Title: Carrollian amplitudes and their role in flat space holography

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Series: Quantum Gravity

Date: February 01, 2024 - 2:30 PM

URL: https://pirsa.org/24020045

Abstract: Carrollian holography aims to express gravity in asymptotically flat space-time in terms of a dual Carrollian CFT living at null infinity. In this talk, I will review some aspects of Carrollian holography and argue that this approach is naturally related to the AdS/CFT correspondence via a flat limit procedure. I will then introduce the notion of Carrollian amplitude, which allows to encode massless scattering amplitudes into boundary correlators, and explain its connection to celestial amplitudes. Finally, I will present recent results concerning Carrollian OPEs and deduce how soft symmetries act at null infinity.

Zoom link TBA

Carrollian Amplitudes and Their Role in Flat Space Holography

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Quantum Gravity Seminar, Perimeter Institute, February 1, 2024

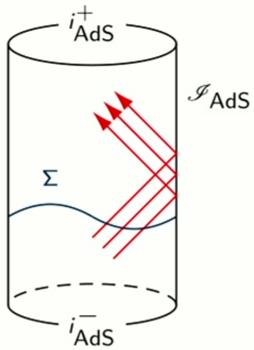
Motivations

- Holographic principle:

  Gravity in a given spacetime region can be encoded on a lower-dimensional boundary of that region.

  $\Longrightarrow$ Explicit realization: AdS/CFT correspondence.

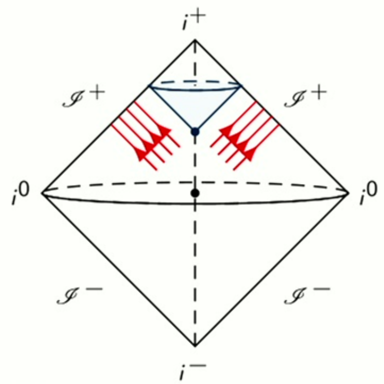
- Bottom-up approach: use what we know from gravity in the bulk to construct a dual theory.
- Interesting observations:
  1. Asymptotic symmetries: conformal symmetries at the timelike boundary.
     $\Longrightarrow$ Asymptotic symmetries of the bulk theory = global symmetries of the dual theory.
  2. Closed system (implemented by Dirichlet boundary conditions).
     $\Longrightarrow$ Quantum gravity in a box.
- How general is the holographic principle? Does it extend to asymptotically flat spacetimes?



Flat space holography program

(Early attempts: [Susskind '99] [Polchinski '99] [Giddings '00] [de Boer-Solodukhin '03] [Arcioni-Dappiaggi '03] [Mann-Marolf '06]).

- Interest: model for a large range of phenomena (from collider physics to astrophysics).
- Important obstructions to flat space holography:
  1. Asymptotic symmetries: Bondi-Metzner-Sachs (BMS) symmetries at the null boundary.
     [Bondi-van der Burg-Metzner '62] [Sachs '62] [Newman-Unti '62] [Penrose '65] [Geroch '77] [Ashtekar-Streubel '81] [Barnich-Troessaert '10] [Strominger '13]
  2. Open system (standard asymptotically flat boundary conditions are leaky).



How to formulate flat space holography?

- Correspondence between **gravity in asymptotically flat spacetimes** and a **lower-dimensional field theory** without gravity.
- Two proposals for flat space holography in 4d:

  $\Longrightarrow$ **Celestial holography**: the dual theory is a **2d CFT** living on the **celestial sphere** S^2.

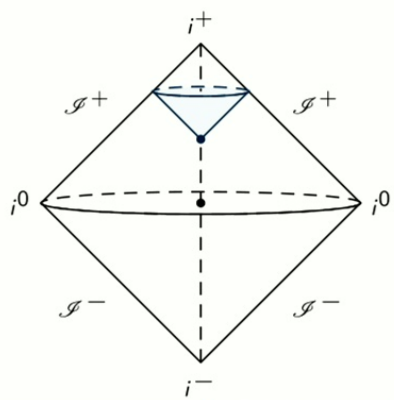
  [de Boer-Solodukhin '03] [He-Mitra-Strominger '15] [Kapec-Mitra-Raclariu-Strominger '16] [Cheung-de la Fuente-Sundrum '16] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17] [Donnay-Puhm-Strominger '18] [Stieberger-Taylor '18] [Pate-Raclariu-Strominger-Yuan '19] [Adamo-Mason-Sharma '21] ...

  $\Longrightarrow$ **Carrollian holography**: the dual theory is a **3d Carrollian CFT** living at **null infinity** $\mathscr{I} \simeq \mathbb{R} \times S^2$.

  [Arcioni-Dappiaggi '03] [Dappiaggi-Moretti-Pinamonti '06] [Barnich-Compère '07] [Bagchi '10] [Barnich '12] [Bagchi-Detournay-Fareghbal-Simon '12] [Barnich-Gomberoff-Gonzalez '12] [Bagchi-Basu-Grumiller-Riegler '15] [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [Donnay-Fiorucci-Herfray-Ruzziconi '22] ...
- The two proposals are **related** [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Bagchi-Banerjee-Basu-Dutta '22].
- Objectives of this talk:

  $\Longrightarrow$ Carrollian holography is useful, **relation with AdS/CFT**.

  $\Longrightarrow$ Definition of **Carrollian amplitudes**. [Mason-Ruzziconi-Yelleshpur Srikant '23].



What is Carrollian holography?

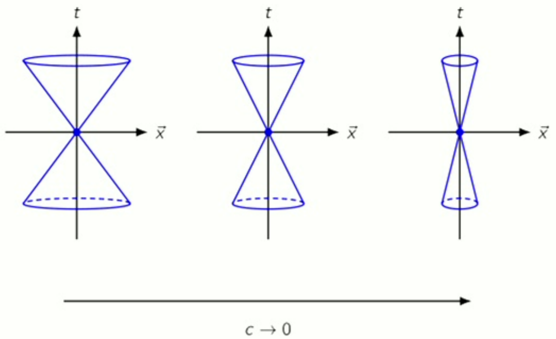
Carrollian algebra

- "Carroll" refers to the limit $c \to 0$ where c is the speed of light [Lévy-Leblond '65].
- Carrollian limit of the Poincaré algebra:
  $\Longrightarrow$ Translations $H = \partial_t$, $P_i = \partial_i$ and rotations $J_{ij} = x_i \partial_j - x_j \partial_i$ are unchanged.
  $\Longrightarrow$ Boosts are affected: $B_i = c^2 t \partial_i - x_i \partial_t \xrightarrow{c \to 0} B_i = -x_i \partial_t$.
- Carrollian boosts shift time but do not affect space:

$$t' = t - \vec{b}.\vec{x}, \qquad \vec{x}' = \vec{x}$$

  $\Longrightarrow$ Space becomes absolute (see diagram).
  $\Longrightarrow$ Opposite to the usual Galilean limit ($c \to \infty$) where time becomes absolute:

$$t' = t, \qquad \vec{x}' = \vec{x} - \vec{v}t$$



- Carrollian algebra (Ionu-Wigner contraction of Poincaré algebra when $c \to 0$):

$$[B_i, H] = 0, \quad [B_i, B_j] = 0, \quad [B_i, P_j] = \delta_{ij} H, \quad [B_k, J_{ij}] = \delta_{k[i} B_{j]}, \quad [P_k, J_{ij}] = \delta_{k[i} P_{j]}$$

- (Global) conformal Carrollian algebra (Ionu-Wigner contraction of the conformal algebra $SO(d,2)$ when $c \to 0$):
  $\Longrightarrow$ Add the dilatation: $D = (t\partial_t + x^i \partial_i)$, and the Carrollian special conformal generators: $K = x^2 \partial_u$ and $K_i = x^2 \partial_i - 2x_i x^j \partial_j - 2x_i t \partial_t$.

