Title: Horizons and Null Infinity: A Fugue in 4 voices

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Series: Quantum Gravity

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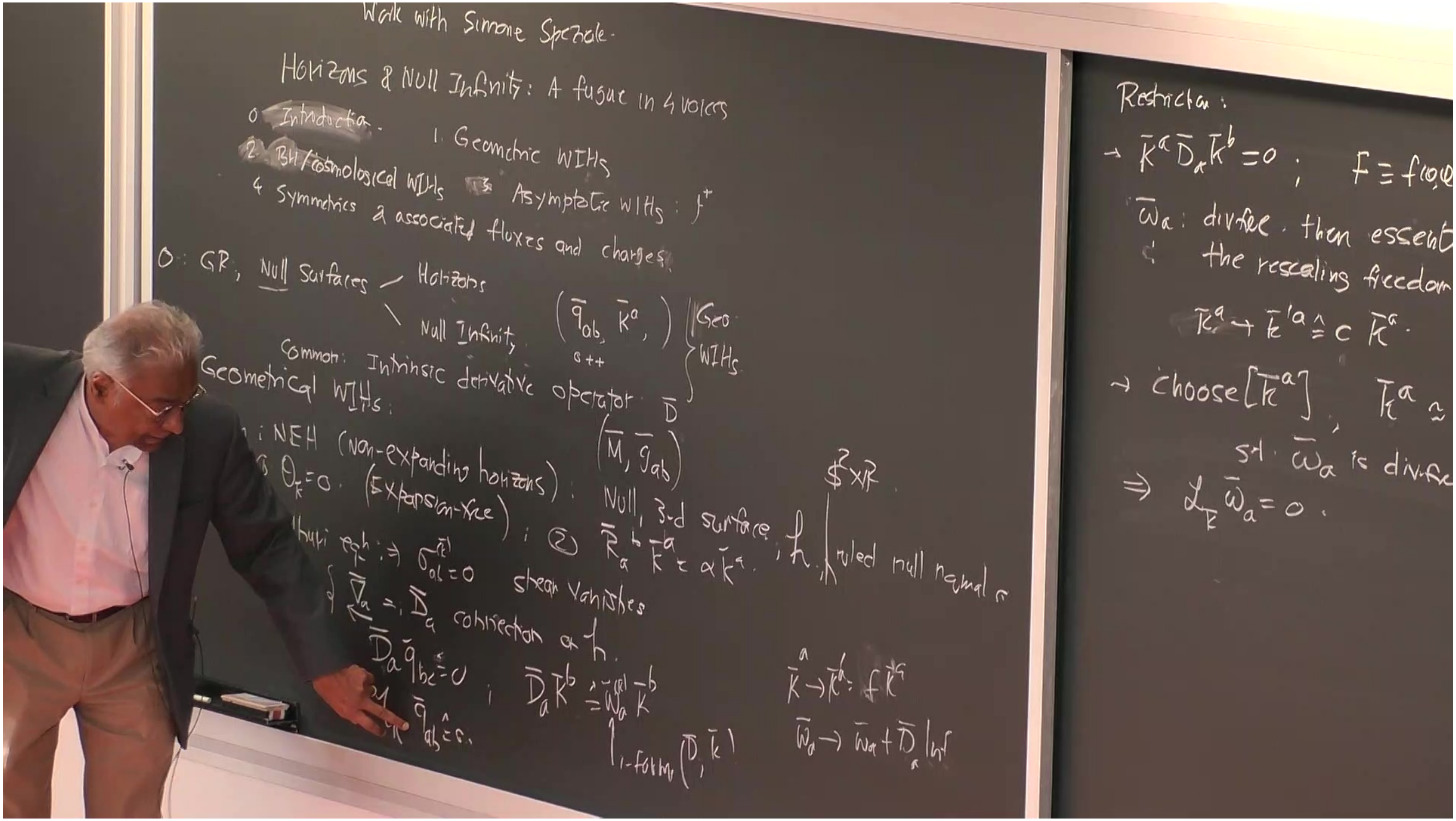
Abstract: Horizons of black holes in equilibrium and null infinity of asymptotically flat space-times are null 3-manifolds but have very different physical connotations. We first show that they share a large number of geometric properties, making them both weakly isolated horizons. We then use this new unified perspective to unravel the origin of the drastic differences in the physics they contain. Interestingly, the themes are woven together in a manner reminiscent of voices in a fugue.The talk is based on joint work with Simone Speziale (arXiv:2401.15618 and arXiv 2401:????).

