Title: Cosmology Lecture

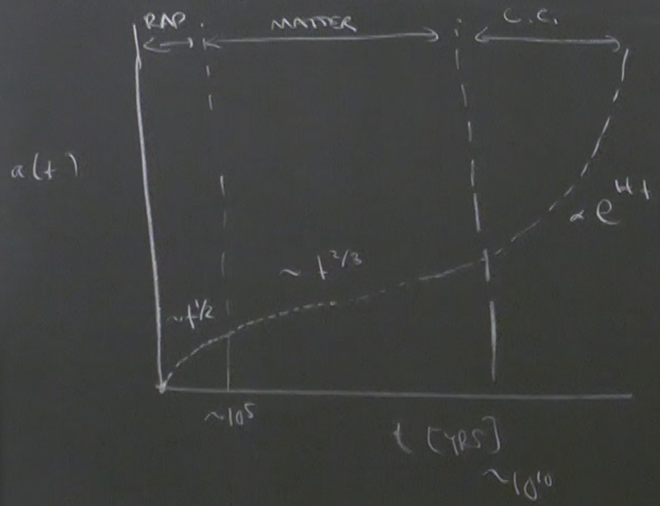
Speakers: Kendrick Smith

Collection: Cosmology 2023/24

Date: February 29, 2024 - 1:00 PM

URL: https://pirsa.org/24020036

$$ds^2 = -dt^2 + a(t)^2 dx^2$$



$$H^2 = \frac{8\pi G}{3}\rho \qquad H \equiv \frac{d\log a}{dt}$$

$$\dot{\rho} = -3H(\rho + p) \qquad T_{\mu\nu} = \begin{pmatrix} \rho(t) & \\ & a(t)^2 p(t)\,\delta_{ij} \end{pmatrix}$$

$$\Longleftrightarrow \quad \frac{d\rho}{d\log a} = -3(\rho + p)$$

CONSTANT - W

$$W = \frac{p(t)}{\rho(t)} = \text{CONST.} \quad \text{"EQUATION OF STATE"}$$

$$= \begin{cases} 1/3 & \text{RADIATION} \\ 0 & \text{MATTER} \\ -1 & \text{COSMOLOGICAL CONSTANT} \end{cases}$$

$$\frac{d\rho}{d\log a} = -3(\rho + p) = -3(1+w)\rho$$

$$\rho = \rho_0\, a^{-3(1+w)} \qquad (\cdot)_0 = \text{“TODAY”} \quad [\text{AT } a = 1]$$

$$= \begin{cases} a^{-3} & \text{MATTER } [w=0] \\ a^{-4} & \text{RADIATION } [w=\tfrac{1}{3}] \\ a^{0} & \text{COSMOLOGICAL CONSTANT } [w=0] \end{cases}$$

COSMOLOGICAL CONST

$$H^2 = \frac{8\pi G}{3} \rho_0 \, a^{-3(1+w)}$$

$$H = H_0 \, a^{-3(1+w)/2} \qquad H_0 \equiv \left[\frac{8\pi G}{3} \rho_0 \right]^{1/2}$$