- Why the name “Carroll”?
  $\Longrightarrow$ Refers to Lewis Carroll, the author of Alice in Wonderland. In Levy-Leblond’s words:

  “Since absence of causality as well as arbitrarinesses in the length of time intervals is especially clear in Alice’s adventures (in particular in the Mad Tea-Party) this did not seem out of place to associate L. Carroll’s name”

- Besides its interest for flat space holography, Carrollian physics is interesting to study black holes, cosmology, fractons, fuids, ...

Carrollian geometry

- Metric degenerates to spatial metric in the limit $c \to 0$:

$$ds^2 = \eta_{ab} dx^a dx^b = -c^2 dt^2 + \delta_{ij} dx^i dx^j \quad \xrightarrow{c \to 0} \quad ds^2 = \delta_{ij} dx^i dx^j$$

- Inverse metric degenerates to temporal bi-vector:

$$-c^2 \eta^{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -c^2 \delta^{ij} \end{pmatrix} \quad \xrightarrow{c \to 0} \quad n^a n^b \text{ with } n^a = \delta^a_t$$

- Carrollian geometry: degenerate metric q_{ab} and vector field n^a such that $q_{ab} n^b = 0$. [Henneaux '79] [Duval-Gibbons-Horvathy-Zhang '14]
- Carrollian algebra $\mathfrak{Carr}_d$ = symmetries of the Carrollian geometry:

$$\mathcal{L}_{\bar{\xi}} q_{ab} = 0, \quad \mathcal{L}_{\bar{\xi}} n^a = 0$$

- Conformal Carrollian algebra $\mathfrak{CCarr}_d$ = conformal symmetries of the Carrollian geometry:

$$\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab}, \qquad \mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a$$

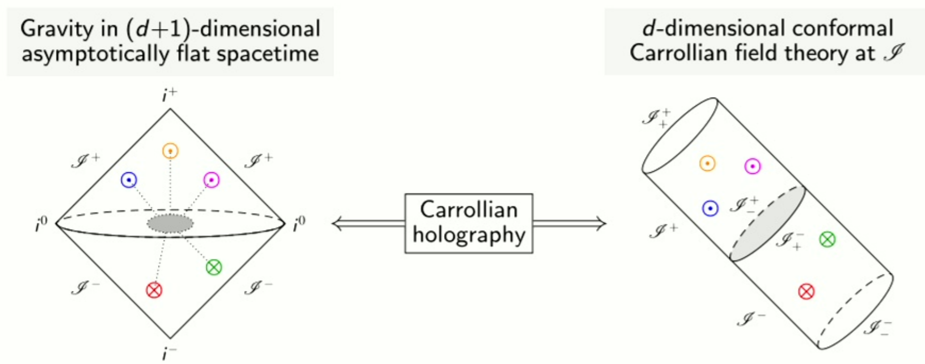
- Why is it relevant at null infinity?
  $\Longrightarrow$ $\mathscr{I}$ being a null hypersurface, the induced metric is degenerate!
  $\Longrightarrow$ Conformal Carrollian structure at $\mathscr{I}$ induced by conformal compactification [Geroch '77] [Ashtekar '14]

$$q_{ab} \sim \omega^2 q_{ab}, \quad n^a \sim \omega^{-1} n^a$$

  $\Longrightarrow$ BMS symmetries = Asymptotic symmetries in flat space [Bondi-van der Burg-Metzner '62] [Sachs '62]
  = Conformal symmetries of the Carrollian structure at $\mathscr{I}$.

- Isomorphism: $\boxed{\mathfrak{bms}_{d+1} \simeq \mathfrak{CCarr}_d.}$ [Duval-Gibbons-Horvathy '14] and Poincaré$_{d+1} \simeq$ Global $\mathfrak{CCarr}_d$

Carrollian holography



- Standard holographic correspondence: Asymptotic symmetries of the bulk theory = global symmetries in the dual theory.
- The dual theory is a d-dimensional Carrollian CFT (= theory exhibiting conformal Carroll/BMS spacetime symmetries)
  $\Longrightarrow$ Can be constructed by taking $c \to 0$ of standard relativistic CFTs, see e.g.
  [Schild '77] [Isberg-Lindstrom-Sundborg-Theodoridis '94] [Barnich-Gomberoff-Gonzalez '12] [Duval-Gibbons-Horvathy '14] [Bagchi-Mehra-Nandi '19]
- Carrollian holography follows a similar pattern than AdS/CFT correspondence: $(d+1)$-dimensional bulk / d-dimensional boundary duality.
  $\Longrightarrow$ Naturally arises from a flat limit procedure ($\Lambda \to 0$).
  $\Longrightarrow$ The flat limit in the bulk induces a Carrollian limit ($c \to 0$) at the boundary.

  [Bagchi '10] [Barnich-Gomberoff-Gonzalez '12] [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [Compère-Fiorucci-Ruzziconi '19] [Campoleoni-Delfante-Pekar-Petropoulos-Rivera Betancour '23]

An overview of previous works...

- Valid for all spacetime dimensions.
- Clear link with AdS/CFT through the flat limit $\Lambda \to 0$, implying $c \to 0$ at the boundary.
- Flat limit successfully exploited in 3d gravity:
  1. Gravitational solution space and symmetries (Witt$\oplus$Witt $\Longrightarrow$ $\mathfrak{bms}_3$) [Barnich-Gomberoff-Gonzalez '12] [Bagchi-Fareghbal '12]
  2. Entropy matching between flat space cosmologies and Carrollian CFT [Barnich '12] [Bagchi-Detournay-Fareghbal-Simon '13]
  3. Entanglement entropy formulae [Li-Takayanagi '11] [Bagchi-Basu-Grumiller-Riegler '14] [Jiang-Song-Wen '17]
  4. Holographic computation of boundary Carrollian stress tensor correlators [Detournay-Grumiller-Scholler-Simon '14] [Bagchi-Grumiller-Merbis '15] [Hartong '16]
  5. Effective Carrollian CFT action at $\mathscr{I}$ [Barnich-Gomberoff-Gonzalez '13]
  6. Holographic anomaly in flat space [Campoleoni-Ciambelli-Delfante-Marteau-Petropoulos-Ruzziconi '22]

  ...
- Flat limit of the gravitational phase space and symmetries also works in 4d.
  [Compère-Fiorucci-Ruzziconi '19] [Compère-Fiorucci-Ruzziconi '20] [Geiller-Zwikel '22]
- Flat limit in the fluid/gravity correspondence. [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [Freidel-Jai-akson '22] [Campoleoni-Delfante-Pekar-Petropoulos-Rivera Betancour '23]
- Carrollian CFT actions at the boundary of 4d flat space. [Adamo-Casali-Skinner '14] [Barnich-Nguyen-Ruzziconi '22]
- Relation with 4d $\mathcal{S}$-matrix and celestial amplitudes?
  [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Bagchi-Banerjee-Basu-Dutta '22] [Mason-Ruzziconi-Yelleshpur Srikant '23]
  $\Longrightarrow$ To be clarified in this talk.

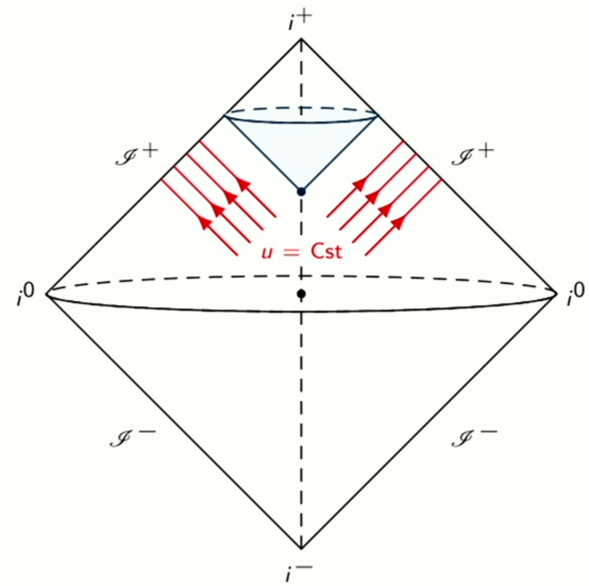
Carrollian stress tensor in 4d asymptotically flat spacetime

