Title: Cosmology Lecture

Speakers: Kendrick Smith

Collection: Cosmology 2023/24

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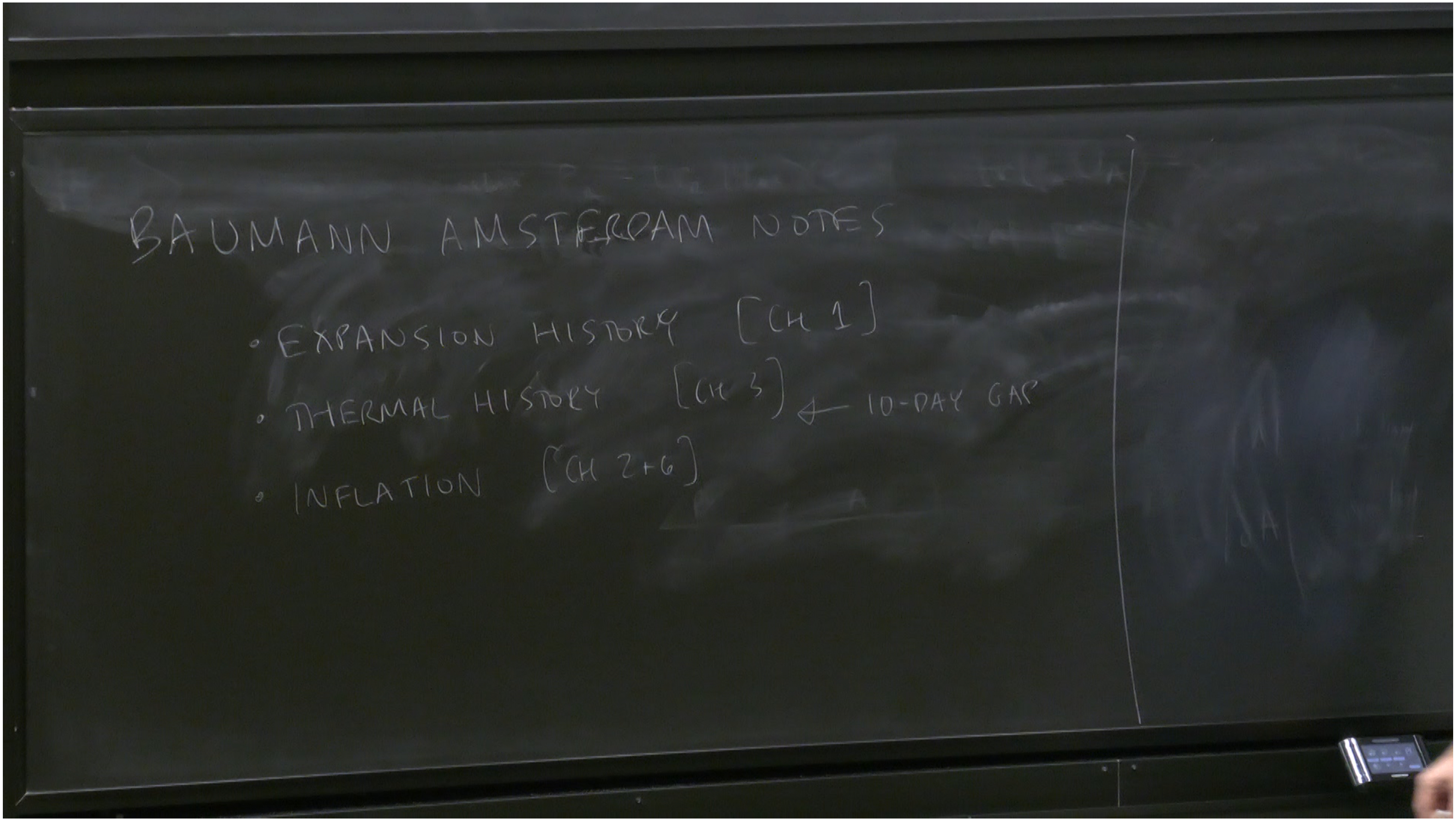
URL: https://pirsa.org/24020033

