Title: Hearts of Darkness: probing the regularization of space-time singularities

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Series: Cosmology & Gravitation

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Abstract:

In a complete theory of gravity we expect space-time singularities to be regularized by quantum effects. From this point of view, we have two possible regular alternatives to singular black holes (BHs) to describe the ultracompact objects that we saw in our universe: regular black holes and horizonless compact objects. I will talk about the possible structures of these black hole mimickers and the gravitational waves ringdown signal that we expect from their coalescence. In particular I will focus on the deviations in the spectrum of QNMs with respect to singular BHs, their possible detectability and the structure of echoes when backreaction effects are taken into account.

Zoom link

Hearts of Darkness:
probing the regularization of space-time singularities

Vania Vellucci

Perimeter Institute, 01 Feb. 2024

SISSA

General Relativity

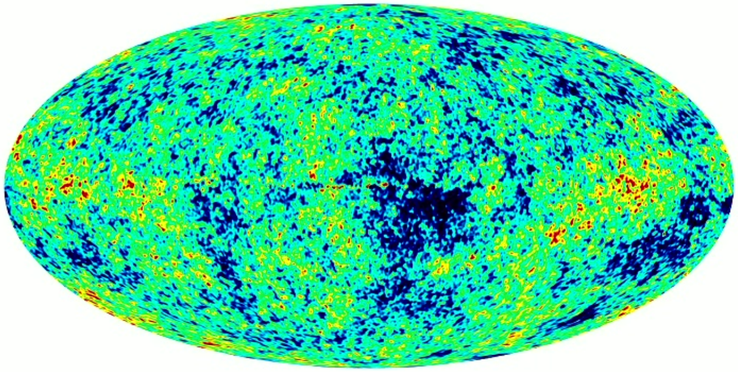
Gravitational collapse

Cosmological evolution

Black Hole singularities

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Big Bang singularity



$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + R^2(r)\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

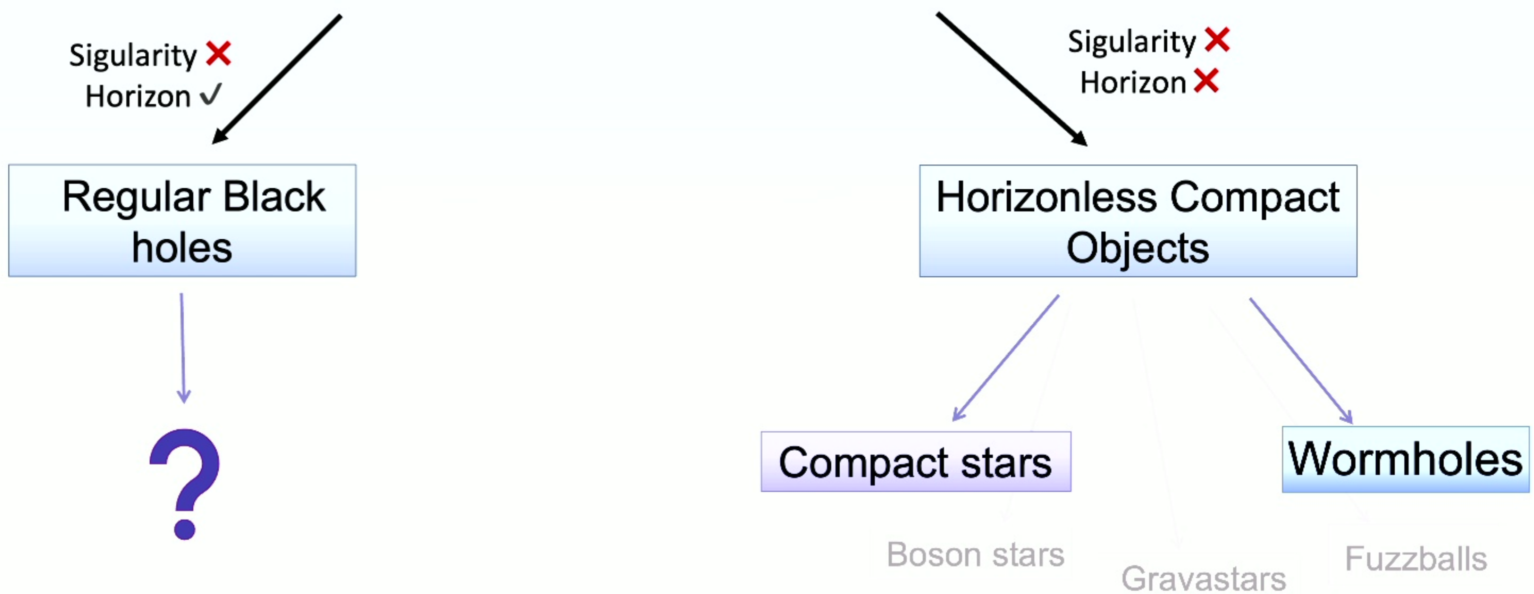
Singularities represent the demise of General Relativity

The theory is no more predictive and the spacetime is not defined!

Black holes Mimickers

In a complete theory of **quantum gravity** we expect **spacetime singularities to be regularized**

There are basically two possible **alternatives to singular black holes** to describe the ultra-compact objects that we see in the sky



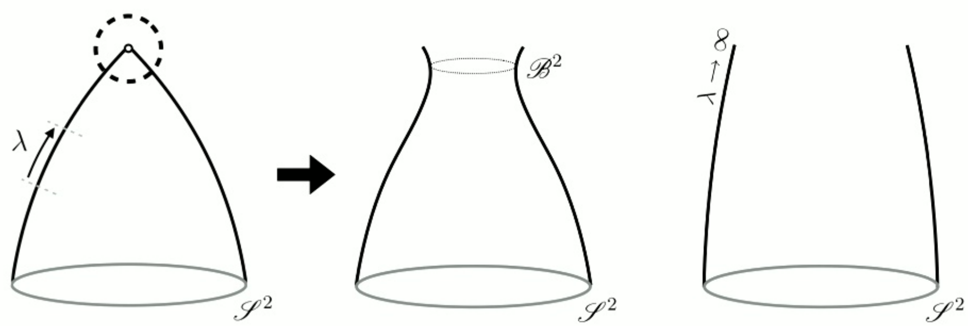
*we are considering spherical symmetric objects

Can we describe these **regular black holes** with the tools of general relativity? (with some effective geometries)

A singular space-time is **geodesic incomplete**:

there exist at least one geodesic that cannot be extended beyond a certain proper time or affine parameter

To describe regular solutions with a smooth mainfold we must **avoid the formation of a focusing point**

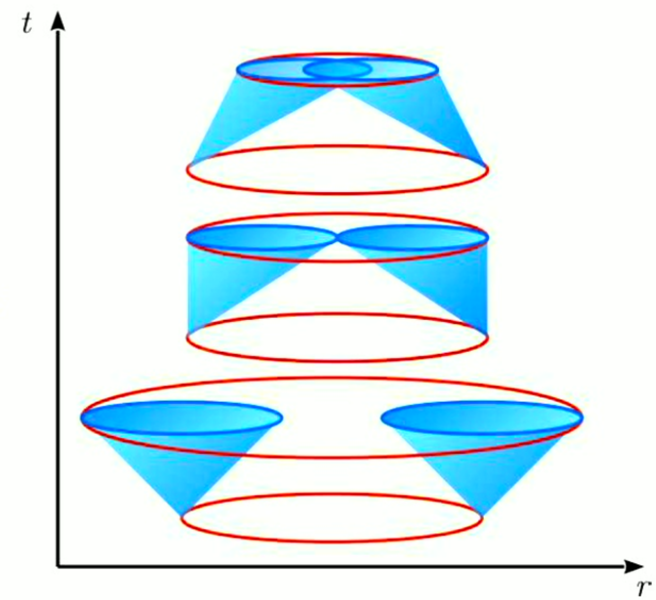


Geodesically complete black holes, Carballo Rubio et al.,2019

More rigorously…

The expansion θ of a congruence of geodesics tells you how much a cloud of particles expands or contracts isotropically as it moves along the congruence