$$\frac{da}{dt} = H_0 \, a^{-(1+3w)/2}$$

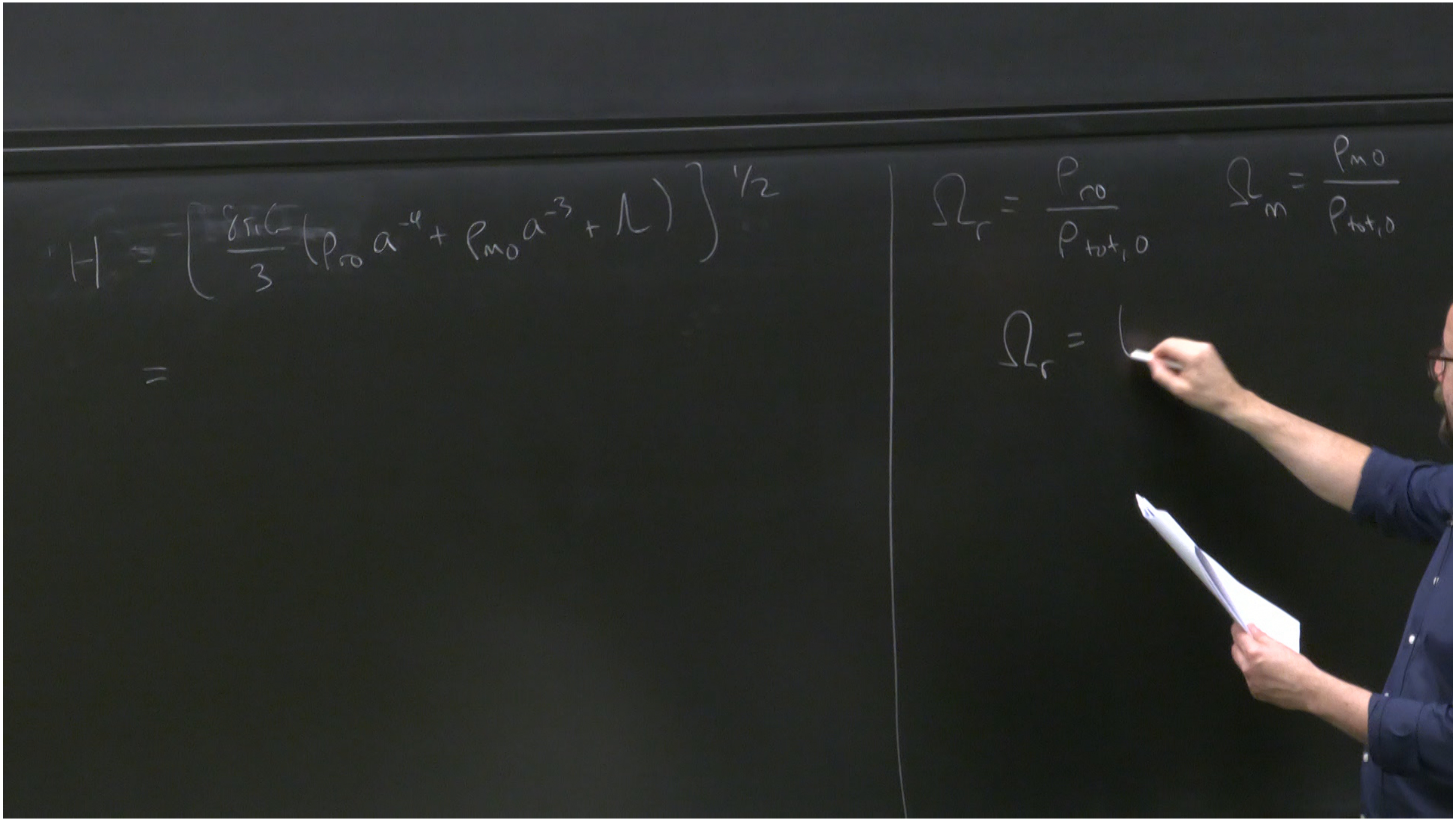
$$\Rightarrow \; a(t) = \begin{cases} \left[\frac{3(1+w)}{2} H_0 t \right]^{\frac{2}{3+3w}} & w > -1 \\ e^{H_0 t} & w = -1 \end{cases} \qquad \propto \begin{cases} t^{1/2} & w = \frac{1}{3} \; [\text{RAD}] \\ t^{2/3} & w = 0 \; [\text{MATTER}] \\ e^{H_0 t} & w = -1 \; [\text{COSM. CONSTANT}] \end{cases}$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

ΛCDM

CONST. $w \Rightarrow \begin{cases} \rho = \rho_0 a^{-3(1+w)} \\ p = w\rho_0 a^{-3(1+w)} \end{cases}$

ΛCDM $\Rightarrow \begin{cases} \rho = \rho_{r0} a^{-4} + \rho_{m0} a^{-3} + \Lambda \\ p = \frac{1}{3}\rho_{r0} a^{-4} + 0 - \Lambda \end{cases}$



$H = \left[\frac{8\pi G}{3} \left(\rho_{r0} a^{-4} + \rho_{m0} a^{-3} + \Lambda \right) \right]^{1/2}$
$=$
$\Omega_r = \frac{\rho_{r0}}{\rho_{tot,0}}$
$\Omega_m = \frac{\rho_{m0}}{\rho_{tot,0}}$
$\Omega_r =$

$^{1/2}$

$$\Omega_r = \frac{\rho_{r0}}{\rho_{tot,0}} \qquad \Omega_m = \frac{\rho_{m0}}{\rho_{tot,0}} \qquad \Omega_\Lambda = \frac{\Lambda}{\rho_{tot,0}}$$

$$\Omega_r = (8.5 \pm 0.7) \times 10^{-5}$$

$$\Omega_m = 0.311 \pm 0.006 \quad \left\langle \begin{array}{l} \Omega_b = 0.05 \\ \Omega_c = 0.25 \end{array} \right.$$

$$\Omega_\Lambda = 0.689 \pm 0.006$$

$$H = \left[\frac{8\pi G}{3}\left(\rho_{r0}\, a^{-4} + \rho_{m0}\, a^{-3} + \Lambda\right)\right]^{1/2}$$

$$= \left[\frac{8\pi G}{3}\rho_{tot}\left(\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda\right)\right]^{1/2}$$

