Title: Conformal Colliders Meet the LHC

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Series: Particle Physics

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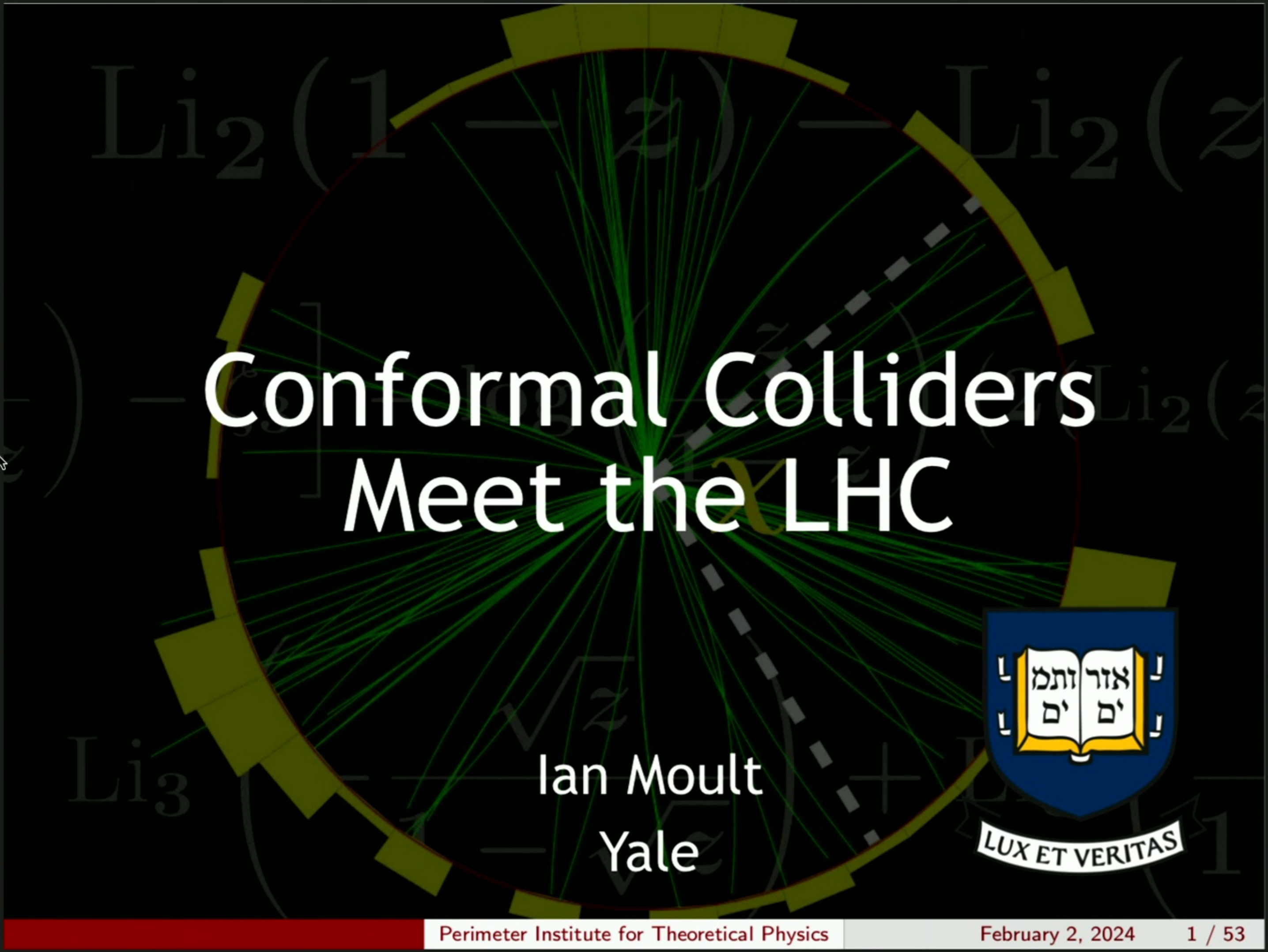
Abstract: Jets of hadrons produced at high-energy colliders provide experimental access to the dynamics of asymptotically free quarks and gluons and their confinement into hadrons. Motivated by recent developments in conformal field theory, we show that questions of interest in collider physics can be reformulated as the study of correlation functions of a specific class of light-ray operators and their associated operator product expansion (OPE). We show that multi-point correlation functions of these operators can be measured in real collider data, allowing us to experimentally verify the scaling properties associated with the OPE, and providing new insights into the dynamics of the confinement transition.

Zoom link



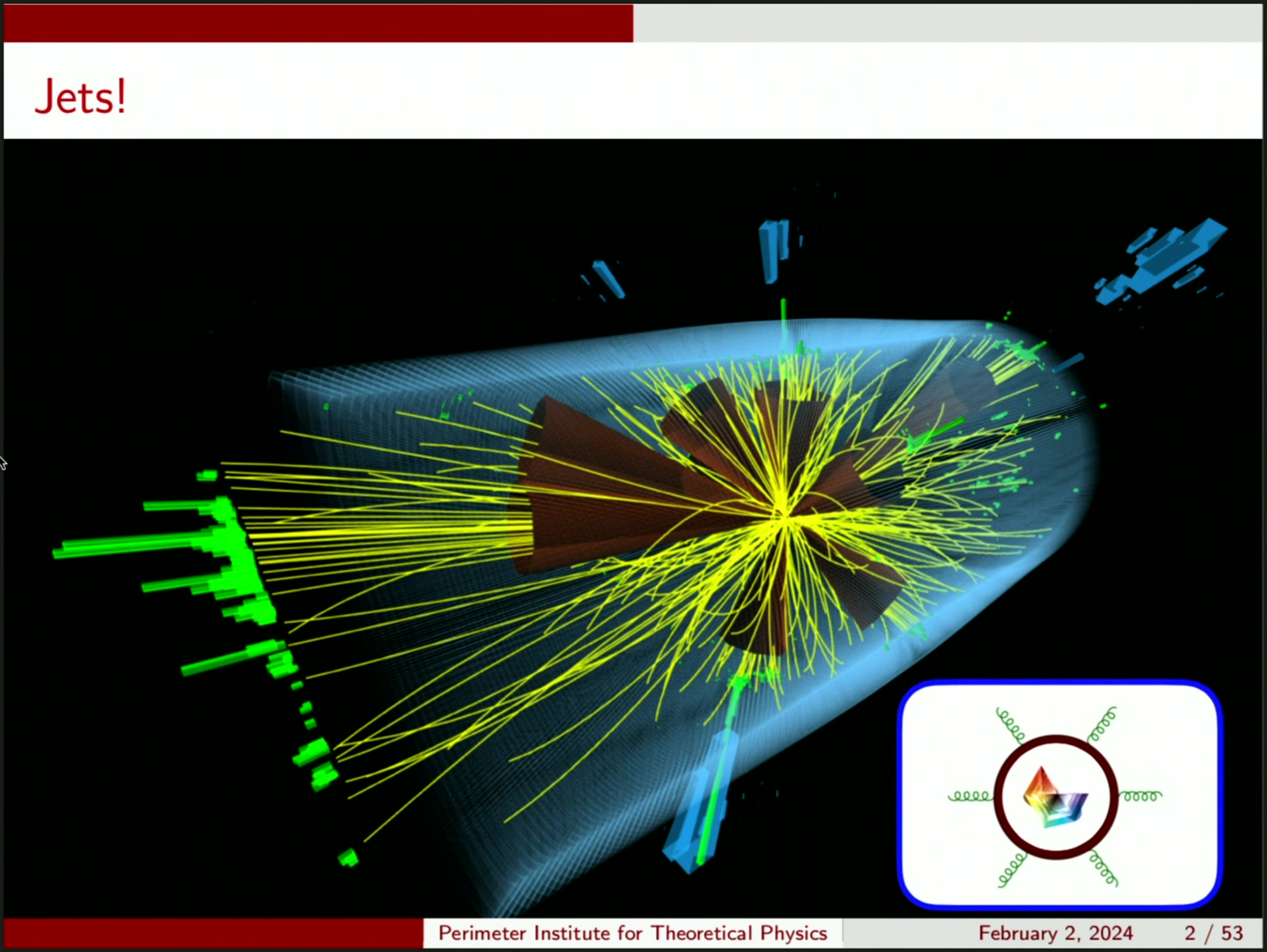
Conformal Colliders Meet the LHC

Ian Moult

Yale

Perimeter Institute for Theoretical Physics

February 2, 2024

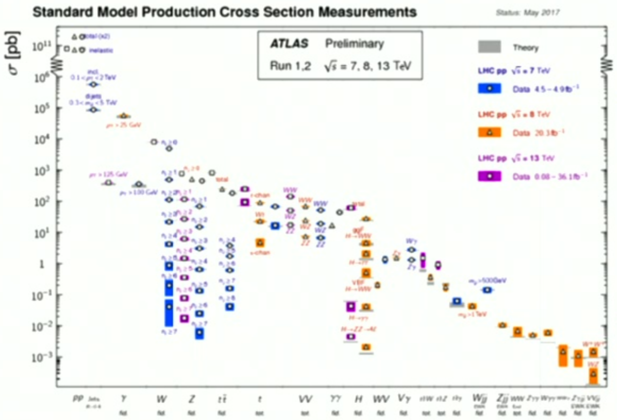


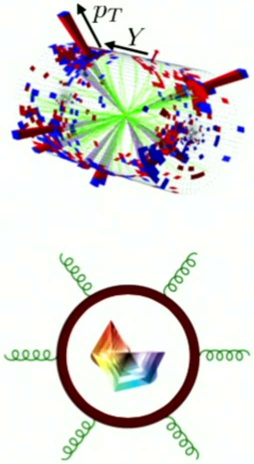
Jets!

Jets at the LHC

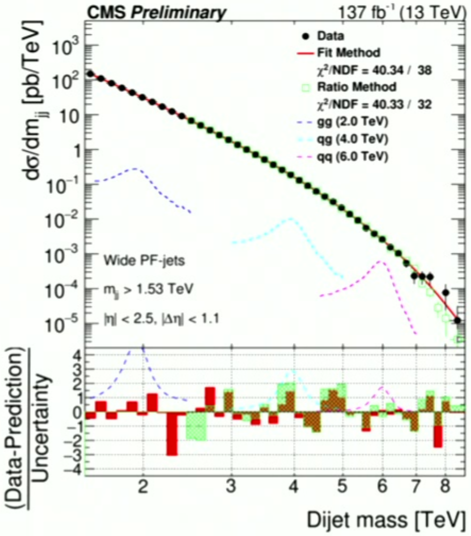
- Obtaining a precise description of jet cross sections has been a significant driver of theory developments in Quantum Field Theory.

Jet Kinematic Distributions



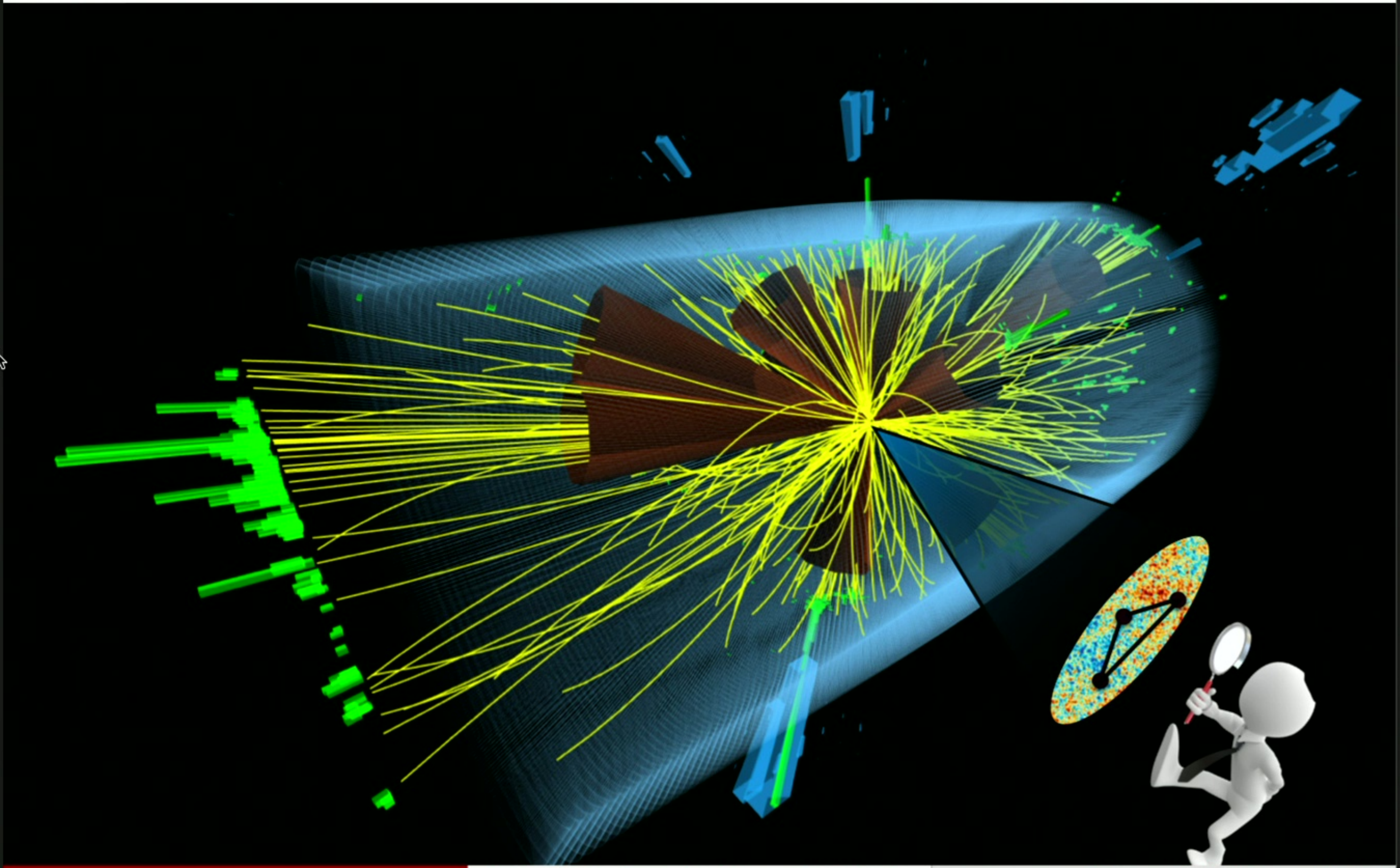


Dijet Mass



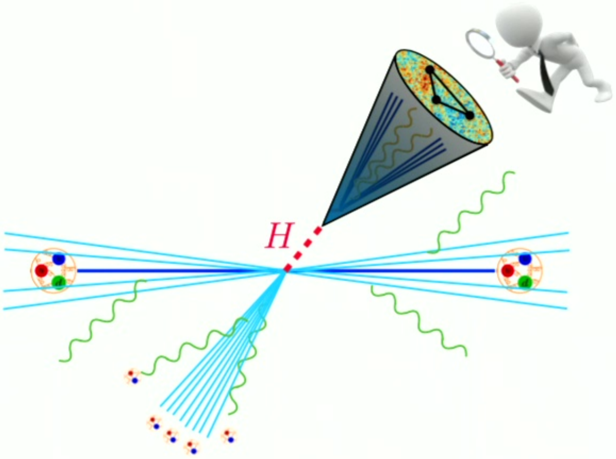
- Enables precision tests of QCD and searches for new physics.

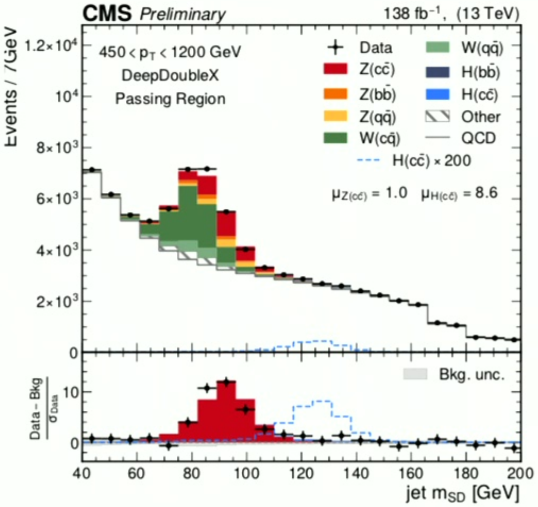
Jet Substructure!



Jet Substructure: Searches

- Jet Substructure uses the internal structure of jets to provide qualitatively new ways to study physics at the LHC.

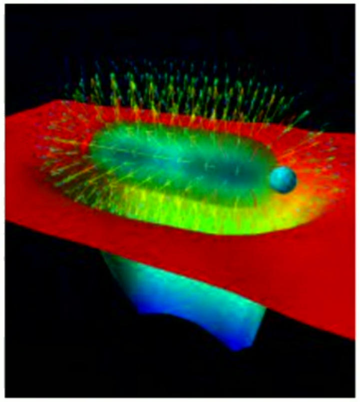


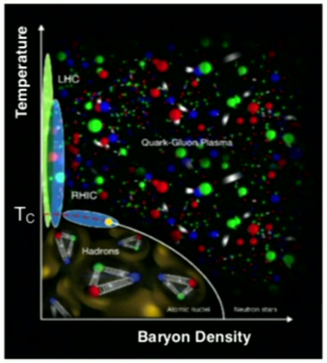


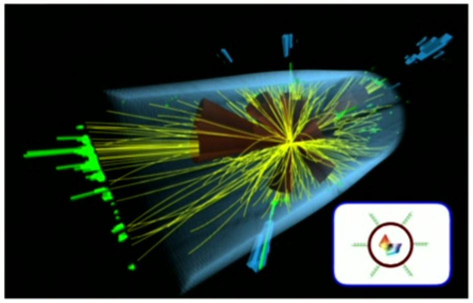
- Its introduction in 2008 by Butterworth, Davison, Rubin and Salam, along with anti-k_T by Cacciari, Soyez, Salam reinvigorated the study of jets in QCD.

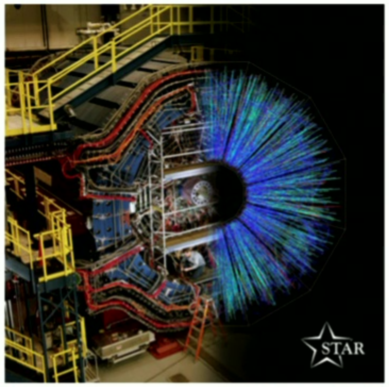
Jet Substructure: Quantum Field Theory

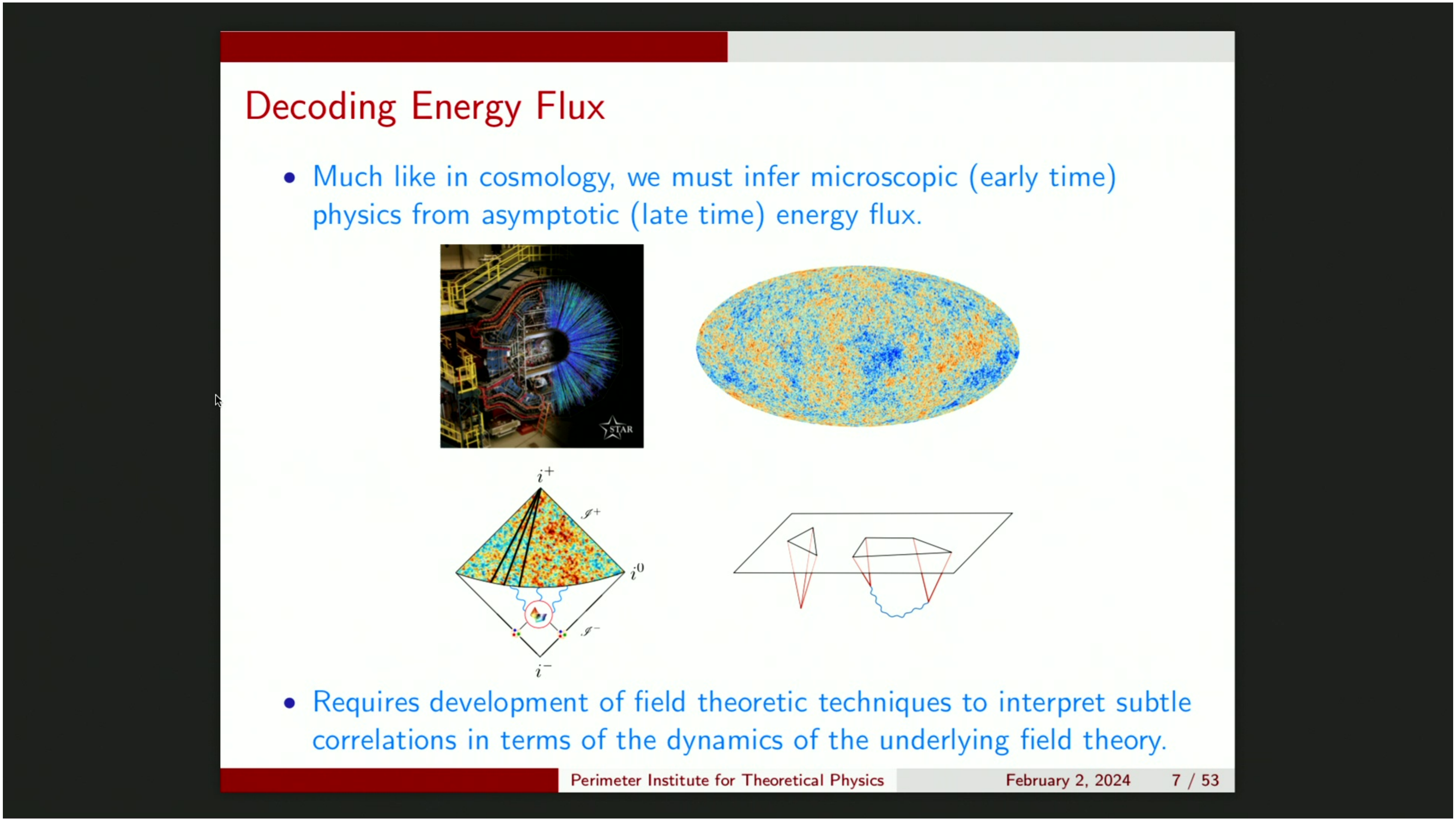
- Beyond searching for new physics, much more subtle questions about QCD are imprinted in collider energy flux:







