Title: Cosmology Lecture

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v_{phys} = "PECULIAR VELOCITY" = $\dfrac{\text{PHYSICAL DISTANCE}}{\text{PHYSICAL TIME}}$

$= a \dfrac{dx}{dt} = a \dfrac{\text{COMOVING DISTANCE}}{\text{PHYSICAL TIME}}$

$q_{phys} \propto \dfrac{1}{a(t)}$

"

$\dfrac{m_0 v_{phys}}{(1 - v_{phys}^2)^{1/2}}$

$$x^i = \text{CONSTANT}$$

$$\frac{dx}{dt} \neq 0$$

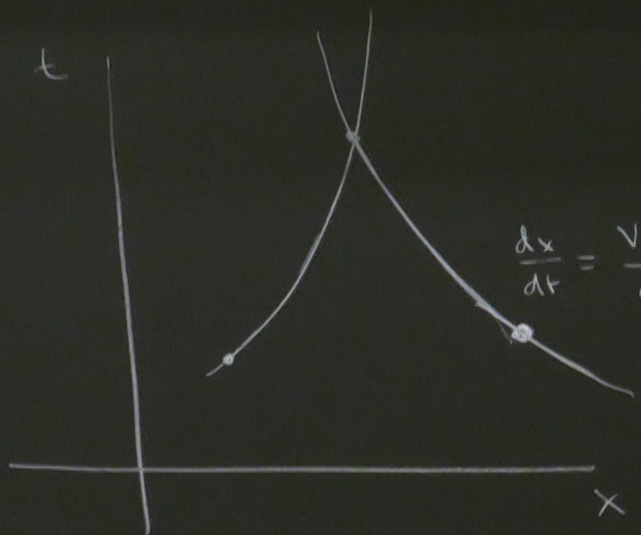
$$d\tau = \sqrt{-ds^2} = \sqrt{dt^2 - a^2 dx^2}$$

$\hbar = c = k_B = 1$ $\qquad M_{pl} = (8\pi G)^{-1/2} \neq 1$ $\qquad (-+++)$

$$ds^2 = -dt^2 + \underbrace{a(t)^2}_{\text{SCALE FACTOR}} dx \qquad (\Delta x)_{phys} = a(t)\cdot(\Delta x)$$

$$H(t) = \frac{\dot{a}(t)}{a(t)} \qquad \text{"HUBBLE EXPANSION RATE"}$$

$$\Gamma^0_{ij} = a^2 H \delta_{ij} \qquad \Gamma^j_{i0} = H \delta^j_i$$



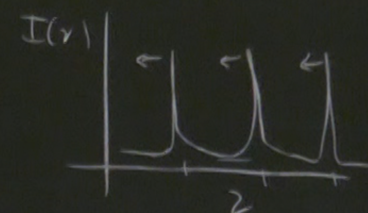
$$\frac{dx}{dt} = \frac{v_{phys}}{a(t)} = \frac{1}{a(t)}$$

$$a(t) = \frac{1}{1+z(t)}$$

$$q_{phys} \propto \frac{1}{a(t)}$$

PHOTON $\nu \propto \frac{1}{a(t)}$

$$\nu_{obs} = \nu_{src} \frac{a_{src}}{a_{obs}} \leftarrow a = 1$$



$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

$$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu) T^{\rho_1 \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} = -\sum_{i=1}^{m} R_{\mu\nu\lambda}{}^{\rho_i} \, T^{\rho_1 \cdots \lambda \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} + \sum_{j=1}^{n} R_{\mu\nu\sigma_i}{}^{\lambda} \, T^{\rho_1 \cdots \rho_m}{}_{\sigma_1 \cdots \lambda \cdots \sigma_n}$$