Λ

Λ

$\Omega_r =$

Ω_r

Ω_m

Ω_Λ

H

$$H(a) = \left[\frac{8\pi G}{3}\left(\rho_{r0}\, a^{-4} + \rho_{m0}\, a^{-3} + \Lambda\right)\right]^{1/2}$$

$$= \left[\frac{8\pi G}{3}\rho_{tot,0}\left(\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda\right)\right]^{1/2}$$

$$= H_0\left[\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda\right]^{1/2}$$

$$\Omega_r + \Omega_m + \Omega_\Lambda = 1$$ BY DEFINITION

$$\Omega_r = \frac{\rho_{r0}}{\rho_{tot,0}} \qquad \Omega_m = \frac{\rho_{m0}}{\rho_{tot,0}} \qquad \Omega_\Lambda = \frac{\Lambda}{\rho_{tot,0}}$$

$$\Omega_r = (8.5 \pm 0.7) \times 10^{-5}$$

$$\Omega_m = 0.311 \pm 0.006 \quad \begin{cases} \Omega_b = 0.05 \\ \Omega_c = 0.25 \end{cases}$$

$$\Omega_\Lambda = 0.689 \pm 0.006$$

$$H_0 = \left[\frac{8\pi G}{3}\rho_{tot,0}\right]^{1/2} = (70 \pm 3)\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$
$$= (0.072 \pm 0.003)\ \mathrm{Gyr^{-1}}$$

