Fundamental Idea:



Black Holes are nothing, simply boundaries of space-time.

Can we tidally squeeze BHs?

How are tidal deformations for BHs characterized?

Can we explain universal features of tidal deformations of black holes?

What can we learn from BH tidal deformations?

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Tidal deformations are a gravitational phenomenon that causes a body to stretch along the line pointing towards and away from the center of mass of another compact object.

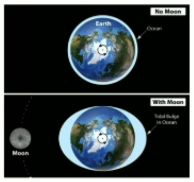
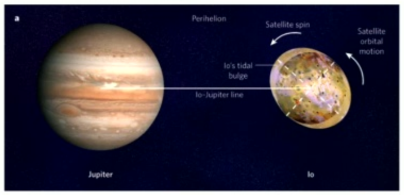
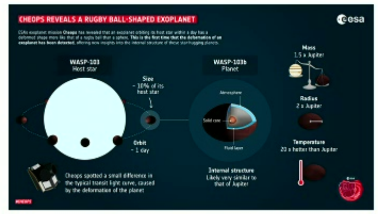


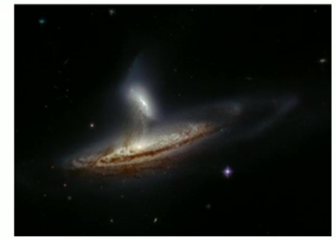
Figure 1: Jupiter–Io tidal interaction.



Tidal Squeezing in the Solar System



Tidal Squeezing beyond the Solar System



Galactic Tidal Deformations

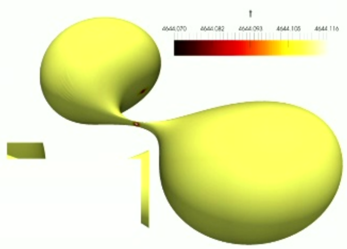


FIG. 1. Event horizon with a toroidal topology, shown in a different time slicing than the one used in the SpEC simulation.

Bohn, Kidder Teukolsky

Tidal Squeezing in Binary Black Hole Mergers

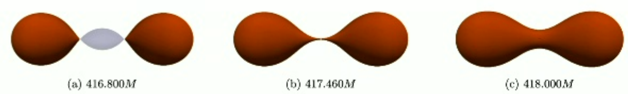


FIG. 8. Event horizon generator surfaces for the equal mass head-on binary. The t slicing in the top row is almost identical to

Bohn, Kidder Teukolsky

Tidal Squeezing in Head-on Black Hole Collisions

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Significance: compact objects with distinct internal compositions undergo distinct deformations.

Binary Black Holes

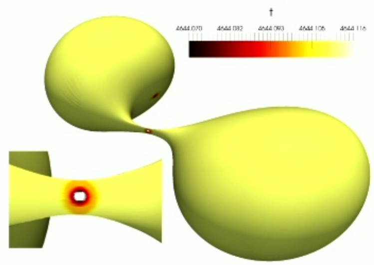

FIG. 1. Event horizon with a toroidal topology, shown in a different time slicing than the one used in the SpEC simulation.

Therefore, the extent of the tidal deformation should be discernible in the gravitational wave and in turn be intricately linked to the inner structure of the entity.

The internal structure of certain objects is governed by he poorly understood nuclear matter in e.g. **NS and new unexpected effects in black holes.**

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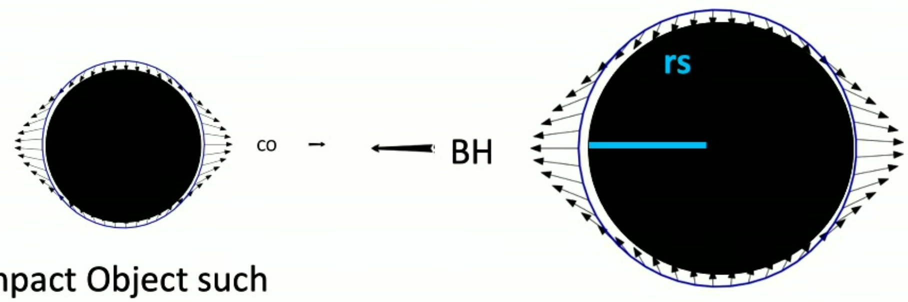
Tidal deformations in General Relativity (GR)