Solution space of 4*d* asymptotically flat spacetimes

- Asymptotically flat metric in Bondi coordinates to study $\mathscr{I}^+$: (u, r, x^A) where $x^A = (z, \bar{z})$ [Bondi-van der Burg-Metzner '62] [Sachs '62]:

$$
\begin{aligned}
ds^2 = & \left(\frac{2M}{r} + \mathcal{O}(r^{-2})\right) du^2 - 2\left(1 + \mathcal{O}(r^{-2})\right) dudr \\
& + \left(r^2 \mathring{q}_{AB} + r\, C_{AB} + \mathcal{O}(r^0)\right) dx^A dx^B \\
& + \left(\frac{1}{2}\partial_B C^B_A + \frac{2}{3r}(N_A + \frac{1}{4} C^B_A \partial_C C^C_B) + \mathcal{O}(r^{-2})\right) dudx^A .
\end{aligned}
$$

- Flat boundary metric: $\mathring{q}_{AB} dx^A dx^B = 2dzd\bar{z}$.
- Minkowski metric: $ds^2_{\text{Mink}} = -2dudr + 2r^2 dzd\bar{z}$.
- Subleading corrections in r with respect to Minkowski metric are obtained by solving the Einstein equations. They involve functions of (u, x^A):
  1. C_{AB} : asymptotic shear,
  2. $N_{AB} = \partial_u C_{AB}$: Bondi news (outgoing radiation),
  3. M : mass aspect,
  4. N_A : angular momentum aspect.
- Similar analysis at $\mathscr{I}^-$ in advanced Bondi coordinates (v, r, x^A).



- *Time evolution/constraint equations* on the mass and angular momentum aspects

$$\partial_u M = -\frac{1}{8} N_{AB} N^{AB} + \frac{1}{4} \partial_A \partial_B N^{AB},$$
$$\partial_u N_A = \partial_A M + \frac{1}{16} \partial_A (N_{BC} C^{BC}) - \frac{1}{4} N^{BC} \partial_A C_{BC} - \frac{1}{4} \partial_B (C^{BC} N_{AC} - N^{BC} C_{AC}) - \frac{1}{4} \partial_B \partial^B \partial^C C_{AC} + \frac{1}{4} \partial_B \partial_A \partial_C C^{BC},$$

with $N_{AB} = \partial_u C_{AB}$ the Bondi news tensor.

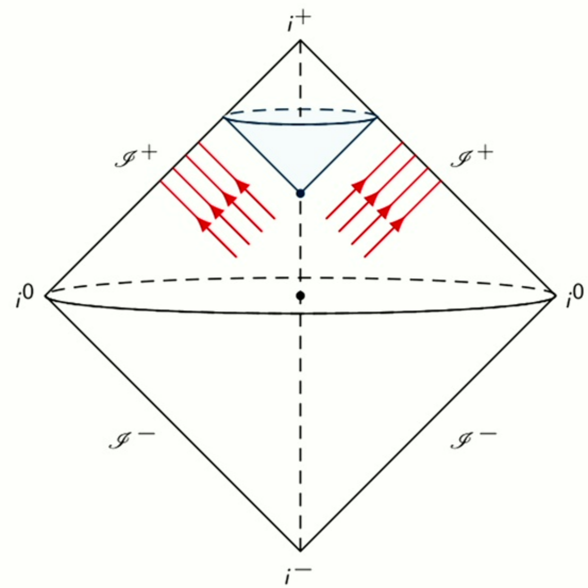
- *Bondi mass loss formula* [Trautman '58] [Bondi-van der Burg-Metzner '62]:

$$\partial_u \left[\int_{S^2_\infty} d^2 z\, M \right] = -\frac{1}{8} \int_{S^2_\infty} d^2 z\, N_{AB} N^{AB} \leq 0.$$

$\Longrightarrow$ The mass decreases in time due to the emission of gravitational waves.

$\Longrightarrow$ The analysis at $\mathscr{I}^+$ provides some information on the dynamics of the system.

- Solution space at leading order: $\{M(x^A), N_A(x^B), C_{AB}(u, x^c)\}$.



Asymptotic symmetries and charges

- Asymptotic symmetries = diffeomorphisms preserving Bondi gauge & asymptotic flatness, with non-trivial action at $\mathscr{I}^+$.
  $\Longrightarrow$ Form the (extended) BMS algebra: $\mathfrak{bms}_4^{\text{ext}} = (\text{Witt} \oplus \text{Witt}) \oplus\!\!\!\!\!\in \mathfrak{supertranslations}^*$ [Bondi-van der Burg-Metzner '62] [Sachs '62] [Barnich-Troessaert '10]
- Restriction at $\mathscr{I}^+$ of the asymptotic symmetries: $\xi\Big|_{\mathscr{I}^+} = \left[\mathcal{T} + \frac{u}{2}(\partial\mathcal{Y} + \bar\partial\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar\partial$ where
  1. $\mathcal{T} = \mathcal{T}(z,\bar z)$ is the supertranslation parameter;
  2. $\mathcal{Y} = \mathcal{Y}(z)$, $\bar{\mathcal{Y}} = \bar{\mathcal{Y}}(\bar z)$ are the superrotation parameters satisfying the conformal Killing equation.
- BMS charges = surface charges defined at a cut $\mathcal{S}_u \equiv \{u = \text{constant}\}$ of $\mathscr{I}^+$. Obtained using covariant phase space methods + compatibility with symmetries [Wald-Zoupas '99] [Barnich-Troessaert '10] [Flanagan-Nichols '15] [Compère-Fiorucci-Ruzziconi '18] [Campiglia-Peraza '20] [Donnay-Ruzziconi '21] [Freidel-Pranzetti '21]

$$\bar H_\xi[g] = \frac{1}{8\pi G}\int_{\mathcal{S}_u} d^2 z\,[2\mathcal{T}\mathscr{M} + \mathcal{Y}\bar{\mathscr{N}} + \bar{\mathcal{Y}}\mathscr{N}]$$

$$\mathscr{M} = M + \frac{1}{8}(C_{zz}N^{zz} + C_{\bar z\bar z}N^{\bar z\bar z}), \qquad \mathscr{N} = N_{\bar z} - u\bar\partial\mathscr{M} + \frac{1}{4}C_{\bar z\bar z}\bar\partial C^{\bar z\bar z} + \frac{3}{16}\bar\partial(C_{zz}C^{zz})\Big]$$
$$+ \frac{u}{4}\bar\partial\left[\left(\partial^2 - \frac{1}{2}N_{zz}\right)C^z_{\bar z} - \left(\bar\partial^2 - \frac{1}{2}N_{\bar z\bar z}\right)C^{\bar z}_z\right].$$

- Remark: $\mathscr{M} = -\text{Re}\Psi_2^0$, $\mathscr{N} = -\Psi_1^0 + u\eth\Psi_2^0$ [Newman-Penrose '62] [Newman-Unti '62].
- BMS flux-balance laws (*e.g.* Bondi mass loss formula):

$$\boxed{\frac{d}{du}\bar H_\xi[g] = \mathcal{F}_\xi[g] \neq 0\,, \quad \mathcal{F}_\xi[g]\Big|_{N_{AB}=0} = 0.}$$

$\Longrightarrow$ Important role in gravitational wave physics.