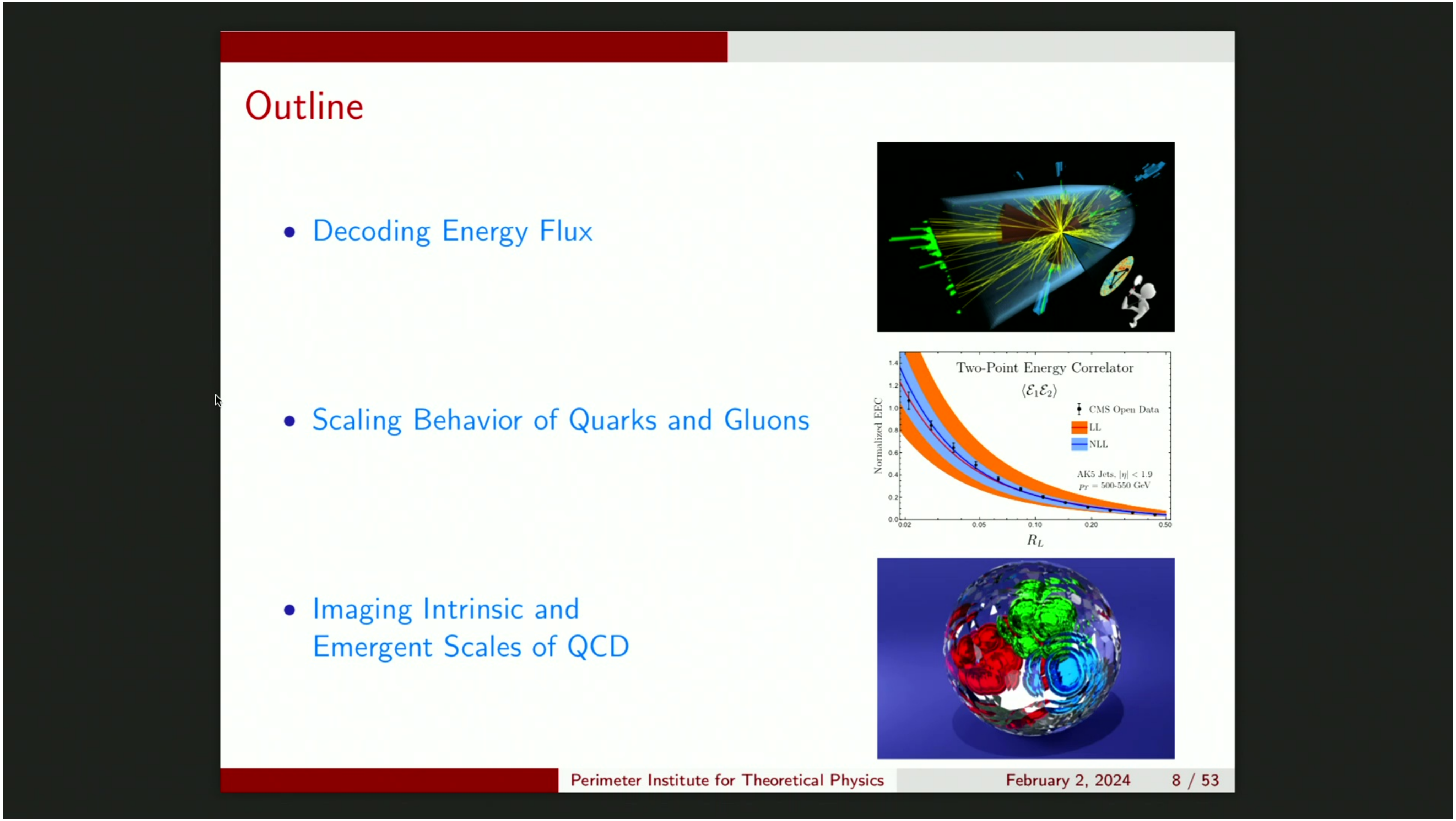




Decoding Energy Flux

- Much like in cosmology, we must infer microscopic (early time) physics from asymptotic (late time) energy flux.

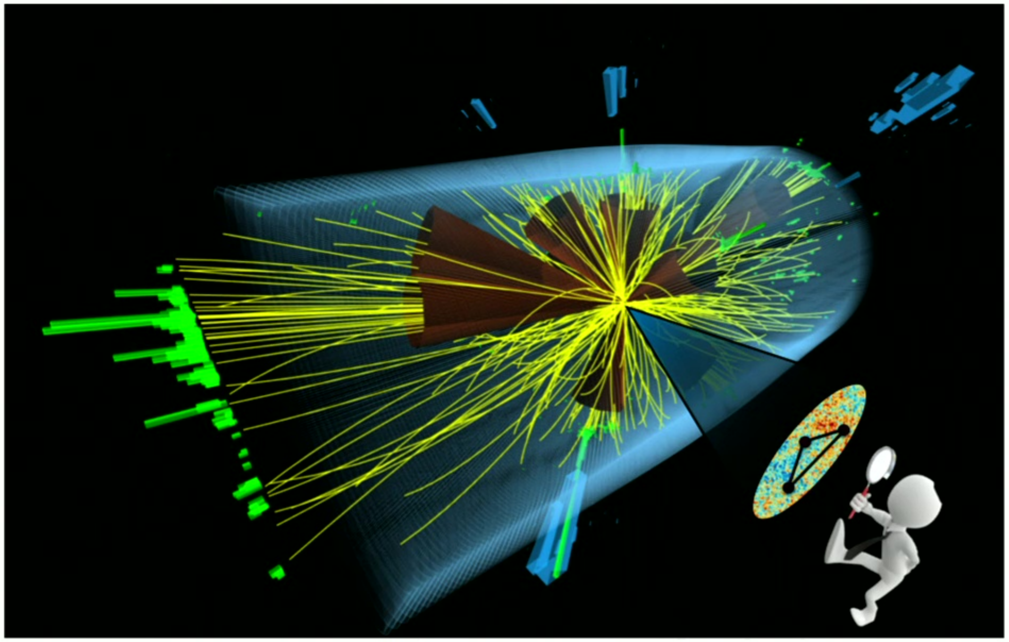
- Requires development of field theoretic techniques to interpret subtle correlations in terms of the dynamics of the underlying field theory.

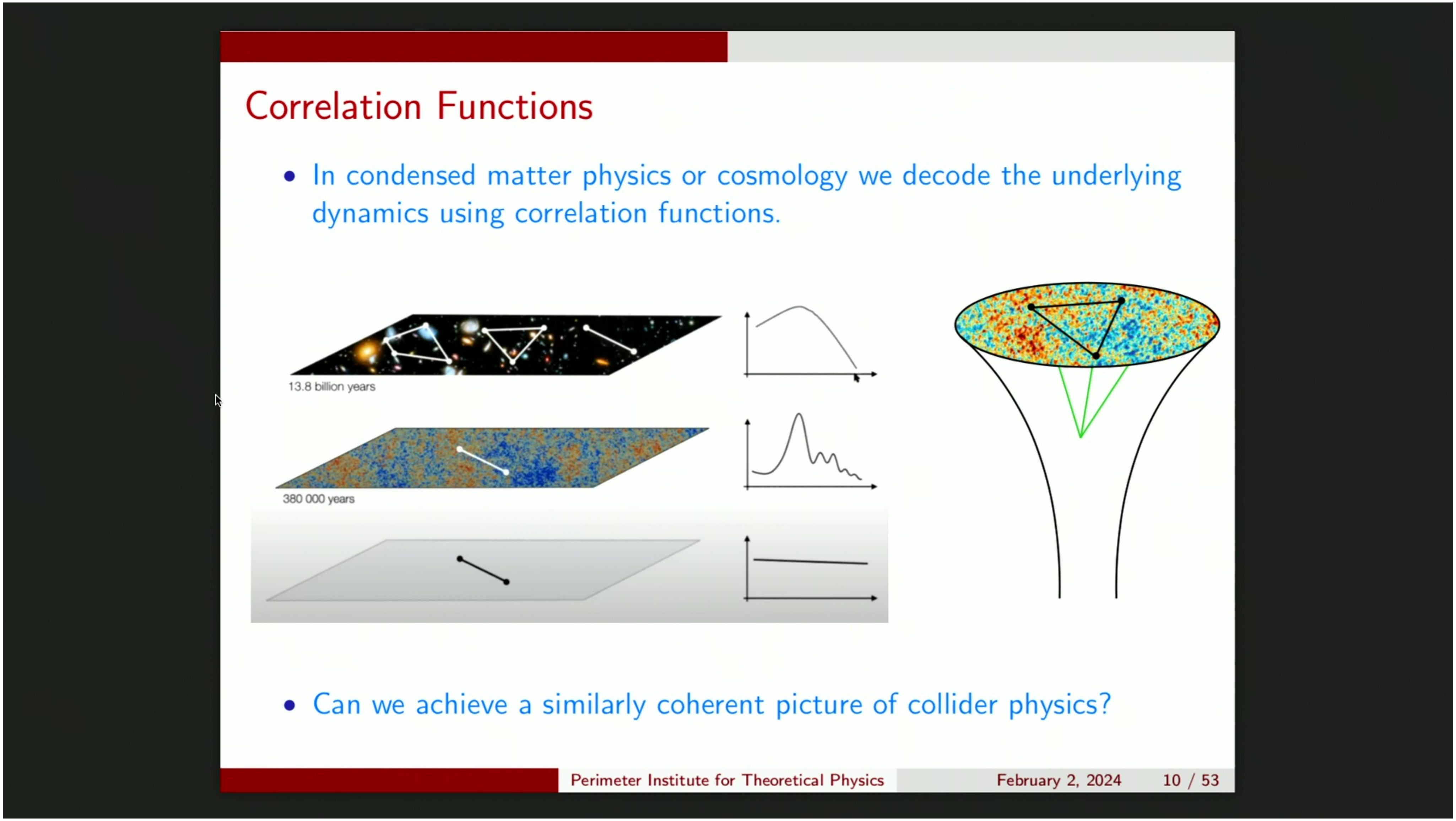


Outline

- Decoding Energy Flux
- Scaling Behavior of Quarks and Gluons
- Imaging Intrinsic and Emergent Scales of QCD

Decoding Energy Flux



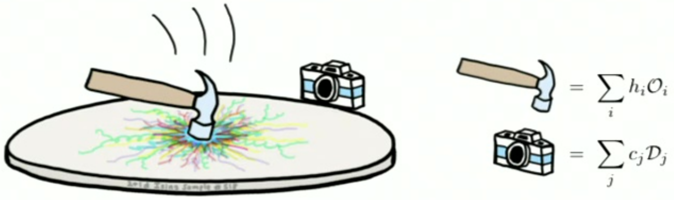


Correlation Functions

- In condensed matter physics or cosmology we decode the underlying dynamics using correlation functions.
- Can we achieve a similarly coherent picture of collider physics?

Defining the Problem

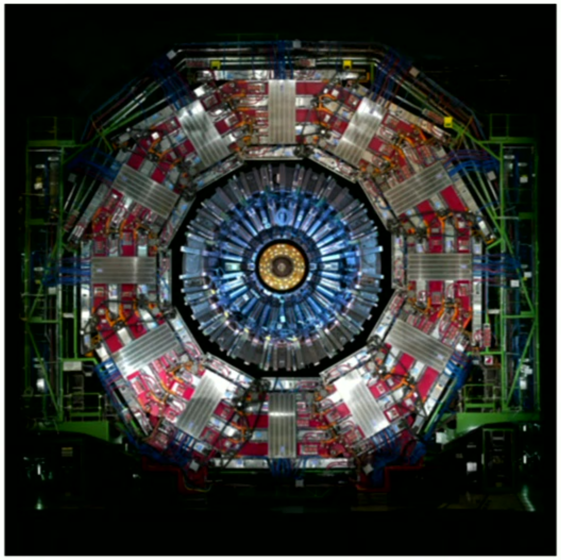
- What is a detector?



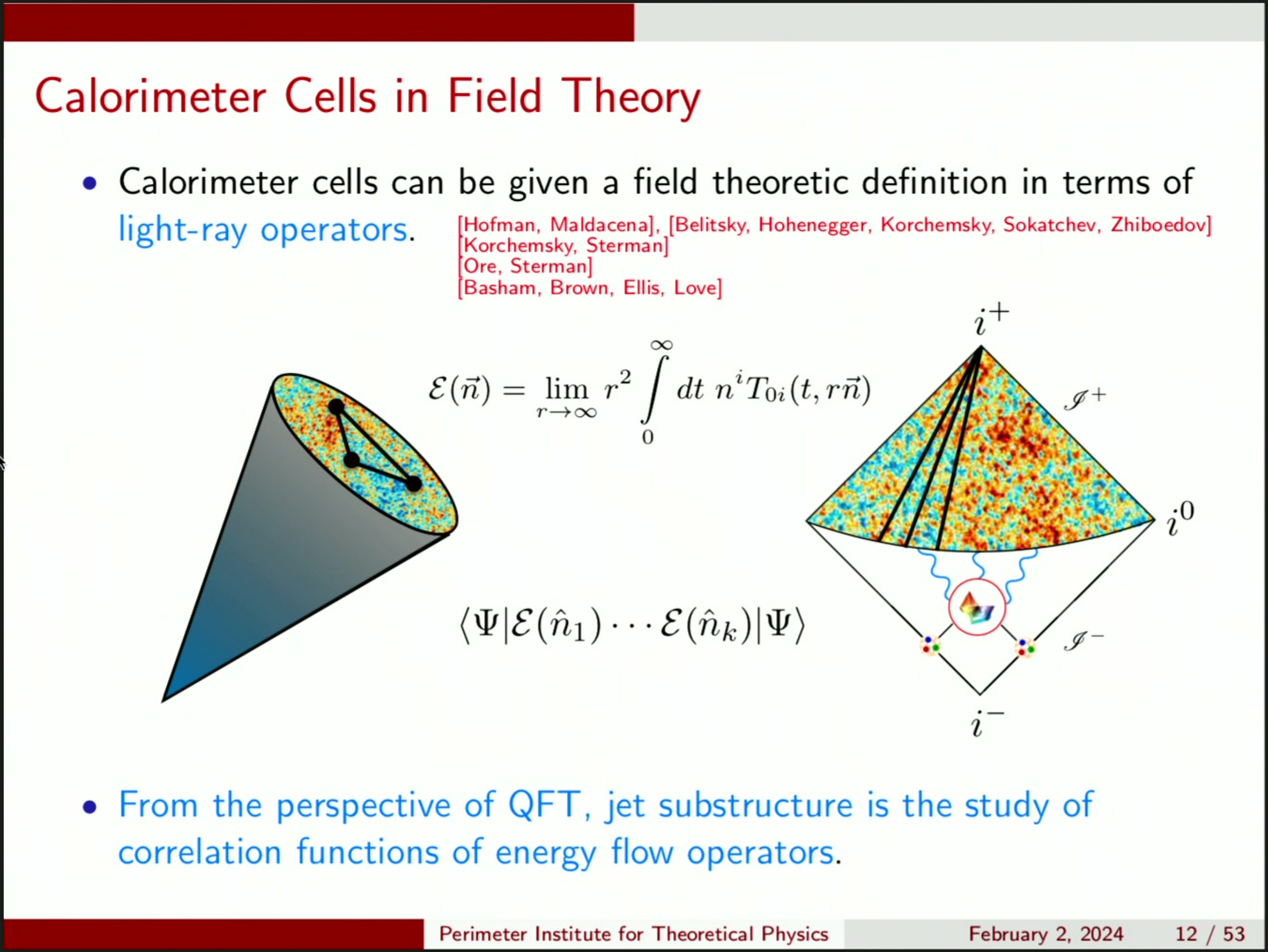
$$= \sum_i h_i \mathcal{O}_i$$

$$= \sum_j c_j \mathcal{D}_j$$

[Caron Huot, Kologlu, Kravchuk, Meltzer, Simmons Duffin]



- To be able to understand subtle signals in energy flux, we must understand what a detector is in Quantum Field Theory.



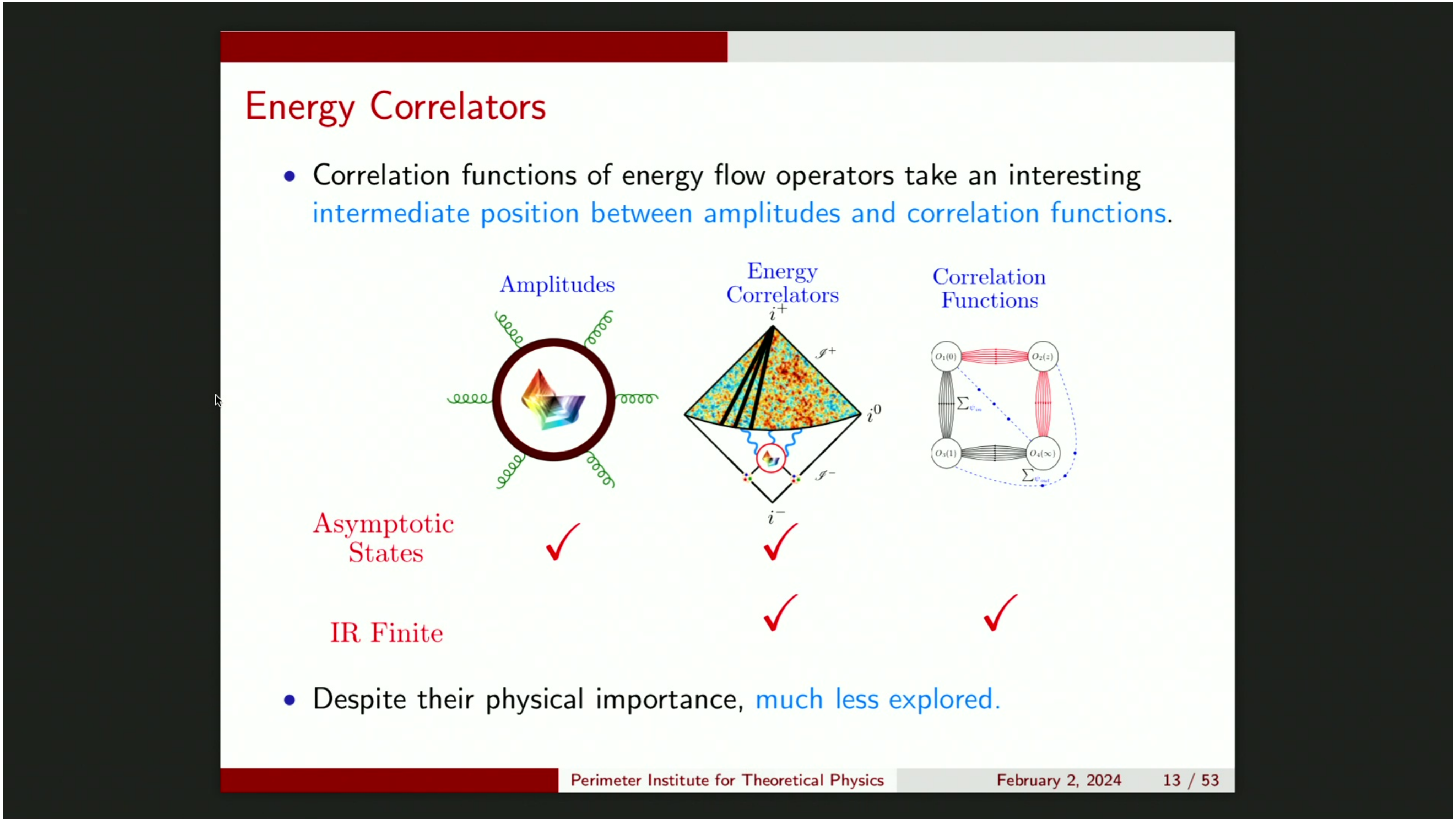
Calorimeter Cells in Field Theory

- Calorimeter cells can be given a field theoretic definition in terms of light-ray operators. [Hofman, Maldacena], [Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov] [Korchemsky, Sterman] [Ore, Sterman] [Basham, Brown, Ellis, Love]

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\; n^i T_{0i}(t, r\vec{n})$$

$$\langle\Psi|\mathcal{E}(\hat{n}_1)\cdots\mathcal{E}(\hat{n}_k)|\Psi\rangle$$

- From the perspective of QFT, jet substructure is the study of correlation functions of energy flow operators.



Energy Correlators

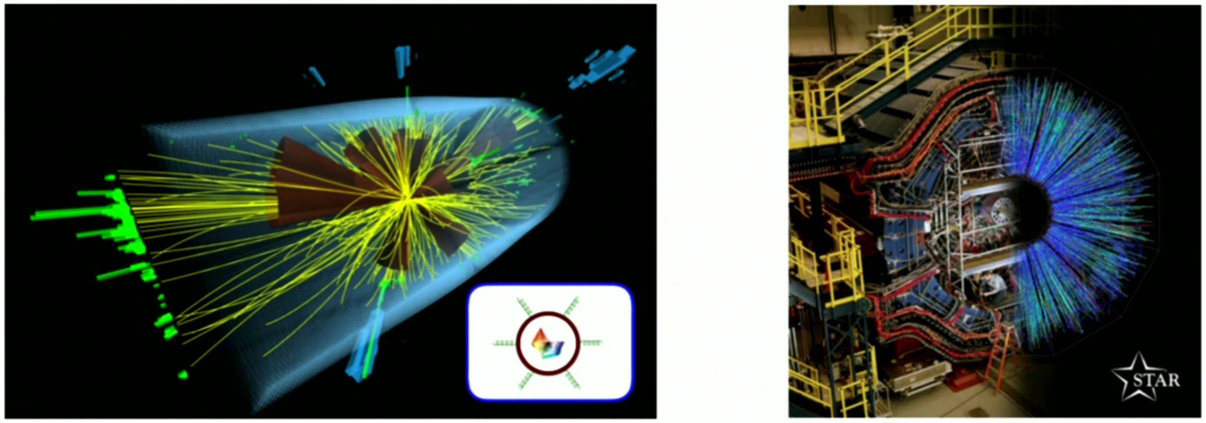
- Correlation functions of energy flow operators take an interesting intermediate position between amplitudes and correlation functions.

	Amplitudes	Energy Correlators	Correlation Functions
Asymptotic States	✓	✓	
IR Finite		✓	✓

- Despite their physical importance, much less explored.