An important observation is that the tidal response coefficients, first identified by Love, k_{lm} can be extracted directly from the solutions of the wave equation for all fields (integer spin fields):



CO = Compact Object such as another BH or NS

Gravitational external potential

$$\Phi = -\frac{M}{r} + \frac{(\ell-2)!}{\ell!} \sum_{\ell=2} \sum_{m=-\ell}^{\ell} Y_{\ell m}\, \mathcal{E}_{\ell m}\, r^{\ell} \left[1 + k_{\ell m} \left(\frac{r}{r_s} \right)^{-2\ell-1} \right] ,$$

r/rs the dimensionless distance to the body

E_{lm} multipole moment

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In the GR picture, it is convenient to look at the metric perturbation for the description of the tidal deformation of the rigid object

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \, .$$

The linearized Einstein equations for a small perturbation around flat Minkowski space

$$\Box \, \bar{h}_{\mu\nu} = -16\pi G \, T_{\mu\nu} \, ,$$

Definition

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \tfrac{1}{2} \, h \, \eta_{\mu\nu}$$

Gauge choice

$$\nabla_\mu \bar{h}^{\mu\nu} = 0.$$

Point like source

$$T_{\mu\nu} = (2/\sqrt{-g})(\delta I_{matter}/\delta \, g^{\mu\nu}),$$

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In the case of black holes in 4D, the temporal metric component perturbations h_{tt}, in the long r-distance limit can be written as

$$2\, h_{tt} = \frac{M}{r} - \frac{(\ell-2)!}{\ell!} \sum_{\ell=1} \sum_{m=-\ell}^{\ell} Y_{\ell m}\, \mathcal{E}_{\ell m}\, r^{\ell} \left[1 + k_{\ell m} \left(\frac{r}{r_s} \right)^{-2\ell-1} \psi_{resp} \right] .$$

By comparison with the Newtonian gravitational potential, one finds a useful way to extract tidal response coefficients from a gravitational potential generated by an external source in GR.

$$\Phi = -\frac{1}{2}\, h_{tt} \,,$$

Therefore, at first glance, this provides us with a practical prescription, such that the tidal response coefficients for Kerr black holes can be computed by using the so called Teukolsky equation

$$\nabla \Phi_s = 0 \,,$$

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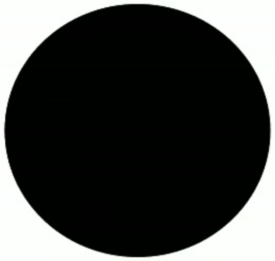
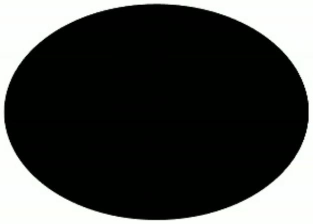
Kerr black holes



Characterized by
mass M and spin parameter a

Tidally Deformed Kerr BHs



Characterized by
mass M and spin parameter a and perturbations $h_{\mu\nu}$

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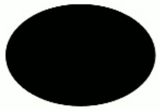
The problem of tidal deformations of Kerr BHs



$$\Phi = -\frac{1}{2}\, h_{tt}\,,$$

Reduces to solving the massless scalar wave-equation equation $\quad \nabla \Phi_s = 0\,, \qquad s = 0, \pm 1, \pm 2,$

$$\Phi_s(t,r,\theta,\phi) = e^{-i\omega t + im\phi} R_s(r) S_s(\theta)\ , \quad \text{with} \quad \omega \in \mathbb{C} \quad \text{and} \quad m \in \mathbb{Z}\ .$$

Boundary conditions. The radial functions must meet the following ingoing boundary conditions at the horizon

$$\hat{R}_s(r) = const \times (r - r_+)^{-i\alpha_+}\,, \text{ with } \alpha_+ > 0 \qquad \text{as} \qquad r \to r_+\,.$$

$$\Omega = a/(2Mr_+)$$

where we defined the coefficient

$$T_+ = (r_+ - r_-)/(8\pi M r_+).$$

$$\alpha_+ \equiv \frac{(\omega - m\,\Omega)}{4\pi\, T_+} \pm \frac{is}{2}$$

$$r_\pm = M \pm \sqrt{M^2 - a^2},$$

Analytic Continuation $\quad \hat{R}_s(r) \xrightarrow[r\to\infty]{} \tilde{c}_1\, r^\ell \left(1 + \left(\frac{r}{r_s}\right)^{-(1+2\ell)} k_{\ell m}\right)$