Zoom link


Work with Simone Speziale.
Horizons & Null Infinity: A fugue in 4 voices
0 Introduction. 1. Geometric WIHs
2. BH/cosmological WIHs 3. Asymptotic WIHs
4 Symmetries & associated fluxes and charges
0: GR, Null Surfaces — Horizons
Null Infinity
Geo WIHs
Common: Intrinsic derivative operator
Geometrical WIHs:
NEH (non-expanding horizons)
(Expansion-free)
shear vanishes
connection on
Null, 3-d surface
ruled null normal
1-form
Restriction:
then essent
the rescaling freedom
choose
s.t.
is divfree

Restriction:

$\to \bar{k}^a \bar{D}_a \bar{k}^b = 0$; $f \equiv f(v, \ell)$

$\bar{\omega}_a$: div-free. then essentially eliminates the rescaling freedom.

$\bar{k}^a \to \bar{k}'^a \hat{=} c\, \bar{k}^a$.

$\to$ choose $[\bar{k}^a]$, $\bar{k}^a \simeq c \bar{k}^a$; $c > 0$

s.t. $\bar{\omega}_a$ is div-free.

$\Rightarrow \mathcal{L}_{\bar{k}} \bar{\omega}_a = 0$.

NEH $\underset{h}{\Delta}$ equipped with $[\bar{k}^a]$ s.t $\mathcal{L}_{\bar{k}} \bar{\omega}_a = 0$ is a WIH.

Is $\bar{D}$ time dependent?

$(\mathcal{L}_{\bar{k}} D_a - D_a \mathcal{L}_{\bar{k}}) = \dot{\bar{D}}_a$

If $\bar{h}_a \bar{k}^a \hat{=} 0$ then $\dot{\bar{D}}_a h_b = 0$

$\underbrace{\cancel{(\mathcal{L}_{\bar{k}} \bar{D}_a - \bar{D}_a \mathcal{L}_{\bar{k}}) \bar{t}^b = 0 \quad \forall\, \bar{t}^b \text{ tgt to } h}}_{\text{Isolated Horizons}}$

suffices to calculate

$\underbrace{\dot{\bar{D}}_a \bar{J}_b}_{\text{Time-depence}}$ $\qquad \bar{J}_b \bar{k}^b = -1$.

$\dot{\bar{D}}_a \bar{J}_b = \bar{D}_a \bar{\omega}_b + \bar{\omega}_a \bar{\omega}_b + \underbrace{\bar{k}^c \bar{C}_{cab}{}^d \bar{J}_d + \frac{1}{2}(\bar{S}_{ab} + \alpha \bar{q}_{ab}}_{\bar{k}^c \bar{R}_{cab}{}^d \bar{J}_d}$

$\bar{S}_{ab}$: $\bar{R}_{ab} - \frac{1}{6} \bar{R}\, \bar{g}_{ab}$

Restriction:

→ $\bar{k}^a \bar{D}_a \bar{k}^b = 0$; $f \equiv f(\theta, \varphi)$

$\bar{\omega}_a$: div-free. then essentially eliminates the rescaling freedom.

$\bar{k}^a \to \bar{k}'^a \hat{=} c\,\bar{k}^a$.

→ Choose $[\bar{k}^a]$, $\bar{k}^a \simeq c\,\bar{k}^a$; $c > 0$

s.t. $\bar{\omega}_a$ is div-free.

$\Rightarrow \mathcal{L}_{\bar{k}} \bar{\omega}_a = 0$.

NEH h equipped with $[\bar{k}^a]$ s.t. $\mathcal{L}_{\bar{k}} \bar{\omega}_a = 0$ is a WIH.

Is $\bar{D}$ time dependent?

$(\mathcal{L}_{\bar{k}} \bar{D}_a - \bar{D}_a \mathcal{L}_{\bar{k}}) \hat{=} \dot{\bar{D}}_a$

If $\bar{h}_a \bar{k}^a \hat{=} 0$ then $\dot{\bar{D}}_a h_b = 0$

~~$(\mathcal{L}_{\bar{k}} \bar{D}_a - \bar{D}_a \mathcal{L}_{\bar{k}}) \bar{t}^b = 0$ $\forall\, \bar{t}^b$ tgt to h.~~