$$\frac{1}{a}\frac{da}{dt} = H(a)$$

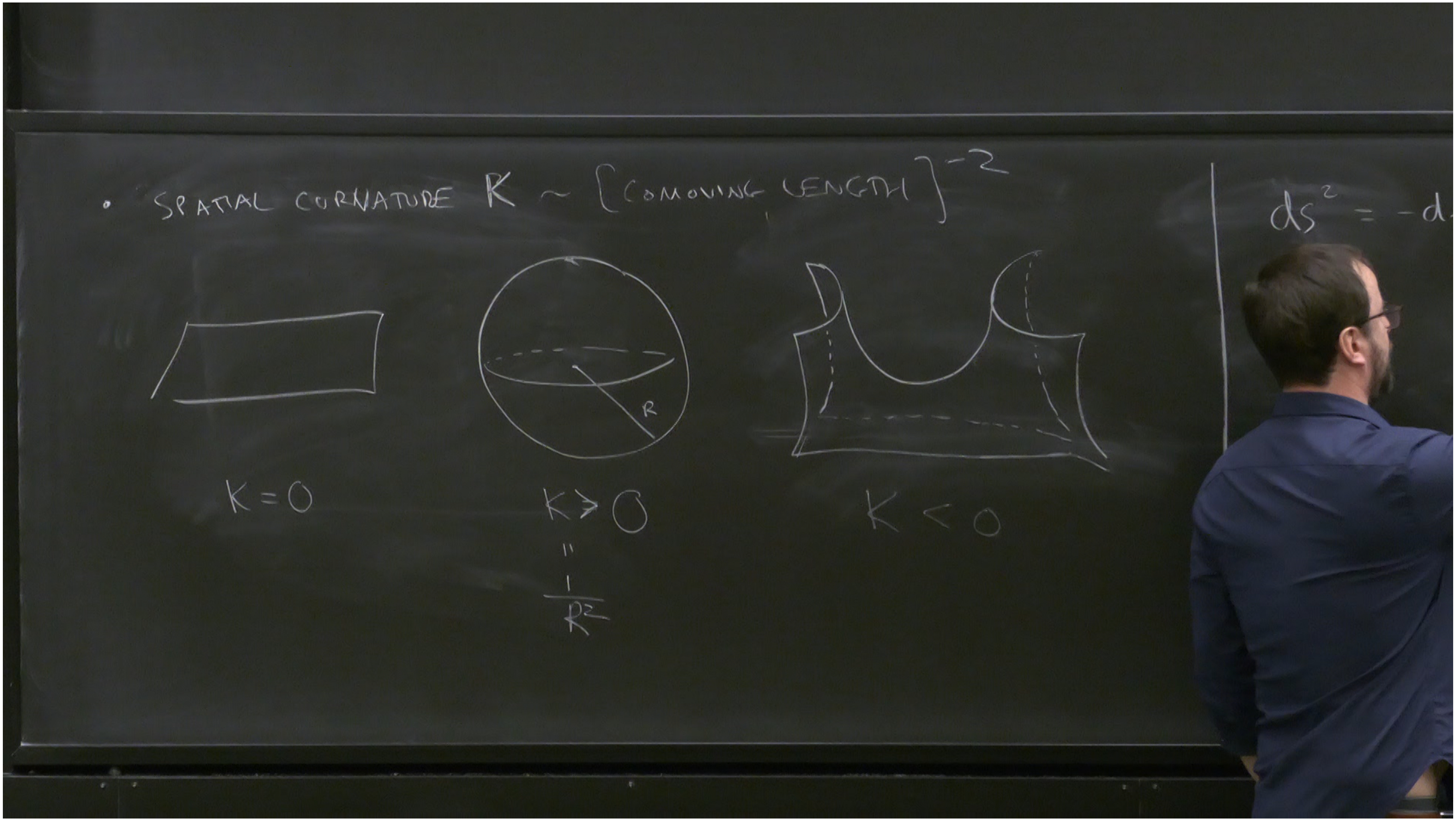
$$\Rightarrow dt = \frac{da}{a\,H(a)}$$

$$t = \int_0^a \frac{da'}{a'\,H(a')} = H_0^{-1}\int_0^a \frac{da'}{a'}\left[\Omega_r a'^{-4} + \Omega_m a'^{-3} + \Omega_\Lambda\right]$$

$$\frac{1}{a}\frac{da}{dt} = H(a)$$

$$\Rightarrow \; dt = \frac{da}{a\,H(a)}$$

$$t = \int_0^a \frac{da'}{a'\,H(a')} = H_0^{-1}\int_0^a \frac{da'}{a'}\left[\Omega_r a'^{-4} + \Omega_m a'^{-3} + \Omega_\Lambda\right]^{-1/2}$$