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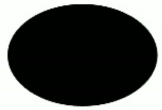
The problem of tidal deformations of Kerr BHs



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Tidal (Love) Coefficients

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Love Numbers for Kerr Black Holes (BHs)

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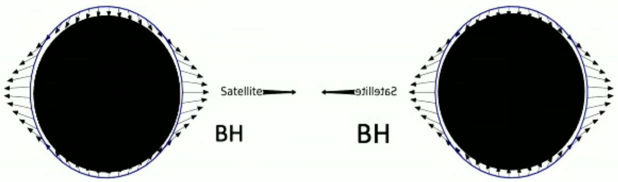
Tidal deformations of BHs



Static tidal Love numbers , $k_{lm} = 0$ for static gravitational deformations ($\omega = 0$)

Dynamical tidal Love numbers , $k_{lm} \neq 0$ for dynamical gravitational deformations ($\omega \neq 0$)

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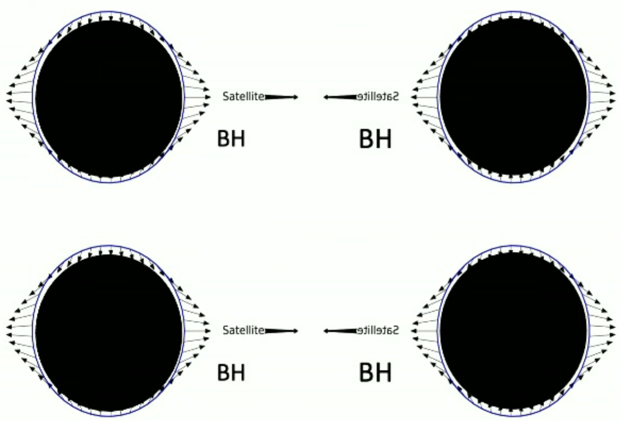
Tidal deformations of BHs



Static tidal Love numbers , $k_{lm} = 0$ for static gravitational deformations ($\omega = 0$)

Dynamical tidal Love numbers , $k_{lm} \neq 0$ for dynamical gravitational deformations ($\omega \neq 0$)

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An important observation is that the tidal response coefficients, first identified by Love, k_{lm} can be extracted directly from the solutions of the wave equation for all fields (integer spin fields) to all orders in the frequency, including static ($\omega = 0$) and dynamical ($\omega \neq 0$) responses.

The gravitational tidal coefficients, k_{lm}, describe the tidal response of a rigid object e.g. star, planet or black hole.

$$k_{\ell m}(\omega) = \underbrace{\kappa_{\ell m}(\omega)}_{\text{Conservative effects or Love numbers}} + i \underbrace{\nu_{\ell m}(\omega)}_{\text{Dissipative effects}} .$$

Conservative effects or Love numbers

Dissipative effects

Static Love Numbers for Kerr BHs

Kerr BH static tidal deformation coefficients defined by

$$\hat{R}_s(r) \xrightarrow[r\to\infty]{} \tilde{c_1}\, r^{\ell} \left(1 + \left(\frac{r}{r_s}\right)^{-(1+2\ell)} k_{\ell m}\right) ,$$

$$k_{\ell m}(\omega = 0) = \kappa_{\ell m}(\omega = 0) + i\, \nu_{\ell m}(\omega = 0) \, ,$$

where

Static Love Numbers $\quad \kappa_{\ell m}(\omega = 0) \;=\; 0 \, ,$

Static Dissipation Coeff. $\quad \nu_{\ell m}(\omega = 0) \;=\; (-1)^{s+1} m\gamma \frac{(\ell+s)!(\ell-s)!}{(2\ell+1)!(2\ell)!} \left(\prod_{n=1}^{\ell}(n^2 + 4m^2\gamma^2)\right) \left(\frac{r_+ - r_-}{r_+ + r_-}\right)^{(1+2\ell)}$

Love Number vanishes, dissipation does not.
Is this a realization of something more fundamental?
Yes: symmetries. In that case we could observe it in the GW data.