Carrollian CFT

- Coordinates: $x^a = (u, x^A)$, $x^A = (z, \bar{z})$. Carrollian structure: $ds^2 = 0\, du^2 + 2 dz d\bar{z}$ and $n^a \partial_a = \partial_u$.
- BMS/conformal Carroll symmetries: $\bar{\xi}^a \partial_a = \left[\mathcal{T} + \frac{u}{2}(\partial \mathcal{Y} + \bar{\partial} \bar{\mathcal{Y}})\right] \partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}$ with defining property:

$$\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab} \, , \qquad \mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a \, ,$$

  with $\alpha = \frac{1}{2}(\partial \mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})$
- Consider a 3d Carrollian CFT (theory exhibiting conformal Carroll/BMS symmetries).
  $\Longrightarrow$ Putative holographic dual in Carrollian holography.
- Noether currents:

$$j^a_{\bar{\xi}} = \mathcal{C}^a{}_b \bar{\xi}^b, \qquad \mathcal{C}^a{}_b = \begin{bmatrix} \mathcal{C}^u{}_u & \mathcal{C}^u{}_B \\ \mathcal{C}^A{}_u & \mathcal{C}^A{}_B \end{bmatrix} .$$

  $\Longrightarrow$ $\mathcal{C}^a{}_b(u, z, \bar{z})$: Carrollian stress tensor locally defined at $\mathscr{I}^+$.
  [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [de Boer, Hartong, Obers, Sybesma, Vandoren '18] [Ciambelli-Marteau '18] [Donnay-Marteau '19] [Chandrasekaran-Flanagan-Shehzad-Speranza '21] [Freidel-Pranzetti '21] [Donnay-Herfray-Fiorucci-Ruzziconi '22]
- Ward identities: $\boxed{\partial_a \langle j^a_{\bar{\xi}}(x) \rangle = 0}$

$$\begin{array}{rcccl} \textit{Carrollian translations} & : & \partial_b & \Rightarrow & \partial_a \langle \mathcal{C}^a{}_b \rangle = 0 \, , \\ \textit{Carrollian rotation} & : & -z\partial + \bar{z}\bar{\partial} & \Rightarrow & \langle \mathcal{C}^z{}_z \rangle - \langle \mathcal{C}^{\bar{z}}{}_{\bar{z}} \rangle = 0 \, , \\ \textit{Carrollian boosts} & : & \bar{x}^A \partial_u & \Rightarrow & \langle \mathcal{C}^A{}_u \rangle = 0 \, , \\ \textit{Carrollian dilatation} & : & x^a \partial_a & \Rightarrow & \langle \mathcal{C}^a{}_a \rangle = 0 \, , \end{array}$$

- Global conformal Carrollian symmetries ($\simeq$ 4d Poincaré symmetries) are enough to constrain $\mathcal{C}^a{}_b$.
  $\Longrightarrow$ No further constraints coming from supertranslations and superrotations.

Holographic correspondence

- Correspondence between boundary Carrollian momenta and bulk gravitational data at $\mathscr{I}^+$ [Donnay-Herfray-Fiorucci-Ruzziconi '22]:

$$\langle \mathcal{C}^u{}_u \rangle = \frac{\mathscr{M}}{4\pi G}\,, \quad \langle \mathcal{C}^A{}_B \rangle + \frac{1}{2}\delta^A{}_B \langle \mathcal{C}^u{}_u \rangle = 0\,,$$

$$\langle \mathcal{C}^u{}_A \rangle = \frac{1}{8\pi G}\left(\mathscr{N}_A + u\,\partial_A \mathscr{M}\right), \quad \mathscr{N}_A = (\bar{\mathscr{N}}, \mathscr{N})\,.$$

- Fixed by requiring compatibility between boundary Noether currents and bulk gravitational charges:

$$\bar{H}_\xi[g] = \underbrace{\int_{\mathcal{S}_u} d^2z\, \langle C^u{}_a \rangle \bar{\xi}^a}_{\text{Boundary Noether charges}} \overset{!}{=} \underbrace{\frac{1}{8\pi G}\int_{\mathcal{S}_u} d^2z\, [2\mathcal{T}\mathscr{M} + \mathcal{Y}\bar{\mathscr{N}} + \bar{\mathcal{Y}}\mathscr{N}]}_{\text{Surface charges for bulk asymptotic symmetries}}$$

- Similar to the AdS/CFT dictionary where the holographic stress-energy tensor of the CFT is identified with some subleading order in the expansion of the bulk metric. [Balasubramanian-Kraus '99] [de Haro-Solodukhin-Skenderis '01]

$$ds^2 = \frac{\ell^2}{\rho^2}d\rho^2 + \frac{1}{\rho^2}\left(g^{(0)}_{ab} + \rho^2 g^{(2)}_{ab} + \rho^3 g^{(3)}_{ab} + \mathcal{O}(\rho^4)\right)dx^a dx^b, \qquad \langle T_{ab} \rangle = \frac{3}{16\pi G\ell} g^{(3)}_{ab}$$

Comparison between AdS and flat

- Interesting sub-sector: 4d non-radiative ($N_{AB} = 0$) asymptotically flat spacetimes.
(Precise definition of non-radiative: $\Psi_4^0 = 0$, $\Psi_3^0 = 0$, $\text{Im}\Psi_2^0 = 0$)
- Situation similar to 3d gravity:
  1. The BMS charges are conserved, $\frac{d}{du}\bar{H}_\xi = 0$.
  2. $(\mathscr{M}, \mathscr{N}_A)$ transforms in the coadjoint representation of BMS. [Barnich-Ruzziconi '21]
  3. Boundary Carrollian CFT action ("BMS geometric action") describing the asymptotic dynamics of the bulk.
  $\Longrightarrow$ Associated Carrollian stress-tensor $\mathcal{C}^a{}_b$ encodes $(\mathscr{M}, \mathscr{N}_A)$. [Barnich-Nguyen-Ruzziconi '22]
- Carrollian holography for 4d non-radiative asymptotically flat spacetimes is qualitatively similar to AdS/CFT ($\Lambda \to 0$):
[Donnay-Herfray-Fiorucci-Ruzziconi '22]

AdS	**Flat**
Timelike boundary	Null boundary $\mathscr{I}^{\pm}$
Fefferman-Graham gauge	Bondi gauge
Conformal symmetries $\bar{\zeta}^a$	BMS symmetries $\bar{\xi}^a$
Relativistic stress tensor $T^a{}_b$	Carrollian stress tensor $\mathcal{C}^a{}_b$
Conserved charges: $\bar{H}_\xi = \int_{S^2} \langle T^t{}_a \rangle \bar{\zeta}^a, \quad \frac{d}{dt}\bar{H}_\xi = 0$	Conserved charges: $\bar{H}_\xi = \int_{S^2} \langle \mathcal{C}^u{}_a \rangle \bar{\xi}^a, \quad \frac{d}{du}\bar{H}_\xi = 0$
Dual: 3d CFT	Dual: 3d Carrollian CFT

External sources and radiation

How to include the radiation?

- **Problem:** No longer correct to assume $\partial_a j^a_\xi = 0$:
  $\Longrightarrow$ This is in contradiction with the flux-balance laws!

$$\frac{d}{du}\bar{H}_\xi[g] = \mathcal{F}_\xi[g] \neq 0\,, \quad \mathcal{F}_\xi[g]\Big|_{N_{AB}=0} = 0.$$

- **Interpretation:** couple the Carrollian CFT with external sources $\sigma(x)$ at the boundary.
- External sources identified with the Bondi news: $\sigma_{AB} \sim N_{AB}$.
  $\Longrightarrow$ The sourced Carrollian Ward identities reproduce the BMS flux-balance laws [Donnay-Fiorucci-Herfray-Ruzziconi '22].
- How to include the sources in the holographic picture?
  $\Longrightarrow$ The sources encode the radiative sector describing initial/final states of a massless scattering.
  $\Longrightarrow$ Investigate massless scattering in a Carrollian language.

Carrollian and celestial amplitudes

Massless scattering in flat space

Can we encode the bulk $\mathcal{S}$-matrix into boundary Carrollian CFT correlators?