When both the expansion of null ingoing θ_- and outgoing θ_+ geodesics become negative, a trapped surface is formed



From the **Penrose Theorem**

If $\theta_+ < 0$ in some points
(a trapping region is present)

$\xrightarrow[\text{GR + NEC + non-compact Cauchy surface}]{}$

Singularity!
$\theta_+ \to -\infty$

To avoid the formation of a singularity we assume that some of the assumptions of the theorem are violated

If $\theta_+ < 0$ in some points
(a trapping region is present)

$\longrightarrow$ ~~GR + NEC + non-compact Cauchy surface~~

Defocusing point at which the expansion changes sign
$\theta_+ = 0$

Possible spherically symmetric Regular Black holes

- Defocusing point at finite affine distance:

Evanescent double horizons or **Hidden wormhole**

(θ_- remains negative)
The singularity is replaced by an inner horizon shelding a non-singular core.

(θ_- changes sign too)
The singularity is replaced by a wormhole throat hidden inside a trapping horizon

- Defocusing point at infinite affine distance:

Everlasting double horizons or **Asymptotic hidden wormhole**

(θ_- remains negative)
Same structure of evanescent horizons but inner and outer horizons never merge

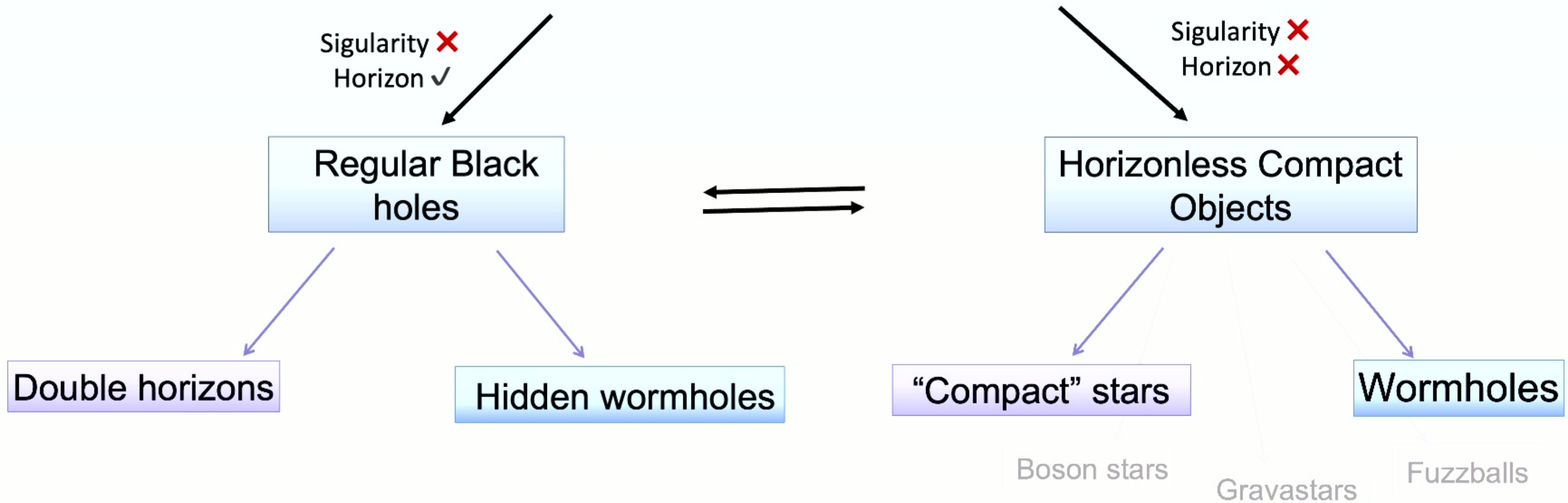
(θ_- changes sign too)
Same structure of the hidden but the throat is pushed to an infinite affine distance

Geodesically complete black holes, Carballo Rubio et al.,2019

Black holes Mimickers

In a complete theory of gravity we expect **spacetime singularities to be regularized** by **quantum effects**

There are basically two possible **alternatives to singular black holes** to describe the ultra-compact objects that we see in the sky



*we are considering spherical symmetric objects

Two families of (spherically symmetric) Black holes Mimickers

both can interpolate between regular black holes and horizonless compact objects*

$$ds^2 = -e^{-2\phi(r)} f(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right), \quad f(r) = 1 - \frac{2m(r)}{r}.$$

From Double horizons to Compact stars (varying ℓ)

$$\phi(r) = 0$$

and

$$m(r) = M\,\frac{r^3}{r^3 + 2M\,\ell^2} \quad \text{(Hayward metric)}$$

or

$$m(r) = M\,\frac{r^3}{(r^2 + \ell^2)^{3/2}} \quad \text{(Bardeen metric)}$$

or…

From Hidden wormholes to traversable Wormholes (varying ℓ)

$$\phi(r) = \frac{1}{2}\log\left(1 - \frac{\ell^2}{r^2}\right)$$

and

$$m(r) = M\left(1 - \frac{\ell^2}{r^2}\right) + \frac{\ell^2}{2\,r} \quad \text{(Simpson-Visser)}$$

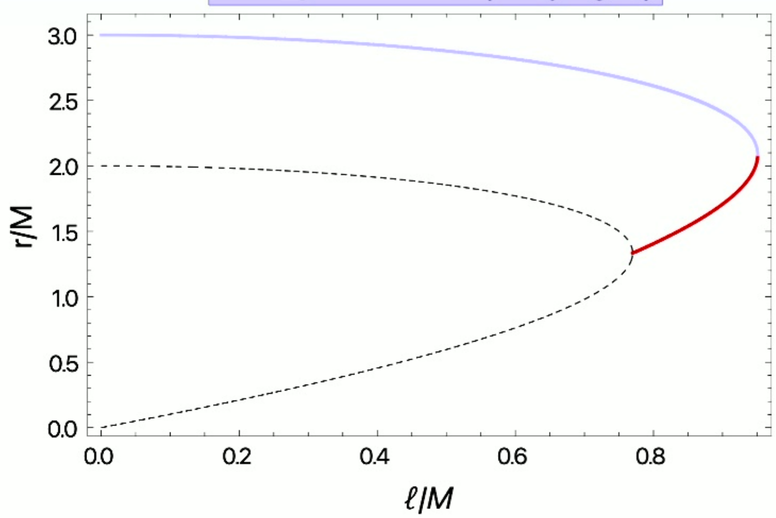
*Carballo-Rubio et al. 2023

Two families of Black holes Mimickers

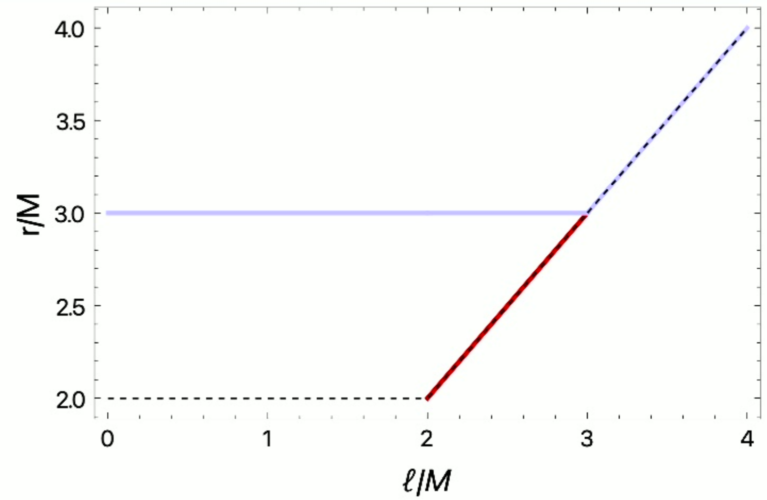
both can interpolate between the two alternatives

$$ds^2 = -e^{-2\phi(r)} f(r)\, \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\varphi^2\right), \quad f(r) = 1 - \frac{2m(r)}{r}.$$