Towards the World of Hadron Colliders

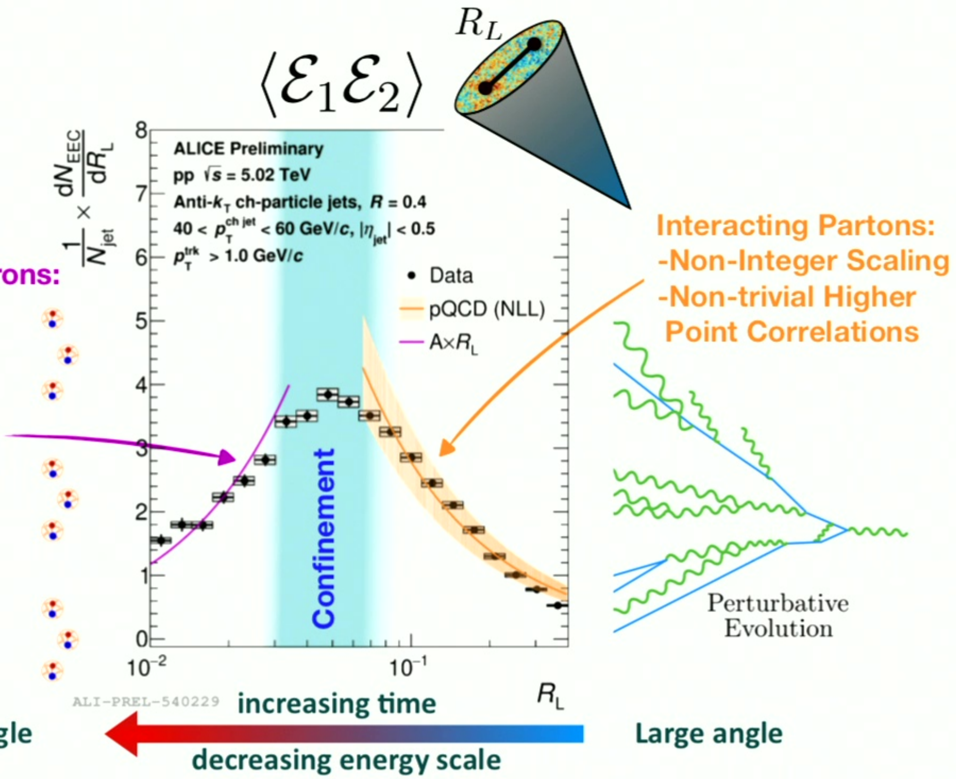
- Can this theoretical idealization possibly work in the messy world of hadron colliders?



- Can it provide new ways of understanding these complex collisions?

Energy Correlators: Present

Figure: Wenqing Fan



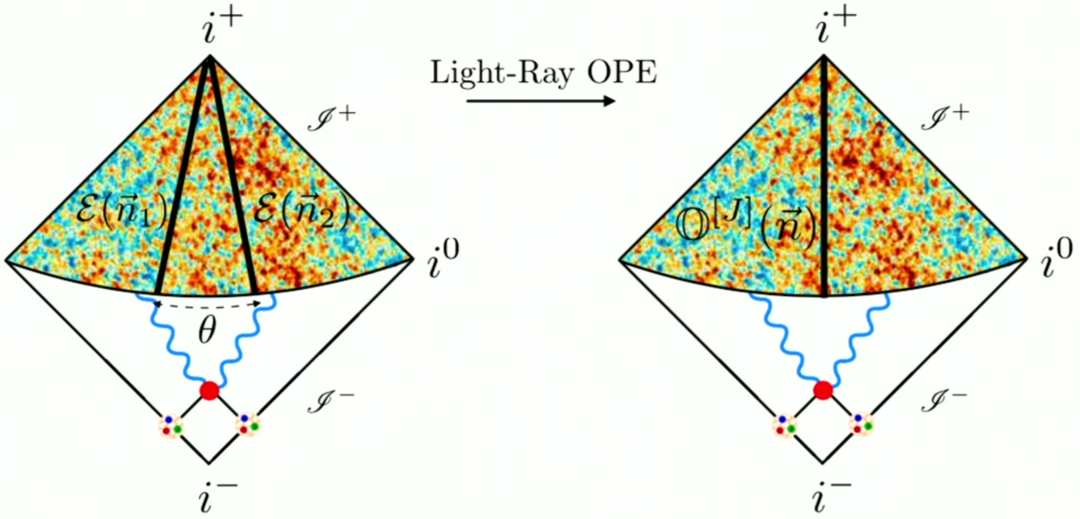
Non-Interacting Hadrons:
-Integer Scaling
-Trivial Higher Point Correlations

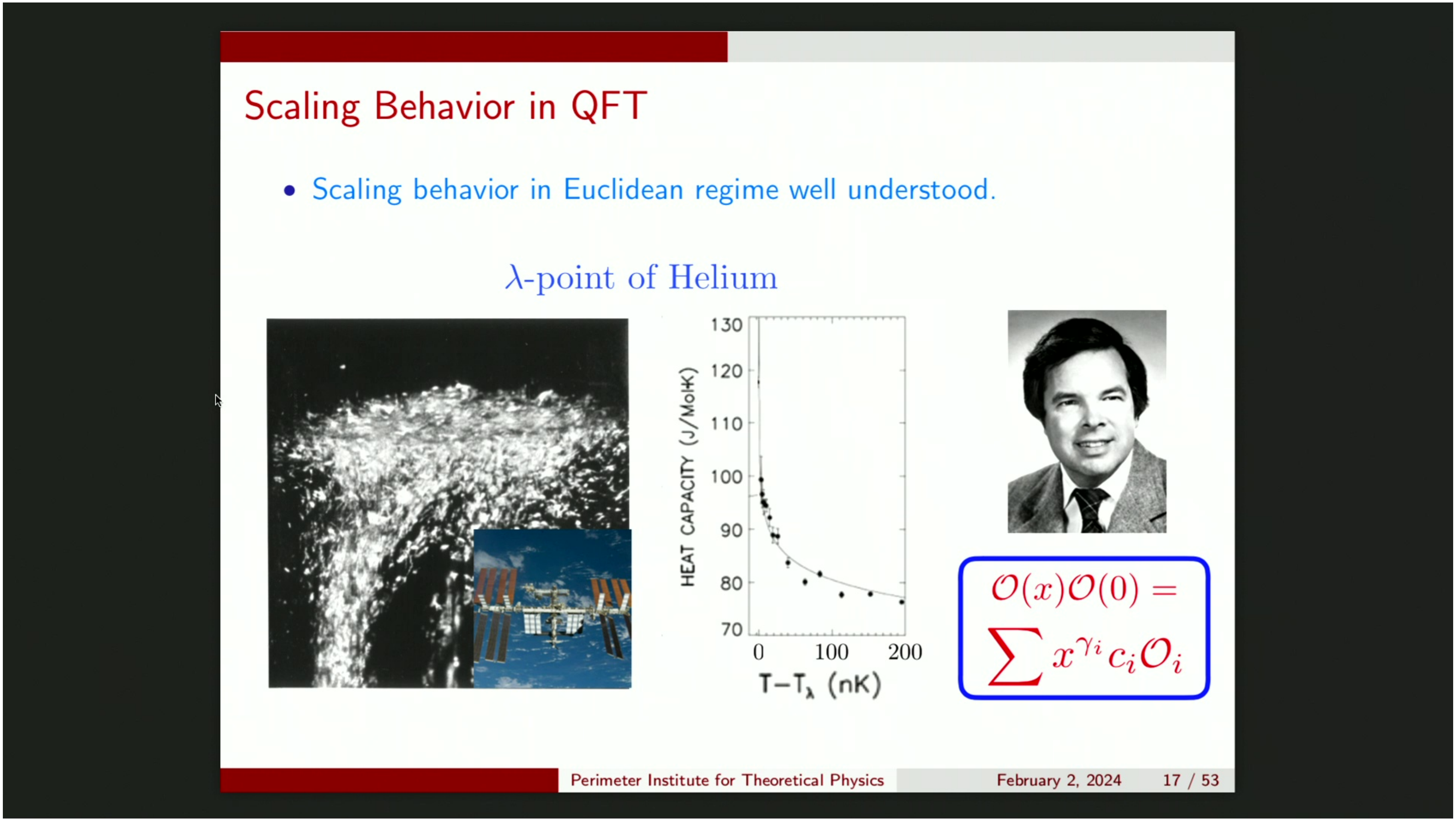
Interacting Partons:
-Non-Integer Scaling
-Non-trivial Higher Point Correlations

Small angle **increasing time** **decreasing energy scale** **Large angle**

- Asymptotic Freedom and Confinement in one plot!

Scaling Behavior of Quarks and Gluons



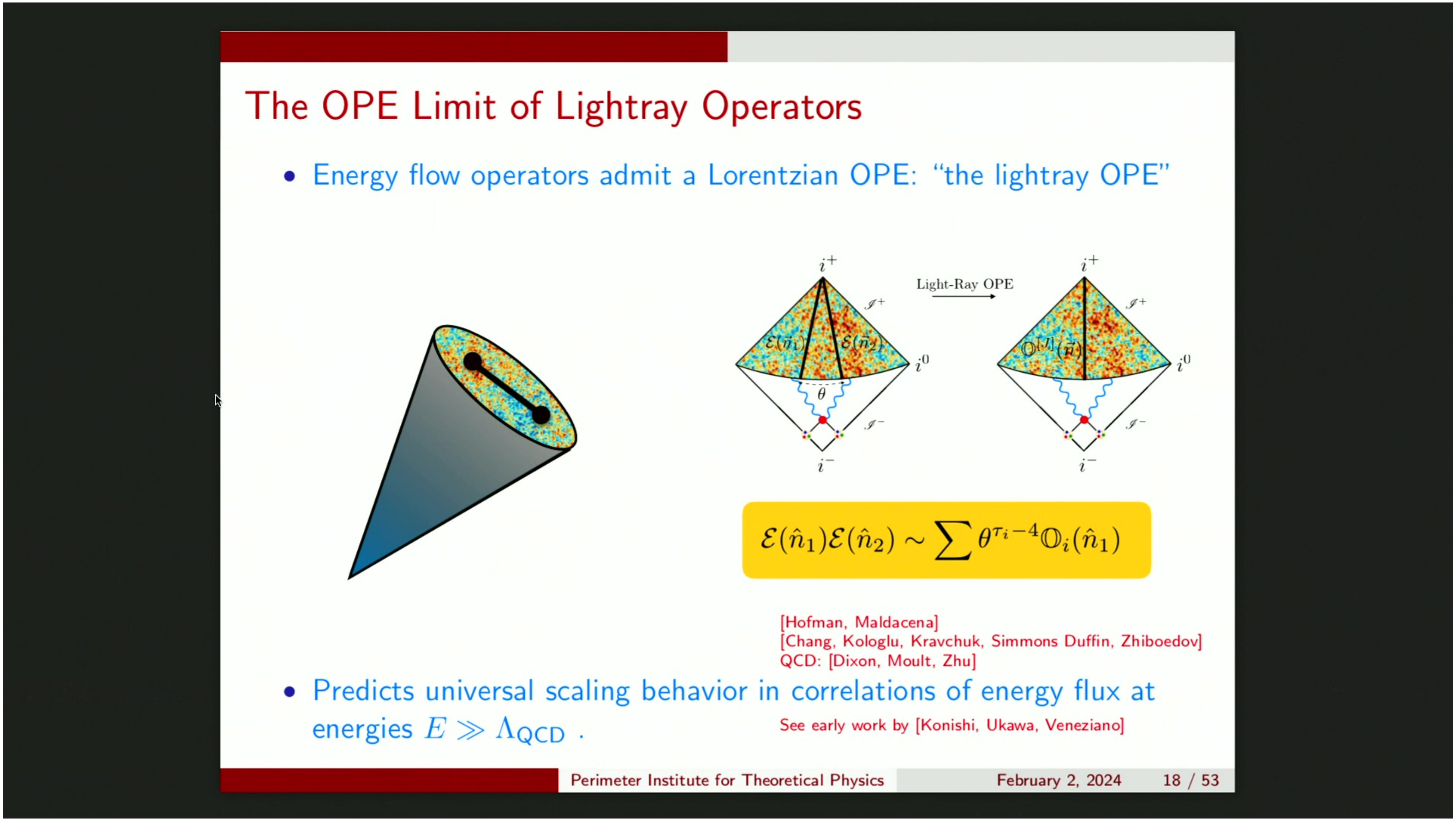


Scaling Behavior in QFT

- Scaling behavior in Euclidean regime well understood.

λ-point of Helium

$$\boxed{\mathcal{O}(x)\mathcal{O}(0) = \sum x^{\gamma_i} c_i \mathcal{O}_i}$$



The OPE Limit of Lightray Operators

- Energy flow operators admit a Lorentzian OPE: "the lightray OPE"

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i - 4}\mathbb{O}_i(\hat{n}_1)$$

[Hofman, Maldacena]
[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]
QCD: [Dixon, Moult, Zhu]

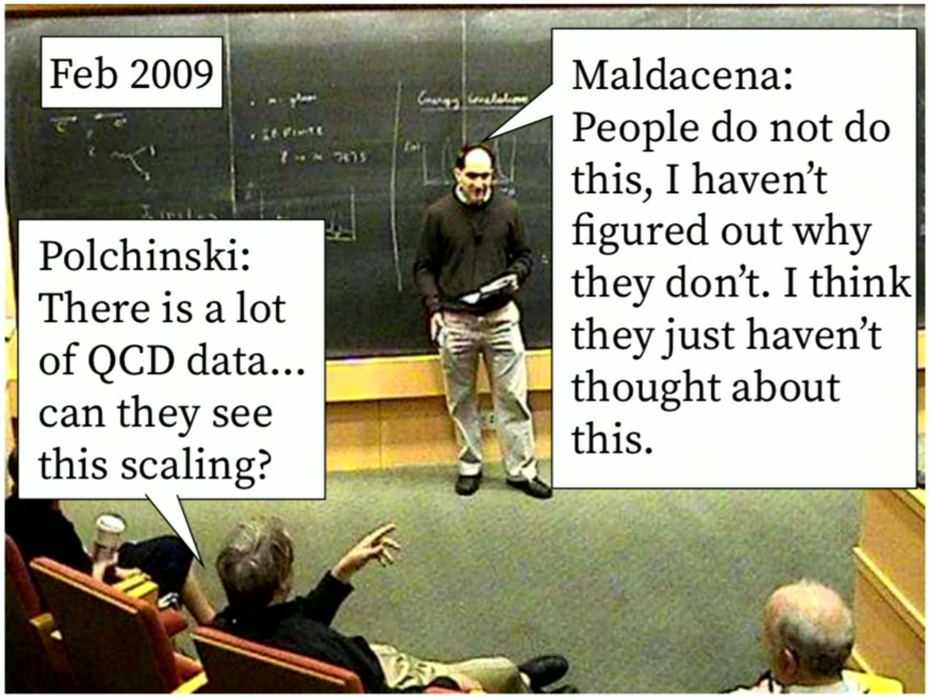
- Predicts universal scaling behavior in correlations of energy flux at energies $E \gg \Lambda_{\text{QCD}}$.

See early work by [Konishi, Ukawa, Veneziano]

Theory-Experiment Gap

- OPE scaling is the most basic prediction of QFT for jet substructure.

[Basham, Brown, Ellis, Love]

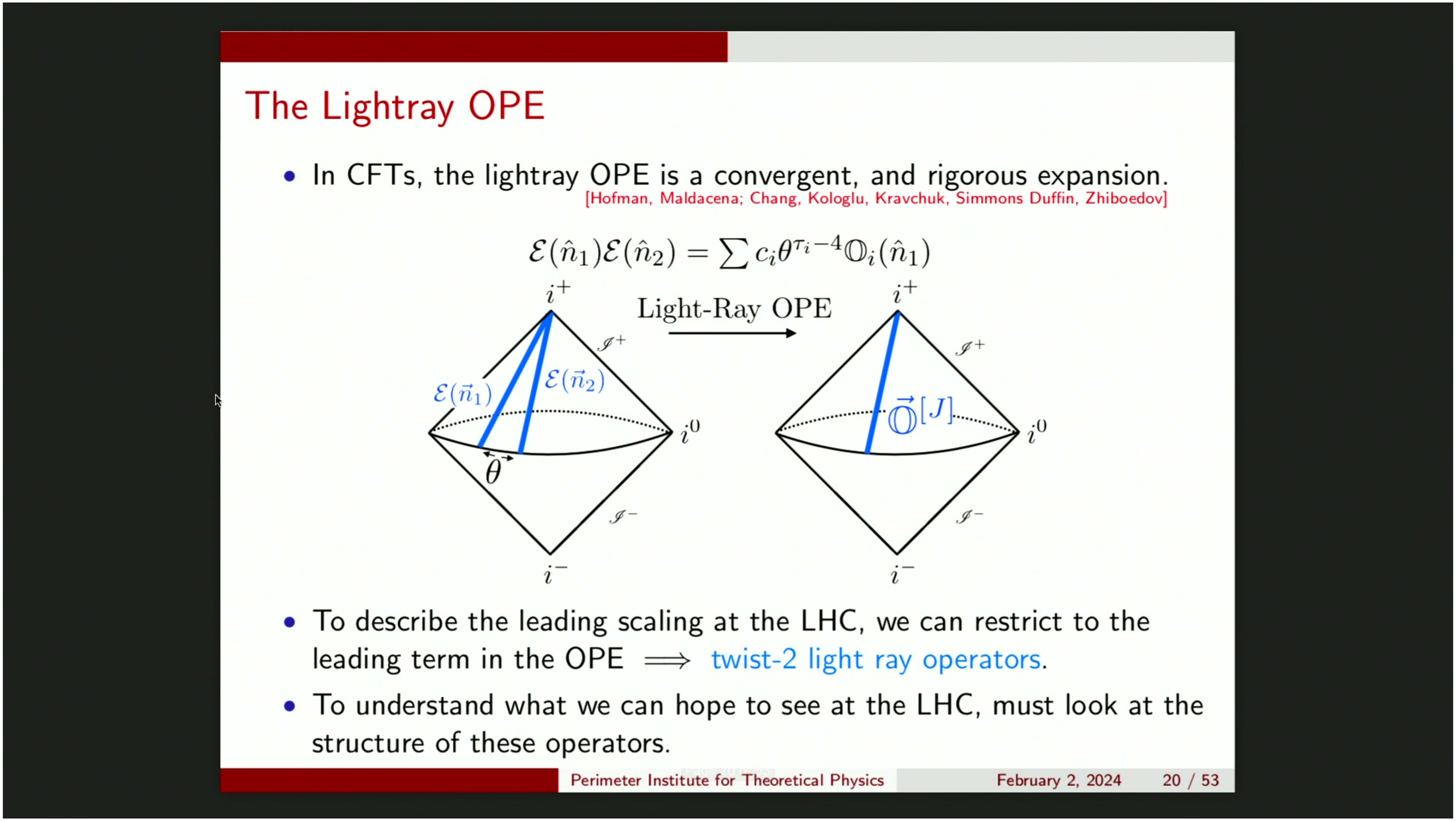


Feb 2009

Polchinski: There is a lot of QCD data... can they see this scaling?

Maldacena: People do not do this, I haven't figured out why they don't. I think they just haven't thought about this.

- Shockingly, still true as of 2022...

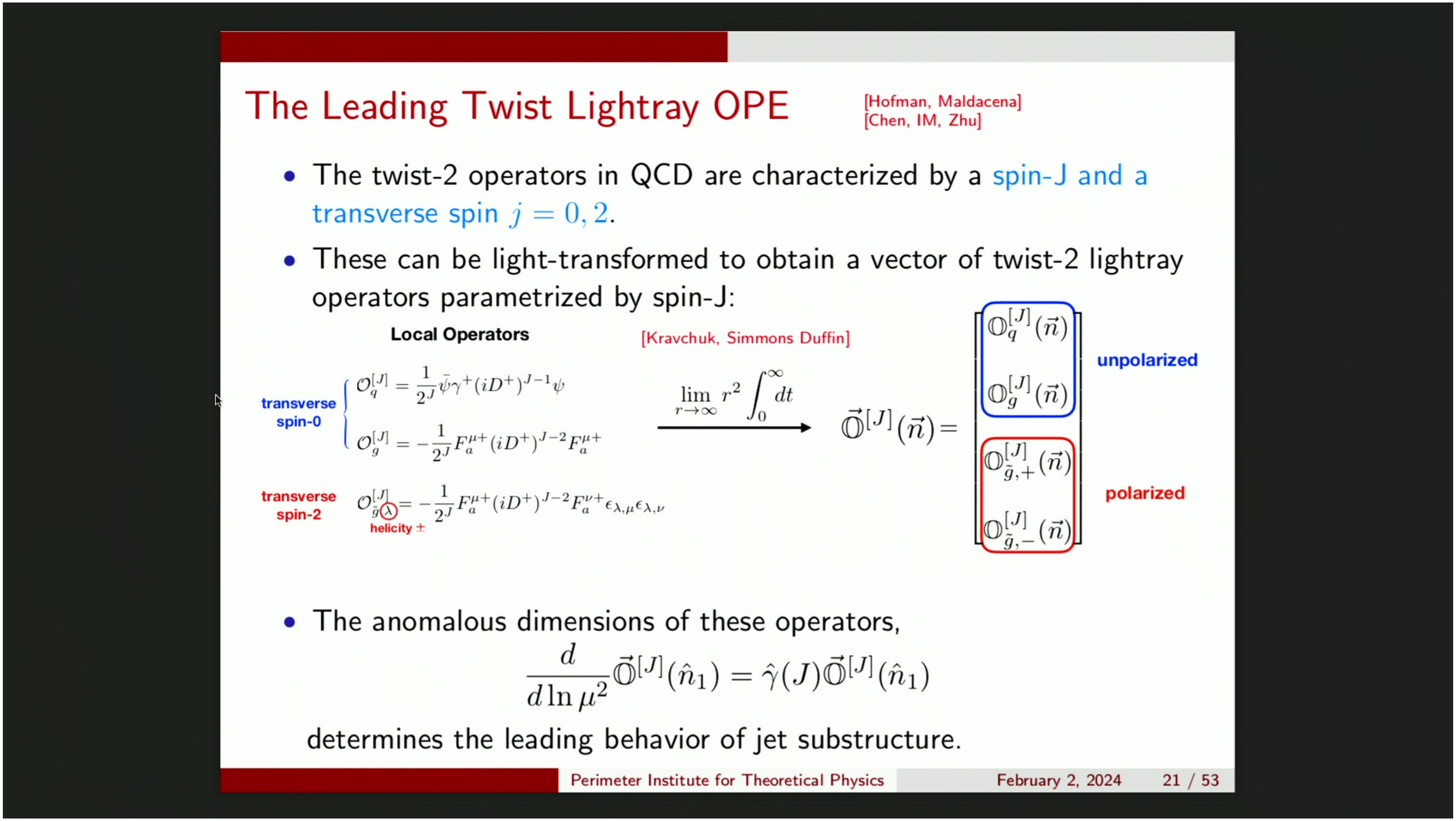


The Lightray OPE

- In CFTs, the lightray OPE is a convergent, and rigorous expansion. [Hofman, Maldacena; Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = \sum c_i \theta^{\tau_i - 4} \mathbb{O}_i(\hat{n}_1)$$

- To describe the leading scaling at the LHC, we can restrict to the leading term in the OPE $\Longrightarrow$ twist-2 light ray operators.
- To understand what we can hope to see at the LHC, must look at the structure of these operators.