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Dissipative Tidal Coefficients for Kerr BH

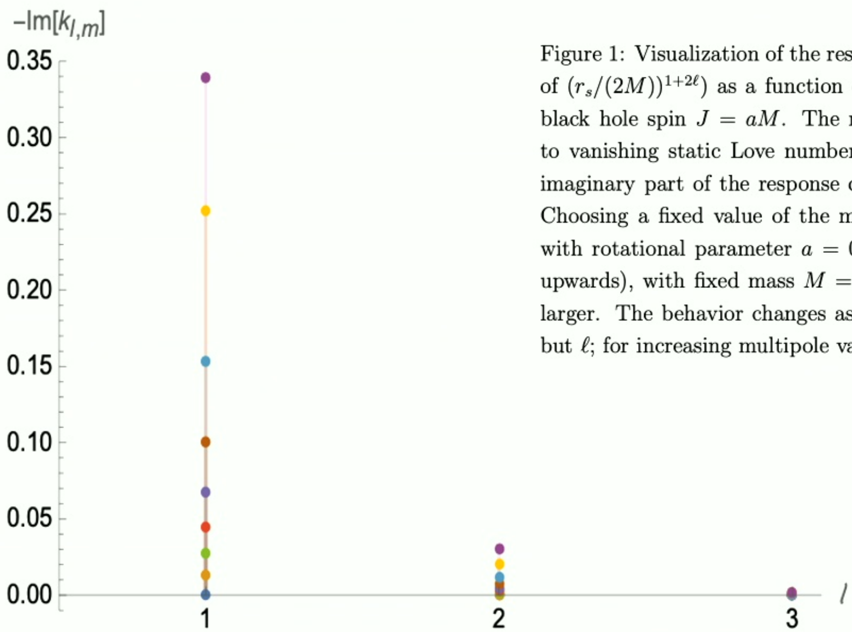

Figure 1: Visualization of the response coefficients $\lambda_{\ell m}^{Kerr}$ for Kerr black holes (3.11) (in units of $(r_s/(2M))^{1+2\ell}$) as a function of the multipole moments ℓ for various values of the Kerr black hole spin $J = aM$. The real part of the coefficients vanish $Re(\lambda_{\ell m}^{Kerr}) = 0$, leading to vanishing static Love numbers. The non trivial dissipation coefficients, defined as the imaginary part of the response coefficients $Im(\lambda_{\ell m}^{Kerr}) = \nu_{lm}(\omega = 0)$, are represented here. Choosing a fixed value of the multipole moment l, as the Kerr black hole spin increases, with rotational parameter $a = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$ (from *lightblue* to *purple* or upwards), with fixed mass $M = 1$, $s = 0$ and $m = 1$, the dissipation parameters becomes larger. The behavior changes as one compares the dissipation coefficients fixing all values but ℓ; for increasing multipole value ℓ the dissipation decreases.

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Hidden symmerties for vanishing Love numbers for Kerr BHs

SL(2,R) x U(1) → Love Symmetry

arXiv:2209.02091 [hep-th]

SO(4,2) → Starobisnky Symmetry

arXiv:2203.08832 [hep-th]

Dynamical Tidal Coefficients from Starobinsky symmetry

$$k^{Eff}_{\ell m} = \frac{\Gamma(-2\ell-1)\Gamma(1+\ell-s)\Gamma(1+\ell+2i\bar{Q})}{\Gamma(2\ell+1)\,\Gamma(-\ell-s)\,\Gamma(-\ell+2i\bar{Q})}\left(\frac{r_+-r_-}{r_++r_-}\right)^{(1+2\ell)}$$

$$\bar{Q} = Q - M\omega$$

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Hidden symmerties for vanishing Love numbers for Kerr BHs

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SO(4,2) → Starobisnky Symmetry
arXiv:2203.08832 [hep-th]

$$\bar{Q} = Q - M\omega$$

Low frequency solutions

Mano and E. Takasugi arXiv:gr-qc/9603020 [gr-qc]
Dubowsky et al arXiv:2209.02091 [hep-th].