From Double horizons to Compact stars (varyng ℓ)



From Hidden wormholes
To traversable Wormholes (varying ℓ)



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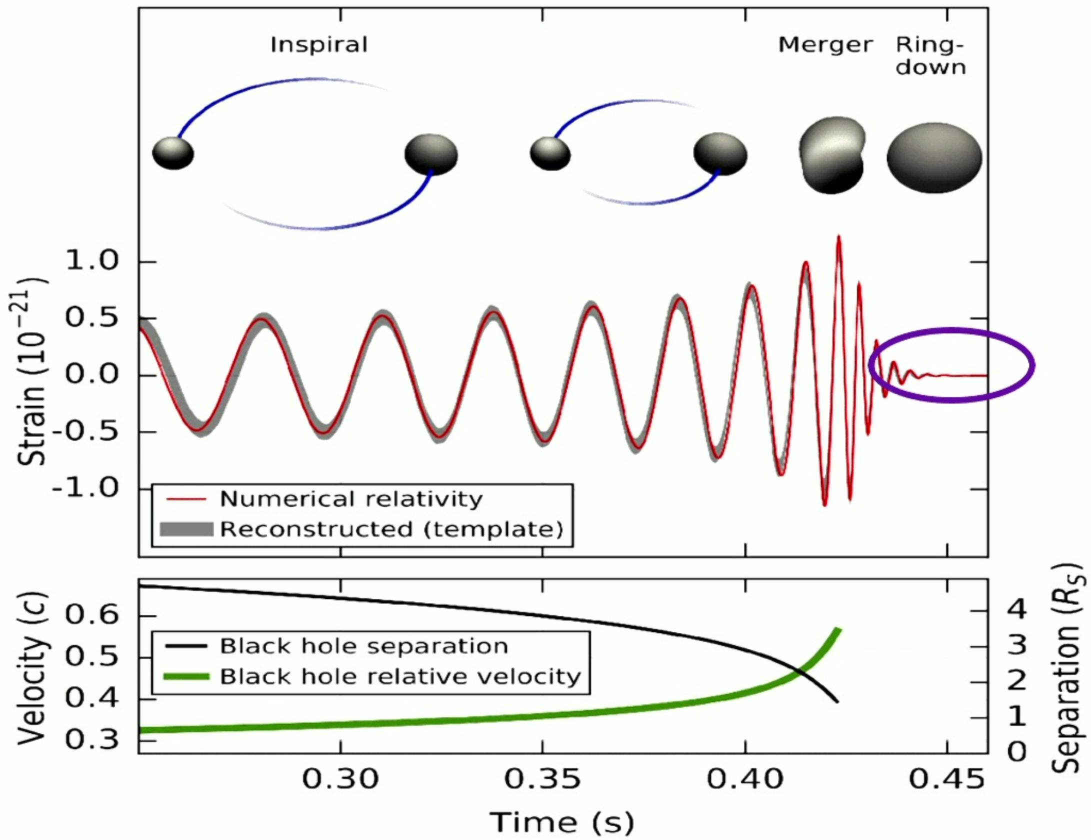
QNMs, instabilities and spectroscopy

E. Franzin, S. Liberati, V. Vellucci, 2023

The ringdown signal

The last phase of the GW signal coming from the coalescence of two compact objects is the ringdown.

It is caused by the characteristic oscillations of the final **peturbed** object



Study of gravitational perturbations

We considered the metric as a solution of Einstein equations sourced by a non-linear coupled magnetic field

$$\mathcal{S} = \int d^4x\, \sqrt{-g}\left(\frac{1}{16\pi}R - \frac{1}{4\pi}\mathcal{L}(F)\right),$$

With

$$F = \frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \frac{\ell^2}{2\,r^4}$$

and

$$\mathcal{L}(F) = \frac{m\prime(r)}{r^2} \neq F$$

$$\nabla_\mu \left(\mathcal{L}_F F^{\alpha\mu}\right) = 0\,,$$

$$G_{\mu\nu} = 2\left(\mathcal{L}_F F_\mu{}^\lambda F_{\nu\lambda} - g_{\mu\nu}\mathcal{L}\right),$$

We perturbed both the metric and the matter fields and we solved the field equations linearly in the perturbation

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}\,,$$

$$A_\mu = A_\mu^{(0)} + \delta A_\mu$$

(We report the computation for "Double horizons" metrics)

Harmonic perturbations

$g_{\mu\nu}$(t,r)= $e^{-i\omega t} g_{\mu\nu}$(r) A_{μ}(t,r)= $e^{-i\omega t} A_{\mu}$(r)

Important issue: the magnetic field has opposite polarity to respect to the metric.

↓

axial gravitational perturbations are actually coupled with polar perturbations of the magnetic field

(as well as polar gravitational perturbations are coupled with axial perturbations of the magnetic field)

$$\frac{d^2\psi_{ij}(r)}{dr_*^2} + \left(\omega^2 - V_{ij}(r)\right)\psi_{ij}(r) = 0$$

+

Boundary conditions

↓

Discrete set of frequencies (QNMs)

Where $\psi_{ij}(\mathrm{r})$ are related to the metric and matter perturbation $h_{\mu\nu}$(t,r) and δA_{μ} and the tortoise coordinate is defined as

$$dr_* = \frac{e^{\phi(r)}}{f(r)} dr$$

Study of test field perturbations

Test field:
we obtain the equations of motion from the Einstein equations
demanding that the perturbations do not change the stress-energy tensor (at least) to first order

↓

- No need to interpret the stress-energy tensor as some form of matter (no need to interpret the spacetime as a solution in GR)

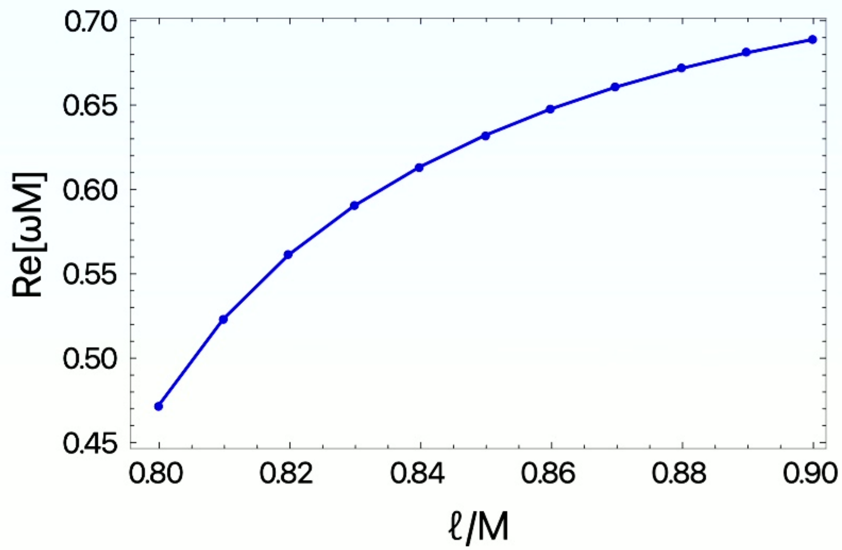
$$\frac{d^2\psi(r)}{dr_*^2} + \left(\omega^2 - V(r)\right)\psi(r) = 0$$