The Leading Twist Lightray OPE

[Hofman, Maldacena]
[Chen, IM, Zhu]

- The twist-2 operators in QCD are characterized by a spin-J and a transverse spin $j = 0, 2$.
- These can be light-transformed to obtain a vector of twist-2 lightray operators parametrized by spin-J:

Local Operators

[Kravchuk, Simmons Duffin]

transverse spin-0

$$\begin{cases} \mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi \\ \mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+} \end{cases}$$

transverse spin-2

$$\mathcal{O}_{\tilde{g}\textcircled{\lambda}}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

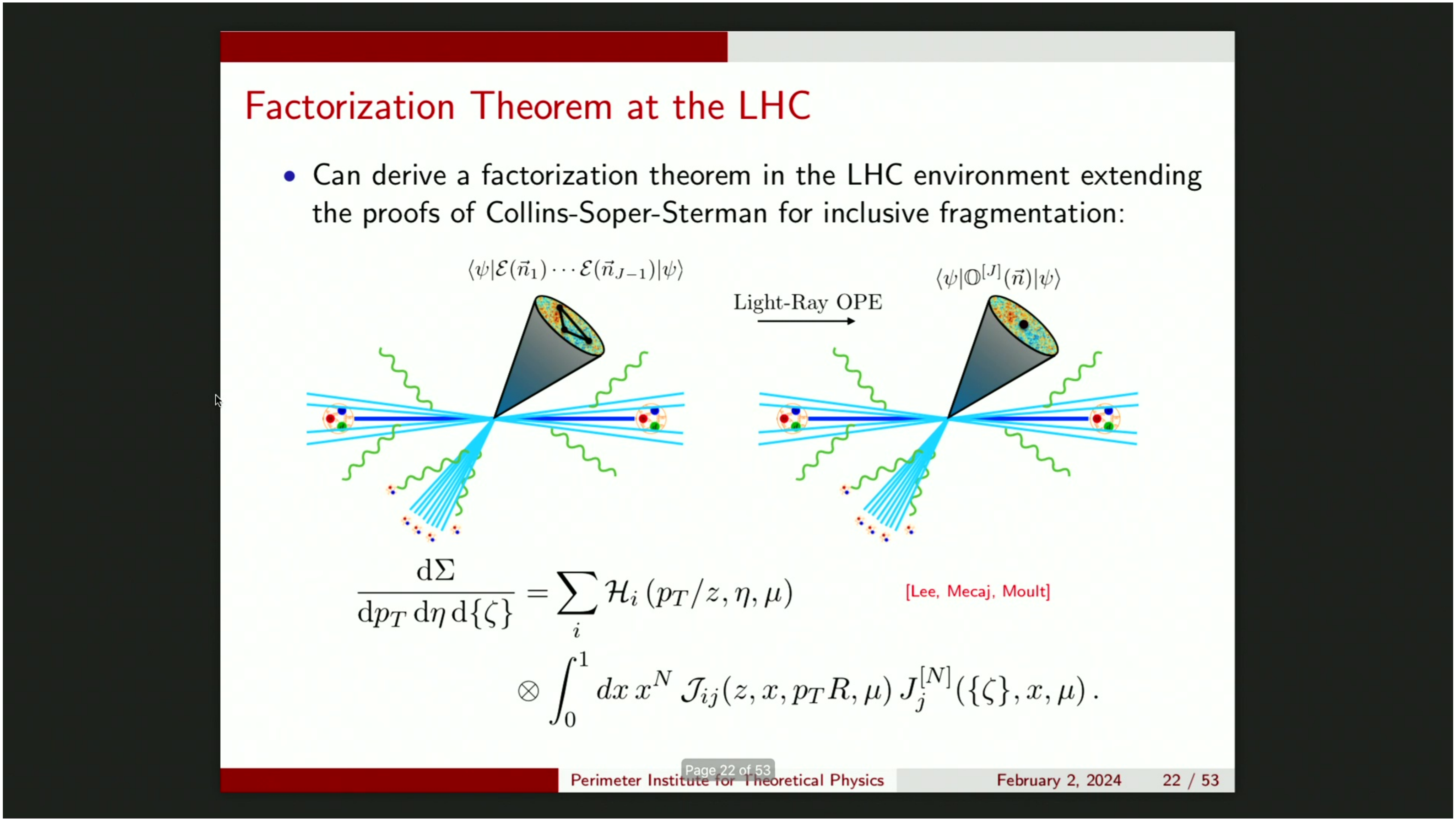
helicity ±

$$\xrightarrow{\lim_{r\to\infty} r^2\int_0^\infty dt}\quad \vec{\mathbb{O}}^{[J]}(\vec{n}) = \begin{array}{cc} \left[\begin{array}{c} \boxed{\begin{matrix}\mathbb{O}_q^{[J]}(\vec{n}) \\ \mathbb{O}_g^{[J]}(\vec{n})\end{matrix}} \\ \boxed{\begin{matrix}\mathbb{O}_{\tilde{g},+}^{[J]}(\vec{n}) \\ \mathbb{O}_{\tilde{g},-}^{[J]}(\vec{n})\end{matrix}} \end{array}\right] & \begin{array}{c} \textbf{unpolarized} \\ \\ \\ \textbf{polarized} \end{array} \end{array}$$

- The anomalous dimensions of these operators,

$$\frac{d}{d\ln\mu^2}\vec{\mathbb{O}}^{[J]}(\hat{n}_1) = \hat{\gamma}(J)\vec{\mathbb{O}}^{[J]}(\hat{n}_1)$$

determines the leading behavior of jet substructure.



Factorization Theorem at the LHC

- Can derive a factorization theorem in the LHC environment extending the proofs of Collins-Soper-Sterman for inclusive fragmentation:

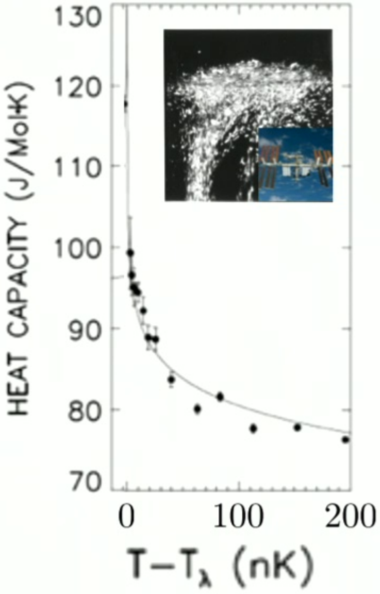
$$\frac{d\Sigma}{dp_T\,d\eta\,d\{\zeta\}} = \sum_i \mathcal{H}_i\left(p_T/z,\eta,\mu\right) \otimes \int_0^1 dx\, x^N\, \mathcal{J}_{ij}(z,x,p_T R,\mu)\, J_j^{[N]}(\{\zeta\},x,\mu)\,.$$

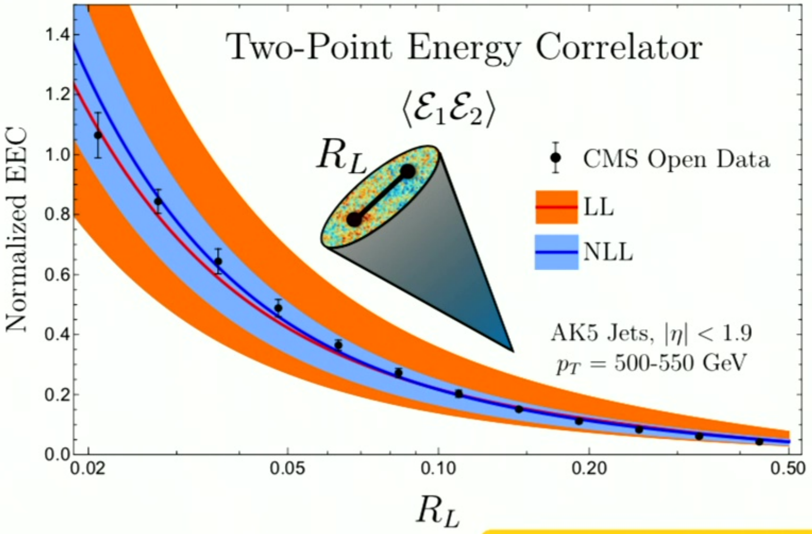
[Lee, Mecaj, Moult]

Scaling Behavior in Jets

[Komiske, Moult, Thaler, Zhu]
[Dixon, Moult, Zhu]
[Lee, Mecaj, Moult]

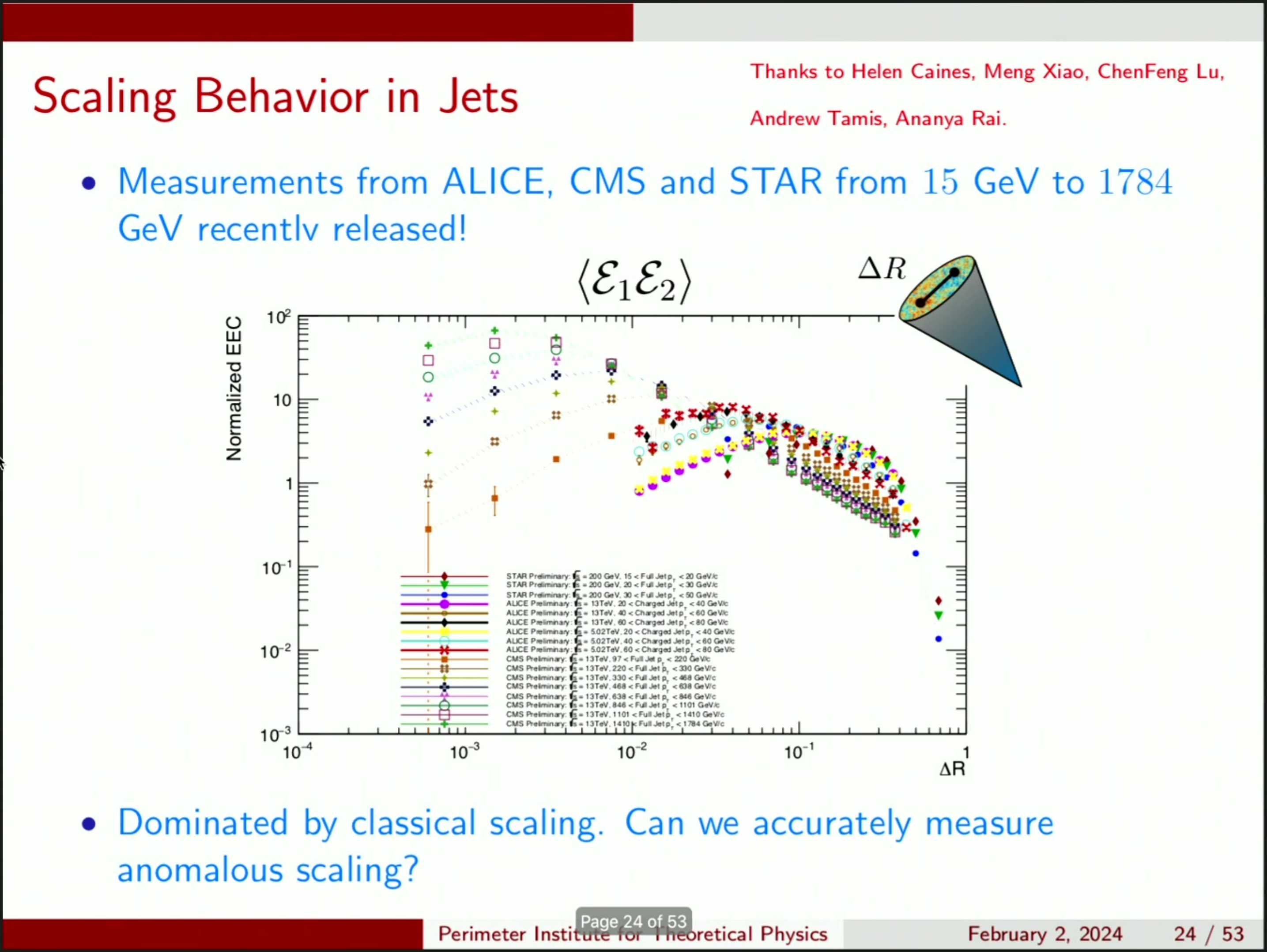
- The $\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)$ OPE inside high-energy jets!





$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i - 4}\mathbb{O}_i(\hat{n}_1)$$

- Beautiful scaling behavior in energy flux!



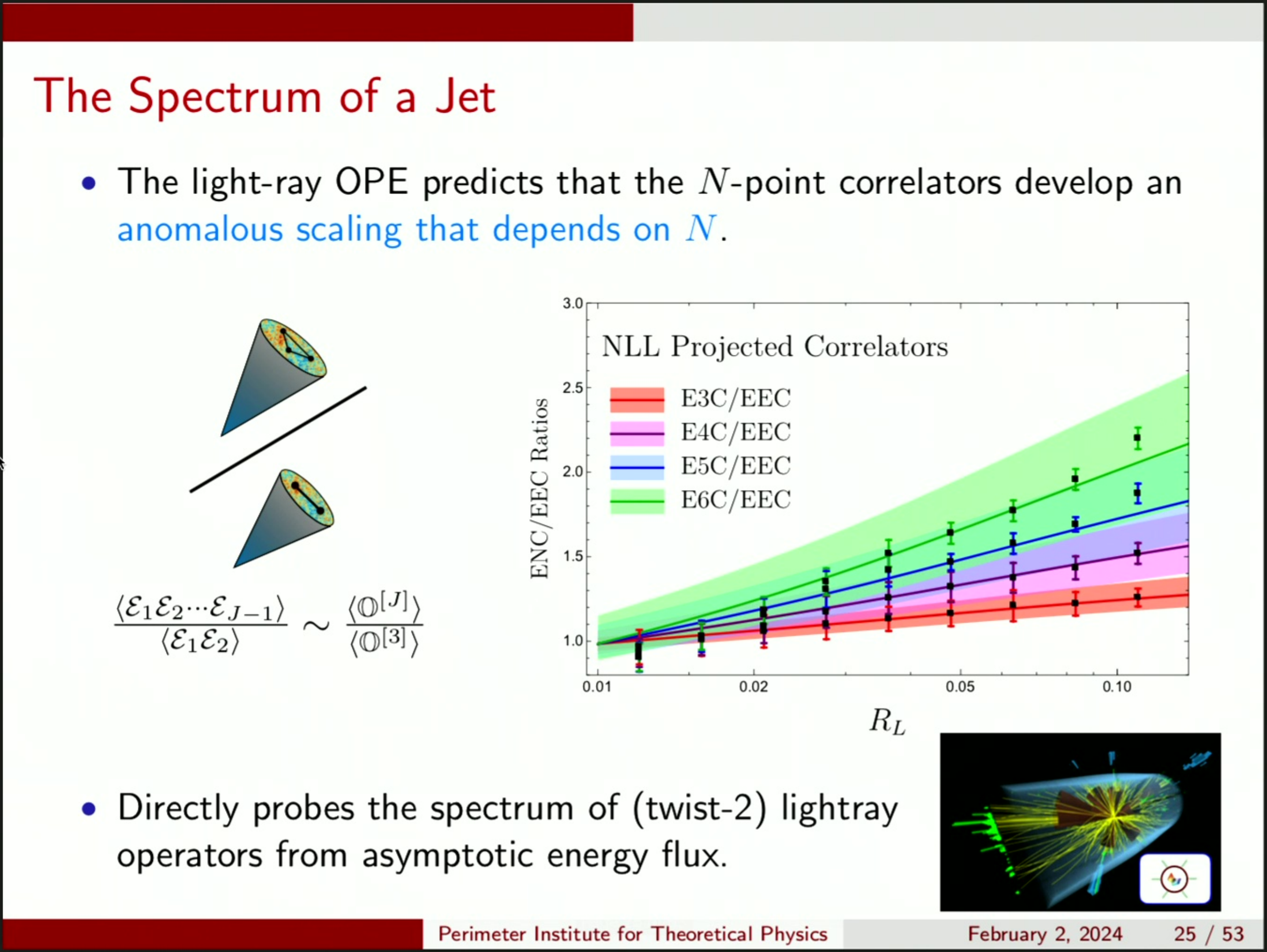
Scaling Behavior in Jets

Thanks to Helen Caines, Meng Xiao, ChenFeng Lu, Andrew Tamis, Ananya Rai.

- Measurements from ALICE, CMS and STAR from 15 GeV to 1784 GeV recentlv released!

$\langle \mathcal{E}_1 \mathcal{E}_2 \rangle$

- Dominated by classical scaling. Can we accurately measure anomalous scaling?



The Spectrum of a Jet

- The light-ray OPE predicts that the N-point correlators develop an anomalous scaling that depends on N.

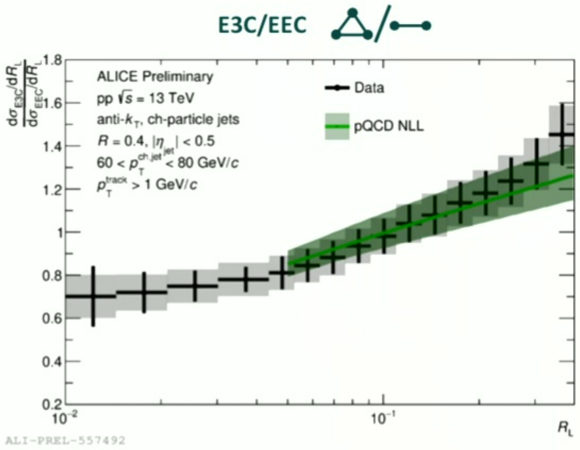
$$\frac{\langle \mathcal{E}_1\mathcal{E}_2\cdots\mathcal{E}_{J-1}\rangle}{\langle \mathcal{E}_1\mathcal{E}_2\rangle} \sim \frac{\langle \mathbb{O}^{[J]}\rangle}{\langle \mathbb{O}^{[3]}\rangle}$$

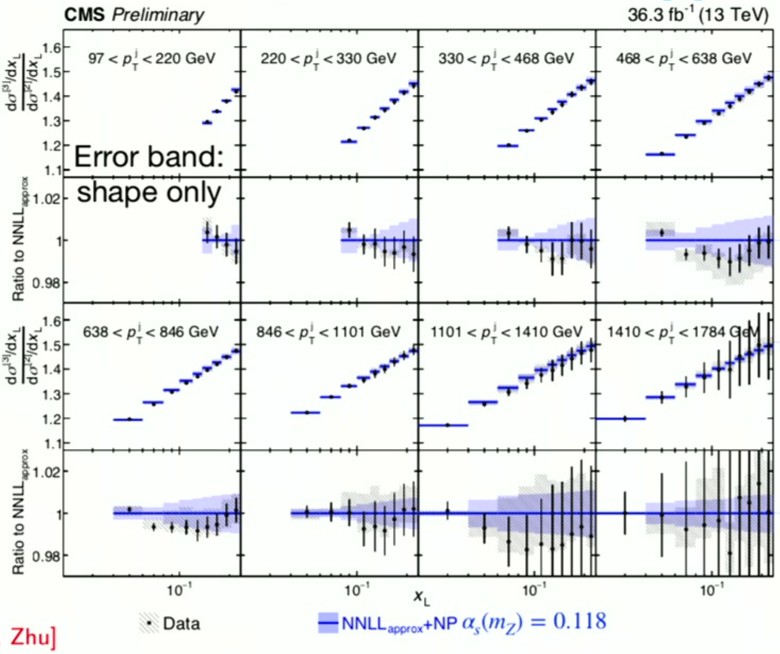
- Directly probes the spectrum of (twist-2) lightray operators from asymptotic energy flux.

Anomalous Scaling of $3/2$ Ratio

- Anomalous scaling measured from 15 GeV to 1784 GeV!

$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \mathcal{E}_3 \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim \frac{\langle \mathbb{O}^{[3]} \rangle}{\langle \mathbb{O}^{[3]} \rangle} \sim R_L^{\gamma(4)-\gamma(3)}$$



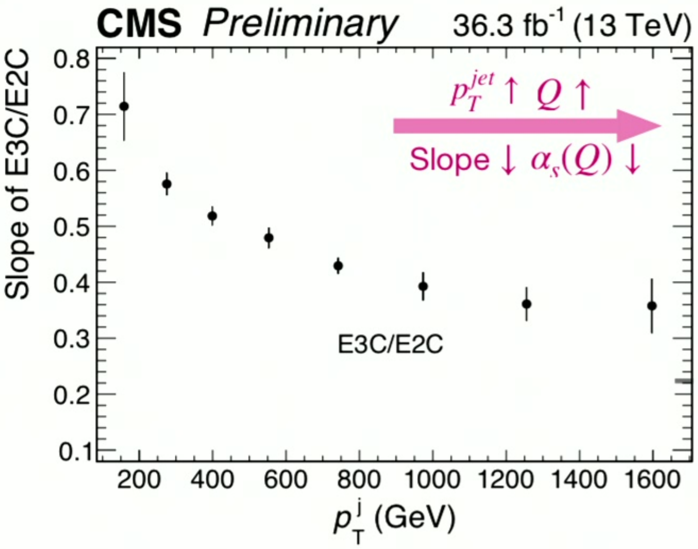


Error band: shape only

Using [Lee, Mecaj, Moult], [Chen, Gao, Li, Xu, Zhang, Zhu]

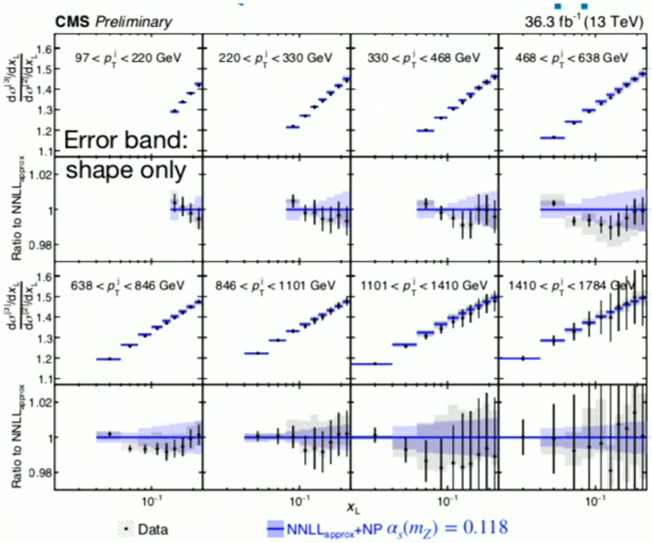
Asymptotic Freedom

- Scaling exponents are proportional to $\alpha_s \implies$ run with p_T.
- Asymptotic Freedom by eye!