$$R_{\mu\nu\rho}{}^{\sigma} = -\partial_\mu \Gamma^\sigma_{\nu\rho} + \partial_\nu \Gamma^\sigma_{\mu\rho} - \Gamma^\sigma_{\mu\lambda} \Gamma^\lambda_{\nu\rho} + \Gamma^\sigma_{\nu\lambda} \Gamma^\lambda_{\mu\rho}$$

$$R_{\mu\nu\rho\sigma} = -R_{\nu\mu\rho\sigma} = -R_{\mu\nu\sigma\rho} = R_{\rho\sigma\mu\nu}$$

$$R_{\underbrace{\mu\nu\rho}\sigma} + R_{\nu\rho\mu\sigma} + R_{\rho\mu\nu\sigma} = 0$$

$$\nabla_\mu R_{\nu\rho\sigma\lambda} + \nabla_\nu R_{\rho\sigma\mu\lambda} + \nabla_\rho R_{\mu\nu\sigma\lambda} = 0$$

$(-+++)$

$(+---)$

$$R_{\mu\nu} \equiv R_{\mu\rho\nu}{}^{\rho} \qquad R_{\mu\nu} = R_{\nu\mu}$$

$$R_{\mu\nu} = \partial_\rho \Gamma^{\rho}_{\mu\nu} - \partial_\mu \Gamma^{\rho}_{\nu\rho} + \Gamma^{\rho}_{\mu\nu} \Gamma^{\lambda}_{\rho\lambda} - \Gamma^{\lambda}_{\mu\rho} \Gamma^{\rho}_{\nu\lambda}$$

$$R \equiv g^{\mu\nu} R_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \qquad \nabla_\mu G^{\mu\nu} = 0$$

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

"STRESS-ENERGY IS CONSERVED"

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} - 8\pi G \Lambda g_{\mu\nu}$$

$$T_{\mu\nu} = \cdots - \Lambda g_{\mu\nu}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

"STRESS-ENERGY IS CONSERVED"

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

$$T_{\mu\nu} = \cdots - \Lambda g_{\mu\nu}$$

"STRESS-ENERGY IS CONSERVED"

$$\nabla_\mu T^{\mu\nu} = 0$$

$$R_{\mu\nu} = \partial_\rho \Gamma^\rho_{\mu\nu} - \partial_\mu \Gamma^\rho_{\nu\rho} + \Gamma^\rho_{\mu\nu}\Gamma^\lambda_{\rho\lambda} - \Gamma^\lambda_{\mu\rho}\Gamma^\rho_{\nu\lambda}$$

$$R_{00} = \partial_\rho \Gamma^\rho_{00} - \partial_0 \Gamma^\rho_{0\rho} + \Gamma^\rho_{00}\Gamma^\lambda_{0\lambda} - \Gamma^\lambda_{0\rho}\Gamma^\rho_{0\lambda}$$

$$= 0 - \partial_0(H\delta^i_i) + 0 - (H\delta^j_i)(H\delta^i_j)$$

$$= -3\dot{H} - 3H^2$$

$$R_{0i} = 0$$

$$R_{ij} = a^2(\dot{H} + 3H^2)\delta_{ij}$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$= (-1) R_{00} + (a^{-2}\delta^{ij}) R_{ij}$$

$$= (-1)(-3\dot{H} - 3H^2) + (a^{-2}\delta^{ij})\, a^2(\dot{H} + 3H^2)\delta_{ij}$$

$$= 6\dot{H} + 12H^2$$

$$\boxed{\begin{aligned} G_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \\ G_{00} &= 3H^2 \\ G_{ij} &= -a^2\left(2\dot{H} + 3H^2\right) \end{aligned}}$$