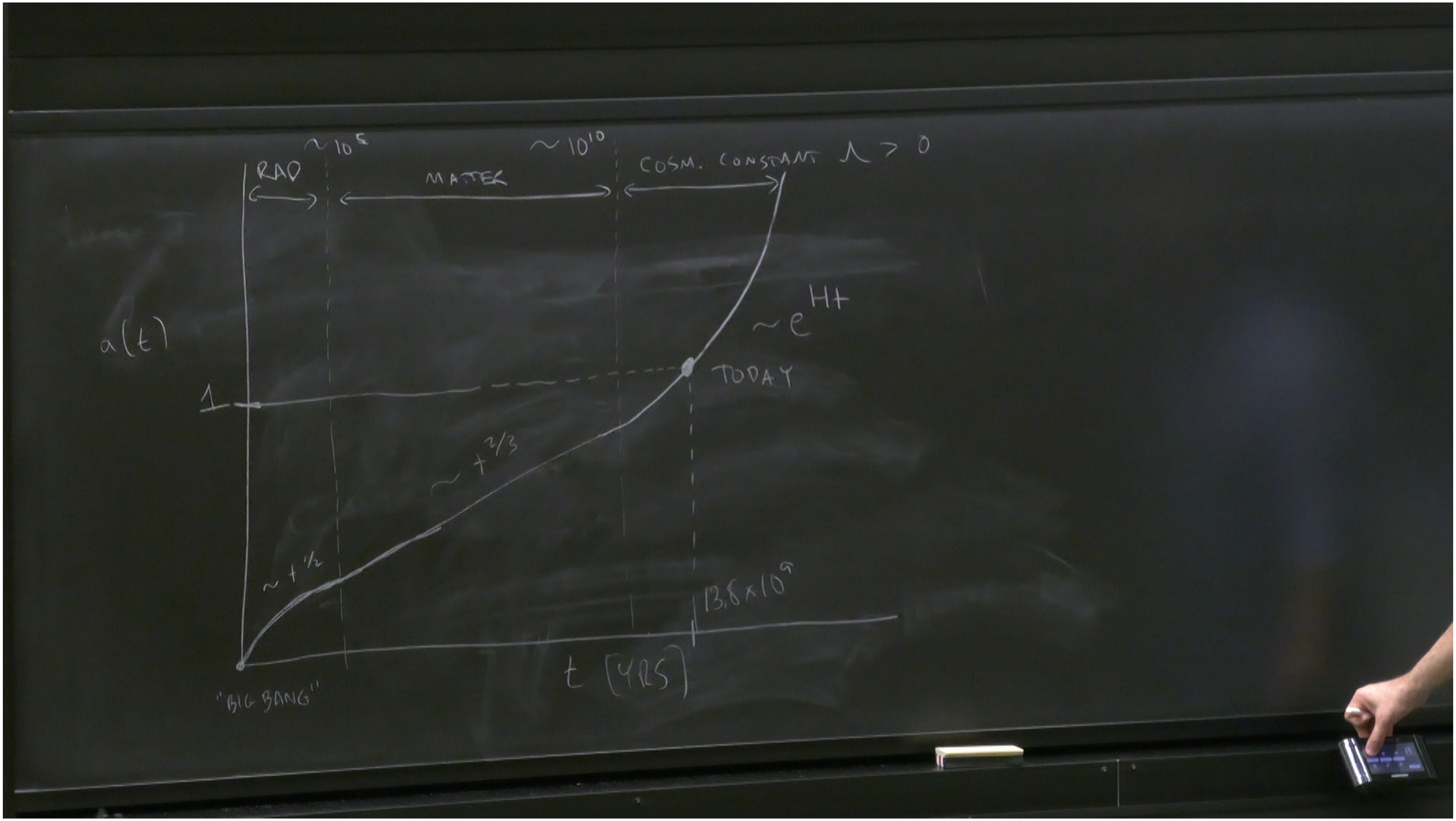

BAUMANN AMSTERDAM NOTES
• EXPANSION HISTORY [CH 1]
• THERMAL HISTORY [CH 3] ← 10-DAY GAP
• INFLATION [CH 2+6]

FLRW METRIC: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

$d\vec{x}^2 \equiv \delta_{ij} dx^i dx^j$

$$g_{\mu\nu} = \begin{pmatrix} -1 & \\ & a(t)^2 \delta_{ij} \end{pmatrix}$$

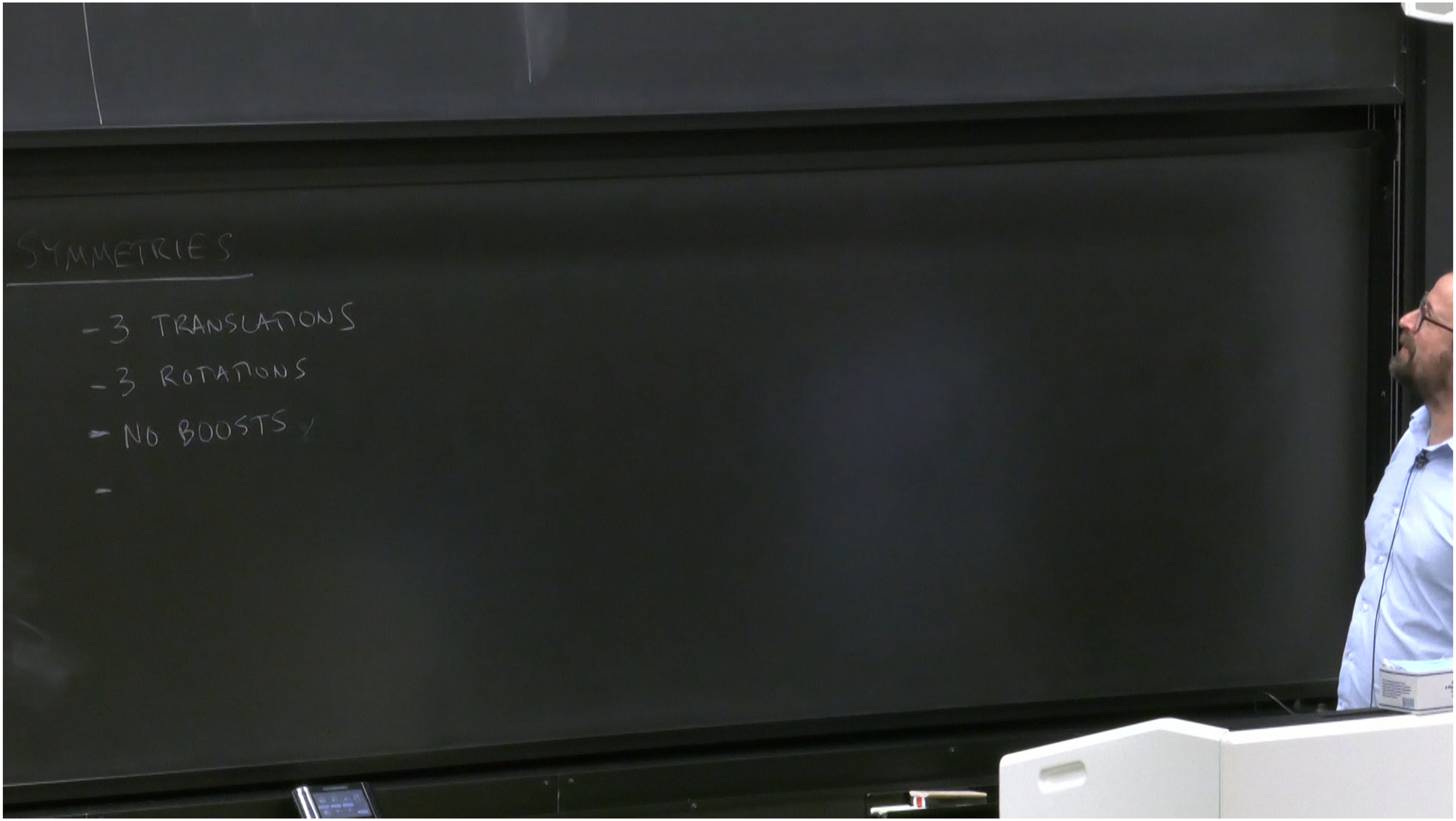
$a(t) \longrightarrow$ "SCALE FACTOR"