$p_T^{jet} \uparrow \; Q \uparrow$

Slope ↓ $\alpha_s(Q) \downarrow$



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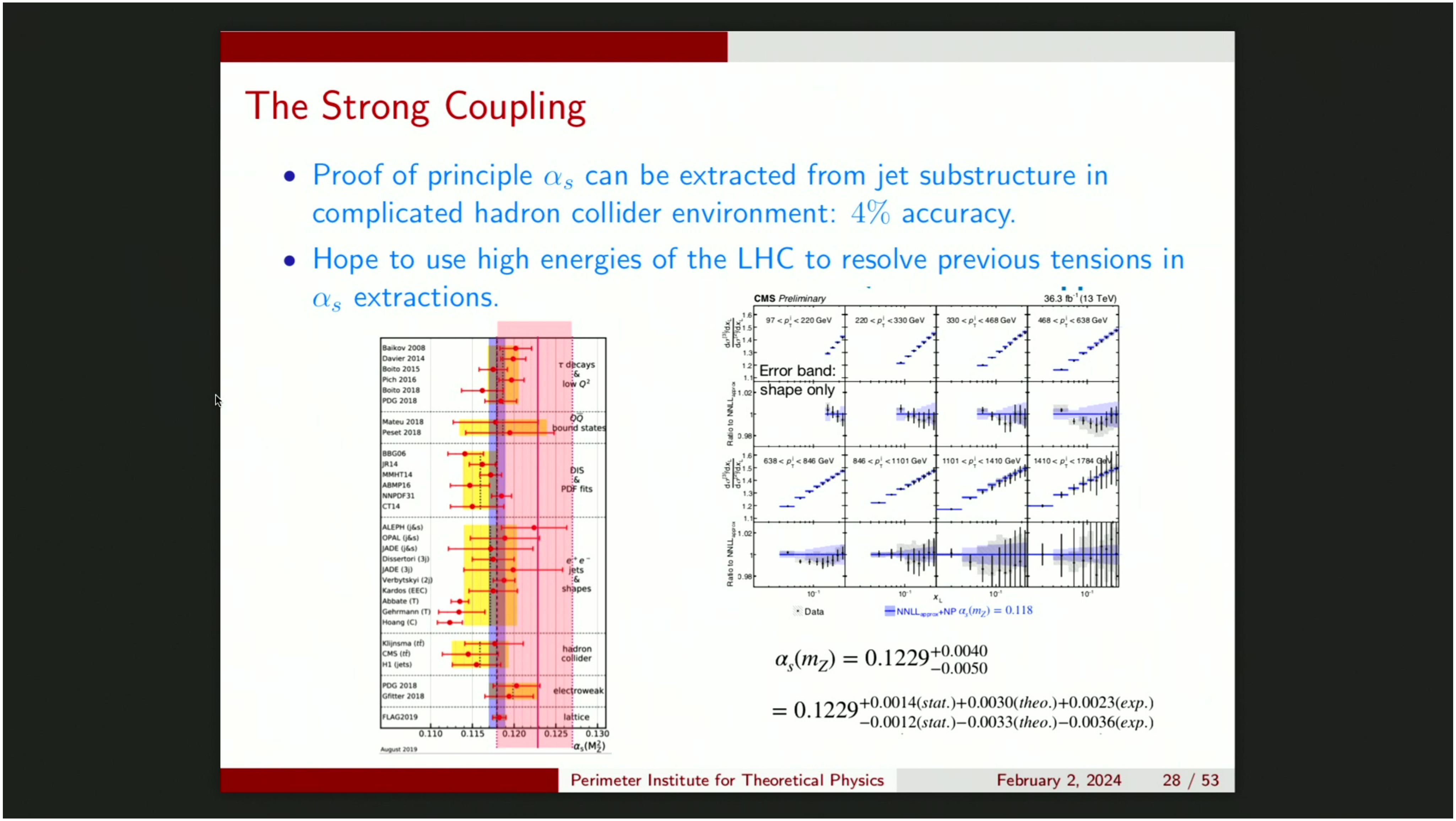


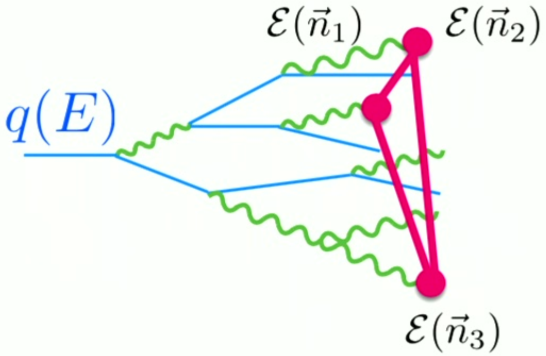
The Strong Coupling

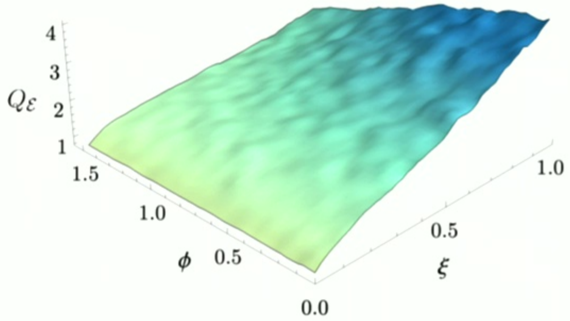
- Proof of principle α_s can be extracted from jet substructure in complicated hadron collider environment: 4% accuracy.
- Hope to use high energies of the LHC to resolve previous tensions in α_s extractions.

$$\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050}$$

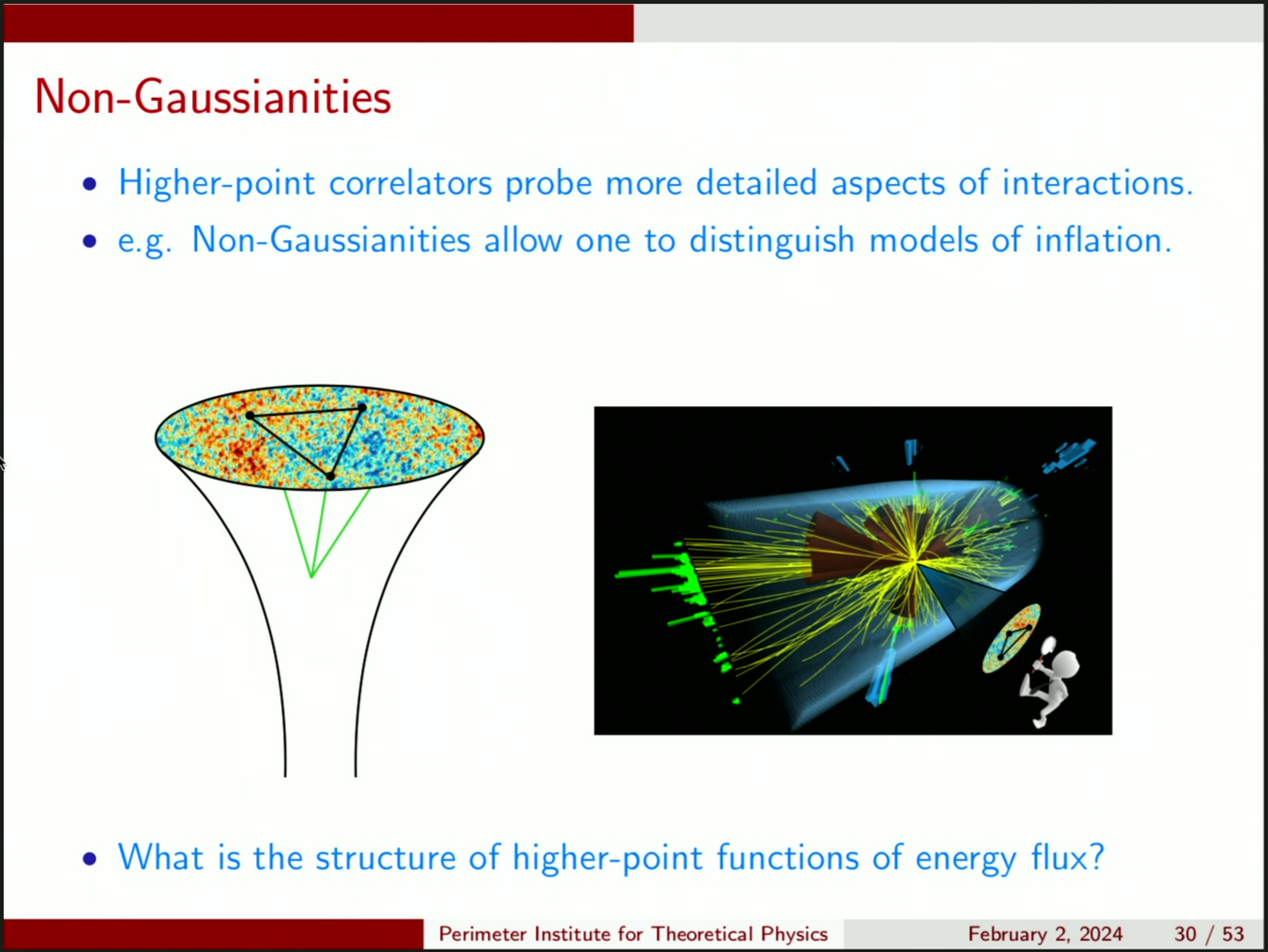
$$= 0.1229^{+0.0014(stat.)+0.0030(theo.)+0.0023(exp.)}_{-0.0012(stat.)-0.0033(theo.)-0.0036(exp.)}$$

Non-Gaussianities in Energy Flux





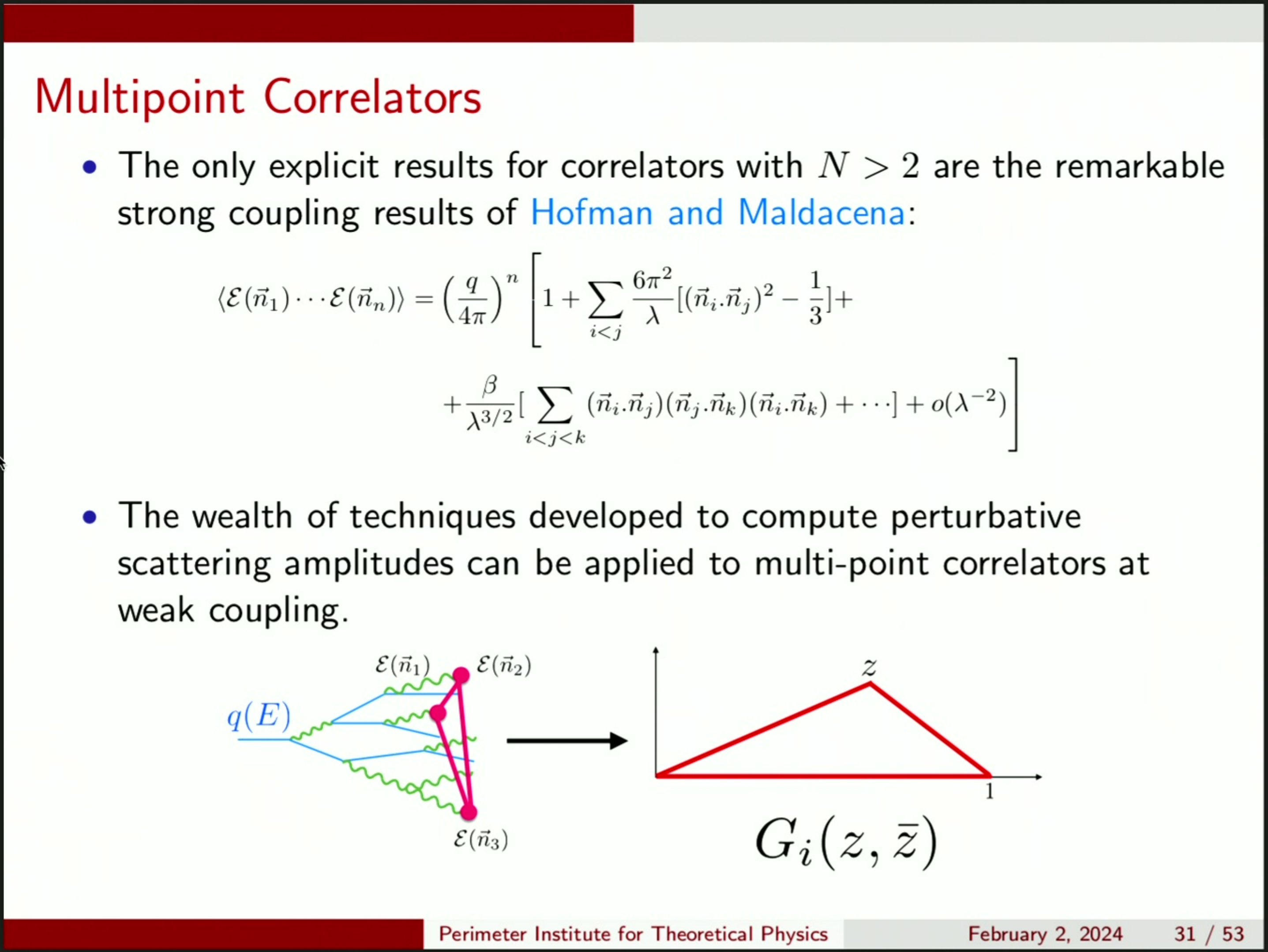
[Chen, Moult, Thaler, Zhu]



Non-Gaussianities

- Higher-point correlators probe more detailed aspects of interactions.
- e.g. Non-Gaussianities allow one to distinguish models of inflation.

- What is the structure of higher-point functions of energy flux?

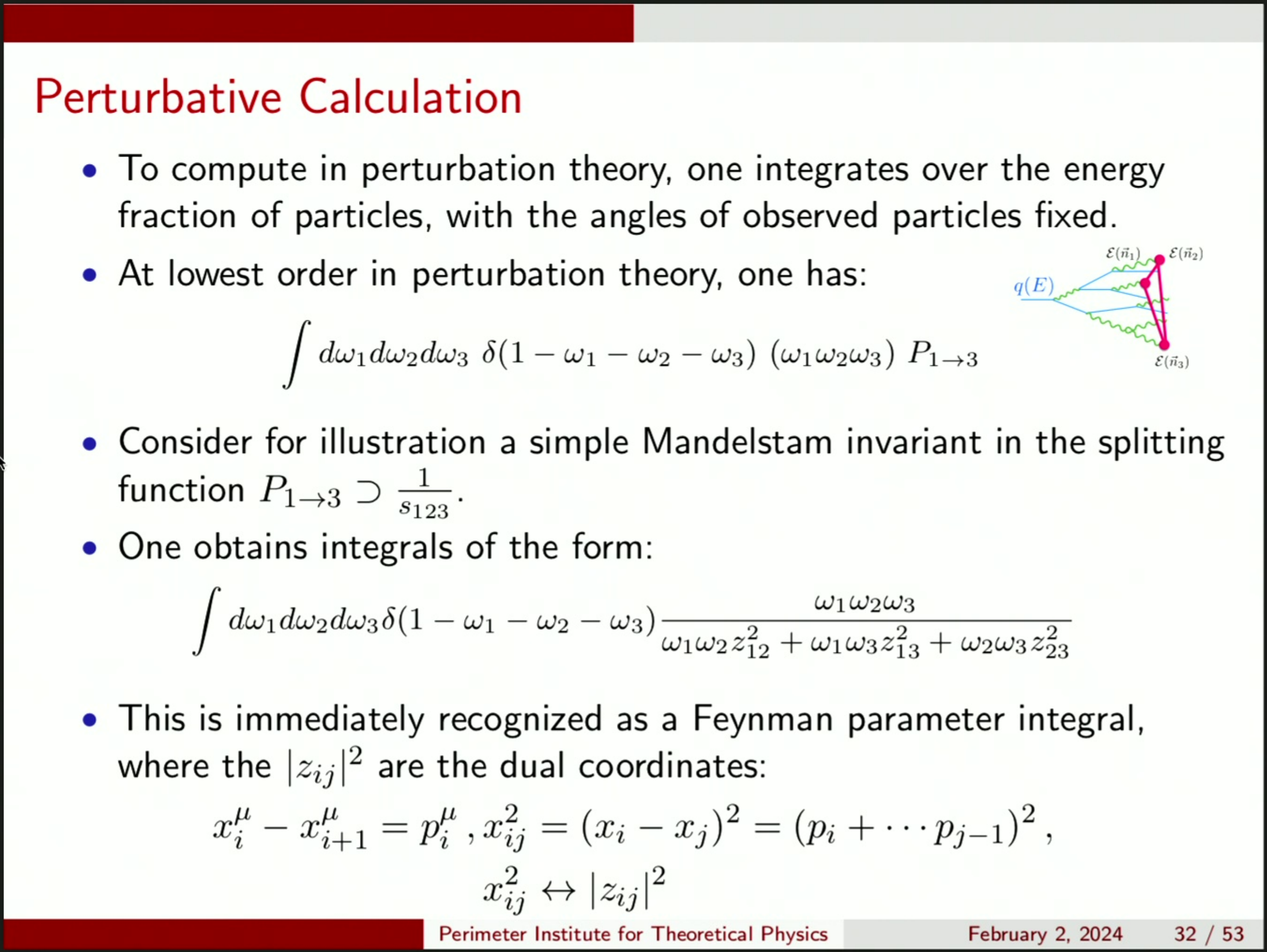


Multipoint Correlators

- The only explicit results for correlators with $N > 2$ are the remarkable strong coupling results of Hofman and Maldacena:

$$\langle \mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_n)\rangle = \left(\frac{q}{4\pi}\right)^n \Bigg[1+\sum_{i<j}\frac{6\pi^2}{\lambda}[(\vec{n}_i.\vec{n}_j)^2-\frac{1}{3}]+ \\ +\frac{\beta}{\lambda^{3/2}}[\sum_{i<j<k}(\vec{n}_i.\vec{n}_j)(\vec{n}_j.\vec{n}_k)(\vec{n}_i.\vec{n}_k)+\cdots]+o(\lambda^{-2})\Bigg]$$

- The wealth of techniques developed to compute perturbative scattering amplitudes can be applied to multi-point correlators at weak coupling.