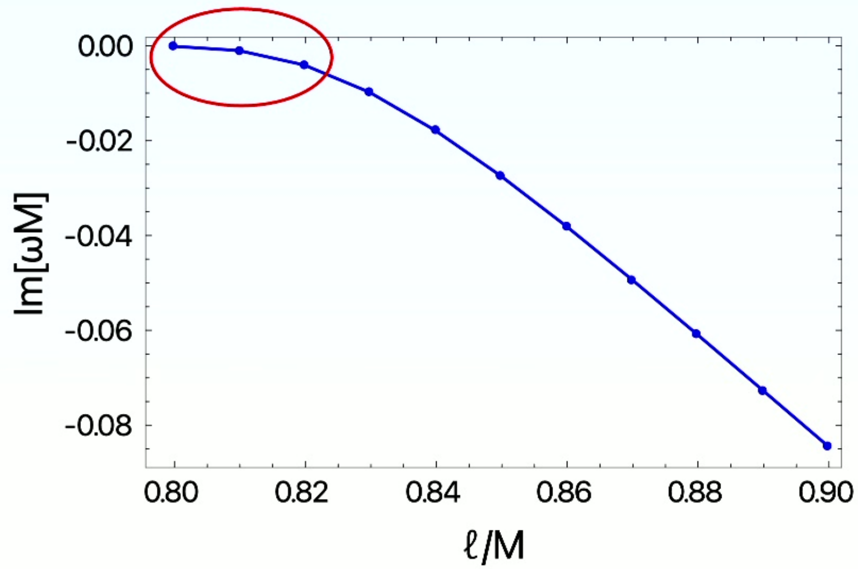
$$V = f(r)\left(e^{-2\,\phi(r)}\,\frac{l(l+1)}{r^2} + \frac{2(1-s^2)\,m(r)}{r^3} - (1-s)\left(\frac{2\,m'(r)}{r^2} + \frac{f(r)\phi'(r)}{r}\right)\right)$$

where s is the spin of the perturbation
(s=0 for scalr perturbations, s=1 for vector perturbations and so on..).

Horizonless compact object branch

Boundary conditions: regular at the center, purely outgoing at infinity



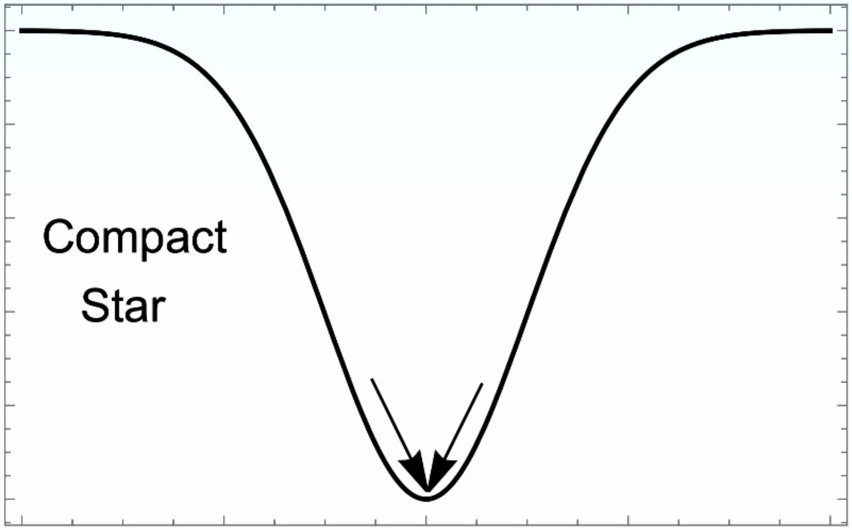


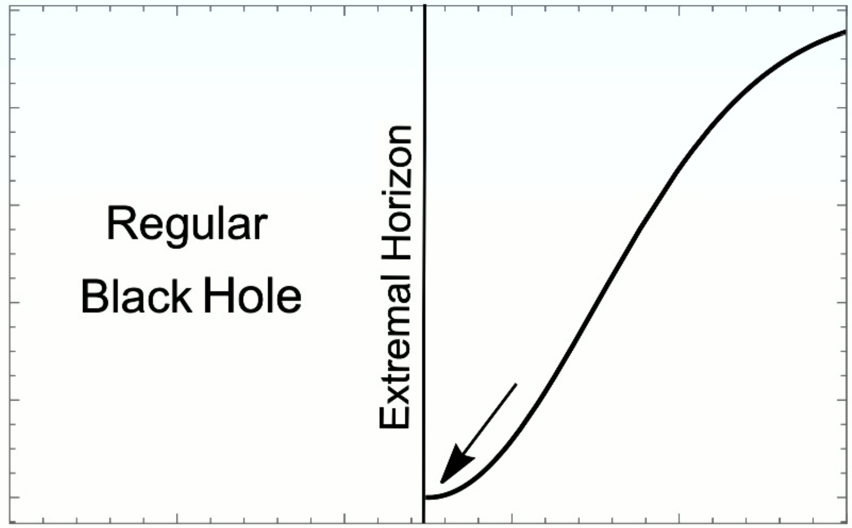
$$e^{i\omega t} = e^{iRe[\omega]t}\, e^{-Im[\omega]t}$$

$\tau = \frac{1}{Im[\omega]}$ is the damping time

Small $Im[\omega] \rightarrow$ **long living modes** connected with the presence of a **stable lightring!**

Lightring and Aretakis instabilities





The perturbation accumulates near the minimum of the potential causing possible non-linear instabilities

The "stable lightring" is already present in the extremal RBH case!
Connection to the Aretakis Instability?

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Detectability

Parspec framework* at order 0 in the spin

A data analysis framework for the GW ringdown of BHs in modified theories of gravity

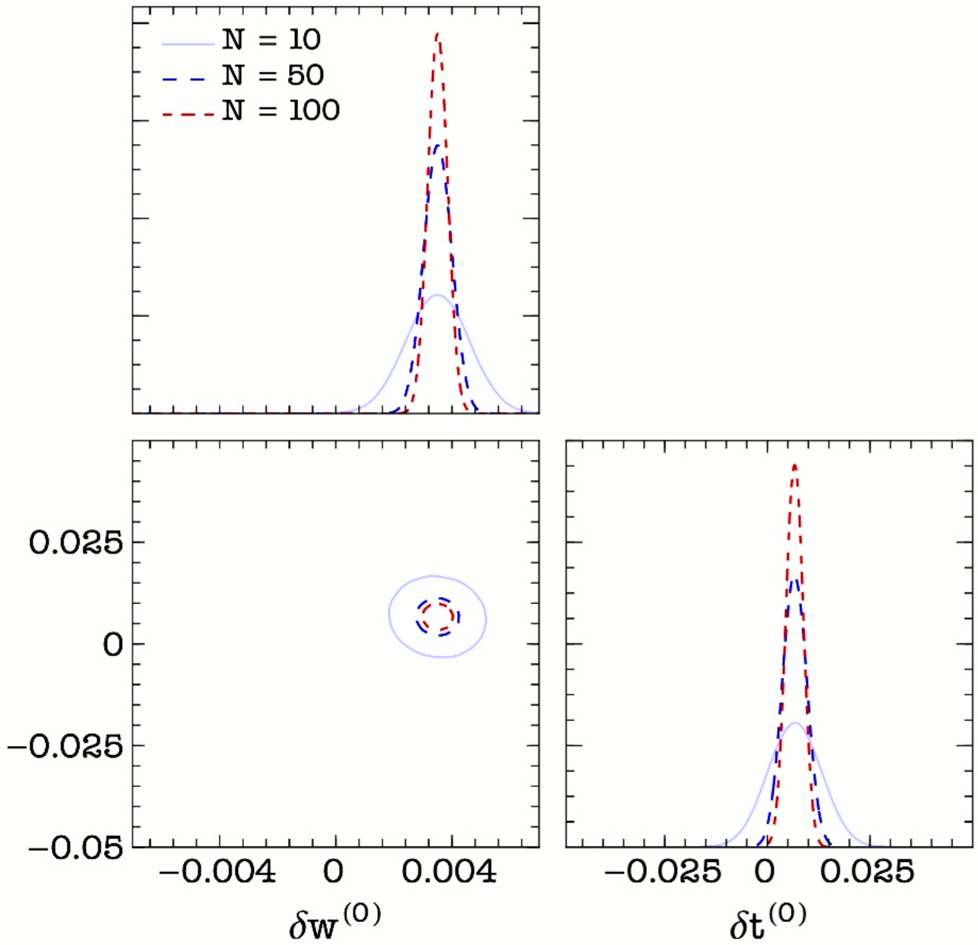
$$\omega_i := Re\,[\omega_i] = \frac{1}{M_i}\omega_{Kerr}^{(0)}\,(1+\gamma_i\delta\omega^{(0)})$$