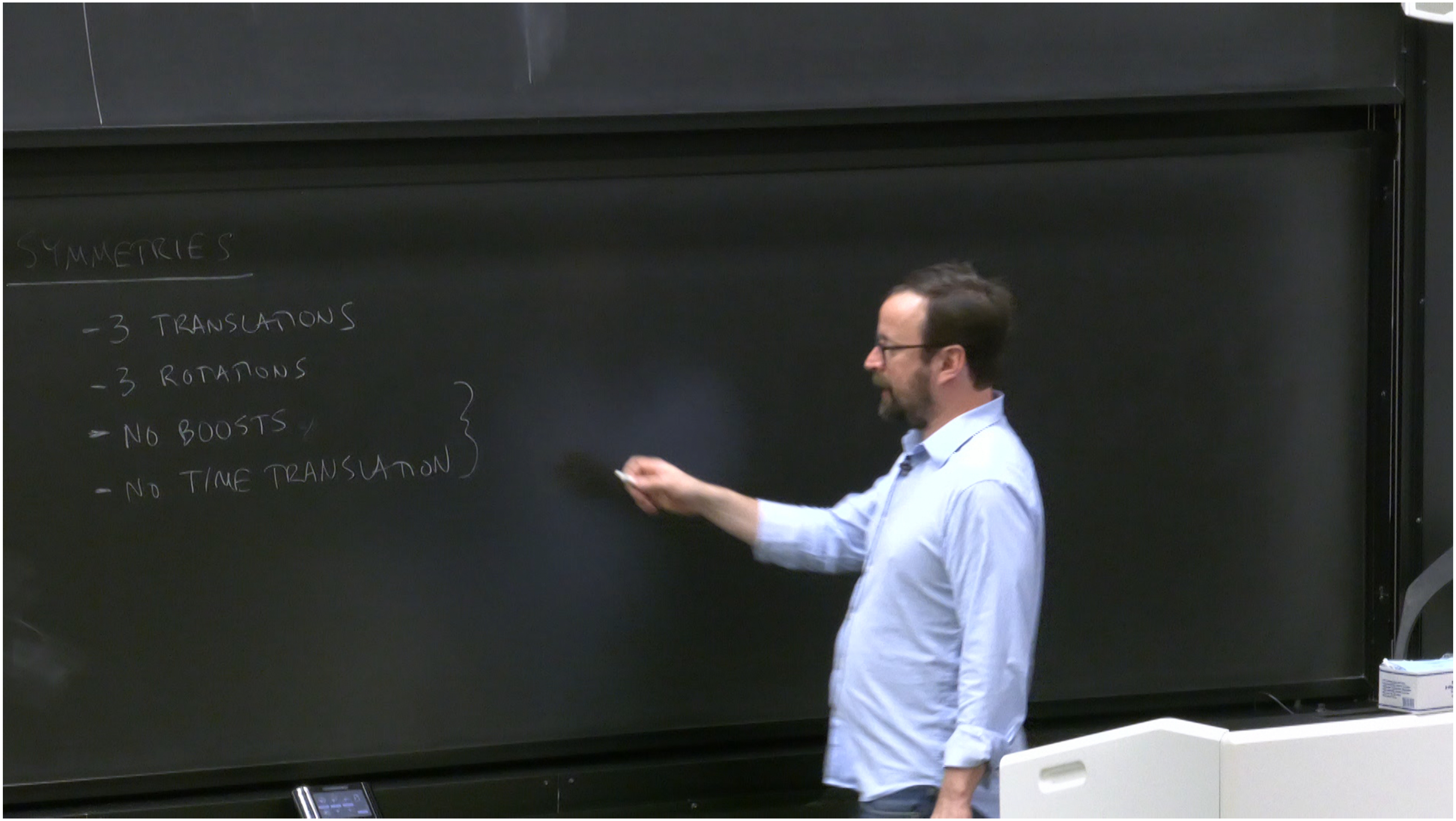

~10^5
~10^10
RAD
MATTER
COSM. CONSTANT Λ > 0
a(t)
~e^{Ht}
TODAY
1
~t^{2/3}
~t^{1/2}
13.8×10^9
t [YRS]
"BIG BANG"