- Strategy: start form the bulk operators, and deduce the boundary operators at $\mathscr{I}$. [Ashtekar '81] [Arcioni-Dappiaggi '03] [Strominger '17] [Donnay-Fiorucci-Herfray-Ruzziconi '22]
- Consider a spin-s ($s = 0, 1, 2, \dots$) massless field in flat space:

$$\phi_I^{(s)}(X) = \frac{K_{(s)}}{16\pi^3} \sum_{\alpha=\pm} \int \omega\, d\omega\, d^2 w \Big[a_\alpha^{(s)}(\omega, w, \bar{w}) \varepsilon_I^{*\alpha}(w, \bar{w})\, e^{i\omega q^\mu X_\mu} + a_\alpha^{(s)}(\omega, w, \bar{w})^\dagger \varepsilon_I^\alpha(w, \bar{w})\, e^{-i\omega q^\mu X_\mu} \Big]$$

with $I = (\mu_1 \mu_2 \dots \mu_s)$ and

$$p^\mu(\omega, w, \bar{w}) = \omega\, q^\mu(w, \bar{w}), \quad q^\mu(w, \bar{w}) = \frac{1}{\sqrt{2}} \Big(1 + w\bar{w}, w + \bar{w}, -i(w - \bar{w}), 1 - w\bar{w} \Big),$$

$$\varepsilon^\pm_{\mu_1 \dots \mu_s}(\vec{q}) = \varepsilon^\pm_{\mu_1}(\vec{q}) \varepsilon^\pm_{\mu_2}(\vec{q}) \dots \varepsilon^\pm_{\mu_s}(\vec{q}), \qquad \varepsilon^+_\mu(\vec{q}) = \partial_w q_\mu = \frac{1}{\sqrt{2}} (-\bar{w}, 1, -i, -\bar{w}), \qquad \varepsilon^-_\mu(\vec{q}) = [\varepsilon^+_\mu(\vec{q})]^* .$$

- Taking $r \to \infty$ (stationary phase approximation), we find the boundary values:

$$\begin{aligned}
\bar{\phi}^{(s)}_{z\dots z}(u, z, \bar{z})^{\text{out}} &= \lim_{r \to +\infty} \Big(r^{1-s} \phi^{(s)}_{z\dots z}(u, r, z, \bar{z}) \Big) \\
&= -\frac{iK_{(s)}}{8\pi^2} \int_0^{+\infty} d\omega \left[a_+^{(s)\,\text{out}}(\omega, z, \bar{z}) e^{-i\omega u} - a_-^{(s)\,\text{out}}(\omega, z, \bar{z})^\dagger e^{i\omega u} \right] \quad \text{at } \mathscr{I}^+, \\
\bar{\phi}^{(s)}_{z\dots z}(v, z, \bar{z})^{\text{in}} &= -\frac{iK_{(s)}}{8\pi^2} \int_0^{+\infty} d\omega \left[a_+^{(s)\,\text{in}}(\omega, z, \bar{z}) e^{-i\omega v} - a_-^{(s)\,\text{in}}(\omega, z, \bar{z})^\dagger e^{i\omega v} \right] \quad \text{at } \mathscr{I}^- .
\end{aligned}$$

$\Longrightarrow$ Insertion operators for a massless scattering between $\mathscr{I}^-$ (in) and $\mathscr{I}^+$ (out).

Amplitudes in position space

- Scattering amplitudes in position space at $\mathscr{I}$:

$$\mathcal{C}_n = \langle 0|\bar{\phi}^{(s)}_{l_1}(x_1)^{\text{out}} \dots \bar{\phi}^{(s)}_{l_n}(x_n)^{\text{out}} \bar{\phi}^{(s)}_{l_{n+1}}(x_{n+1})^{\text{in}\,\dagger} \dots \bar{\phi}^{(s)}_{l_N}(x_N)^{\text{in}\,\dagger}|0\rangle$$
$$= \frac{1}{(2\pi)^N}\prod_{k=1}^{n}\int_0^{+\infty} d\omega_k\, e^{-i\omega_k u_k}\prod_{\ell=n+1}^{N}\int_0^{+\infty} d\omega_\ell\, e^{i\omega_\ell v_\ell}\mathcal{A}_N(\{\omega_i\},\{z_i\},\{\bar{z}_i\}).$$

where $x_i^{\text{out/in}} = (u_i/v_i, z_i, \bar{z}_i)$.

Can we re-interpret these as correlators of Carrollian operators at $\mathscr{I}$?

- Conformal Carrollian primary field $\Phi_{(k,\bar{k})}(u,z,\bar{z})$:

$$\delta_{\bar{\xi}}\Phi_{(k,\bar{k})} = \left[\left(\mathcal{T} + \frac{u}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + k\,\partial\mathcal{Y} + \bar{k}\,\bar{\partial}\bar{\mathcal{Y}}\right]\Phi_{(k,\bar{k})}, \qquad (k,\bar{k})\text{: Carrollian weights.}$$

(analogue of primary field in CFT)

$\Longrightarrow$ Can be deduced from Carrollian highest-representation [Bagchi-Dhivakar-Dutta '23]:

$$[L_0, \Phi(0,0,0)] = k\Phi(0,0,0), \quad [\bar{L}_0, \Phi(0,0,0)] = \bar{k}\Phi(0,0,0)$$
$$[L_n, \Phi(0,0,0)] = 0 = [\bar{L}_n, \Phi(0,0,0)], \quad [M_{r,s}, \Phi(0,0,0)] = 0, \quad \forall n,r,s > 0$$

where

$$L_n = z^{n+1}\partial_z + \frac{1}{2}(n+1)z^n u\partial_u, \qquad \bar{L}_n = \bar{z}^{n+1}\partial_{\bar{z}} + \frac{1}{2}(n+1)\bar{z}^n u\partial_u, \qquad M_{r,s} = z^r\bar{z}^s\partial_u, \qquad n,r,s\in\mathbb{Z}$$

$\Longrightarrow$ The Carrollian primaries transform in unitary representations of global $\mathfrak{CCarr}_3 \simeq$ Poincaré$_4$ [Nguyen-West '23] [Nguyen '23].

- Remark: if $\Phi_{(k,\bar{k})}(u,z,\bar{z})$ is a Carrollian primary, then $\partial_u\Phi_{(k,\bar{k})}(u,z,\bar{z})$ is also a Carrollian primary with shifted weights $(k+\frac{1}{2}, \bar{k}+\frac{1}{2})$.

Carrollian holography identification

- Carrollian operators = boundary value of bulk operators:
  [Arcioni-Dappiaggi '03] [Dappiaggi-Moretti-Pinamonti '05] [Donnay-Fiorucci-Herfray-Ruzziconi '22]

$$\Phi^{\epsilon=+1}_{(k,\bar{k})}(u,z,\bar{z}) = \bar{\phi}^{(s)\,\text{out}}_{z\dots z}(u,z,\bar{z}), \qquad \Phi^{\epsilon=-1}_{(k,\bar{k})}(v,z,\bar{z}) = \bar{\phi}^{(s)\,\text{in}}_{z\dots z}(v,z,\bar{z})^\dagger$$

$\Longrightarrow$ This implies $k = \frac{1+\epsilon J}{2}$, $\bar{k} = \frac{1-\epsilon J}{2}$ with $\epsilon = \pm 1$ for out/in.

$\Longrightarrow$ For gravity ($s = 2$), $\Phi^{\epsilon=+1}_{(k,\bar{k})}(u,z,\bar{z}) \equiv C_{zz}(u,z,\bar{z})$ ($J = 2$ and $(k,\bar{k}) = (\frac{3}{2}, -\frac{1}{2})$).