$$\tau_i := \frac{1}{Im[\omega_i]} = M_i\,\tau_{Kerr}^{(0)}\,(1+\gamma_i\delta\tau^{(0)})$$

(γ_i= 1 in our case)

- You simulate N observations of ringdown signals from regular BHs binary merger
- Isolating the dependence of the corrections on the masses of the sources you can combine different observations to obtain more precise results on $\delta\omega$ and $\delta\tau$
- Through a Monte Carlo Markov chain you obtain the posterior probability distribution for $\delta\omega$ and $\delta\tau$

*Maselli et al. 2019



From the observations of the ringdown **of 0(100) RBHs** with **SNR~100** we can exclude the GR hypothesis at **90% confidence** level for macroscopic values of ℓ

but remember this is at **order 0 in the spin**…

Echoes:
the effect of breakreaction

V. Vellucci, E. Franzin, S. Liberati, 2022

Echoes

Consider a perturbation around a spherically symmetric BH or an Horizonless Compact Object

$$\Phi = \sum_{lm} Y_{lm}(\theta, \phi)\Psi_{lm}(r)/r$$

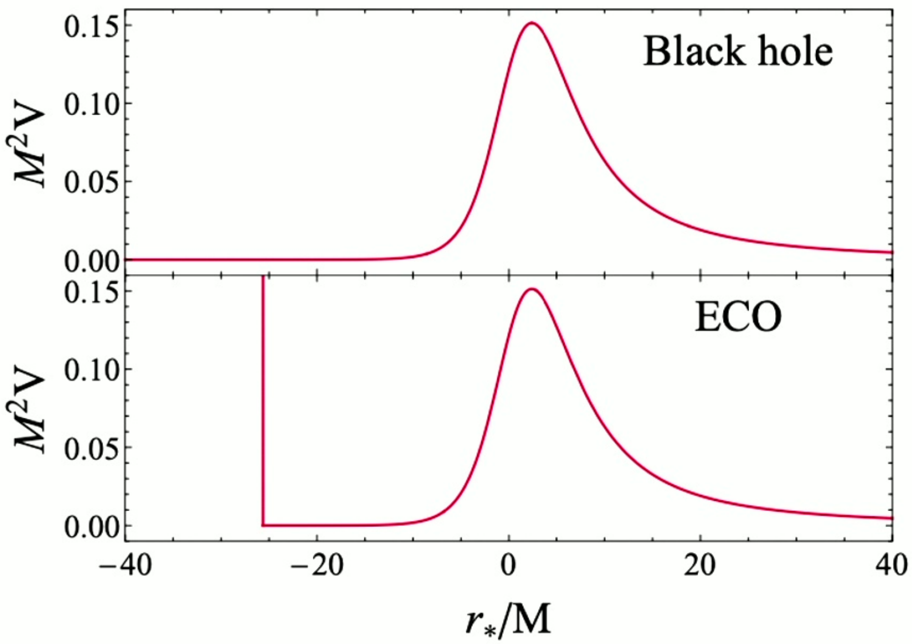
At linear level the field equation is:

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_l(r)\right]\Psi_{lm}(t, r) = 0$$

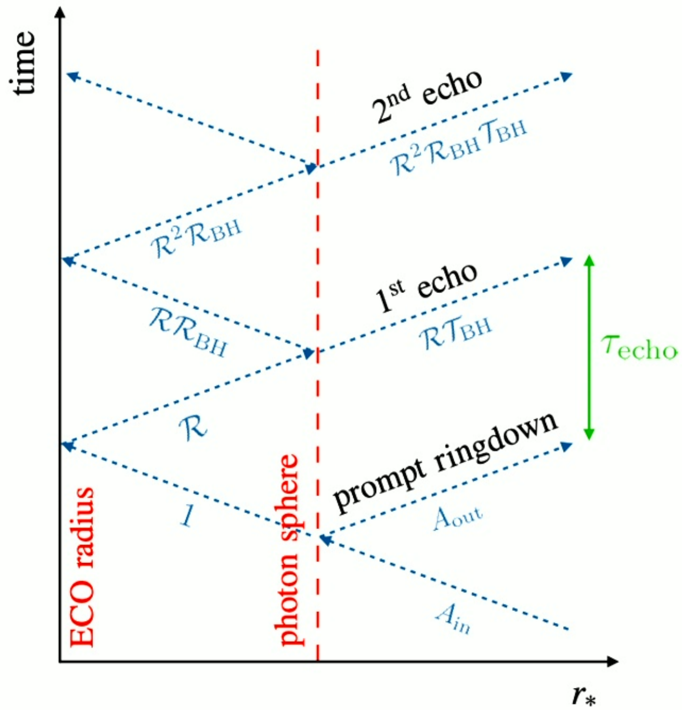
But the potential is very different in the two cases