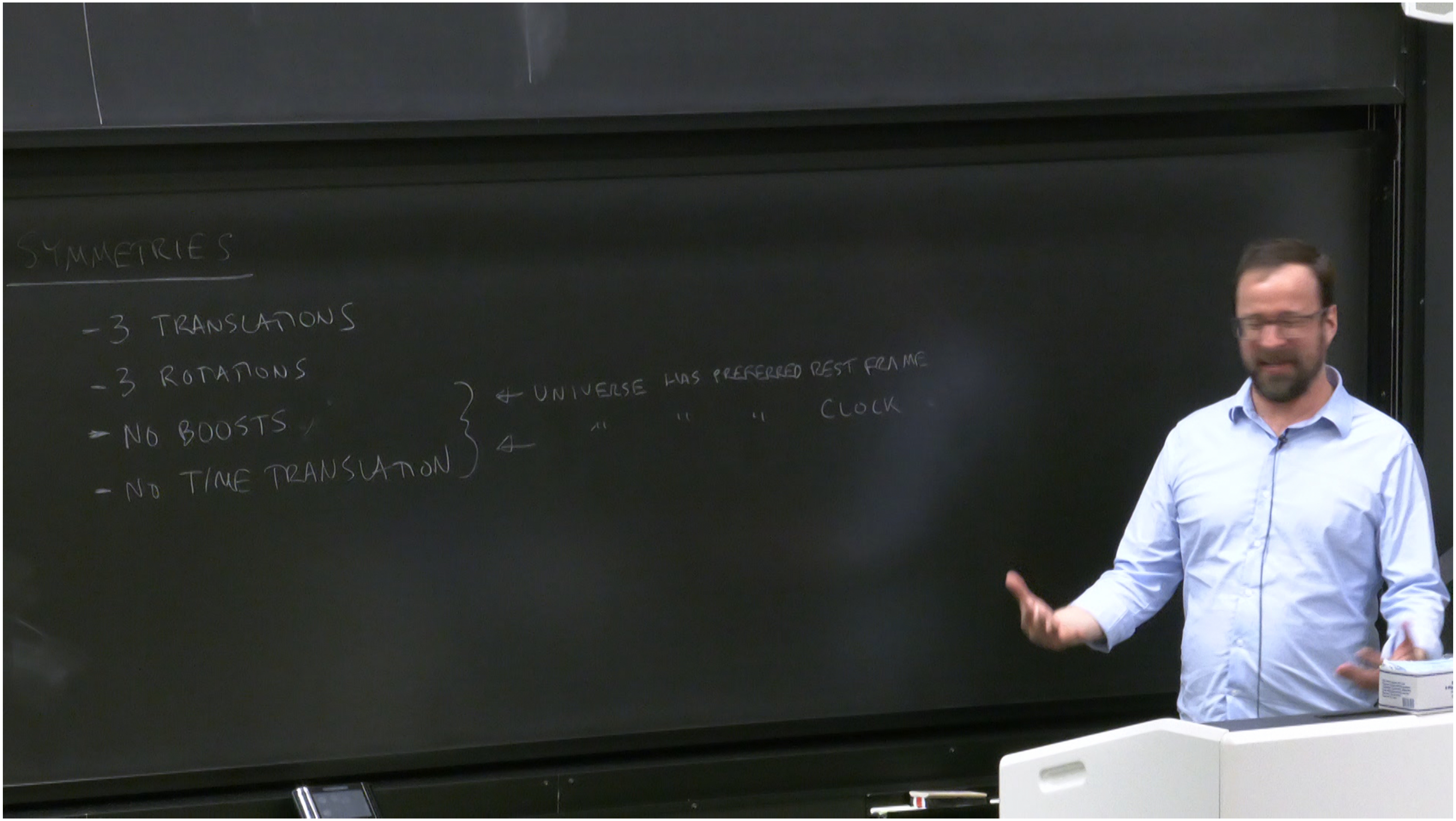
FLAT FLRW METRIC: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$ $\qquad d\vec{x}^2 \equiv \delta_{ij}\, dx^i dx^j$

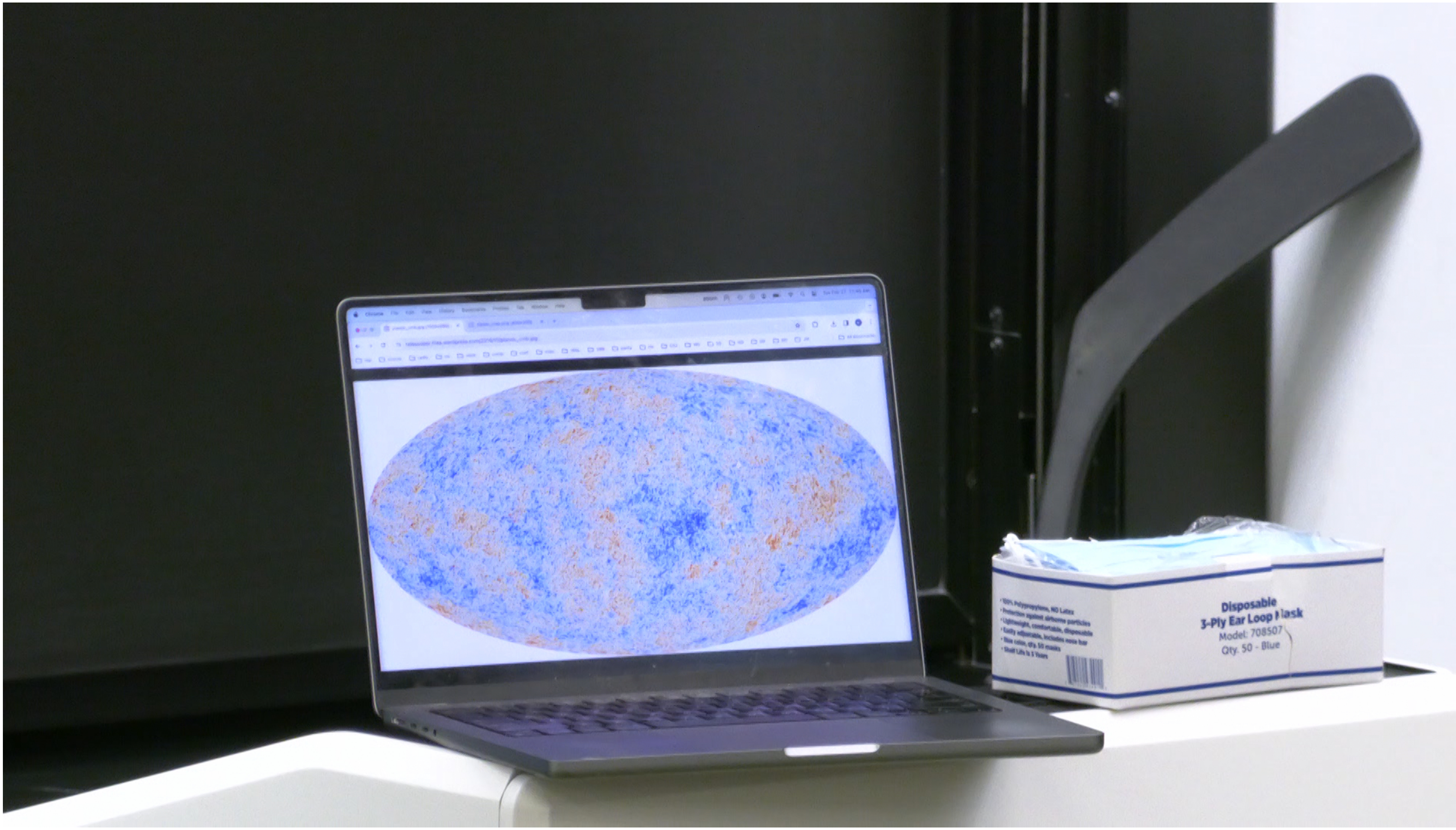
$$g_{\mu\nu} = \begin{pmatrix} -1 & \\ & a(t)^2 \delta_{ij} \end{pmatrix}$$