- Carrollian correlators = scattering amplitudes in position space at $\mathscr{I}$:

$$\begin{aligned}\langle \Phi^{\epsilon_1}_{(k_1,\bar{k}_1)}(u_1,z_1,\bar{z}_1)\dots\Phi^{\epsilon_n}_{(k_n,\bar{k}_n)}(u_n,z_n,\bar{z}_n)\rangle &= \prod_{i=1}^{n}\left(\int_0^{+\infty}\frac{d\omega_i}{2\pi}\, e^{i\epsilon_i\omega_i u_i}\right)\mathcal{A}_n\left(\{\omega_1,z_1,\bar{z}_1\}^{\epsilon_1}_{J_1},\dots,\{\omega_n,z_n,\bar{z}_n\}^{\epsilon_n}_{J_n}\right)\\ &= \mathcal{C}_n\left(\{u_1,z_1,\bar{z}_1\}^{\epsilon_1}_{J_1},\dots,\{u_n,z_n,\bar{z}_n\}^{\epsilon_n}_{J_n}\right)\end{aligned}$$

$\Longrightarrow$ Amplitudes in position space at $\mathscr{I}$ = Carrollian amplitudes. [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Mason-Ruzziconi-Yelleshpur Srikant '23]

- Extrapolate dictionary for Carrollian holography:

$$\langle \Phi^{\epsilon_1}_{(k_1,\bar{k}_1)}(u_1,z_1,\bar{z}_1)\dots\Phi^{\epsilon_n}_{(k_n,\bar{k}_n)}(u_n,z_n,\bar{z}_n)\rangle_{\text{Boundary}} = \lim_{r\to\epsilon\infty}\langle r^{1-s_1}\phi^{(s_1)}(u_1,r_1,z_1,\bar{z}_1)\dots r^{1-s_n}\phi^{(s_n)}(u_n,r_n,z_n,\bar{z}_n)\rangle_{\text{Bulk}}$$

Carrollian Ward identities

- Consistent with the (global) conformal Carrollian Ward identities:

$$\sum_{i=0}^{n} \Big[\Big(\mathcal{T}(z_i, \bar{z}_i) + \frac{u_i}{2} (\partial_{z_i} \mathcal{Y}(z_i) + \partial_{\bar{z}_i} \bar{\mathcal{Y}}(\bar{z}_i)) \Big) \partial_{u_i} + \mathcal{Y}(z_i) \partial_{z_i} + \bar{\mathcal{Y}}(\bar{z}_i) \partial_{\bar{z}_i} + k_i \partial_{z_i} \mathcal{Y}(z_i) + \bar{k}_i \partial_{\bar{z}_i} \bar{\mathcal{Y}}(\bar{z}_i) \Big] \langle \Phi_{(k_1, \bar{k}_1)}(u_1, z_1, \bar{z}_1) \dots \Phi_{(k_n, \bar{k}_n)}(u_n, z_n, \bar{z}_n) \rangle = 0$$

where

$$\mathcal{T}(z, \bar{z}) = 1, z, \bar{z}, z\bar{z}, \qquad \mathcal{Y}(z) = 1, z, z^2, \qquad \bar{\mathcal{Y}}(\bar{z}) = 1, \bar{z}, \bar{z}^2$$

$\Longrightarrow$ The low-point correlation functions are fixed by the conformal Carrollian symmetries.

$\Longrightarrow$ The higher-point functions provide non-trivial information on the dynamics.

- In particular, for the 2-point function [Chen-Liu-Zheng, '21]:

$$\langle \Phi_{(k_1, \bar{k}_1)}(u_1, z_1, \bar{z}_1) \Phi_{(k_2, \bar{k}_2)}(u_2, z_2, \bar{z}_2) \rangle = \begin{cases} \dfrac{\alpha}{(u_1 - u_2)^{k_1 + k_2 + \bar{k}_1 + \bar{k}_2 - 2}} \delta^{(2)}(z_1 - z_2) \delta_{k_1 + k_2, \bar{k}_1 + \bar{k}_2} & \text{(Electric branch)} \\ \dfrac{\beta}{(z_1 - z_2)^{k_1 + k_2} (\bar{z}_1 - \bar{z}_2)^{\bar{k}_1 + \bar{k}_2}} \delta_{k_1, k_2} \delta_{\bar{k}_1, \bar{k}_2} & \text{(Magnetic branch)} \end{cases}$$

$\Longrightarrow$ Electric branch relevant for massless scattering.

[Donnay-Fiorucci-Herfray-Ruzziconi '22] [Bagchi-Banerjee-Basu-Dutta '22] [Mason-Ruzziconi-Yelleshpur Srikant '23]

$\Longrightarrow$ The $\delta^{(2)}(z_1 - z_2)$ distribution is a standard feature of Carrollian CFT (light cones shrink into lines), not undesirable.

Relation between Carrollian CFT and celestial CFT

- Celestial amplitudes obtained by Mellin transform [de Boer-Solodukhin '03] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17]:

$$\mathcal{M}_n\left(\{\Delta_1, z_1, \bar{z}_1\}^{\epsilon_1}_{J_1}, \dots, \{\Delta_n, z_n, \bar{z}_n\}^{\epsilon_n}_{J_n}\right) = \prod_{i=1}^{n}\left(\int_0^{+\infty} d\omega_i\, \omega_i^{\Delta_i - 1}\right) \mathcal{A}_n\left(\{\omega_1, z_1, \bar{z}_1\}^{\epsilon_1}_{J_1}, \dots, \{\omega_n, z_n, \bar{z}_n\}^{\epsilon_n}_{J_n}\right)$$
$$\equiv \langle \mathcal{O}^{\epsilon_1}_{\Delta_1, J_1}(z_1, \bar{z}_1) \dots \mathcal{O}^{\epsilon_n}_{\Delta_n, J_n}(z_n, \bar{z}_n)\rangle$$

- Relation between Carrollian and celestial amplitudes [Donnay-Fiorucci-Herfray-Ruzziconi '22]:

$$\mathcal{M}_n\left(\{\Delta_1, z_1, \bar{z}_1\}^{\epsilon_1}_{J_1}, \dots, \{\Delta_n, z_n, \bar{z}_n\}^{\epsilon_n}_{J_n}\right) = \prod_{i=1}^{n}\left((-i\epsilon_i)^{\Delta_i}\Gamma[\Delta_i]\int_{-\infty}^{+\infty}\frac{du_i}{(u_i - i\epsilon_i\varepsilon)^{\Delta_i}}\right) \mathcal{C}_n\left(\{u_1, z_1, \bar{z}_1\}^{\epsilon_1}_{J_1}, \dots, \{u_n, z_n, \bar{z}_n\}^{\epsilon_n}_{J_n}\right)$$

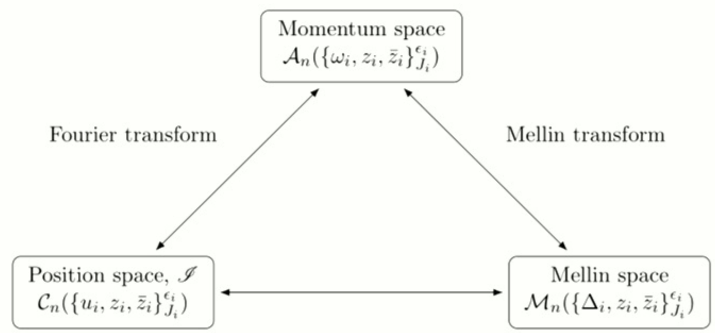
- Relation between Carrollian and celestial operators:

$$\boxed{\mathcal{O}^{\epsilon}_{\Delta_i, J_i}(z_i, \bar{z}_i) = (-i\epsilon_i)^{\Delta_i}\Gamma[\Delta_i]\int_{-\infty}^{+\infty}\frac{du_i}{(u_i - i\epsilon_i\varepsilon)^{\Delta_i}}\,\Phi^{\epsilon}_{(k_i, \bar{k}_i)}(u_i, z_i, \bar{z}_i)}$$

$\Longrightarrow$ Exchange between time and conformal dimension.

$\Longrightarrow$ Three scattering bases (ω, u, Δ) [Donnay-Pasterski-Puhm '22].

$\Longrightarrow$ Extrapolate dictionary in celestial holography [Pasterski-Puhm-Trevisani '21].