Perturbative Calculation

- To compute in perturbation theory, one integrates over the energy fraction of particles, with the angles of observed particles fixed.
- At lowest order in perturbation theory, one has:

$$\int d\omega_1 d\omega_2 d\omega_3 \ \delta(1-\omega_1-\omega_2-\omega_3) \ (\omega_1\omega_2\omega_3) \ P_{1\to 3}$$

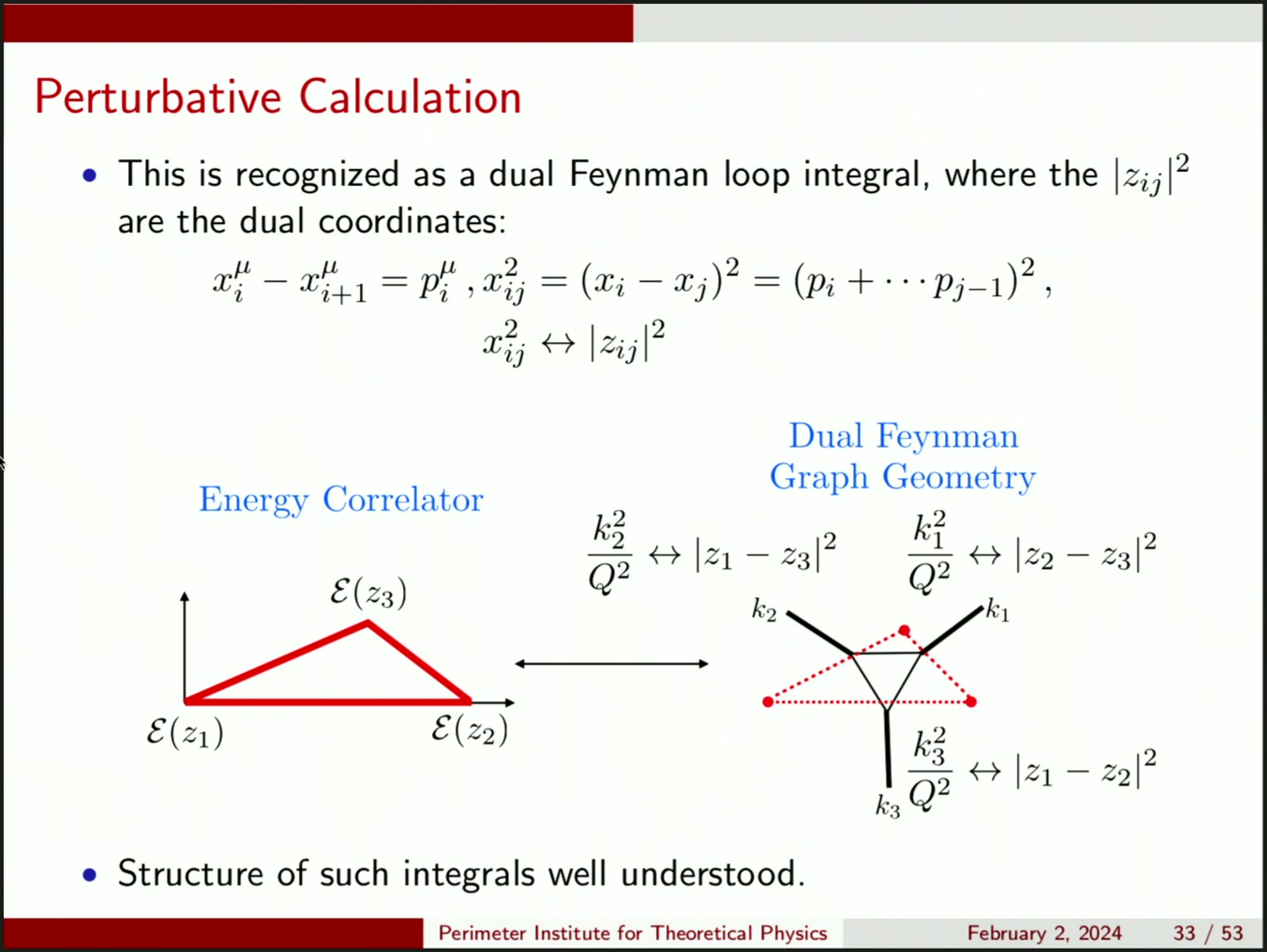
- Consider for illustration a simple Mandelstam invariant in the splitting function $P_{1\to 3} \supset \frac{1}{s_{123}}$.
- One obtains integrals of the form:

$$\int d\omega_1 d\omega_2 d\omega_3 \delta(1-\omega_1-\omega_2-\omega_3) \frac{\omega_1\omega_2\omega_3}{\omega_1\omega_2 z_{12}^2+\omega_1\omega_3 z_{13}^2+\omega_2\omega_3 z_{23}^2}$$

- This is immediately recognized as a Feynman parameter integral, where the $|z_{ij}|^2$ are the dual coordinates:

$$x_i^\mu - x_{i+1}^\mu = p_i^\mu \, , x_{ij}^2 = (x_i - x_j)^2 = (p_i + \cdots p_{j-1})^2 \, ,$$

$$x_{ij}^2 \leftrightarrow |z_{ij}|^2$$

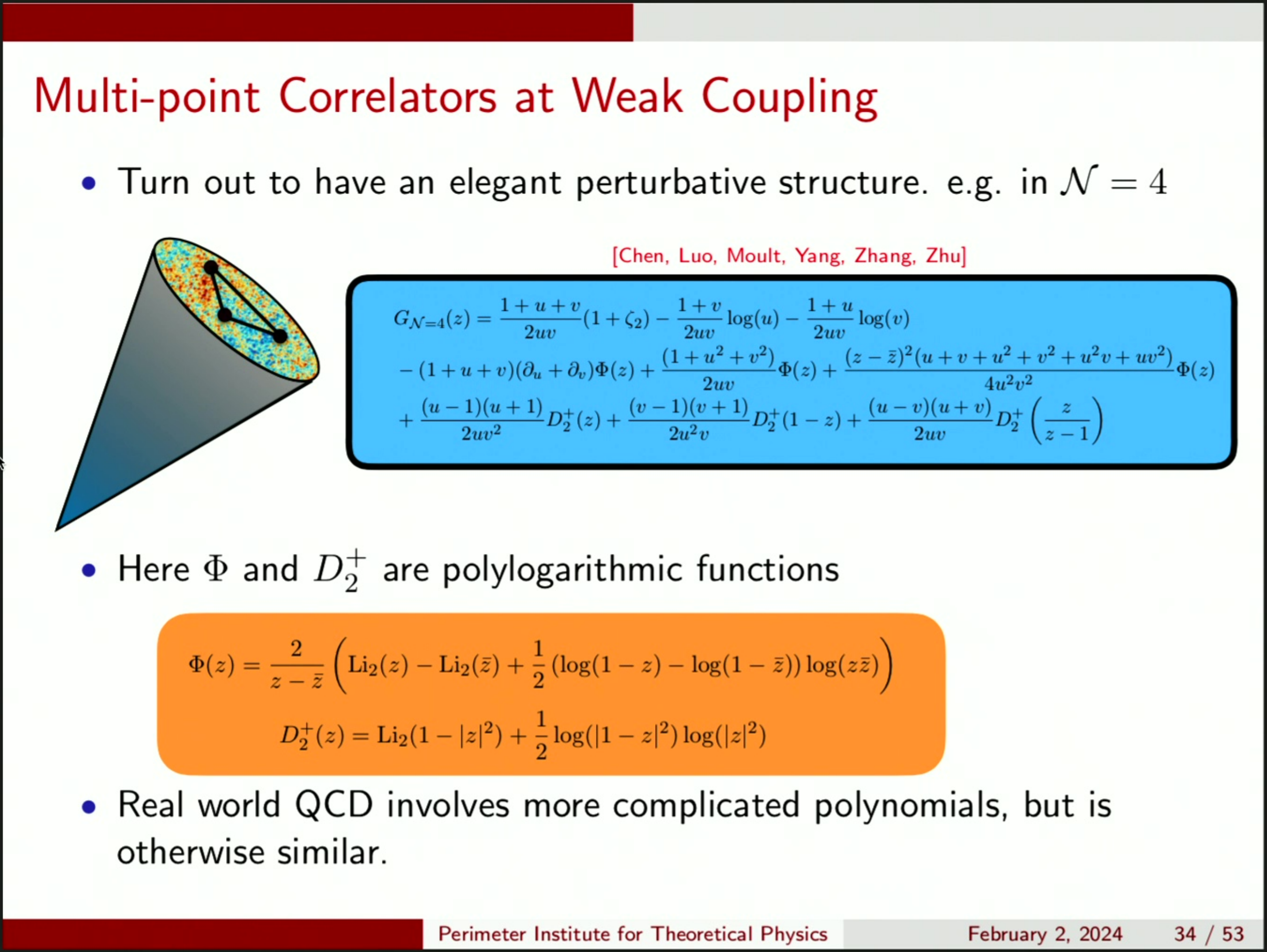


Perturbative Calculation

- This is recognized as a dual Feynman loop integral, where the $|z_{ij}|^2$ are the dual coordinates:

$$x_i^\mu - x_{i+1}^\mu = p_i^\mu \,, x_{ij}^2 = (x_i - x_j)^2 = (p_i + \cdots p_{j-1})^2 \,,$$
$$x_{ij}^2 \leftrightarrow |z_{ij}|^2$$

- Structure of such integrals well understood.



Multi-point Correlators at Weak Coupling

- Turn out to have an elegant perturbative structure. e.g. in $\mathcal{N}=4$

[Chen, Luo, Moult, Yang, Zhang, Zhu]

$$G_{\mathcal{N}=4}(z) = \frac{1+u+v}{2uv}(1+\zeta_2) - \frac{1+v}{2uv}\log(u) - \frac{1+u}{2uv}\log(v)$$
$$- (1+u+v)(\partial_u+\partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-\bar{z})^2(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z)$$
$$+ \frac{(u-1)(u+1)}{2uv^2}D_2^+(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^+(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^+\left(\frac{z}{z-1}\right)$$

- Here Φ and D_2^+ are polylogarithmic functions

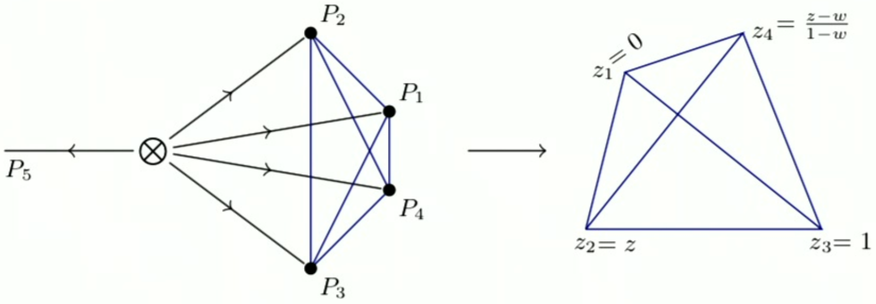
$$\Phi(z) = \frac{2}{z-\bar{z}}\left(\mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z}) + \frac{1}{2}\left(\log(1-z) - \log(1-\bar{z})\right)\log(z\bar{z})\right)$$
$$D_2^+(z) = \mathrm{Li}_2(1-|z|^2) + \frac{1}{2}\log(|1-z|^2)\log(|z|^2)$$

- Real world QCD involves more complicated polynomials, but is otherwise similar.

Multi-point Correlators at Weak Coupling

- Define an interesting class of *finite* integrals.
- Provide a playground for the exploration of the perturbative structure of *physical observables*.

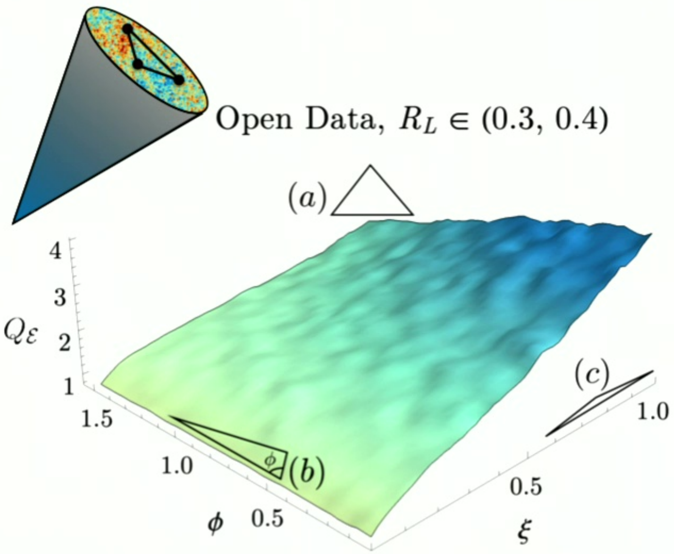
[Chicherin, Moult, Sokatchev, Yan, Zhu]



- Obtained a compact expression in terms of weight-3 polylogarithms for four-point correlator in $\mathcal{N} = 4$ SYM.

Shape Dependence of Non-Gaussianities

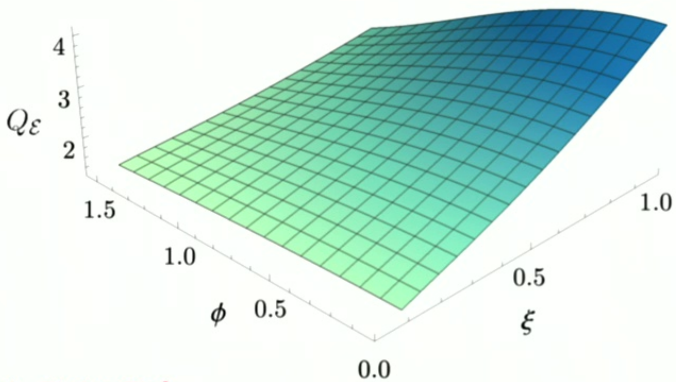
- Can directly study non-gaussianities inside high energy jets.



Open Data, $R_L \in (0.3, 0.4)$

$$G_{\mathcal{N}=4}(z) = \frac{1+u+v}{2uv}(1+\zeta_2) - \frac{1+v}{2uv}\log(u) - \frac{1+u}{2uv}\log(v) - (1+u+v)(\partial_u+\partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-\bar{z})^2(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z) + \frac{(u-1)(u+1)}{2uv^2}D_2^+(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^+(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^+\left(\frac{z}{z-1}\right)$$

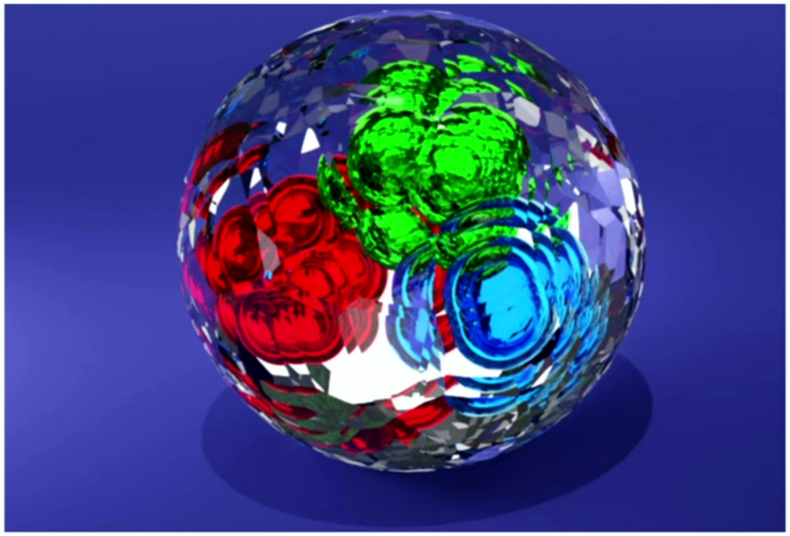
LL + LO prediction, $R_L = 0.35$

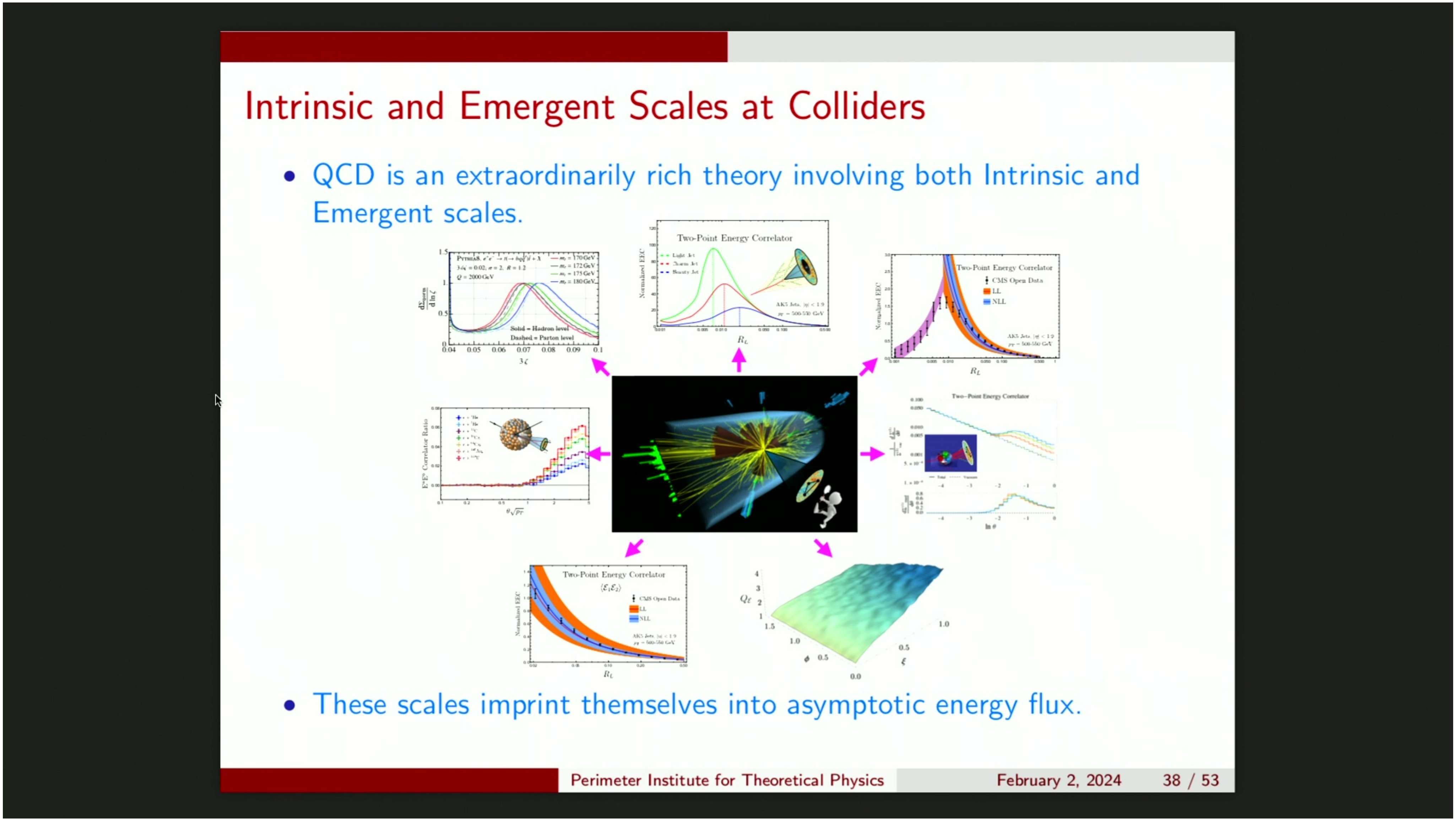


[Chen, Moult, Thaler, Zhu]

- Illustrates theoretical control over multi-point correlations!

Identifying Intrinsic and Emergent Scales of QCD

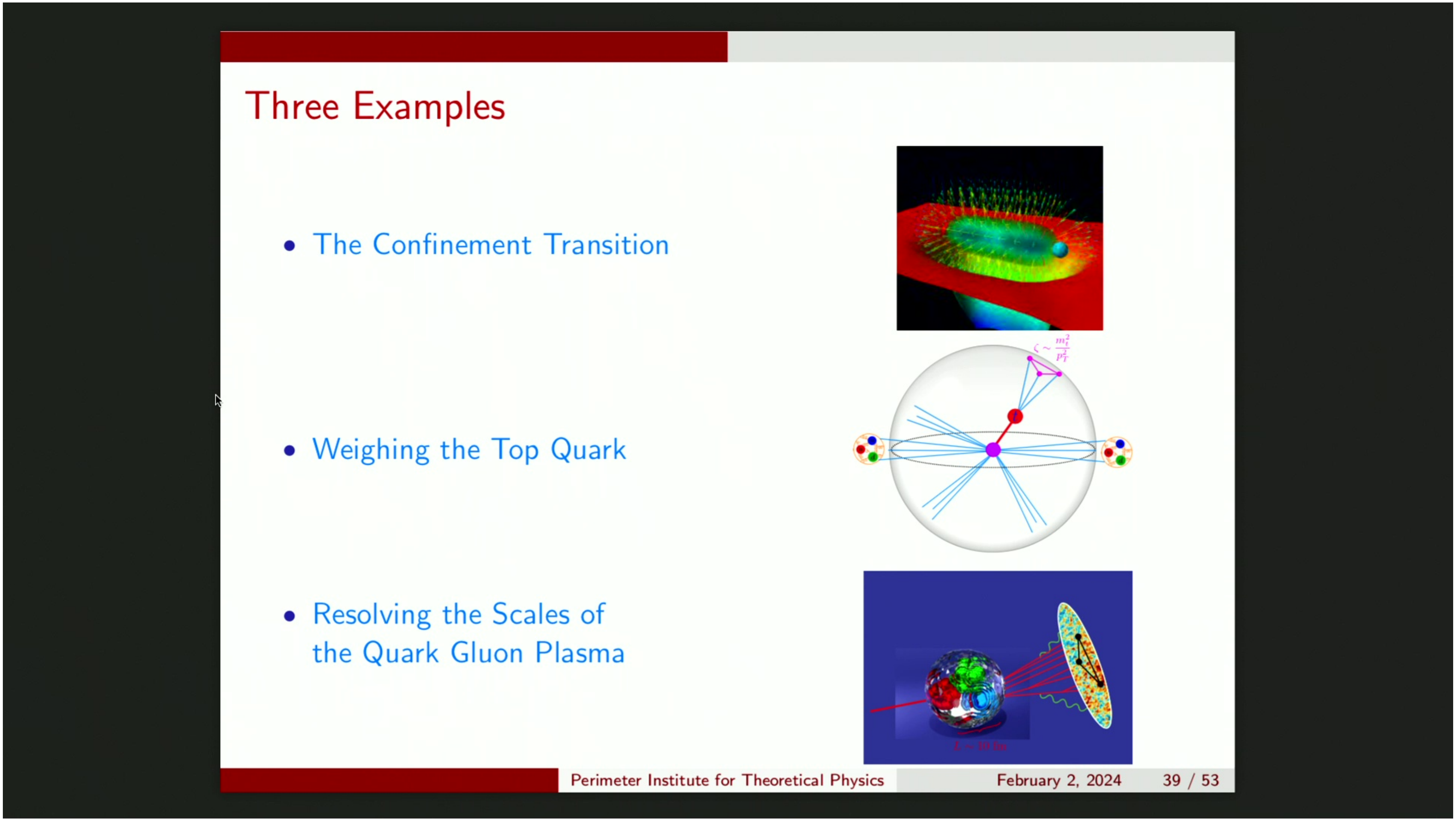




Intrinsic and Emergent Scales at Colliders

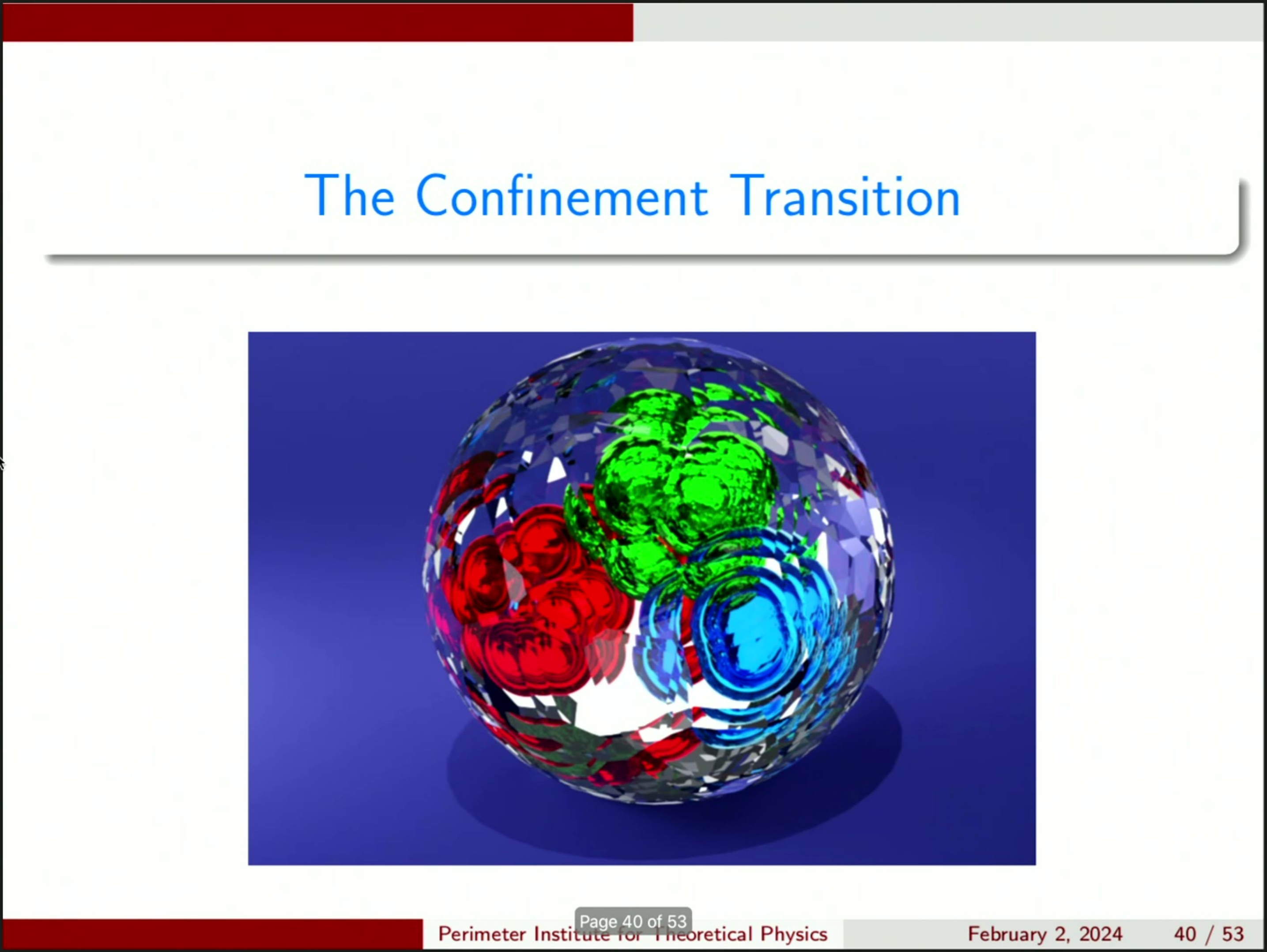
- QCD is an extraordinarily rich theory involving both Intrinsic and Emergent scales.