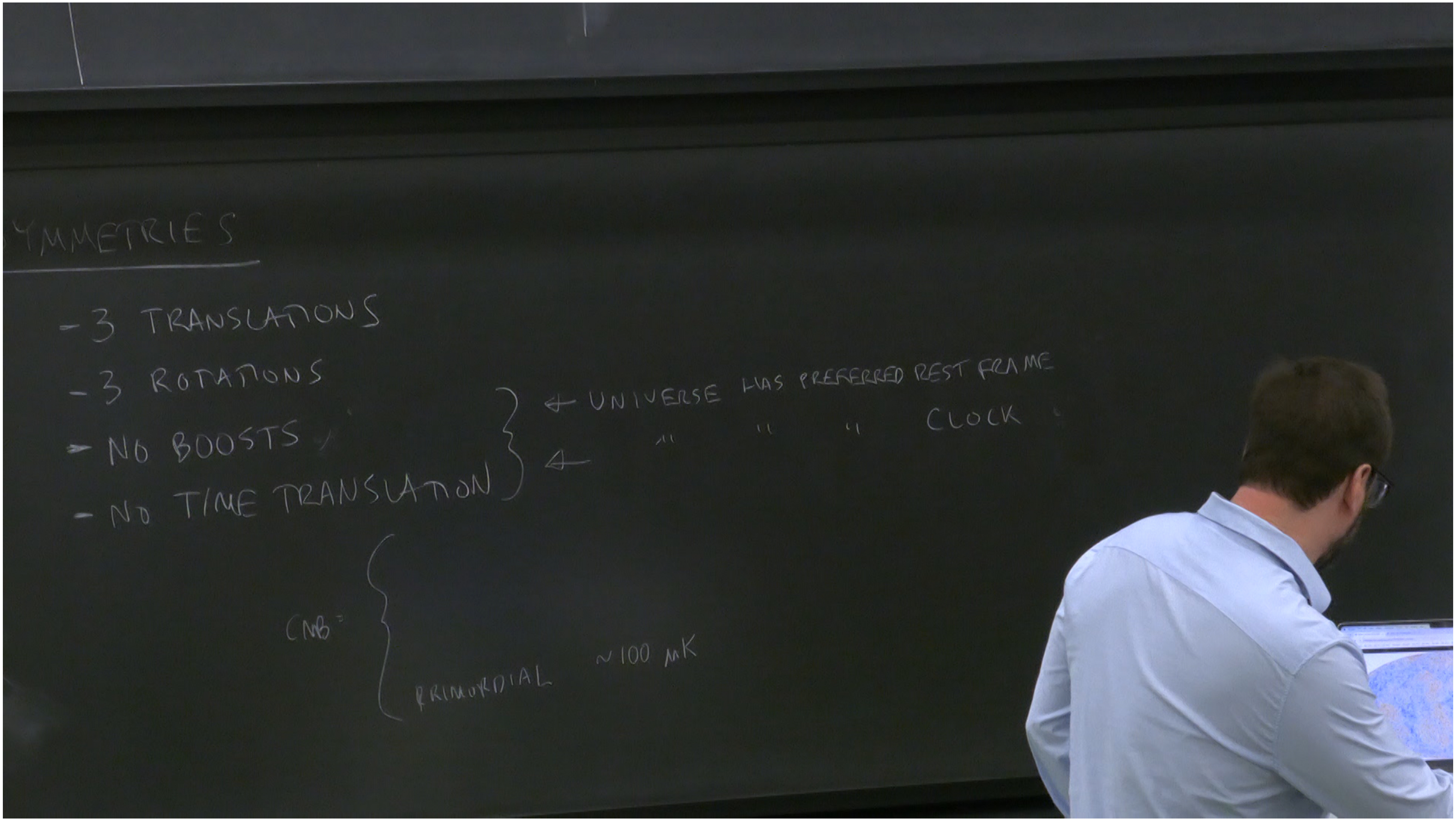
$a(t) \longrightarrow$ "SCALE FACTOR"