Two-point Carrollian amplitude

- Two-point amplitude (one incoming and one outgoing particle):

$$\mathcal{A}_2(\{\omega_1, z_1, \bar{z}_1\}^-_{J_1}, \{\omega_2, z_2, \bar{z}_2\}^+_{J_2}) = \kappa^2_{J_1,J_2}\, \pi\, \frac{\delta(\omega_1 - \omega_2)}{\omega_1}\, \delta^{(2)}(z_1 - z_2)\, \delta_{J_1,J_2},$$

- Carrollian two-point amplitude obtained by Fourier transform [Liu-Long '22] [Donnay-Fiorucci-Herfray-Ruzziconi '22]:

$$\begin{aligned}\mathcal{C}_2(\{u_1, z_1, \bar{z}_1\}^-_{J_1}, \{u_2, z_2, \bar{z}_2\}^+_{J_2}) &= \frac{1}{4\pi^2}\int_0^{+\infty} d\omega_1 \int_0^{+\infty} d\omega_2\, e^{-i\omega_1 u_1} e^{i\omega_2 u_2}\, \mathcal{A}_2(\{\omega_1, z_1, \bar{z}_1\}^-_{J_1}, \{\omega_2, z_2, \bar{z}_2\}^+_{J_2}) \\ &= \frac{\kappa^2_{J_1,J_2}}{4\pi}\int_0^{+\infty} \frac{d\omega}{\omega}\, e^{-i\omega(u_1 - u_2)}\, \delta^{(2)}(z_1 - z_2)\, \delta_{J_1,J_2}\,.\end{aligned}$$

- The divergent integral in the last line can be regularized. Instead, let us consider the correlator of ∂_u-descendants:

$$\tilde{\mathcal{C}}_2(\{u_1, z_1, \bar{z}_1\}^-_{J_1}, \{u_2, z_2, \bar{z}_2\}^+_{J_2}) = \lim_{\varepsilon \to 0^+} \frac{\kappa^2_{J_1,J_2}}{4\pi}\, \frac{1}{(u_{12} - i\varepsilon)^2}\, \delta^{(2)}(z_{12})\, \delta_{J_1,J_2}$$

$\Longrightarrow$ Standard solution of the Carrollian Ward identities (electric branch) for operators with fixed conformal Carrollian dimension $\Delta = k + \bar{k} = 2$.

- Apply the Carroll/celestial relation:

$$\mathcal{M}_2(\{\Delta_1, z_1, \bar{z}_1\}^-_{J_1}, \{\Delta_2, z_2, \bar{z}_2\}^+_{J_2}) = 2\pi^2\, \kappa^2_{J_1,J_2}\, \delta(\Delta_1 + \Delta_2 - 2)\, \delta^{(2)}(z_{12})\, \delta_{J_1,J_2},$$

[Pasterski-Shao-Strominger '17]

$\Longrightarrow$ Price to pay to go from 3d Carrollian CFT to 2d CFT: distributional low-point functions, not standard in CFT.

Three-point Carrollian amplitude

- The three-point amplitude generically vanishes in Lorentzian signature $\implies$ Go to split $(2,2)$ signature.

$$p_i^\mu = \epsilon_i q_i^\mu = \epsilon_i \omega_i \left(1 + z_i \bar{z}_i, z_i + \bar{z}_i, z_i - \bar{z}_i, 1 - z_i \bar{z}_i\right).$$

Here $(z_i, \bar{z}_i)$ are coordinates on a Poincaré patch of $\mathcal{LT}_2$ and $\epsilon_i = \pm 1$ labels the Poincaré patches.

- Using spinor-helicity notations, $p_{\alpha\dot{\alpha}} \equiv \sigma^\mu_{\alpha\dot{\alpha}} p_\mu = \kappa_\alpha \tilde{\kappa}_{\dot{\alpha}}$ and $[ij] = \tilde{\kappa}_{i\dot{\alpha}} \tilde{\kappa}_j^{\dot{\alpha}}$, at tree-level, the three-point amplitude reads as

$$\mathcal{A}_3(1^{J_1}, 2^{J_2}, 3^{J_3}) = \kappa_{J_1,J_2,J_3} [12]^{J_1+J_2-J_3} [23]^{J_2+J_3-J_1} [31]^{J_3+J_1-J_2} \delta^{(4)}(p_1 + p_2 + p_3), \text{ if } J_1 + J_2 + J_3 > 0$$

A similar expression exists for $J_1 + J_2 + J_3 + 2 < 0$.

- Three-point Carrollian amplitude (still determined by Ward identities):

[Banerjee-Ghosh-Pandey-Saha '20] [Salzer '23] [Mason-Ruzziconi-Yelleshpur Srikant '23]

$$\begin{aligned}\tilde{\mathcal{C}}_3 = \kappa_{J_1,J_2,J_3} &\frac{-i\epsilon_1\epsilon_2\epsilon_3\,\delta(z_{12})\,\delta(z_{23})}{4(2\pi)^3} \Theta\left(-\frac{\bar{z}_{13}}{\bar{z}_{23}}\epsilon_1\epsilon_2\right) \Theta\left(\frac{\bar{z}_{12}}{\bar{z}_{23}}\epsilon_1\epsilon_3\right) |\bar{z}_{12}|^{J_1+J_2} |\bar{z}_{23}|^{J_2+J_3} |\bar{z}_{31}|^{J_3+J_1} \\ &\times (\text{sign } \bar{z}_{12})^{J_1+J_2-J_3+1} (\text{sign } \bar{z}_{23})^{J_2+J_3-J_1+1} (\text{sign } \bar{z}_{13})^{J_1+J_3-J_2+1} \frac{(i\,\epsilon_1 \text{ sign}(\bar{z}_{23}))^{J_1+J_2+J_3+2}\,\Gamma(J_1+J_2+J_3+2)}{(\bar{z}_{23}u_1 - \bar{z}_{13}u_2 + \bar{z}_{12}u_3 + i\epsilon_1 \text{ sign}(\bar{z}_{23})\varepsilon)^{J_1+J_2+J_3+2}}\end{aligned}$$

for $J_1 + J_2 + J_3 + 2 > 0$ (similarly for $J_1 + J_2 + J_3 + 2 > 0$).

- Using Carroll/celestial correspondence ($\bar{h}_k = \frac{\Delta_k - J_k}{2}$):

$$\begin{aligned}\mathcal{M}_3 = \frac{(-i)^{J_1+J_2+J_3}\,\pi}{2} &\kappa_{J_1,J_2,J_3}\,\delta(z_{12})\,\delta(z_{23})\,\Theta\left(-\frac{\bar{z}_{13}}{\bar{z}_{23}}\epsilon_1\epsilon_2\right) \Theta\left(\frac{\bar{z}_{12}}{\bar{z}_{23}}\epsilon_1\epsilon_3\right) \frac{1}{\bar{z}_{12}^{\bar{h}_1+\bar{h}_2-\bar{h}_3}\,\bar{z}_{23}^{\bar{h}_2+\bar{h}_3-\bar{h}_1}\,\bar{z}_{12}^{\bar{h}_3+\bar{h}_1-\bar{h}_2}} \\ &\times (\text{sign } \bar{z}_{12})^{J_1+J_2-J_3} (\text{sign } \bar{z}_{23})^{J_2+J_3-J_1} (\text{sign } \bar{z}_{13})^{J_1+J_3-J_2} (\epsilon_1)^{\Delta_1} (\epsilon_2)^{\Delta_2} (\epsilon_3)^{\Delta_3}\,\delta(\Delta_1+\Delta_2+\Delta_3+J_1+J_2+J_3-4).\end{aligned}$$

[Pasterski-Shao-Strominger '17]

Four-point Carrollian amplitude

- At tree-level the 4-point gluon MHV amplitude is given by

$$\mathcal{A}_4\left(1^{+1},2^{-1},3^{-1},4^{+1}\right)=\kappa^2_{1,1,-1}\frac{\langle 23\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}=\kappa^2_{1,1,-1}\frac{\omega_2\omega_3}{\omega_1\omega_4}\frac{z_{23}^3}{z_{12}z_{34}z_{41}},$$

- Applying the Fourier transform yields the corresponding Carrollian amplitude (very similar computation for gravitons):
[Banerjee-Ghosh-Pandey-Saha '20] [Mason-Ruzziconi-Yelleshpur Srikant '23]

$$\tilde{\mathcal{C}}_4\left(1^{+1},2^{-1},3^{-1},4^{+1}\right)=\frac{\kappa^2_{1,1,-1}}{(2\pi)^4}\frac{z_{34}^2\bar{z}_{14}^4\bar{z}_{34}^2}{z^3(1-z)z_{13}^3z_{24}\bar{z}_{13}^5\bar{z}_{24}^3}\delta(z-\bar{z})\,\Theta\left(-z\left|\frac{z_{24}}{z_{12}}\right|^2\epsilon_1\epsilon_4\right)\Theta\left(\frac{1-z}{z}\left|\frac{z_{34}}{z_{23}}\right|^2\epsilon_2\epsilon_4\right)$$
$$\Theta\left(-\frac{1}{1-z}\left|\frac{z_{14}}{z_{13}}\right|^2\epsilon_3\epsilon_4\right)\times\frac{3!}{\left(u_4-u_1z\left|\frac{z_{24}}{z_{12}}\right|^2+u_2\frac{1-z}{z}\left|\frac{z_{34}}{z_{23}}\right|^2-u_3\frac{1}{1-z}\left|\frac{z_{14}}{z_{13}}\right|^2\right)^4},$$