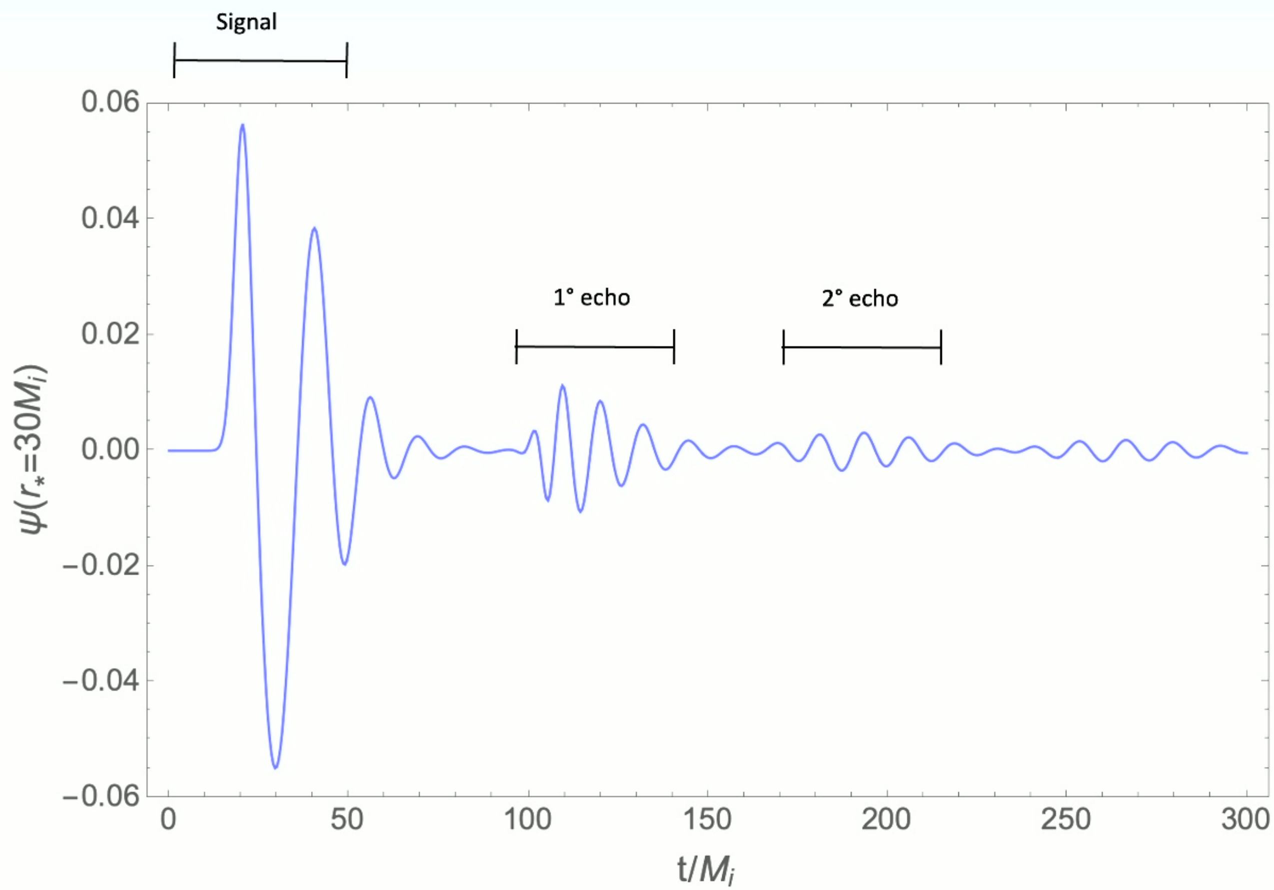
↓

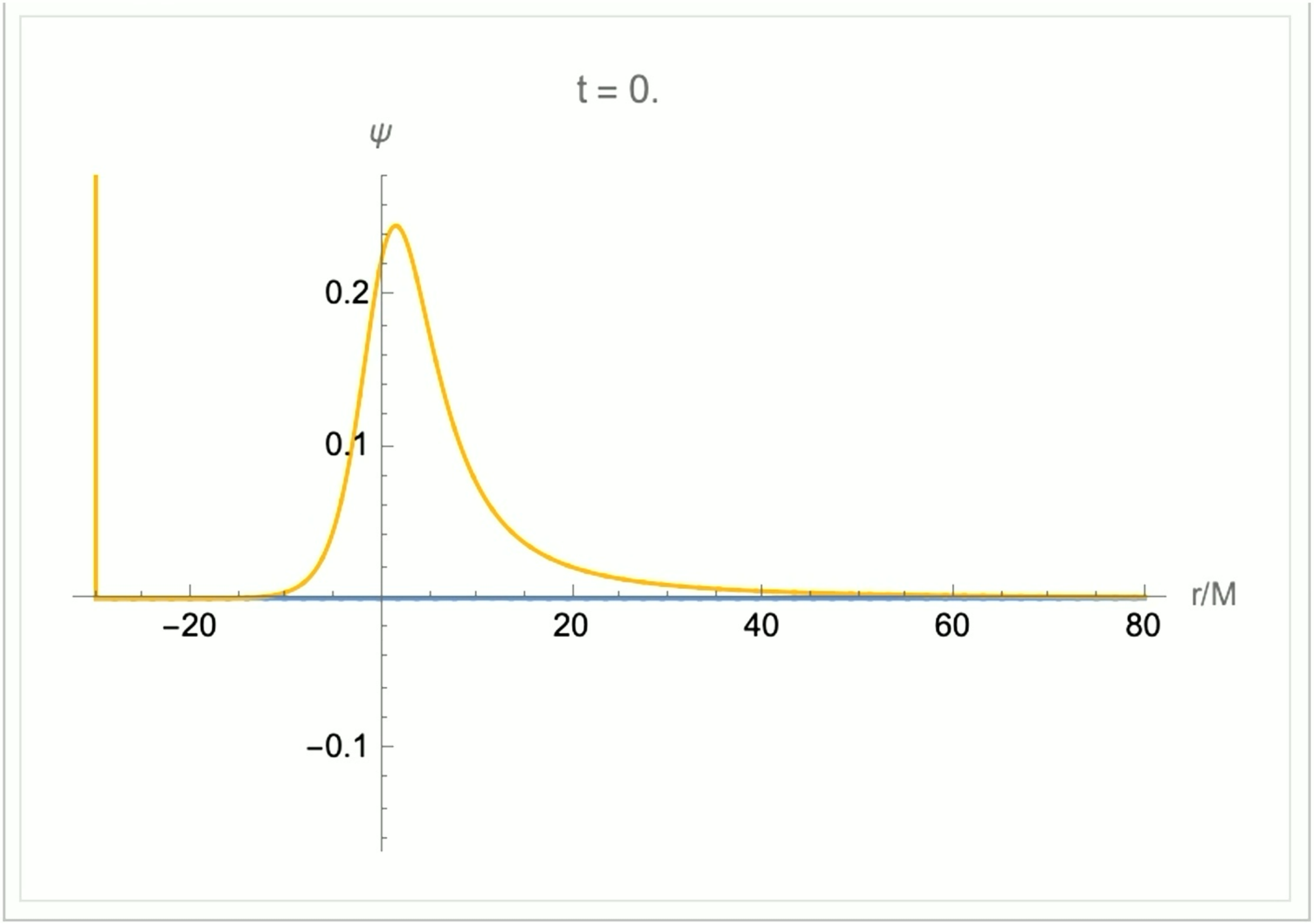
We see echoes of the original signal

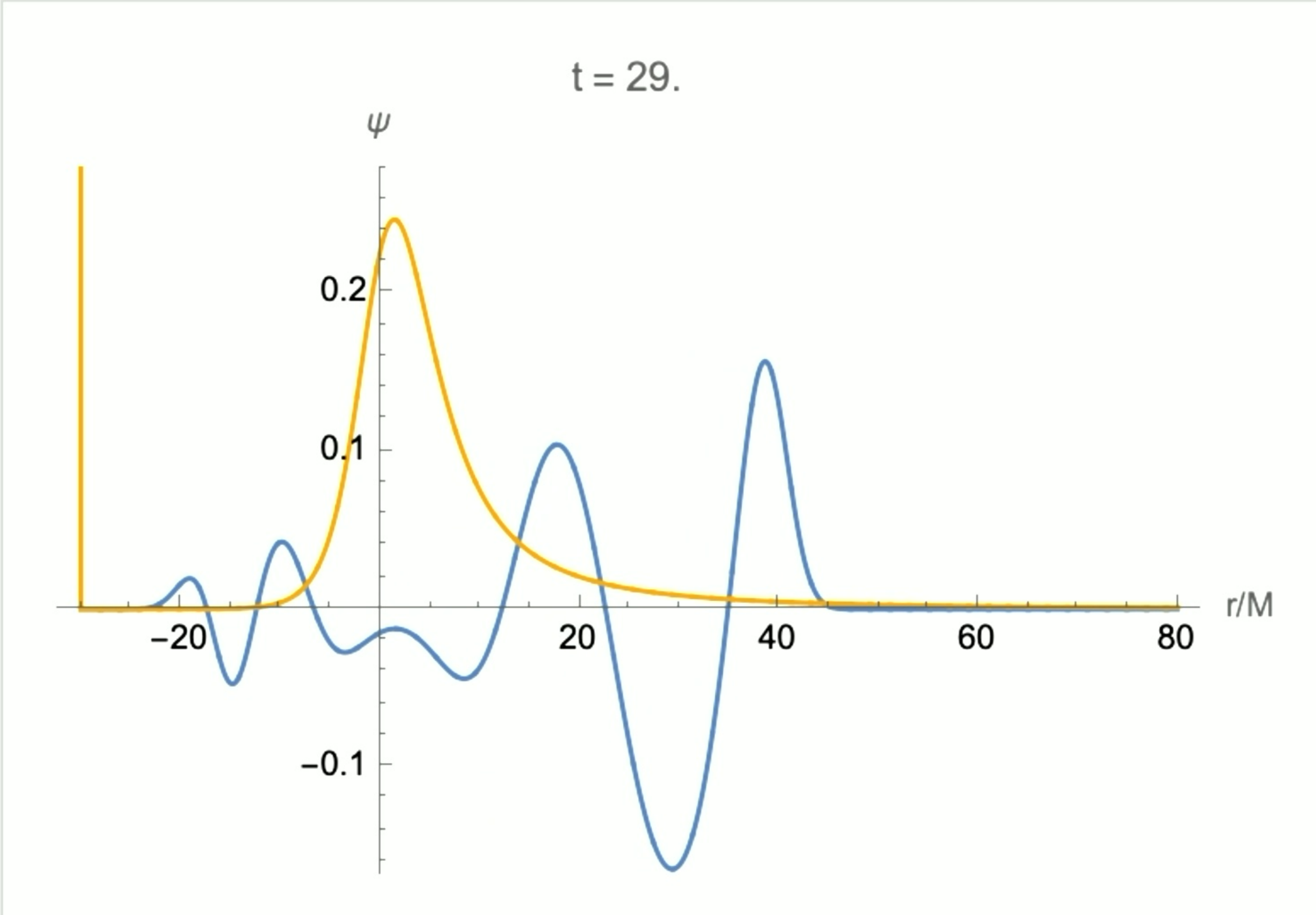


From arXiv:2105.06410
E. Maggio, P. Pani, G. Raposo


time
2nd echo
$\mathcal{R}^2\mathcal{R}_{BH}\mathcal{T}_{BH}$
$\mathcal{R}^2\mathcal{R}_{BH}$
1st echo
$\mathcal{R}\mathcal{R}_{BH}$
$\mathcal{R}\mathcal{T}_{BH}$
τ_{echo}
$\mathcal{R}$
prompt ringdown
ECO radius
photon sphere
1
A_{out}
A_{in}
r_*


Signal
1° echo
2° echo
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
$\psi(r_*=30M_i)$
0
50
100
150
200
250
300
t/M_i


t = 0.
ψ
0.2
0.1
−0.1
−20
20
40
60
80
r/M


t = 29.
ψ
0.2
0.1
−0.1
−20
20
40
60
80
r/M

Time delay