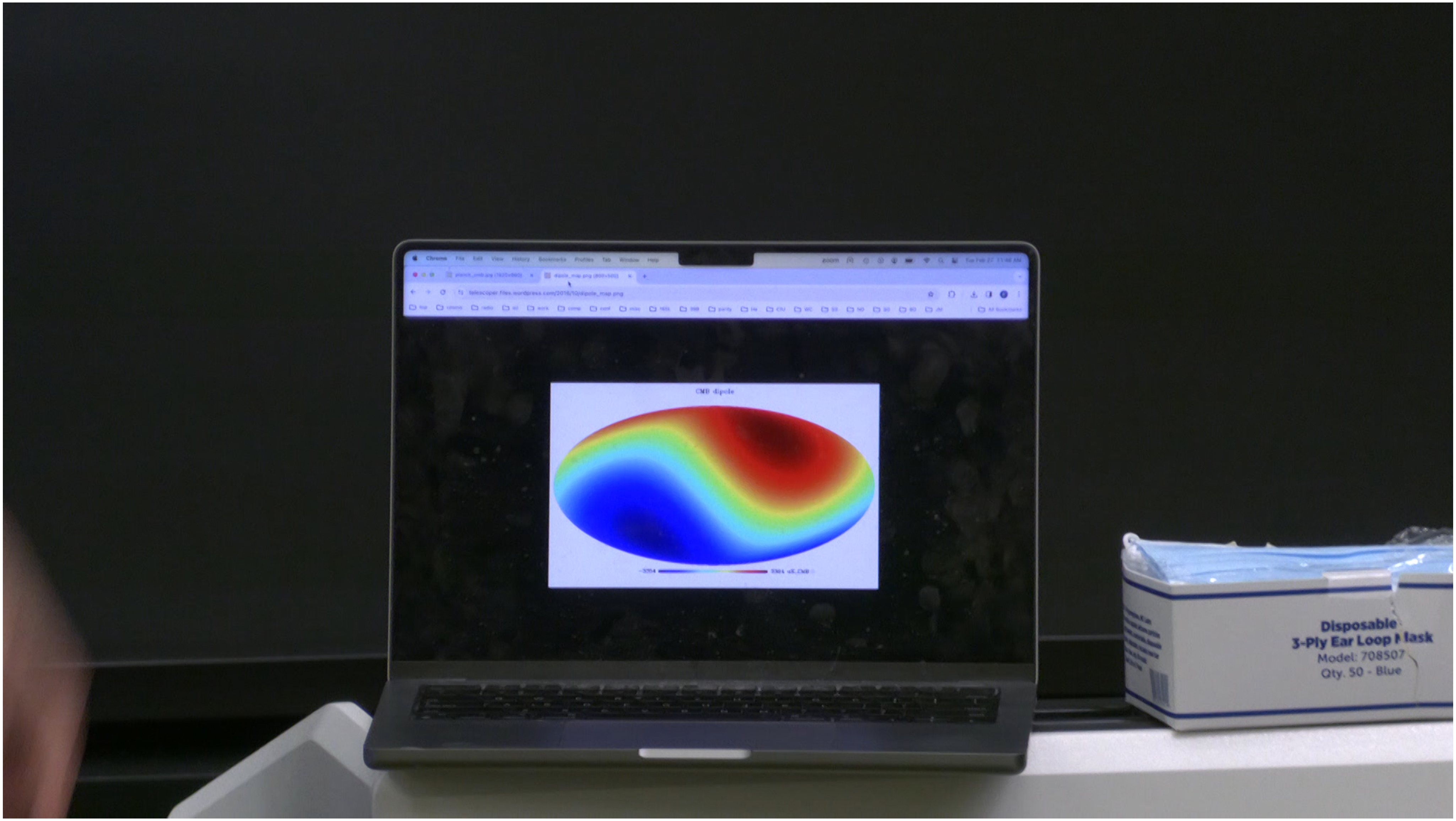

SYMMETRIES
- 3 TRANSLATIONS
- 3 ROTATIONS
- NO BOOSTS
-


SYMMETRIES
- 3 TRANSLATIONS
- 3 ROTATIONS
- NO BOOSTS
- NO TIME TRANSLATION


SYMMETRIES
- 3 TRANSLATIONS
- 3 ROTATIONS
- NO BOOSTS
- NO TIME TRANSLATION
← UNIVERSE HAS PREFERRED REST FRAME
← " " " CLOCK