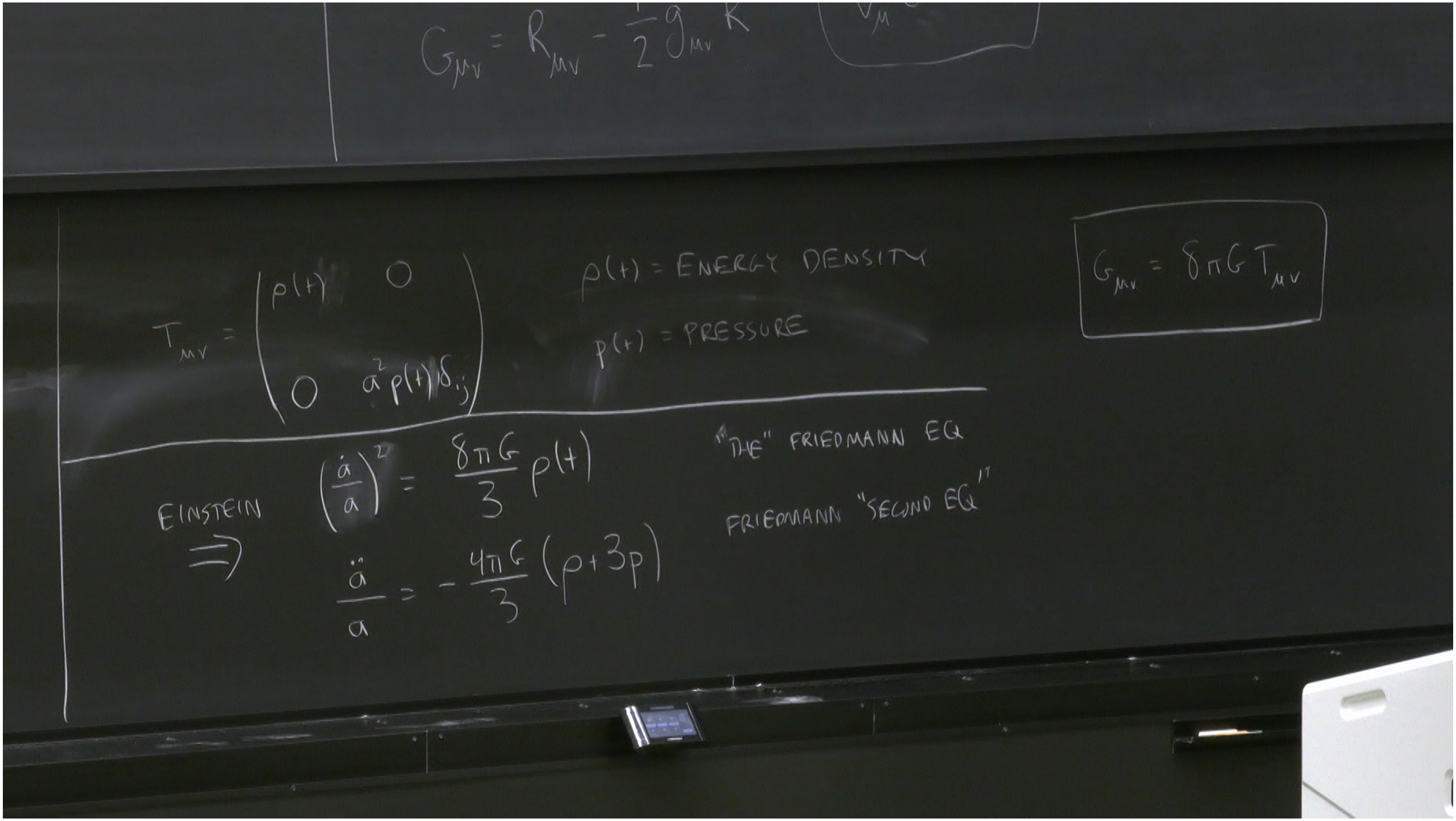
$$T_{\mu\nu} = \begin{pmatrix} \rho(t) & 0 \\ 0 & a^2 p(t)\,\delta_{ij} \end{pmatrix}$$

EINSTEIN

$\Longrightarrow$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(\ldots$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\ldots$$