$$\begin{aligned} k_{\ell m}(\omega) &= \frac{\Gamma(-2\nu-1)\,\Gamma(1+\nu-s-2iM\omega)\,\Gamma\,(1+\nu+2iQ)}{\Gamma(2\nu+1)\,\Gamma(-\nu-s-2iM\omega)\,\Gamma\,(-\nu+2iQ)}\left[1-2\Delta\ell\,\log\left(\frac{r_+-r_-}{r}\right)\right] \\ &\quad \times(1+A_{\ell m}\,\omega)\left(\frac{r_+-r_-}{r_++r_-}\right)^{(1+2l)}+\mathcal{O}(\omega^2) \end{aligned}$$

where

$$Q = -\frac{2M}{r_+-r_-}(M\omega - r_+ m\Omega)\,,$$

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One possibility to compute the Love numbers for Kerr is to work in a regime where

$$\omega M \ll 1\,, \qquad \omega r \ll 1\,.$$

Such that the scalar/ Teukolsky's equation becomes

SL(2,R) x SL(2,R)
Hidden Symmetry

$$\left[\partial_r \Delta \partial_r + \frac{\left(2M\omega r_+ - \frac{i}{2}s(r_+ - r_-) - am\right)^2}{(r-r_+)(r_+-r_-)} - \frac{\left(2M\omega r_- + \frac{i}{2}s(r_+ - r_-) - am\right)^2}{(r-r_-)(r_+-r_-)} - \hat{K}_{\ell,s}\right]\hat{R}_s = 0 \quad (4.2)$$

$$\left[\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta\right) - \frac{(m + s\cos\theta)^2}{\sin^2\theta} + K_{\ell,s}\right] S_s(\theta) = 0\ .$$

Spheroidal eigenvalues

$$K_{\ell,s} = (\ell - s)(\ell + s + 1) + s.$$

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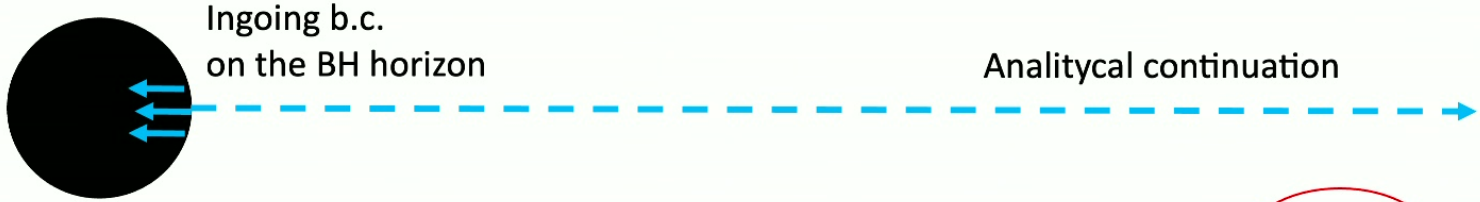
Teukolsky's radial equatio $$z\,(1-z)\,\frac{d^2w}{dz^2}+\left[\mathfrak{c}-(\mathfrak{a}+\mathfrak{b}+1)\,z\right]\frac{dw}{dz}-ab\;w=0$$

SL(2,R) x SL(2,R)
Hidden Symmetry

The solution takes the form

$$\hat{R}_s(z) \;=\; (1-z)^p\,z^q\left(c_1F[\mathfrak{a},\mathfrak{b},\mathfrak{c};z]+c_2\,z^{1-\mathfrak{c}}\,F[\mathfrak{a}-\mathfrak{c}+1,\mathfrak{b}-\mathfrak{c}+1,2-\mathfrak{c};z]\right),$$

Boundary Conditions



$$\hat{R} \;=\; \left(\frac{r-r_+}{r-r_-}\right)^{-i\frac{2Mr_+}{r_+-r_-}(\omega-m\Omega)-s/2}(r-r_-)^{-1-\ell} \tag{4.10}$$
$$F\left(1+\ell-i\frac{4M(M\omega-r_+m\Omega),}{r_+-r_-},1+\ell-2iM\omega-s,1-i\frac{4Mr_+}{r_+-r_-}(\omega-m\Omega)-s;\frac{r-r_+}{r-r_-}\right).$$

$$R\xrightarrow[r\to\infty]{}\tilde{c}_1\left[\frac{\Gamma(k)\Gamma(\mathfrak{a}+\mathfrak{b}-k)}{\Gamma(\mathfrak{a})\Gamma(\mathfrak{b})}r^{\ell}+\frac{\Gamma(\mathfrak{a}+\mathfrak{b}-k)}{k!\,\Gamma(\mathfrak{a}-k)\Gamma(\mathfrak{b}-k)}\log\left(\frac{r_+-r_-}{r}\right)r^{-\ell-1}\right]$$

$$\mathfrak{a}=1+\ell-i\frac{4M}{r_+-r_-}(M\omega-r_+m\Omega)\,,\qquad \mathfrak{b}=1+\ell-2iM\omega-s\,,$$
$$\mathfrak{c}=1-i\frac{4M\,r_+}{r_+-r_-}(\omega-m\Omega)-s\,.$$