Isolated Horizons

suffices to calculate

$$\underbrace{\dot{\bar{D}}_a \bar{J}_b}_{\text{Time-dependence}} \qquad \bar{J}_b \bar{k}^b = -1.$$

$$\dot{\bar{D}}_a \bar{J}_b = \bar{D}_a \bar{\omega}_b + \bar{\omega}_a \bar{\omega}_b + \bar{k}^c \bar{C}_{cab}{}^d \bar{J}_d + \tfrac{1}{2}\left({}^4\bar{S}_{ab} + \alpha \bar{q}_{ab}\right)$$

$${}^4\bar{S}_{ab} = {}^4\bar{R}_{ab} - \tfrac{1}{6}\,{}^4\bar{R}\,\bar{g}_{ab}$$

$$\bar{k}^c \bar{R}_{cab}{}^d \bar{J}_d.$$

WIH

radiation

$\Lambda = 0$

2. BH (cosmological Horizons) $\Lambda > 0$

$h \to \Delta$: WIH for BHs

$\bar{k}^a \to \ell^a$: normal ~~or cosmo~~

$\bar{D} \to D$

calculate

$\bar{J}_b$ $\quad\quad$ $\bar{J}_b \bar{K}^b = -1.$

dependence

$= \bar{D}_a \bar{\omega}_b + \bar{\omega}_a \bar{\omega}_b$

$+ \underbrace{\bar{K}^c \bar{C}_{cab}{}^d \bar{J}_d + \frac{1}{2}({}^4\bar{S}_{ab} + \alpha \bar{q}_{ab})}_{\bar{K}^c \bar{R}_{cab}{}^d \bar{J}_d}$

${}^4\bar{R}_{ab} - \frac{1}{6}{}^4\bar{R}\, \bar{g}_{ab}$

2. BH (cosmological Horizon)

$h \to \Delta$: WIH for BHs

$\bar{K}^a \to \ell^a$: normal or cosmo

$\bar{D} \to D$

$J_b \to n_a$; $n_a \ell^a = -1$

$$\dot{D}_a n_b = \bar{D}_a \bar{\omega}_b + \bar{\omega}_a \bar{\omega}_b + \underbrace{\ell^c C_{cab}{}^d n_d}_{-\frac{1}{4} R\, q_{ab} - D_{[a}\omega_{b]}}.$$

R: Ricci scalar of q_{ab}, 2-d object

$\mathcal{L}_\ell R = 0$; $\mathcal{L}_\ell \omega_a = 0$, $\omega_a \ell^a = 0$.

ω_a : horizontal 1-form

RHS is time independent.

$\Rightarrow \dot{D}_a$: completely determined by data $(\tilde{q}_{ab}, \tilde{\omega}_a,)$

$\Rightarrow$ No 3-d DoF ; No Radiation!!

$$\dot{D}_a n_b = \bar{D}_a \bar{\omega}_b + \bar{\omega}_a \bar{\omega}_b + \underbrace{\ell^c C_{cab}{}^d n_d}_{-\frac{1}{4} R\, q_{ab} - D_{[a}\omega_{b]}}.$$

R: Ricci scalar of q_{ab}, 2-d object

$\mathcal{L}_\ell R = 0$; $\mathcal{L}_\ell \omega_a = 0$, $\omega_a \ell^a = 0$.

ω_a : horizontal 1-form

RHS is time independent.

$\Rightarrow D_a$: completely determined by data $(\tilde{q}_{ab}, \tilde{\omega}_a)$.