$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$
$T_{\mu\nu} = \begin{pmatrix} \rho(t) & 0 \\ 0 & a^2 p(t) \delta_{ij} \end{pmatrix}$
$\rho(t)$ = ENERGY DENSITY
$p(t)$ = PRESSURE
$\boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$
EINSTEIN $\Rightarrow$
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t)$
"THE" FRIEDMANN EQ
$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$
FRIEDMANN "SECOND EQ"

CONSERVATION OF STRESS ENERGY:

$$\nabla_\mu T^{\mu 0} = 0$$

$$\partial_\mu T^{\mu 0} + \Gamma^\mu_{\mu\nu} T^{\nu 0} + \Gamma^0_{\mu\nu} T^{\mu\nu} = 0$$

$$\partial_0 T^{00} + \Gamma^i_{i0} T^{00} + \Gamma^0_{ij} T^{ij} = 0$$

$$\partial_0 \rho + (H\delta^i_i)\rho + (a^2 H \delta_{ij})(a^{-2} p \delta^{ij}) = 0$$

$$\dot\rho + 3H\rho + 3Hp = 0$$

CONSERVATION OF STRESS ENERGY:

$$\nabla_\mu T^{\mu 0} = 0$$

$$\partial_\mu T^{\mu 0} + \Gamma^{\mu}_{\mu\nu} T^{\nu 0} + \Gamma^{0}_{\mu\nu} T^{\mu\nu} = 0$$

$$\partial_0 T^{00} + \Gamma^{i}_{i0} T^{00} + \Gamma^{0}_{ij} T^{ij} = 0$$

$$\partial_0 \rho + (H \delta^{i}_{i}) \rho + (a^2 H \delta_{ij})(a^{-2} p \delta^{ij}) = 0$$

$$\dot{\rho} + 3H\rho + 3Hp = 0$$

CONSERVATION OF STRESS ENERGY :

$$\nabla_\mu T^{\mu 0} = 0$$

$$\partial_\mu T^{\mu 0} + \Gamma^\mu_{\mu\nu} T^{\nu 0} + \Gamma^0_{\mu\nu} T^{\mu\nu} = 0$$

$$\partial_0 T^{00} + \Gamma^i_{i0} T^{00} + \Gamma^0_{ij} T^{ij} = 0$$

$$\partial_0 \rho + (H\delta^i_i)\rho + (a^2 H \delta_{ij})(a^{-2} p \delta^{ij}) = 0$$

$$\dot\rho + 3H\rho + 3Hp = 0$$

$$\boxed{\dot{\rho} = -3H(\rho + p)}$$

"CONTINUITY EQUATION"

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t)$$

"CONSTANT-w COSMOLOGY" = STRESS-ENERGY IS DOMINATED BY A SINGLE COMPONENT WITH CONSTANT w

$\hbar = c$

$$w \equiv \frac{p(t)}{\rho(t)}$$ "EQUATION OF STATE"

$$w = \begin{cases} 0 & \text{"MATTER" = NONREL. PARTICLES} \\ 1/3 & \text{"RADIATION" = RELATIVISTIC PARTICLES} \\ -1 & \text{COSMOLOGICAL CONSTANT} \end{cases}$$

$c = k_B = 1$ $\qquad M_{pl} = (8\pi G)^{-1/2} \neq 1$ $\qquad (-+++)$

$$ds^2 = -dt^2 + \underbrace{a(t)^2}_{\text{SCALE FACTOR}} dx$$

$$(\Delta x)_{phys} = a(t) \cdot (\Delta x)$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

"HUBBLE EXPANSION RATE"

$$\Gamma^0_{ij} = a^2 H \delta_{ij} \quad \Big| \quad \Gamma^j_{i0} = H \delta^j_i$$

$$T_{\mu\nu} \supset -\Lambda g_{\mu\nu}$$

$$\rho = T_{00}$$
$$= -\Lambda g_{00}$$
$$= \Lambda \quad [\text{CONST.}]$$