Disposable
3-Ply Ear Loop
Model: 708507
Qty. 50 - Blue

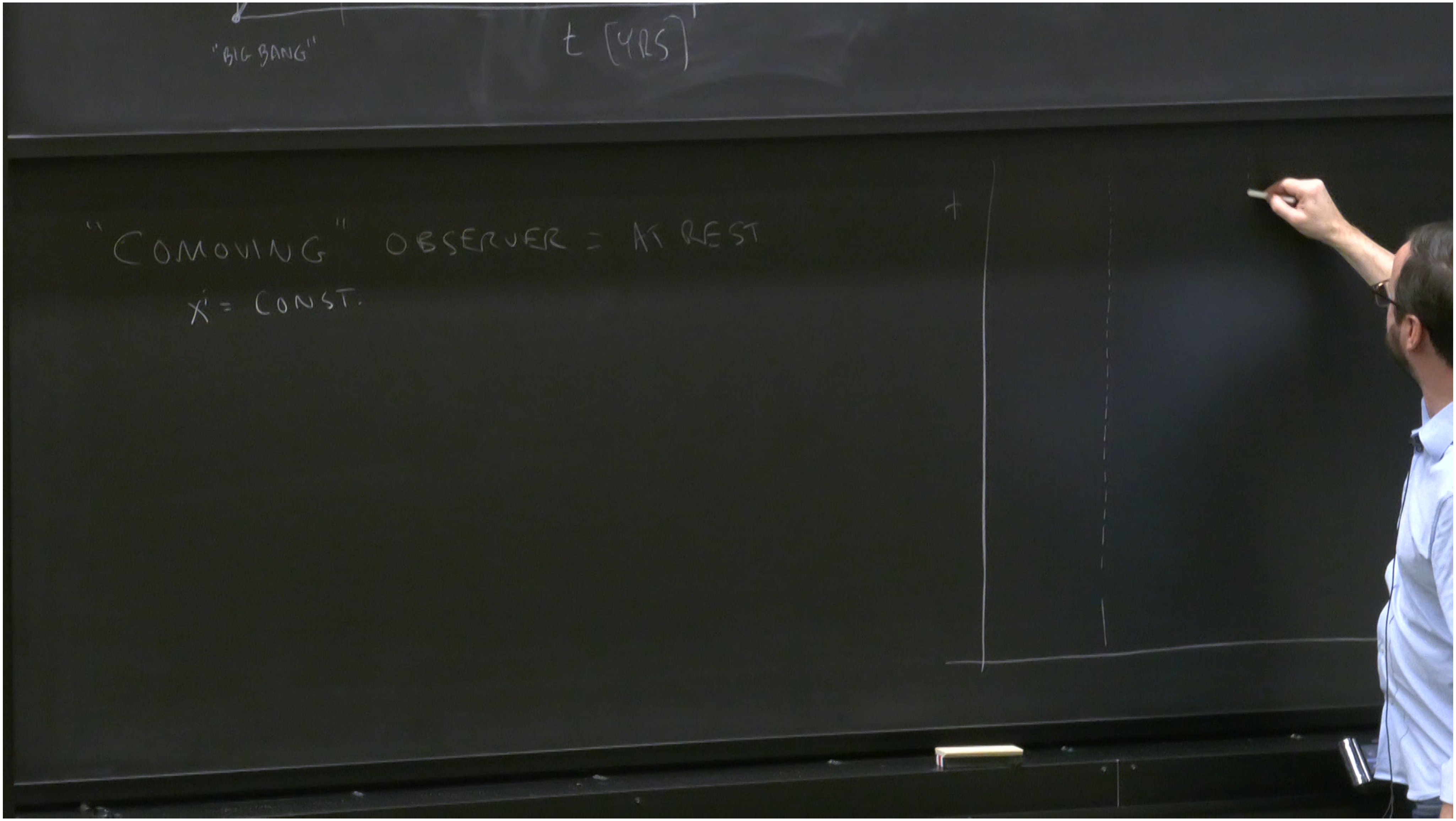

YMMETRIES
- 3 TRANSLATIONS
- 3 ROTATIONS
- NO BOOSTS
- NO TIME TRANSLATION
← UNIVERSE HAS PREFERRED REST FRAME
← " " " CLOCK
CMB =
PRIMORDIAL ~100 μK