Generic dimensionless tidal coefficient

$$k_{\ell m}(\omega)=\frac{\Gamma(\mathfrak{a})\,\Gamma(\mathfrak{b})}{\Gamma(k+1)\,\Gamma(\mathfrak{a}-k)\Gamma(\mathfrak{b}-k)}\log\left(\frac{r_+-r_-}{r}\right).$$

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Tidal coefficients for dynamical external gravitational sources

$$
\begin{aligned}
k_{\ell m}(\omega) \;&=\; \frac{\Gamma\left(1+\ell-i\frac{4M}{r_+-r_-}(M\omega-r_+m\Omega)\right)\,\Gamma(1+\ell-2iM\omega-s)}{(2\ell+1)!\,\Gamma(2\ell+1)\,\Gamma\left(-\ell-i\frac{4M}{r_+-r_-}(M\omega-r_+m\Omega)\right)\Gamma(-\ell-2iM\omega-s)} \\
&\quad\times\left(\frac{r_+-r_-}{r_++r_-}\right)^{(1+2\ell)}\log\left(\frac{r_+-r_-}{r}\right) \\
&=\; \frac{4Mi(M\omega-r_+m\Omega)(2iM\omega+s)}{(2\ell+1)!\,\Gamma(2\ell+1)\,(r_+-r_-)}\left[\prod_{m=1}^{\ell}\left(m^2+\frac{16M^2(M\omega-r_+m\Omega)^2}{(r_+-r_-)^2}\right)\right] \\
&\quad\times\left[\prod_{n=1}^{\ell}(n^2+(2M\omega-is)^2)\right]\left(\frac{r_+-r_-}{r_++r_-}\right)^{(1+2\ell)}\log\left(\frac{r}{r_+-r_-}\right).
\end{aligned}
$$

$$\gamma = a/(r_+-r_-)\,.$$

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Kerr Dynamical Tidal Coefficients

$$k_{lm}(\omega) = \sum_{n=1}^{\infty} k_{lm}^{(n)}\,\omega^n = \sum_{n=1}^{\infty}(\kappa^{(n)} + i\,\nu^{(n)})\left(\frac{r_+ - r_-}{r_+ + r_-}\right)^{(1+2l)}\omega^n\,,$$

where the first few orders yield

$$\begin{aligned}
\kappa^{(1)} &= \frac{(-1)^{l+s+1}(l+s)!\,\Gamma(1+l-s)\,\Gamma(1+l+2mi\gamma)}{(2l+1)!\,\Gamma(2l+1)\,\Gamma(-l+2mi\gamma)}\log\left(\frac{r_+ - r_-}{r}\right)2Mi \\
&= \frac{(-1)^s(l+s)!\,(l-s)!}{(2l+1)!\,(2l)!}\,4M\,m\,\gamma\,\log\left(\frac{r_+ - r_-}{r}\right)\prod_{n=1}^{l}(n^2+4m^2\gamma^2) \\
&= \nu^{(0)}\,4M\log\left(\frac{r_+ - r_-}{r}\right) \\
\kappa^{(2)} &= \kappa^{(1)}\frac{4iM^2}{(r_+ - r_-)}(\psi(-l+2im\gamma) - \psi(1+l+2im\gamma))\,, \\
&= -\kappa^{(1)}\frac{4M^2}{(r_+ - r_-)}\left(\frac{1}{2m\gamma} + 2\sum_{n=1}^{l}\frac{2m\gamma}{(2m\gamma)^2+(n^2)}\right) \\
\nu^{(2)} &= \kappa^{(1)}\,2M(\psi(1+l+s) - \psi(1+l-s)) \\
&= \kappa^{(1)}\,2M\sum_{n=0}^{2s-1}\frac{1}{n+\ell+1-s}\,.
\end{aligned}$$

** Dynamical Love numbers for Kerr are generically not zero at all orders in the frequency ω and exhibit logarithmic running,*