- These scales imprint themselves into asymptotic energy flux.

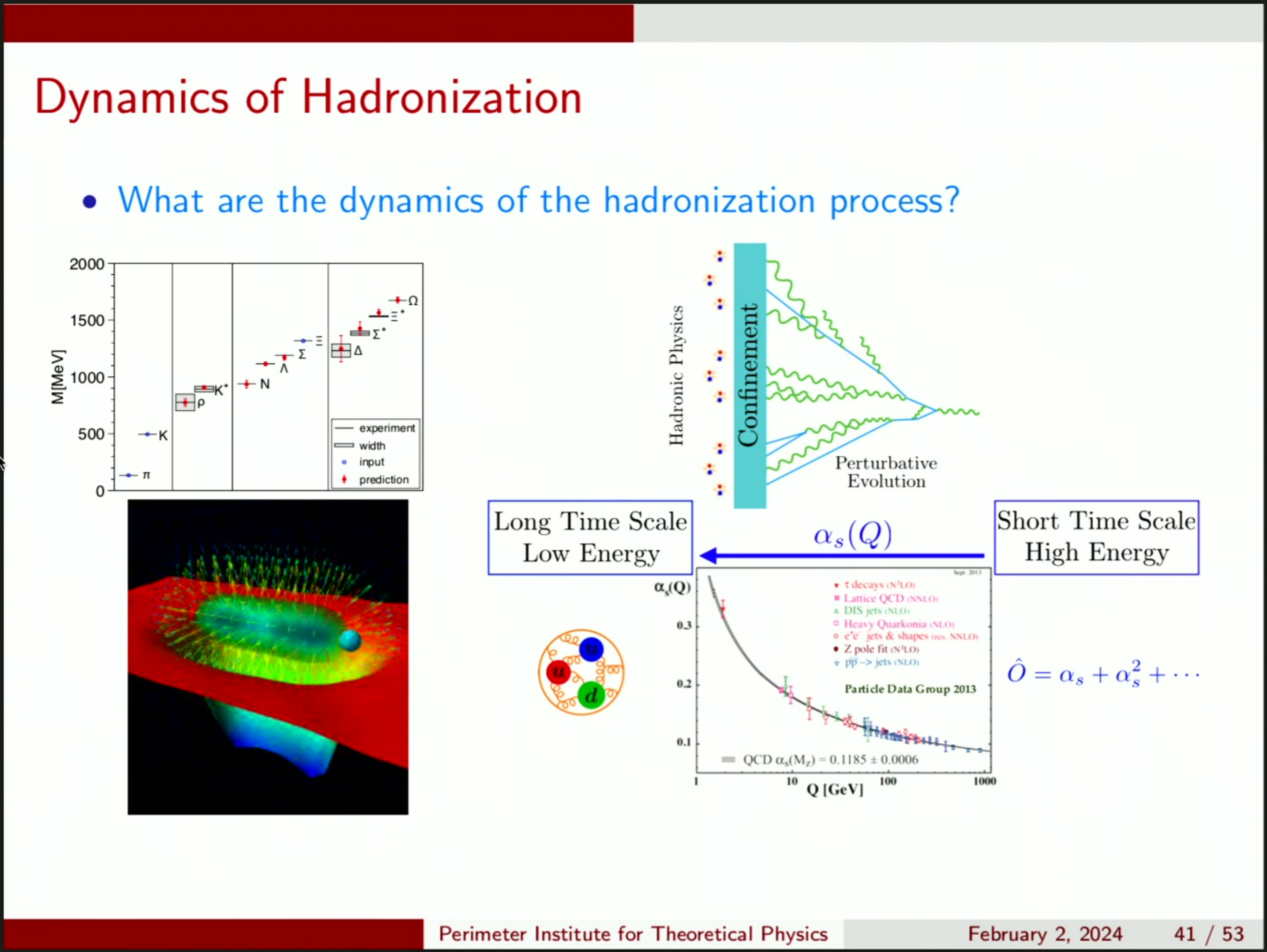


Three Examples

- The Confinement Transition
- Weighing the Top Quark
- Resolving the Scales of the Quark Gluon Plasma



The Confinement Transition



Dynamics of Hadronization

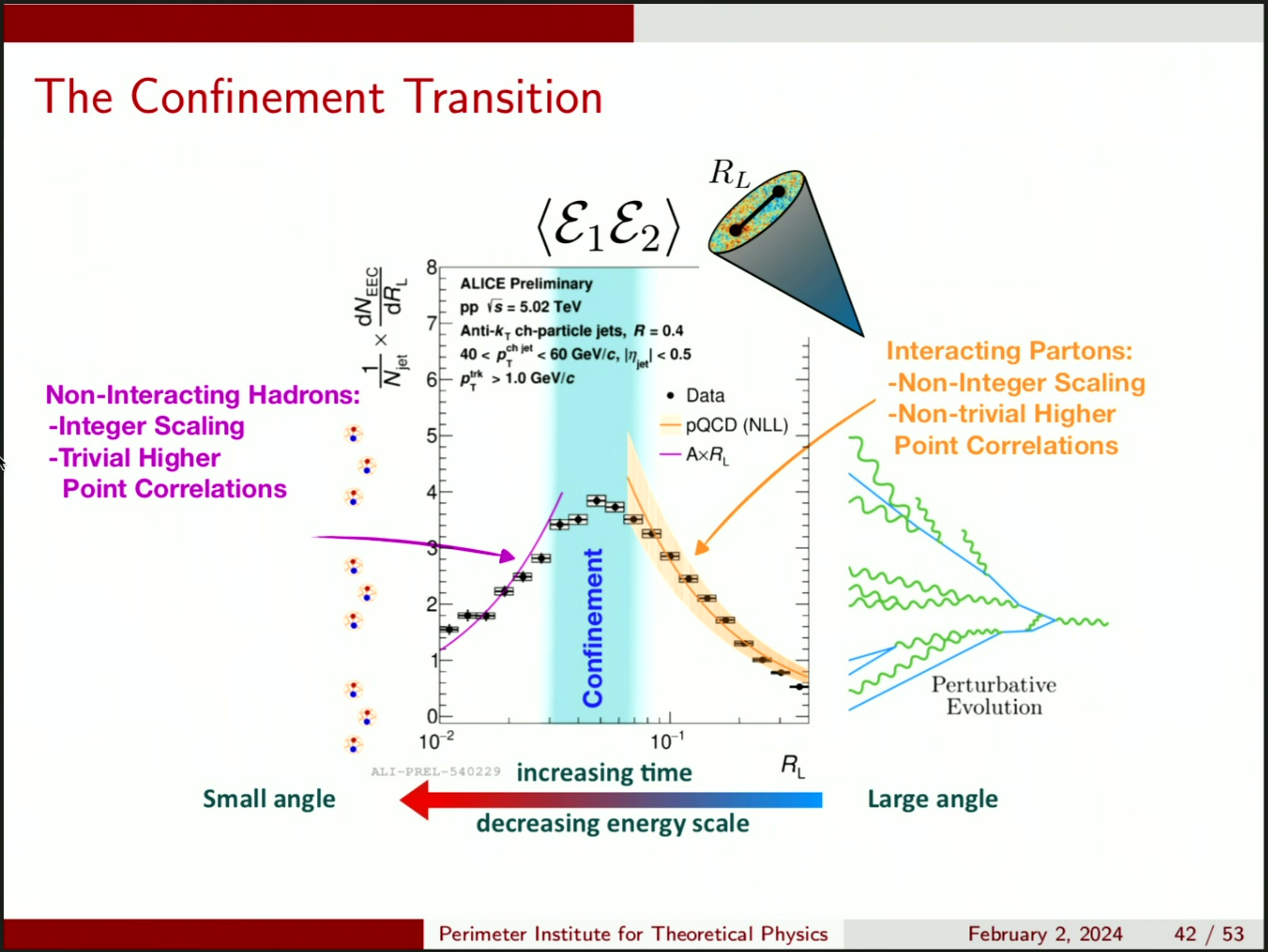
- What are the dynamics of the hadronization process?

Long Time Scale
Low Energy

$\alpha_s(Q)$

Short Time Scale
High Energy

$$\hat{O} = \alpha_s + \alpha_s^2 + \cdots$$



The Confinement Transition

$\langle \mathcal{E}_1 \mathcal{E}_2 \rangle$

Non-Interacting Hadrons:
- Integer Scaling
- Trivial Higher Point Correlations

Interacting Partons:
- Non-Integer Scaling
- Non-trivial Higher Point Correlations

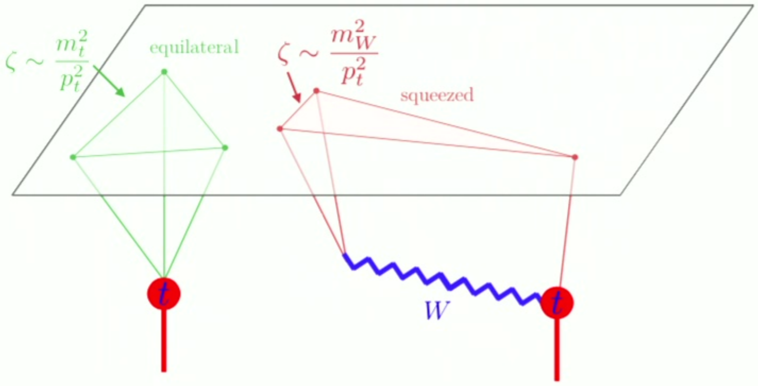
increasing time

Small angle

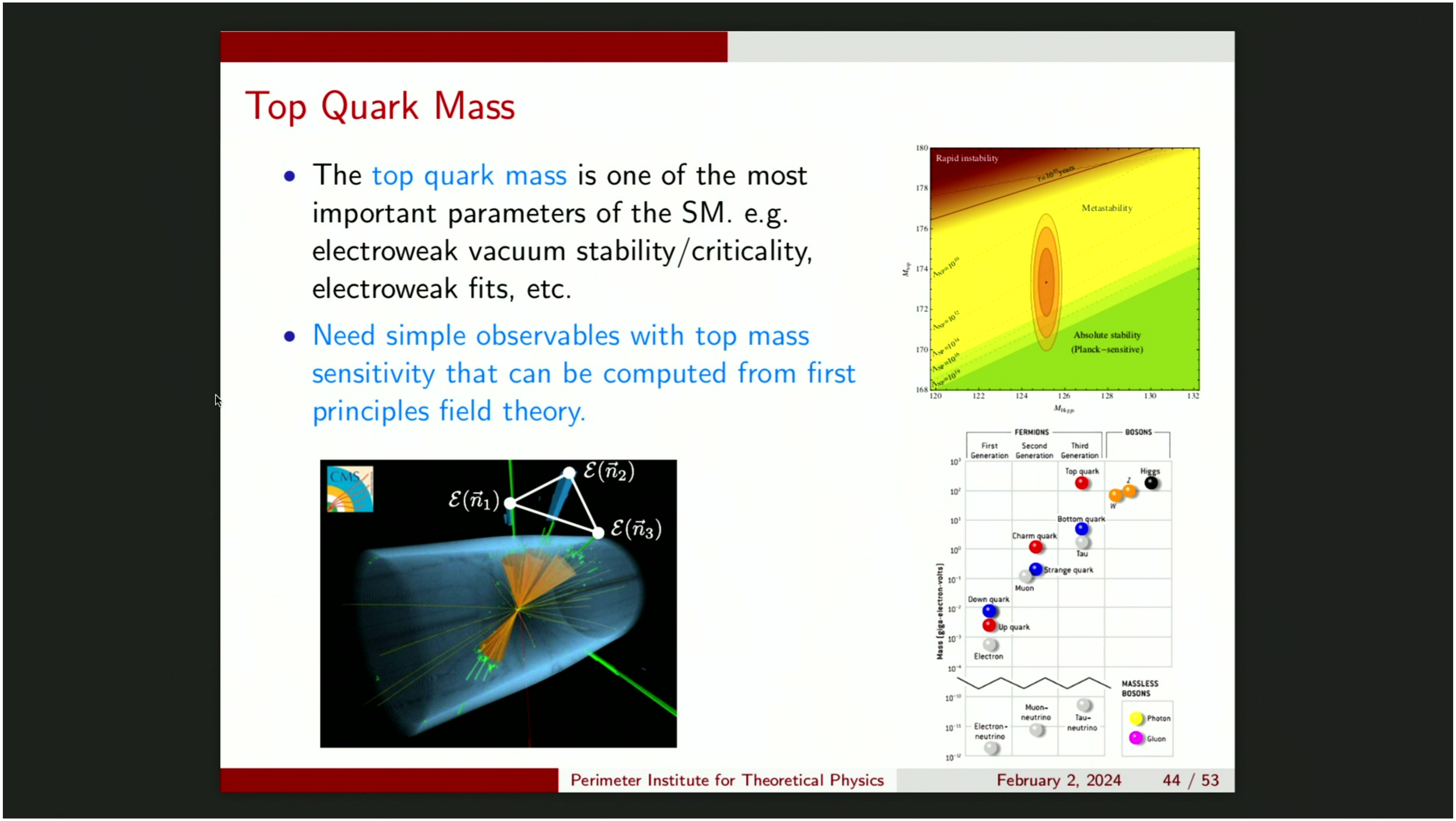
Large angle

decreasing energy scale

Weighing the Top Quark



[Holguin, Moult, Pathak, Procura, Schofbeck, Schwarz]

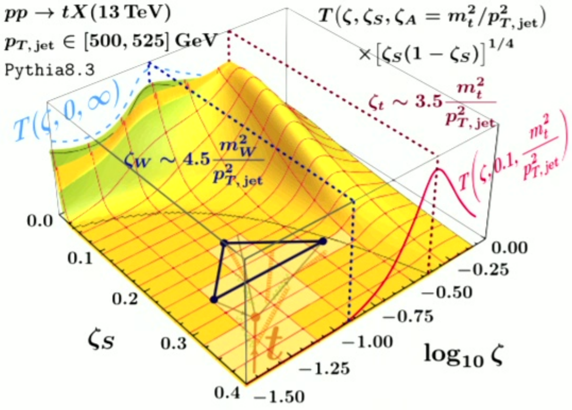


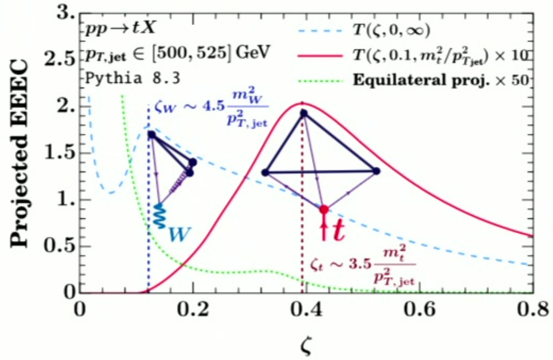
Top Quark Mass

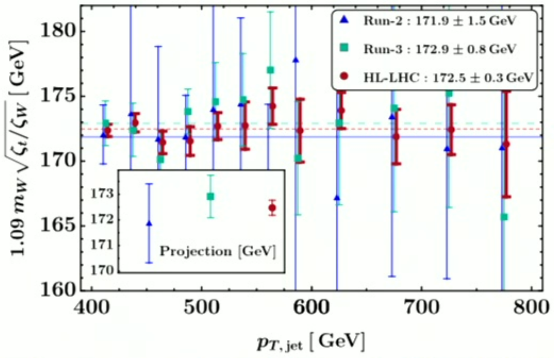
- The top quark mass is one of the most important parameters of the SM. e.g. electroweak vacuum stability/criticality, electroweak fits, etc.
- Need simple observables with top mass sensitivity that can be computed from first principles field theory.

Top Quark Mass Measurement Proposal

- Use the W to convert between angular and mass scales.

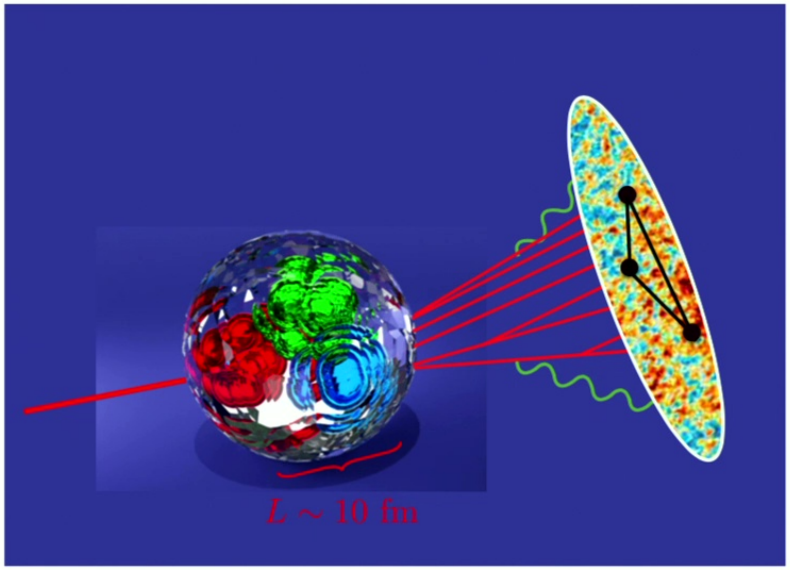




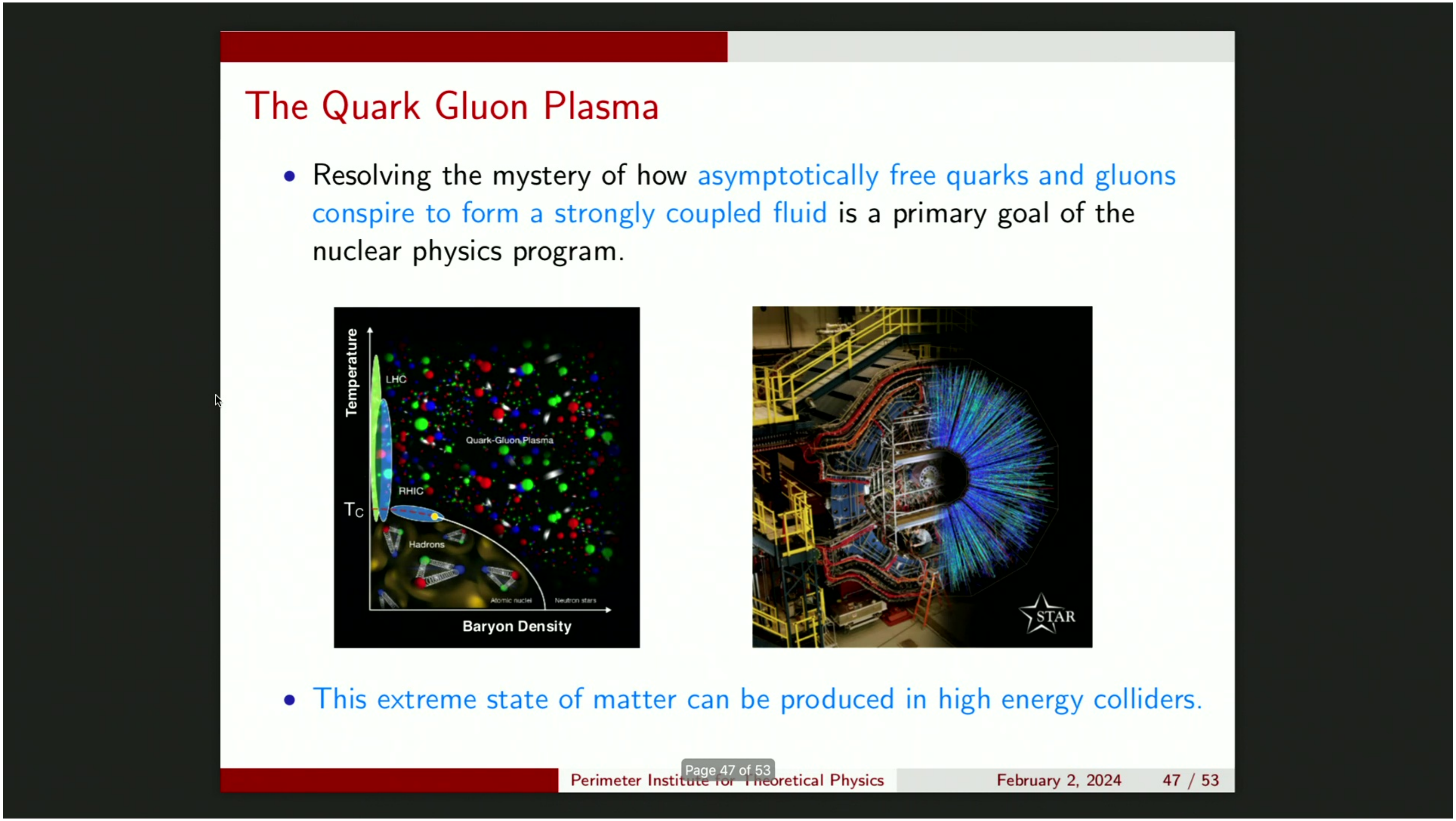


- Optimistic for a precision top quark mass extraction at the LHC.