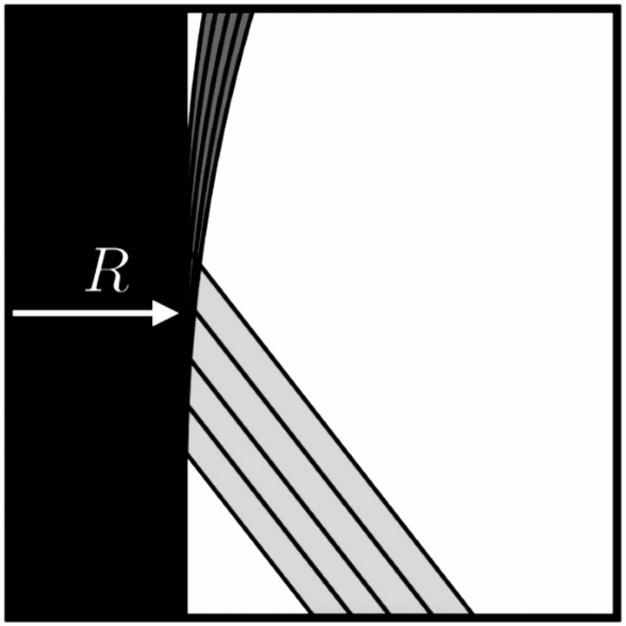
Defining the compactness parameter as:

$$\sigma = \frac{r_0}{2M} - 1$$

$$\Delta t_{echo} = 2 \int_{r_0=2M(\sigma+1)}^{r_{peak}\sim 3M} g_{rr} dr = 2 \int_{r_0=2M(\sigma+1)}^{r_{peak}\sim 3M} \frac{dr}{1 - \frac{2M}{r}} \simeq 2M(1 - 2\sigma - 2\ln(2\sigma))$$

The logarithmic dependence on σ would allow to detect even Planckian corrections ($\sigma \sim l_{Planck}$/M) at the horizon scale

Limits of linear approximation



Peeling of outgoing geodesic
The accumulation of geodesics around the gravitational radius produces high densities

Instability
Lightring and Ergoregion instability!
They can be quenced if **absorption** is taken into account