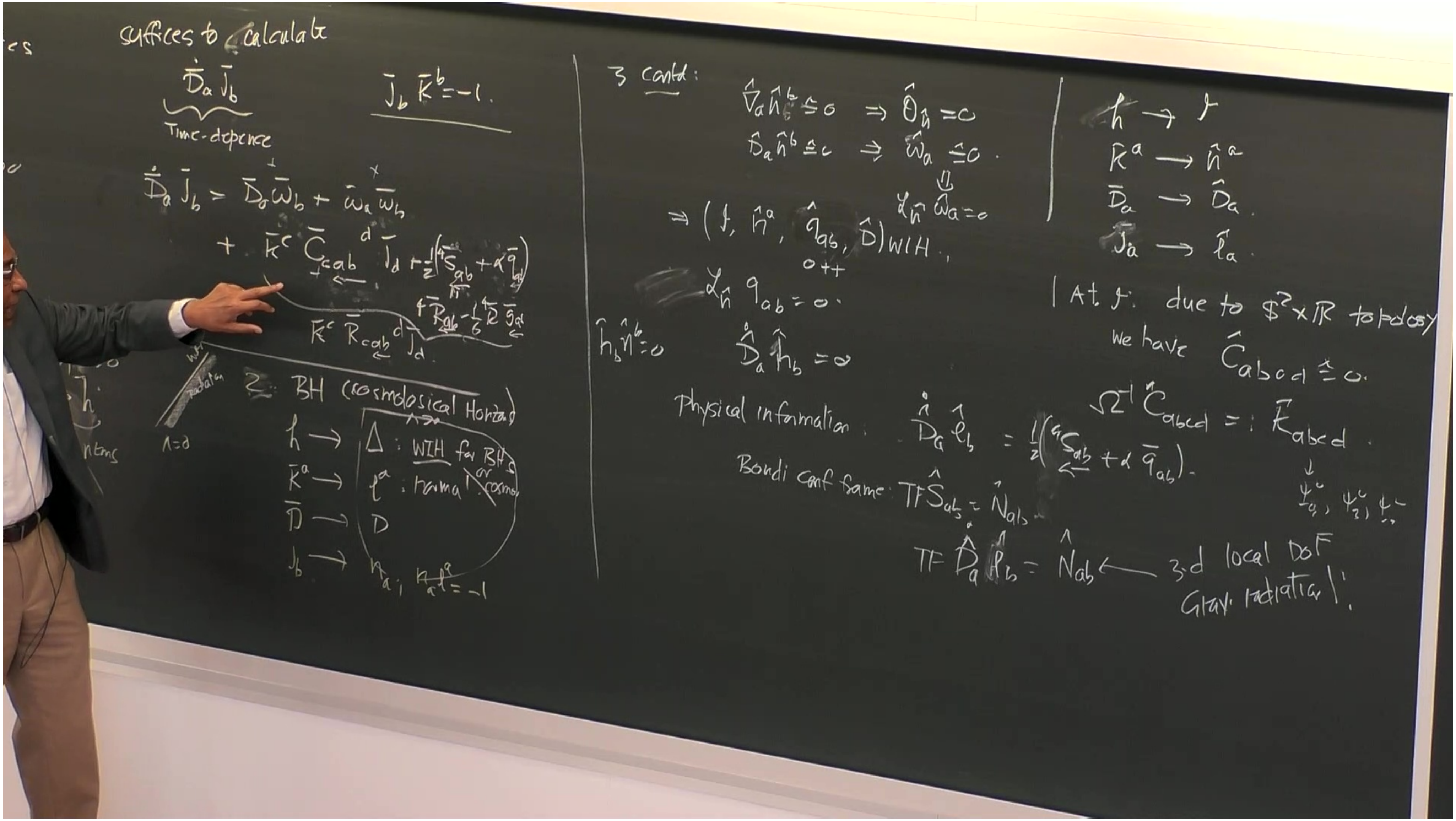
$\Rightarrow$ No 3-d DOF : No Radiation!!

3. Asymptotic WIH: $\mathscr{I}$.

(M, g_{ab}) : physical ; AF @ $\mathscr{I}^+$; $\exists\, \Omega > 0$ on M s.t. $\hat{g}_{ab} = \Omega^2 g_{ab}$ is smooth @ $\Omega \hat{=} 0$ ($\mathscr{I}^+$) ; s.t. $\hat{\nabla}_a \Omega \hat{\neq} 0$, $\Omega^{-1} T_{ab}$ has a limit

Freedom : $\Omega \to \Omega' = \omega \Omega$, ω : no where zero ; Use it with Field eqns (with T_{ab})

(Div free $\mathscr{I}^+$) $\quad \hat{\nabla}_a \hat{n}^a \hat{=} 0$, Field eqn $\Rightarrow \hat{\nabla}_a \hat{n}_b \hat{=} 0$


suffices to calculate
$\dot{\bar{D}}_a \bar{J}_b$
Time-depence
$\bar{J}_b \bar{K}^b = -1$.
$\dot{\bar{D}}_a \bar{J}_b = \bar{D}_a \bar{\omega}_b + \bar{\omega}_a \bar{\omega}_b + \bar{K}^c \bar{C}_{cab}{}^d \bar{J}_d + \frac{1}{2}({}^4S_{ab} + \alpha \bar{q}_{ab})$
$\bar{K}^c \bar{R}_{cab}{}^d \bar{J}_d$
2. BH (cosmological Horizon)
$h \to \Delta$: WIH for BH's or cosmo.
$\bar{K}^a \to \ell^a$: norma
$\bar{D} \to D$
$J_b \to n_a;\ n_a \ell^a = -1$
3 contd:
$\hat{\nabla}_a \hat{n}^b \triangleq 0 \Rightarrow \hat{\theta}_{\hat{n}} = 0$
$\hat{D}_a \hat{n}^b \triangleq 0 \Rightarrow \hat{\omega}_a \triangleq 0$.
$\mathcal{L}_{\hat{n}} \hat{\omega}_a = 0$
$\Rightarrow (\mathcal{I}, \hat{n}^a, \hat{q}_{ab}, \hat{D})$ WIH.
$\mathcal{L}_{\hat{n}} q_{ab} = 0$.
$\hat{n}_b \hat{n}^b = 0$
$\hat{D}_a \hat{n}_b = 0$
Physical information: $\hat{D}_a \hat{\ell}_b = \frac{1}{2}({}^4S_{ab} + \alpha \bar{q}_{ab})$.
Bondi conf frame: $TF\, \hat{S}_{ab} = \hat{N}_{ab}$
$TF\, \hat{D}_a \hat{\ell}_b = \hat{N}_{ab} \leftarrow$ 3-d local DoF
Grav. radiation!
$h \to \mathcal{I}$
$\bar{K}^a \to \hat{n}^a$
$\bar{D}_a \to \hat{D}_a$
$\bar{J}_a \to \hat{\ell}_a$
At $\mathcal{I}$: due to $S^2 \times R$ topology
we have $\hat{C}_{abcd} \triangleq 0$.
$\Omega^{-1} \hat{C}_{abcd} =: \hat{K}_{abcd}$
$\Psi_4^0, \Psi_3^0, \Psi_2^0$