**No frequency-dependent dissipation in Kerr by scalar perturbations (s = 0)*

** Kerr black holes do not universally behave like rigidly rotating dissipative spheres*

** Agreement with low frenquency results*

Maria J Rodriguez

Schwarzschild Dynamical Tidal Coefficients

$$\Omega \to 0,\ r_+ \to 2M \text{ and } r_- \to 0$$

$$\begin{aligned} k_\ell^{Schw}(\omega) &= \frac{\Gamma(1+\ell-s-2iM\omega)\Gamma(1+\ell-2iM\omega)}{(2\ell+1)!\,\Gamma(2\ell+1)\,\Gamma(-\ell-s-2iM\omega)\,\Gamma(-\ell-2iM\omega)} \log\left(\frac{2M}{r}\right) \qquad (4.27) \\ &= \frac{(2iM\omega\ s - 4M^2\omega^2)}{(2\ell+1)!\,\Gamma(2\ell+1)} \left[\prod_{j=1}^{\ell} (j^2 + 4M^2\omega^2)\right] \left[\prod_{n=1}^{\ell} (n^2 + (2M\omega - is)^2)\right] \log\left(\frac{r}{2M}\right) . \end{aligned}$$

$$k_{s=2}^{Schw}(\omega) = -\frac{(\ell-2)!(\ell-1)!\,\ell!(\ell+2)!}{2\,(1+2\ell)!(2\ell-1)!} 4M^2\omega^2 \log\left(\frac{2M}{r}\right) + \mathcal{O}(\omega^3) .$$

Maria J Rodriguez

CFT Interpretation for Kerr tidal coefficients

The dynamical Love numbers measure the response to an incoming wave by the near region black hole, it is proportional to a two-point function in the CFT

$$G_R(\omega) \sim k_{\ell m}(\omega)\,.$$

To compare our results with the CFT we can write the Love numbers in terms of a dual CFT

$$\begin{aligned} k_{\ell m}(\omega_R,\omega_L) &= \frac{\Gamma\left(1+h_R-i\frac{\omega_R}{2\pi T_R}\right)\Gamma(1+h_L-i\frac{\omega_L}{2\pi T_L})}{(2\ell+1)!\,2\ell!\,\Gamma\left(-h_R-i\frac{\omega_R}{2\pi T_R}\right)\Gamma(-h_L-i\frac{\omega_L}{2\pi T_L})}\log\left(\frac{r_+-r_-}{r}\right) \\ &= \sinh\left(\frac{\omega_R}{2T_R}\right)\sinh\left(\frac{\omega_L}{2T_L}\right)\left(h_R^2+\left(\frac{\omega_R}{2\pi T_R}\right)^2\right)\left(h_L^2+\left(\frac{\omega_L}{2\pi T_L}\right)^2\right) \\ &\quad\times\left|\Gamma\left(h_R-i\left(\frac{\omega_R}{2\pi T_R}\right)\right)\right|^2\left|\Gamma\left(h_L-i\left(\frac{\omega_L}{2\pi T_L}\right)\right)\right|^2\frac{\log\left(\frac{r}{r_+-r_-}\right)}{(h_R+h_L+1)!(h_R+h_L)!} \end{aligned}$$

where

$$(T_L,T_R)=(\tfrac{r_++r_-}{4\pi a},\tfrac{r_+-r_-}{4\pi a}),$$

$$(\omega_L,\omega_R)=(2M^2\omega/a,\omega_L-m)$$

Maria J Rodriguez

Tidal Coefficients for mor general D=4 BHs

$$k_{\ell m}(\omega) \;=\; \frac{\Gamma\left(1+\ell - i\frac{4M}{r_+-r_-}(M\omega - r_+ m\Omega)\right)\,\Gamma(1+\ell-2iM\omega - s)}{(2\ell+1)!\,\Gamma(2\ell+1)\,\Gamma\left(-\ell - i\frac{4M}{r_+-r_-}(M\omega - r_+ m\Omega)\right)\Gamma(-\ell-2iM\omega - s)} \times \left(\frac{r_+-r_-}{r_++r_-}\right)^{(1+2\ell)} \log\left(\frac{r_+-r_-}{r}\right) \qquad (8$$