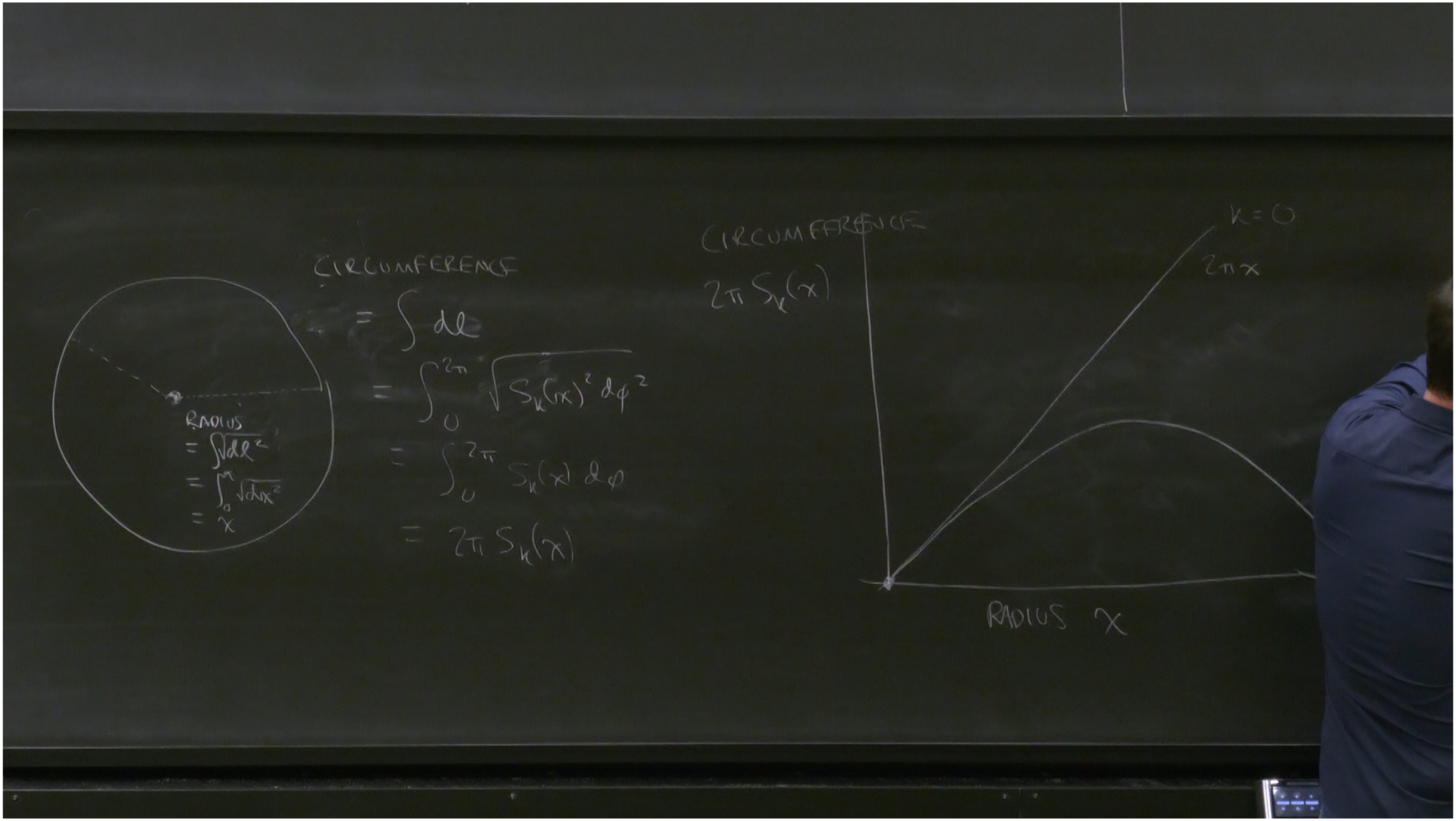


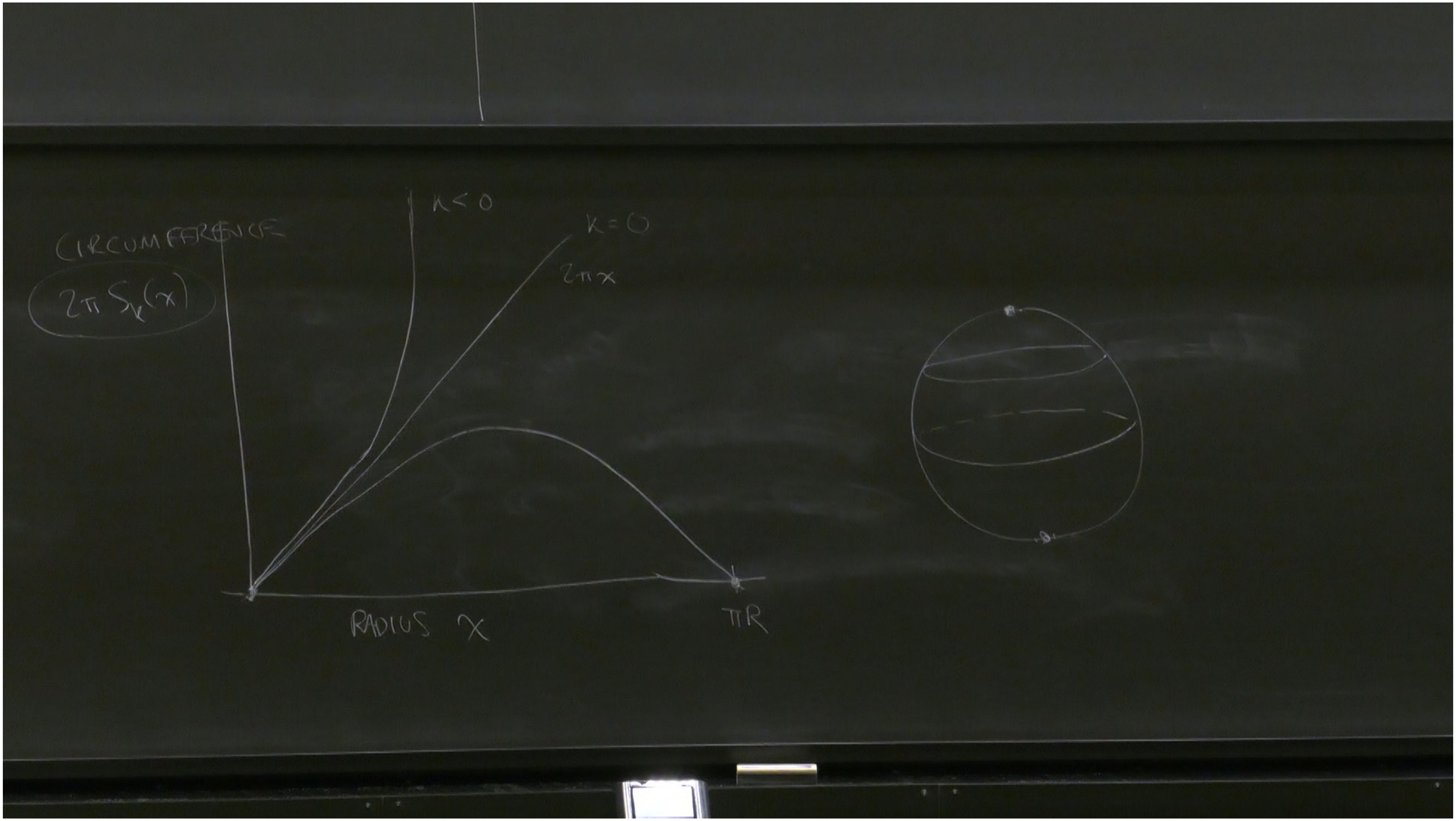
• SPATIAL CURVATURE K ~ [COMOVING LENGTH]^-2
R
K = 0
K > 0
= 1/R^2
K < 0
ds^2 = -d