calculate

$\bar{J}_b \bar{K}^b = -1.$

ence

$\bar{D}_a \bar{\omega}_b + \bar{\omega}_a \bar{\omega}_b$

$\bar{K}^c \bar{C}_{cab}{}^d \bar{J}_d + \frac{1}{2}({}^4\bar{S}_{ab} + \alpha \bar{q}_{ab})$

${}^4\bar{R}_{ab} - \frac{1}{6}\bar{R}\,\bar{g}_{ab}$

$\bar{K}^c \bar{R}_{cab}{}^d \bar{J}_d$

2. BH (cosmological Horizon)

$h \to \Delta$: WIH for BHs or cosmo

$\bar{K}^a \to \ell^a$: normal

$\bar{D} \to D$

$J_b \to n_a ,\ n_a \ell^a = -1$

3 contd:

$\hat{\nabla}_a \hat{n}^b \triangleq 0 \Rightarrow \hat{\Theta}_{\hat{n}} = 0$

$\hat{D}_a \hat{n}^b \triangleq 0 \Rightarrow \hat{\omega}_a \triangleq 0.$

$\Downarrow$

$\mathcal{L}_{\hat{n}} \hat{\omega}_a = 0$

$\Rightarrow (\mathcal{I}, \hat{n}^a, \hat{q}_{ab}, \hat{D})$ WIH.

$\mathcal{L}_{\hat{n}} q_{ab} = 0.$

$\hat{h}_b \hat{n}^b = 0$

$\hat{D}_a \hat{h}_b = 0$

Physical information: $\hat{D}_a \hat{\ell}_b = \frac{1}{2}({}^4S_{ab} + \alpha\, \bar{q}_{ab}).$

Bondi conf frame: $\mathrm{TF}\, \hat{S}_{ab} = \hat{N}_{ab}.$

$\mathrm{TF}\, \hat{D}_a \hat{\ell}_b = \hat{N}_{ab} \longleftarrow$ 3d local DoF

Grav. radiation!

4: symmetry:

WIH: universal structure, group $\mathcal{G} = \mathcal{B} \ltimes \mathcal{D}$

At $\mathcal{I}$: . .

$\mathcal{I}$: $\mathcal{L}_\xi \hat{q}_{ab} = 2\beta \hat{q}_{ab}$

$\mathcal{L}_\xi \hat{n}^a = -\beta \hat{n}^a$

group $\mathcal{B}$

dilations $\mathcal{D}$, generator $\mathcal{L}_d q_{ab} = 0$

$\mathcal{L}_d \bar{K}^a = c \bar{K}^a$

General $\xi^a \in \mathcal{G}$ Lie d.s

$\mathcal{L}_\xi q_{ab} = (2\beta) q_{ab}$

$\mathcal{L}_\xi \bar{K}^a = (-\beta + c) \bar{K}^a.$

$h \to \mathcal{I}$

$\bar{K}^a \longrightarrow \hat{n}^a$

$\bar{D}_a \longrightarrow \hat{D}_a$

$\bar{J}_a \longrightarrow \hat{\ell}_a$

At $\mathcal{I}$: due to $S^2 \times \mathbb{R}$ topology we have $\hat{C}_{abcd} \triangleq 0.$

$\Omega^{-1} \hat{C}_{abcd} =: \bar{K}_{abcd}.$

$\Psi_4^\circ, \Psi_3^\circ, \Psi_2^\circ$

$\hat{D}_a n_b = \bar{D}$

R

$\Rightarrow D_a$

$\Rightarrow$ N

3. Asymptoti

(M, g_{ab}): P

is smooth