Kerr-NUT black holes

$$r_\pm \to r_\pm^{KTN} = M \pm \sqrt{M^2+N^2-a^2}\,,$$

Kerr-MOG black hole of the Scalar Tensor Vector Gravity (STVG),

$$r_\pm \to r_\pm^{MOG} = r(1+\alpha) \pm \sqrt{M^2(1+\alpha)-a^2}\,,$$

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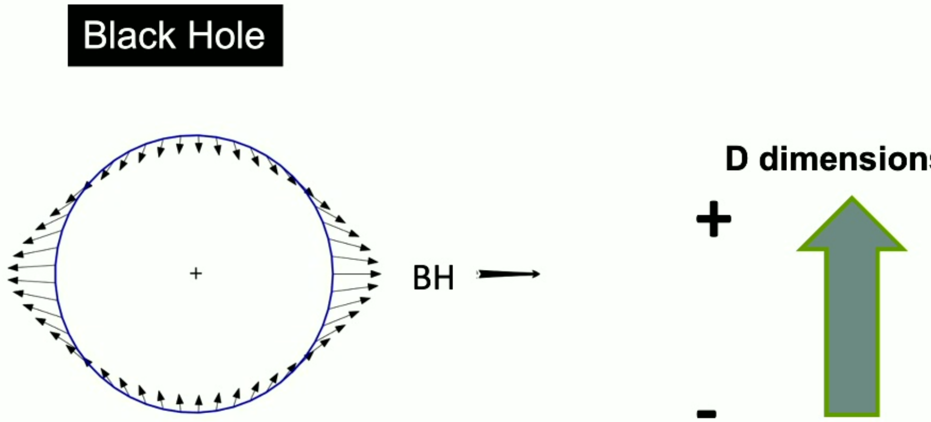
Tidal Coefficients for Higher Dimensional Bhs



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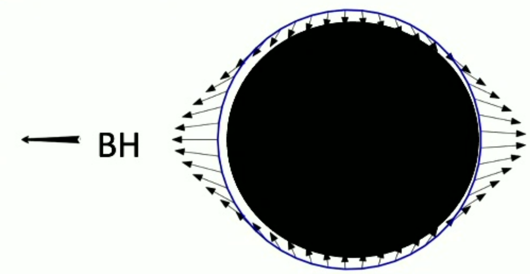
Contributions



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– Offered steps toward a better understanding of the computation of static Love numbers, and discussed the vanishing controversies for BHs

– Determined the dynamical tidal coefficients for Kerr through the study of the tidal deformations of Kerr BHs in dynamical external fields

– Argued that the Love numbers for Kerr have an approximate SL(2,R) x SL(2,R) hidden symmetry and match both, the low frequency regimes and Post-Newtonian computations.

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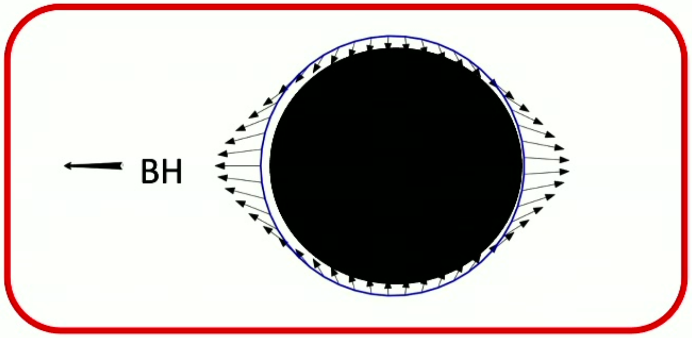

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Tidal squeezing in the farm.

Maria J Rodriguez

One possibility to compute the Love numbers for Kerr is to work in a regime where

$$\omega M \ll 1\,, \qquad \omega r \ll 1\,.$$

Such that the scalar/ Teukolsky's equation becomes

SL(2,R) x SL(2,R)
Hidden Symmetry

$$\left[\partial_r \Delta \partial_r + \frac{\left(2M\omega r_+ - \frac{i}{2}s(r_+ - r_-) - am\right)^2}{(r-r_+)(r_+-r_-)} - \frac{\left(2M\omega r_- + \frac{i}{2}s(r_+ - r_-) - am\right)^2}{(r-r_-)(r_+-r_-)} - \hat{K}_{\ell,s}\right]\hat{R}_s = 0 \quad (4.2)$$

$$\left[\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta\right) - \frac{(m+s\cos\theta)^2}{\sin^2\theta} + K_{\ell,s}\right] S_s(\theta) = 0\ .$$

Spheroidal eigenvalues

$$K_{\ell,s} = (\ell - s)(\ell + s + 1) + s.$$

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