$$ds^2 = -dt^2 + a(t)^2 \, d\ell^2$$

$$d\ell^2 = (dx)^2 + S_k(x)^2 \left[(d\theta)^2 + (\sin\theta)^2 \, d\phi^2 \right]$$

$$S_k(x) = \begin{cases} x & \text{if } k = 0 \\ \frac{1}{\sqrt{k}} \sin(\sqrt{k}\, x) & \text{if } k > 0 \\ \frac{1}{\sqrt{-k}} \sinh(\sqrt{-k}\, x) & \text{if } k < 0 \end{cases}$$


CIRCUMFERENCE
$= \int d\ell$
$= \int_0^{2\pi} \sqrt{S_k(x)^2\, d\phi^2}$
$= \int_0^{2\pi} S_k(x)\, d\phi$
$= 2\pi S_k(x)$
RADIUS
$= \int \sqrt{d\ell^2}$
$= \int_0^{x} \sqrt{dx^2}$
$= x$
CIRCUMFERENCE
$2\pi S_k(x)$
$k=0$
$2\pi x$
RADIUS x


CIRCUMFERENCE
2π S_k(x)
k<0
k=0
2πx
RADIUS x
πR

$$x \leftrightarrow r \qquad r = S_K(x)$$

$$d\ell^2 = \frac{(dr)^2}{1-Kr^2} + r^2\left[(d\theta)^2 + (\sin\theta)^2 d\phi^2\right]$$

CIRCUMFERENCE $= 2\pi r$

$$\text{RADIUS } x = \begin{cases} r & K=0 \\ \frac{1}{\sqrt{K}}\sin^{-1}(\sqrt{K}r) & K>0 \\ \frac{1}{\sqrt{-K}}\sinh^{-1}(\sqrt{K}r) & \end{cases}$$

us x πR

$$\underbrace{H^2 + \frac{K}{a^2}}_{G_{00}/3} = \underbrace{\frac{8\pi G}{3}\rho}_{8\pi G T_{00}/3}$$

$K=0$

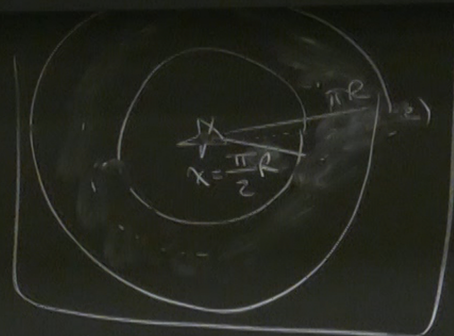
$K>0$

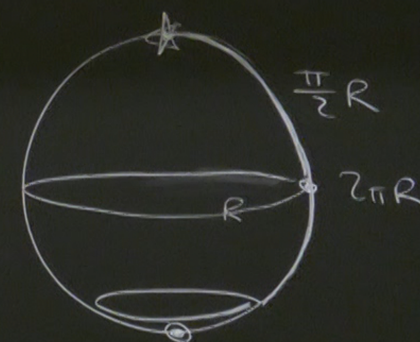
$Kr)$

$$\dot{\rho} = -3H(\rho + p) \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

πR

$$\underbrace{H^2 + \frac{K}{a^2}}_{G_{00}/3} = \underbrace{\frac{8\pi G}{3}\rho}_{8\pi G T_{00}/3}$$





$$\dot{\rho} = -3H(\rho + p)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$