- Using Carroll/celestial correspondence:

$$\mathcal{M}_4=\prod_{i=1}^4\left(\int_{-\infty}^{+\infty}du_i\,(-i\epsilon_i)^{\Delta_i}\Gamma(\Delta_i-1)\,u_i^{1-\Delta_i}\right)\tilde{\mathcal{C}}_4$$
$$=\prod_{i=1}^4(-i\epsilon_i)^{\Delta_i}z^{-\frac{1}{3}}(1-z)^{\frac{5}{3}}\prod_{i<j}z_{ij}^{\frac{h}{3}-h_i-h_j}\bar{z}_{ij}^{\frac{\bar{h}}{3}-\bar{h}_i-\bar{h}_j}(-1)^{\Delta_2+\Delta_4+1}2\pi\delta(\Delta_1+\Delta_2+\Delta_3+\Delta_4-4)$$
$$\delta(z-\bar{z})\,\Theta\left(-z\left|\frac{z_{24}}{z_{12}}\right|^2\epsilon_1\epsilon_4\right)\Theta\left(\frac{1-z}{z}\left|\frac{z_{34}}{z_{23}}\right|^2\epsilon_2\epsilon_4\right)\Theta\left(-\frac{1}{1-z}\left|\frac{z_{14}}{z_{13}}\right|^2\epsilon_3\epsilon_4\right).$$

[Pasterski-Shao-Strominger '17]

n-point MHV Carrollian amplitude

- Colour ordered MHV gluon amplitude (with n + 1 identified with 1):

$$\mathcal{A}_n\left(1^-, 2^-, 3^+, \dots, n^+\right) = \kappa_{1,1,-1}^{n-2} \frac{\langle 12\rangle^4}{\prod_{j=1}^n \langle jj+1\rangle} = \kappa_{1,1,-1}^{n-2} \frac{\omega_1\omega_2}{\prod_{j=3}^n \omega_j} \frac{z_{12}^3}{\prod_{j=2}^n z_{jj+1}}$$

(similar formula for gravitons)

- Use the decomposition of the delta distribution [Schreiber-Volovich-Zlotnikov '17]:

$$\delta^{(4)}\left(\sum_{i=1}^n p_i\right) = \frac{1}{|\mathcal{U}_{1234}|} \prod_{I=1}^4 \delta\left(\omega_I - \omega_I^*\right), \quad \text{with} \quad \omega_I^* = -\frac{1}{\mathcal{U}_{1234}} \sum_{i=5}^n \omega_i \mathcal{U}_{Ii}$$

where

$$\mathcal{U}_{1234} = \det\left(q_1^\mu, \dots, q_4^\mu\right), \qquad \mathcal{U}_{Ii} = \mathcal{U}_{1234}|_{I\to i}, \qquad I = 1,2,3,4; i = 5, \dots, n.$$

- Applying the Fourier transform [Mason-Ruzziconi-Yelleshpur Srikant '23]:

$$\tilde{\mathcal{C}}_n\left(1^-, 2^-, 3^+, \dots, n^+\right) = \frac{\kappa_{1,1,-1}^{n-2}}{(2\pi)^n |\mathcal{U}_{1234}|} \frac{z_{12}^3}{\prod_{j=2}^n z_{jj+1}} \frac{\partial^4}{\partial u_1^2 \partial u_2^2} I_n$$

where the integral I_n can be computed explicitly:

$$I_n = \int \prod_{j=5}^n d\omega_j e^{i\omega_j L_j} \prod_{I=1}^4 \Theta\left(\omega_I^*\right) = (-1)^{n-4} \prod_{j=5}^n \frac{1}{L_j}$$

with $L_j = \left(\epsilon_j u_j - \sum_{J=1}^4 \epsilon_J u_J \frac{\mathcal{U}_{Jj}}{\mathcal{U}_{1234}}\right)$

- Non-trivial dynamical constraints on the dual Carrollian CFT.
- Similar expression for the MHV graviton amplitude (same integral I_n).
- Surprisingly simpler than its celestial counterparts involving Aomoto-Gelfand hypergeometric function. [Schreiber-Volovich-Zlotnikov '17]

Collinear limit and Carrollian OPE

- Collinear limit of two outgoing particles ($\epsilon_1 = \epsilon_2 = +1$):

$$\mathcal{A}_n\left(1^{J_1}, 2^{J_2}, 3^{J_3}, \dots, n^{J_n}\right) \xrightarrow{1||2} \sum_J \mathcal{A}_3\left(1^{J_1}, 2^{J_2}, -P^{-J}\right) \frac{1}{\langle 12\rangle[21]} \mathcal{A}_{n-1}\left(P^J, 3^{J_3}, \dots, n^{J_n}\right)$$

where J is the helicity of the exchanged particle.

- In the limit $z_{12} \to 0$, we obtain the Carrollian OPE block [Mason-Ruzziconi-Yelleshpur Srikant '23]:

$$\Phi_{J_1}(u_1, z_1, \bar{z}_1)\,\Phi_{J_2}(u_2, z_2, \bar{z}_2) \sim -\frac{\kappa_{J_1,J_2,-J}}{2\pi}\frac{\bar{z}_{12}^p}{z_{12}}\int_0^1 dt\, t^{J_2-J-1}(1-t)^{J_1-J-1}\left(\frac{\partial}{\partial u}\right)^p \Phi_J(u, z_2, \bar{z}_2 + t\bar{z}_{12})|_{u=u_2+tu_{12}}.$$

with implicit sum on p with range determined by

$$p \geq 0, \qquad |J_1 + J_2 - p - 1| \leq 2 \quad \text{and} \quad |J_1| \leq 2, \qquad |J_2| \leq 2.$$

- Invariance under global $\mathfrak{CCarr}_3$ explicitly checked $\Longrightarrow$ Starting point for Carrollian CFT bootstrap?
- Action of soft symmetries ($Lw_{1+\infty}$ for gravity) on Carrollian operators and $\mathscr{I}$.
- Relation between Carrollian operators and twistor space ($Lw_{1+\infty}$ has a geometric interpretation).
- Using the Carroll/celestial correspondence, we recover the celestial OPE block

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\,\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim -\kappa_{J_1,J_2,-J}\frac{\bar{z}_{12}^p}{z_{12}}\int_0^1 dt\, t^{2\bar{h}_1+p-1}(1-t)^{2\bar{h}_2+p-1}\mathcal{O}_{\Delta_1+\Delta_2+p-1,J}.$$

[Fan-Fotopoulos-Taylor '19] [Pate-Raclariu-Strominger-Yuan '19]

Summary and perspectives

- Two complementary approaches to flat space holography:

  Carrollian holography $\Longleftrightarrow$ Celestial holography

- The two approaches are related via integral transform ✓
- Carrollian holography is a useful path:
  - $\Longrightarrow$ Naturally related to AdS/CFT via $\Lambda \to 0$ ✓
  - $\Longrightarrow$ Successful in 3d gravity, and beyond ✓
  - $\Longrightarrow$ Standard extrapolate dictionary ✓
  - $\Longrightarrow$ Carrollian physics has applications beyond flat space holography ✓
- Perspectives:
  - $\Longrightarrow$ Bootstrap of Carrollian amplitudes?
  - $\Longrightarrow$ Top-down models for Carrollian holography? via $c \to 0$ limit or relation with twistor theory?

Thank you!