Non linear interactions should be taken into account

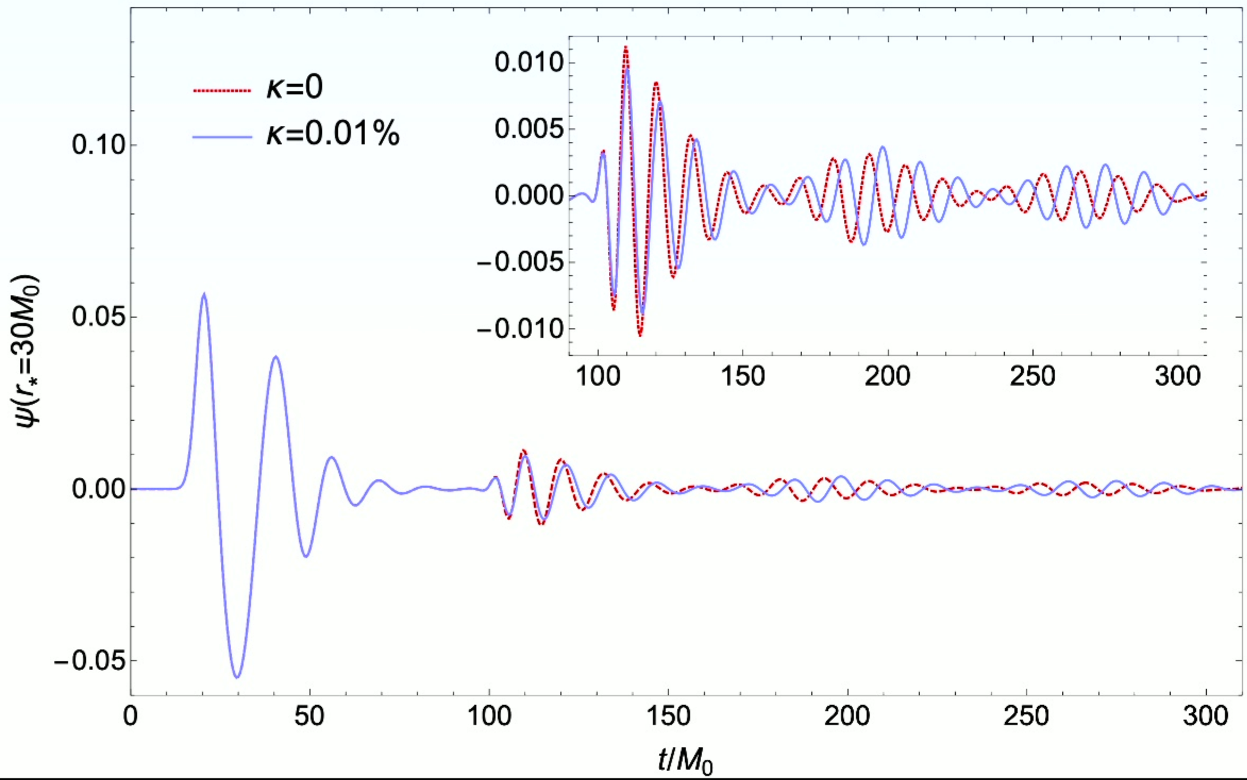
Absorption beyond the test field limit

Partial absorption of the first echo → $M_0 \to M = M_0 + \Delta E_{1st\ ehco}$

→ $\sigma_{2nd\ echo} \ll \sigma_{1st\ echo}$ → $\Delta t_{2nd\ echo} > \Delta t_{1st\ echo}$

For high compact object very small ΔM causes big changes in the compactness!

We lose the main feature of echoes signal: the quasi-periodicity!



How to prevent instability

Expansion at constant compactness

$$\frac{r_0(t)}{2\,M(t)} = \sigma_0 + 1$$

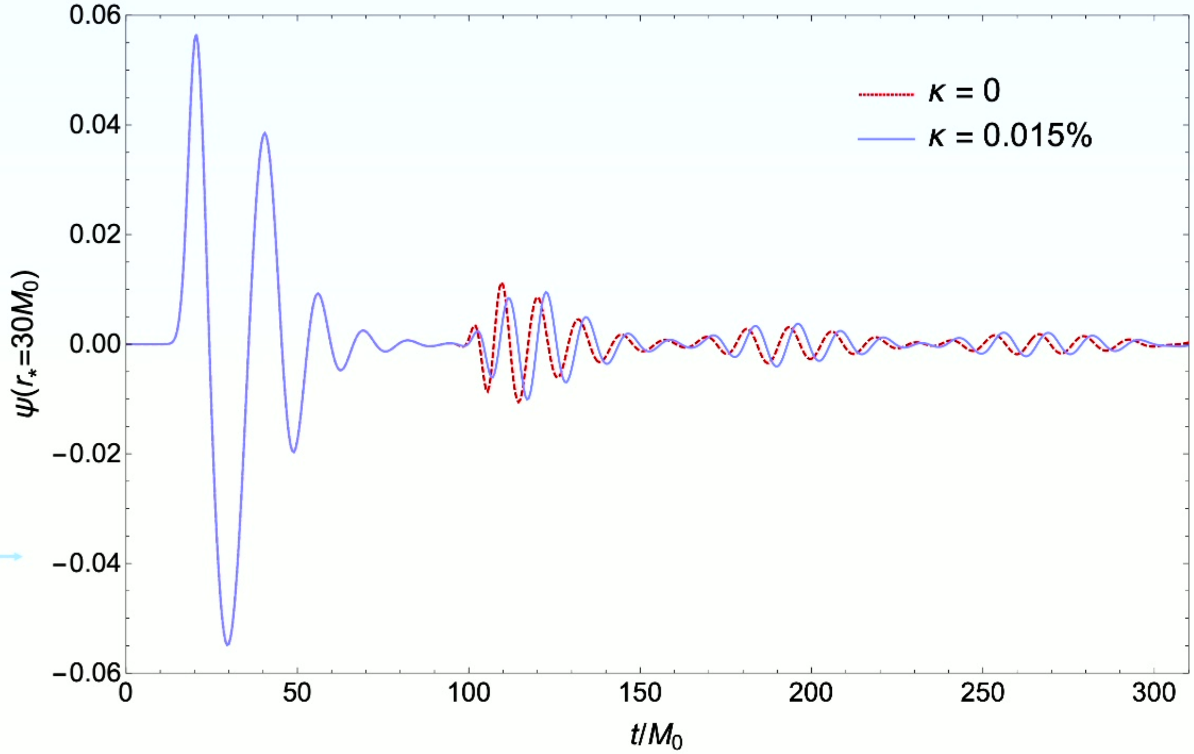
↓

$$\Delta t_{echo} \sim constant$$

BUT

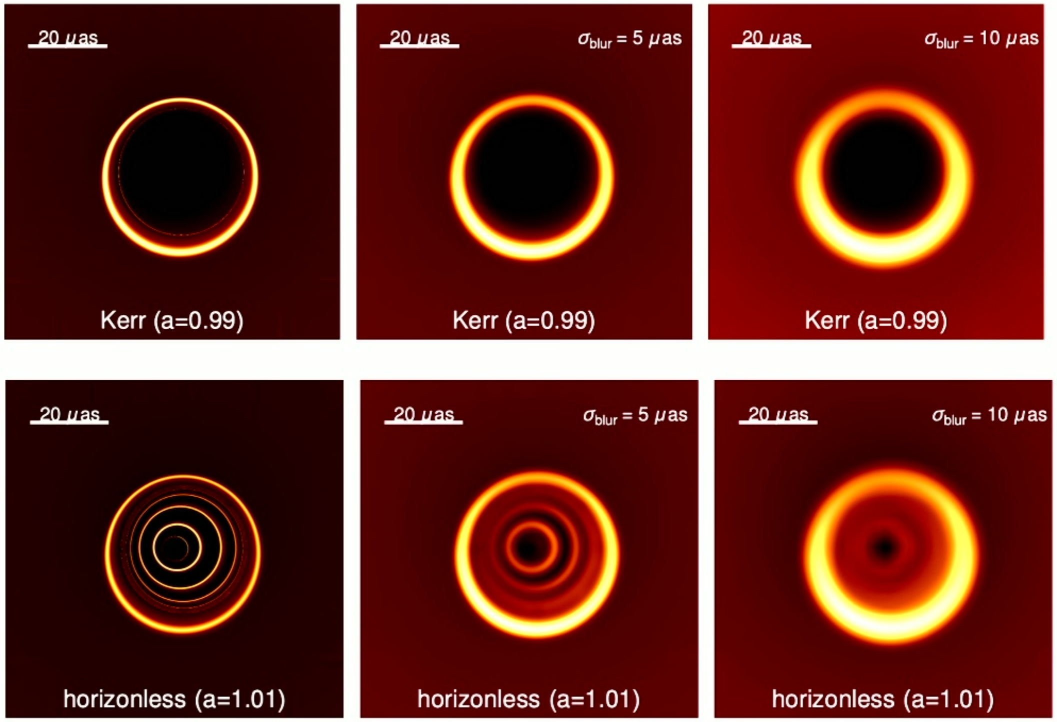
- It can requires superluminal motion of the surface
- There might be a transient phase →
- Other, more general model of expansion can be possible

Transient phase $\tau \sim 65M_0 > \Delta t_{echo}$



What about infrared observations?

Horizonless configurations have additional features while regular black holes have small deviations from Kerr



From Carballo Rubio et al.,2023

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Summarizing

- Both **regular BHs** and **horizonless compact objects** can be described by the same metric.
- We found that the quasi-normal modes of these objects **deviates** from the Schwarschild ones showing that we can potentially probe the inner structure of compact objects
- These deviations from the spectrum of singular BHs seems to be **detectable with the next Generation of GW detectors stacking multiple events**!
- Also **Echoes** of the ringdown can probe even Planckian corrections at horizon but non-linear effects should be taken into account and they can drastically affect the signal

Thank You