Resolving the Scales of the Quark Gluon Plasma



[Andres, Dominguez, Holguin, Kunnawalkam Elayavalli, Marquet, Moult]



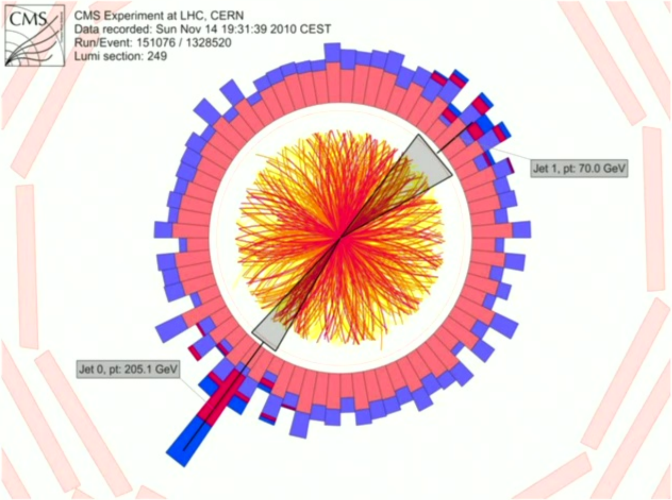
The Quark Gluon Plasma

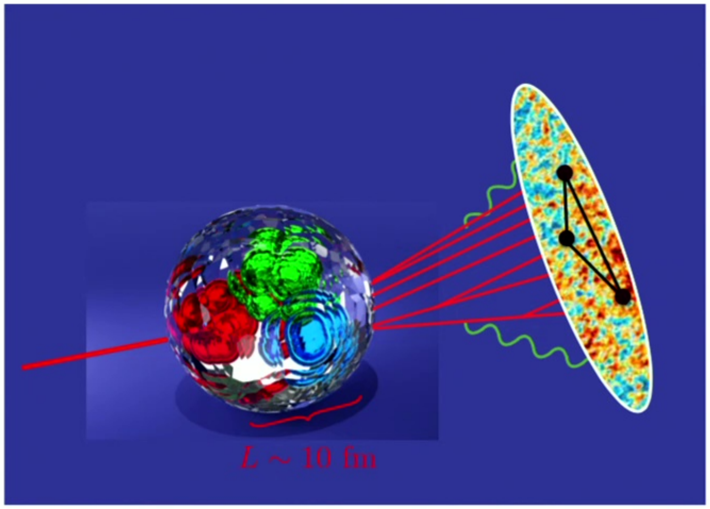
- Resolving the mystery of how asymptotically free quarks and gluons conspire to form a strongly coupled fluid is a primary goal of the nuclear physics program.

- This extreme state of matter can be produced in high energy colliders.

Imaging the Plasma

- Energetic quarks and gluons produced in the collisions shoot through the plasma, much like the classic Rutherford experiment.

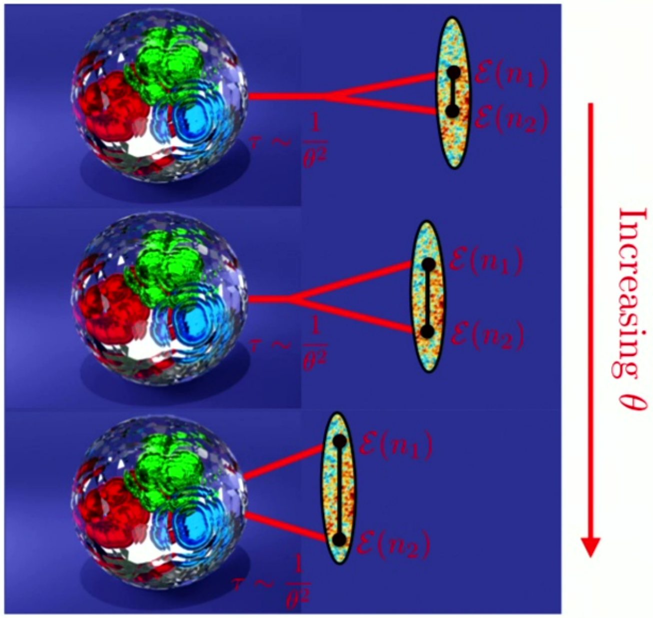


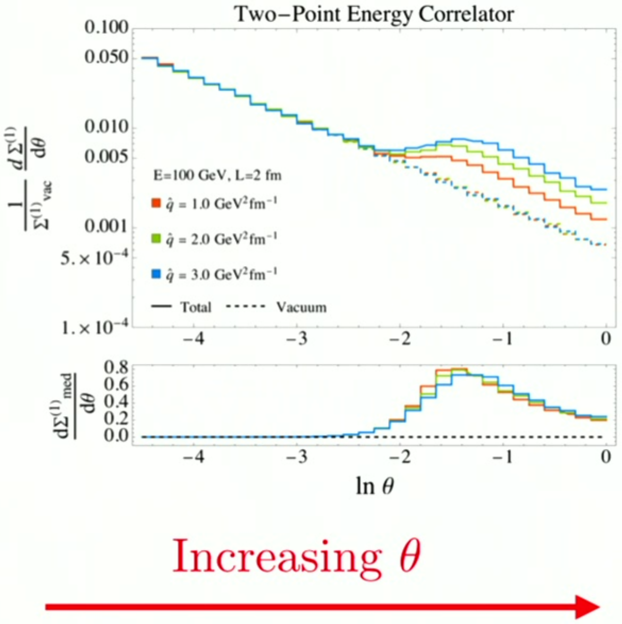


- How can we see there was a 10^{-14}m ball of plasma at the center?

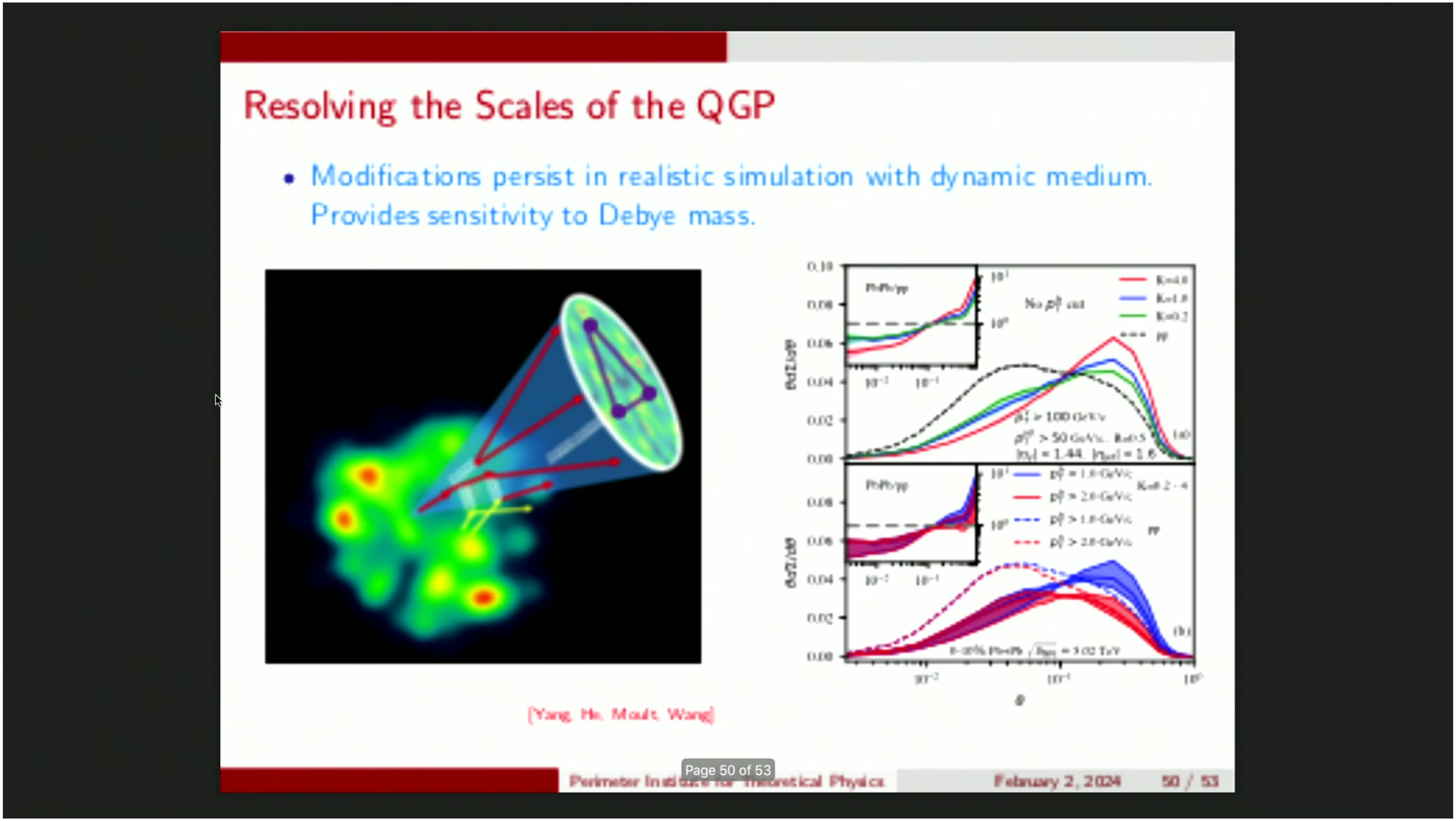
Resolving the Scales of the QGP

- QGP scales cleanly imprinted in two-point correlation!





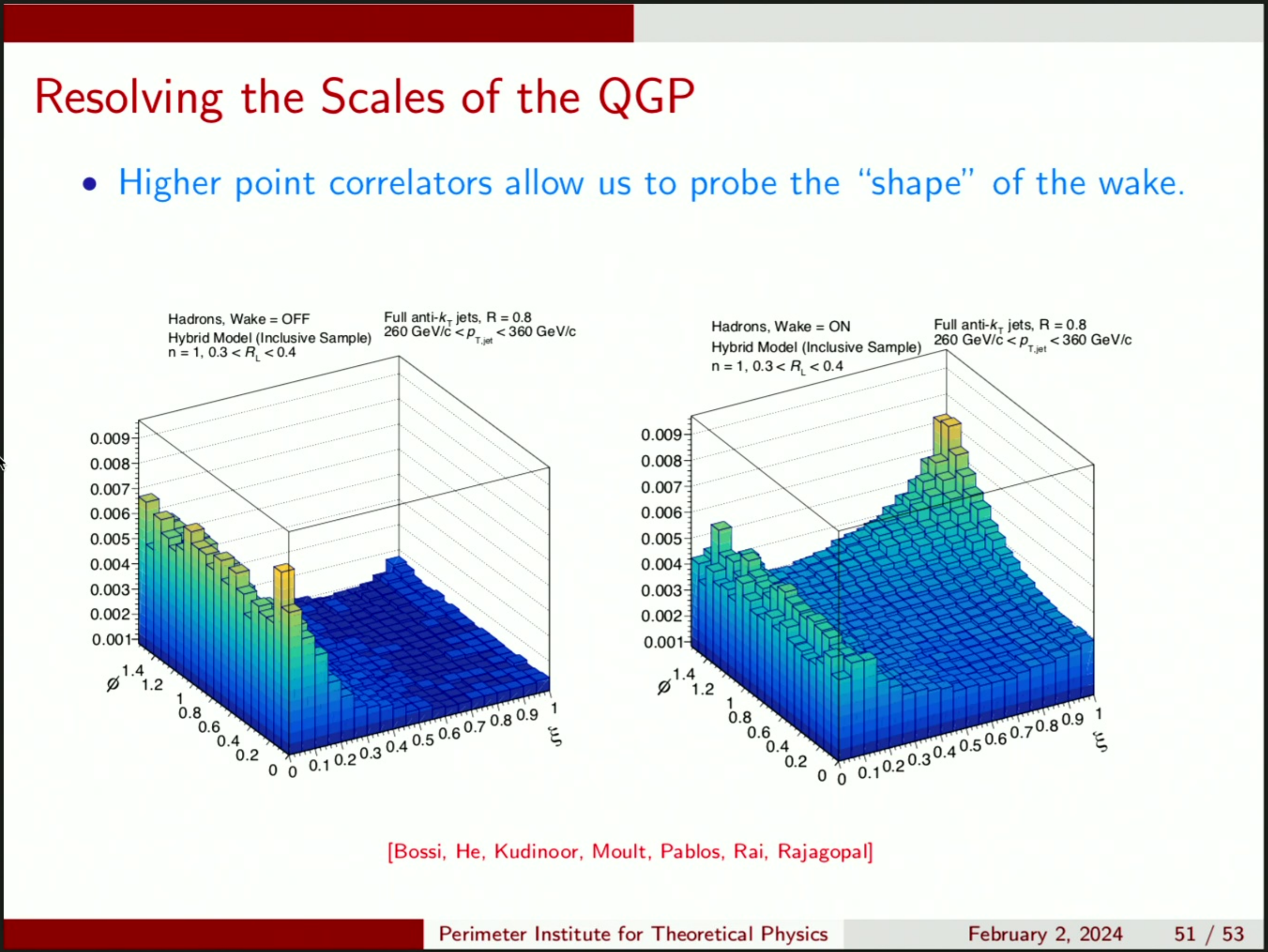
- Resolve Femtometer scales from asymptotic energy flux!



Resolving the Scales of the QGP

- Modifications persist in realistic simulation with dynamic medium. Provides sensitivity to Debye mass.

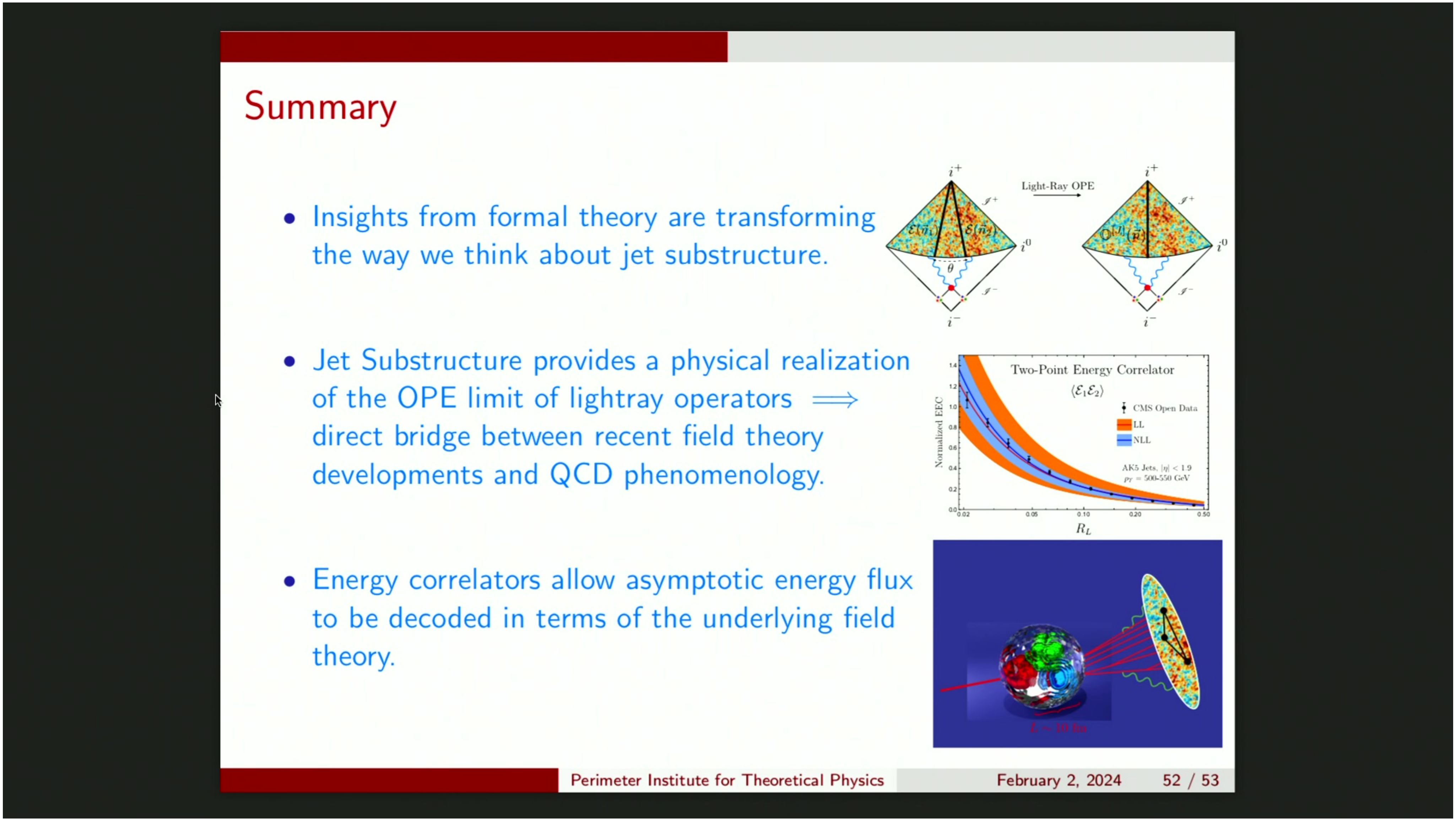
[Yang, He, Moult, Wang]



Resolving the Scales of the QGP

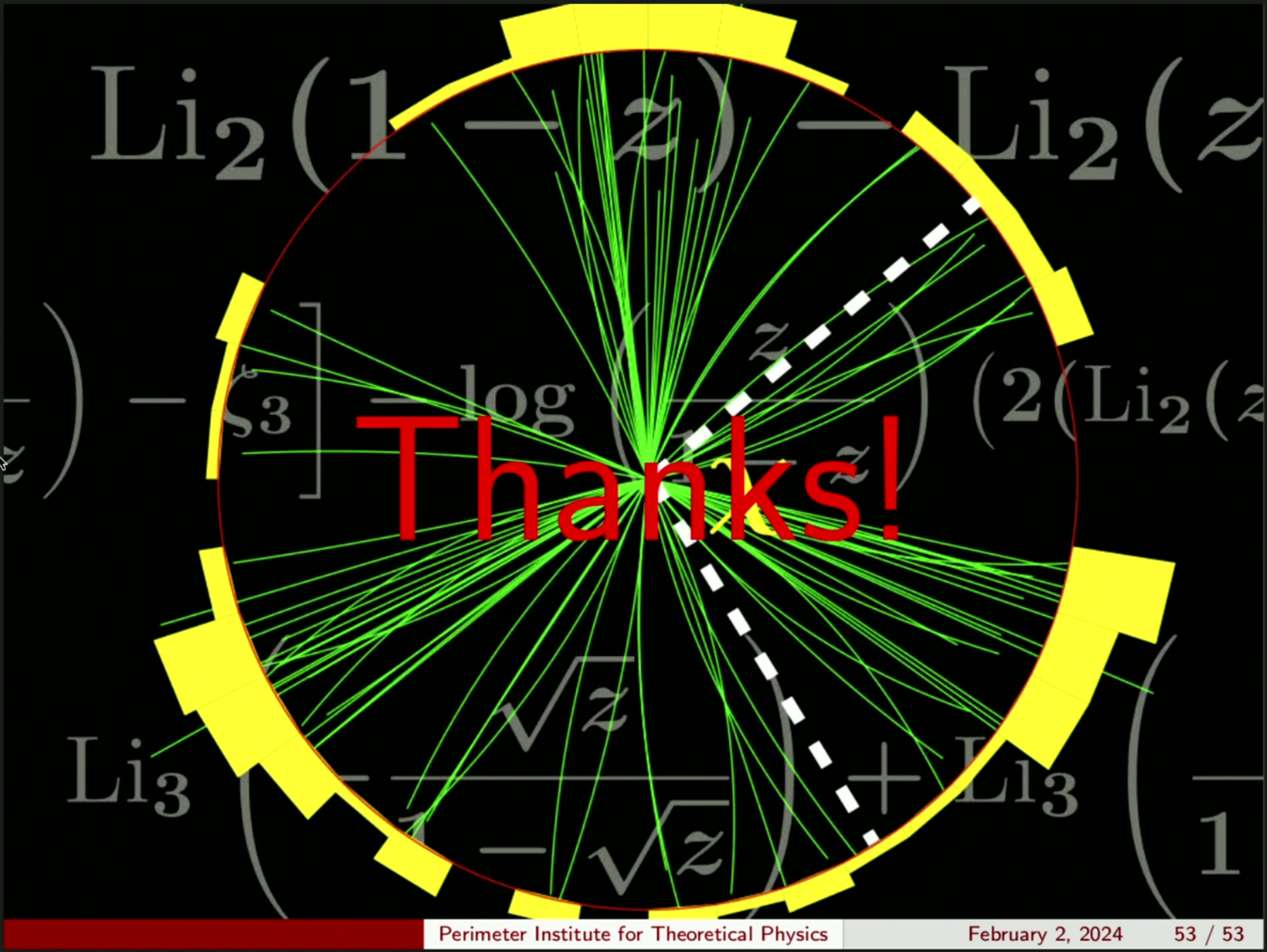
- Higher point correlators allow us to probe the "shape" of the wake.

[Bossi, He, Kudinoor, Moult, Pablos, Rai, Rajagopal]



Summary

- Insights from formal theory are transforming the way we think about jet substructure.
- Jet Substructure provides a physical realization of the OPE limit of lightray operators $\Longrightarrow$ direct bridge between recent field theory developments and QCD phenomenology.
- Energy correlators allow asymptotic energy flux to be decoded in terms of the underlying field theory.



Thanks!