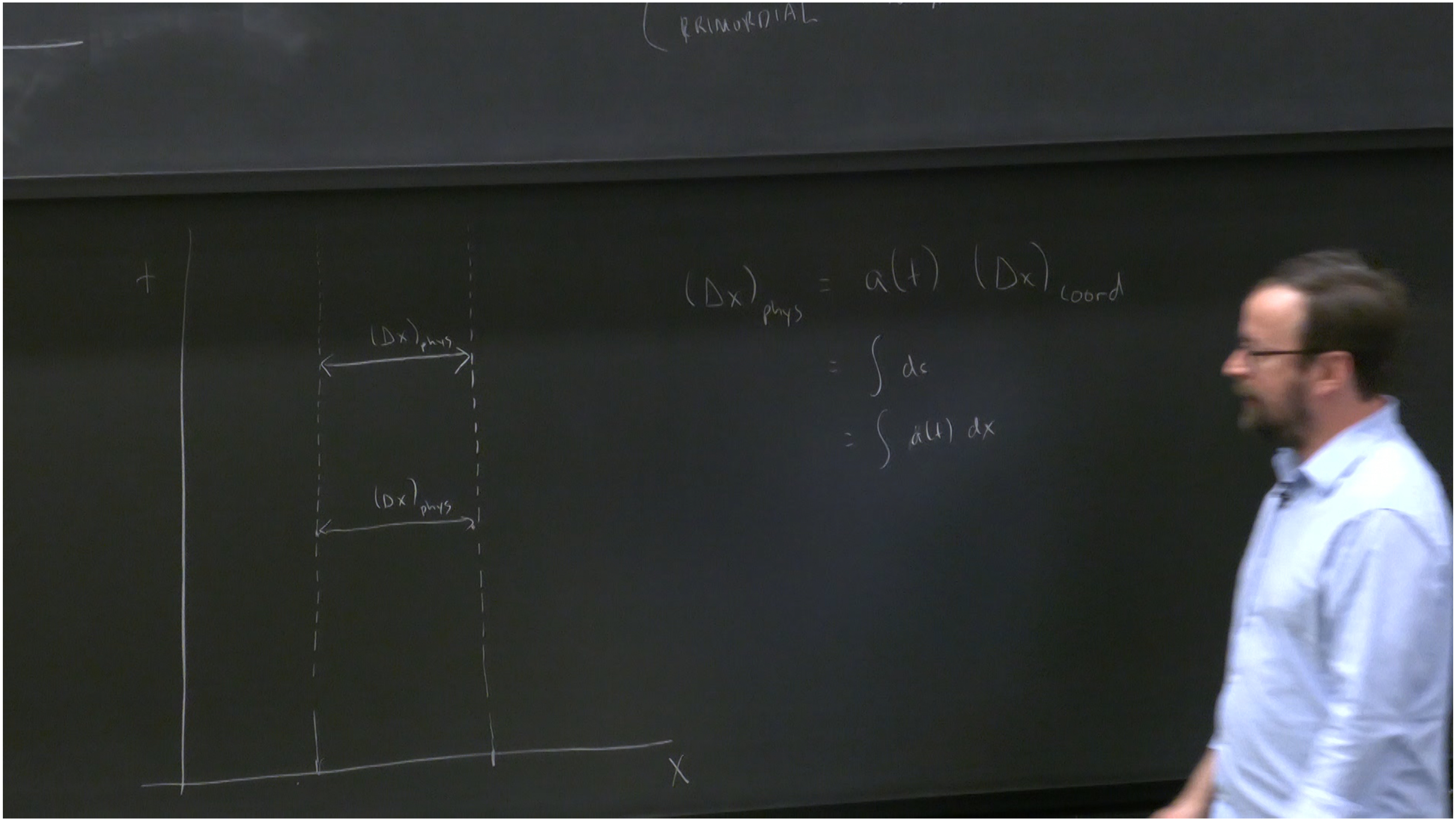

CMB dipole
Disposable
3-Ply Ear Loop
Model: 708507
Qty. 50 - Blue

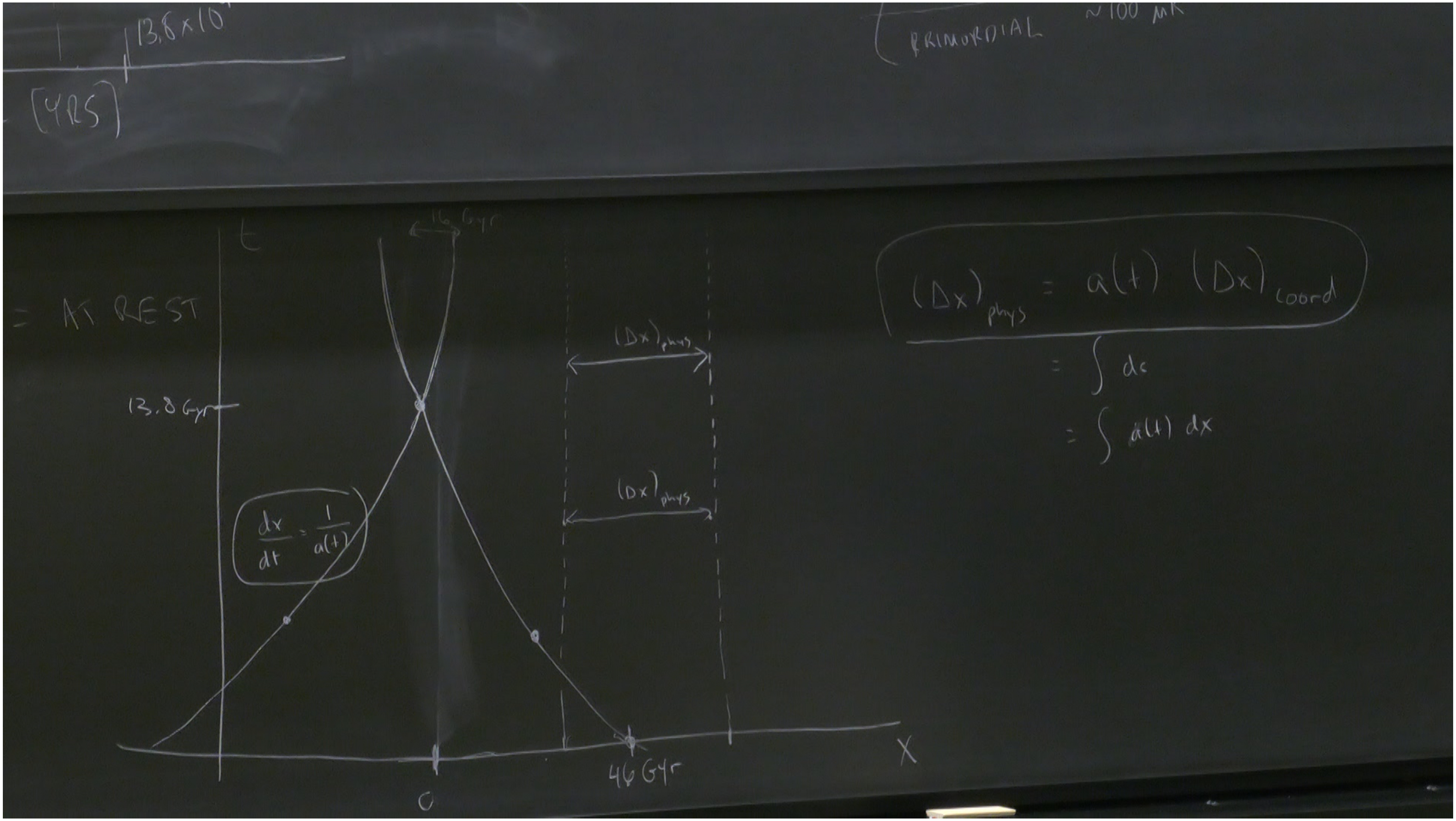
YMMETRIES

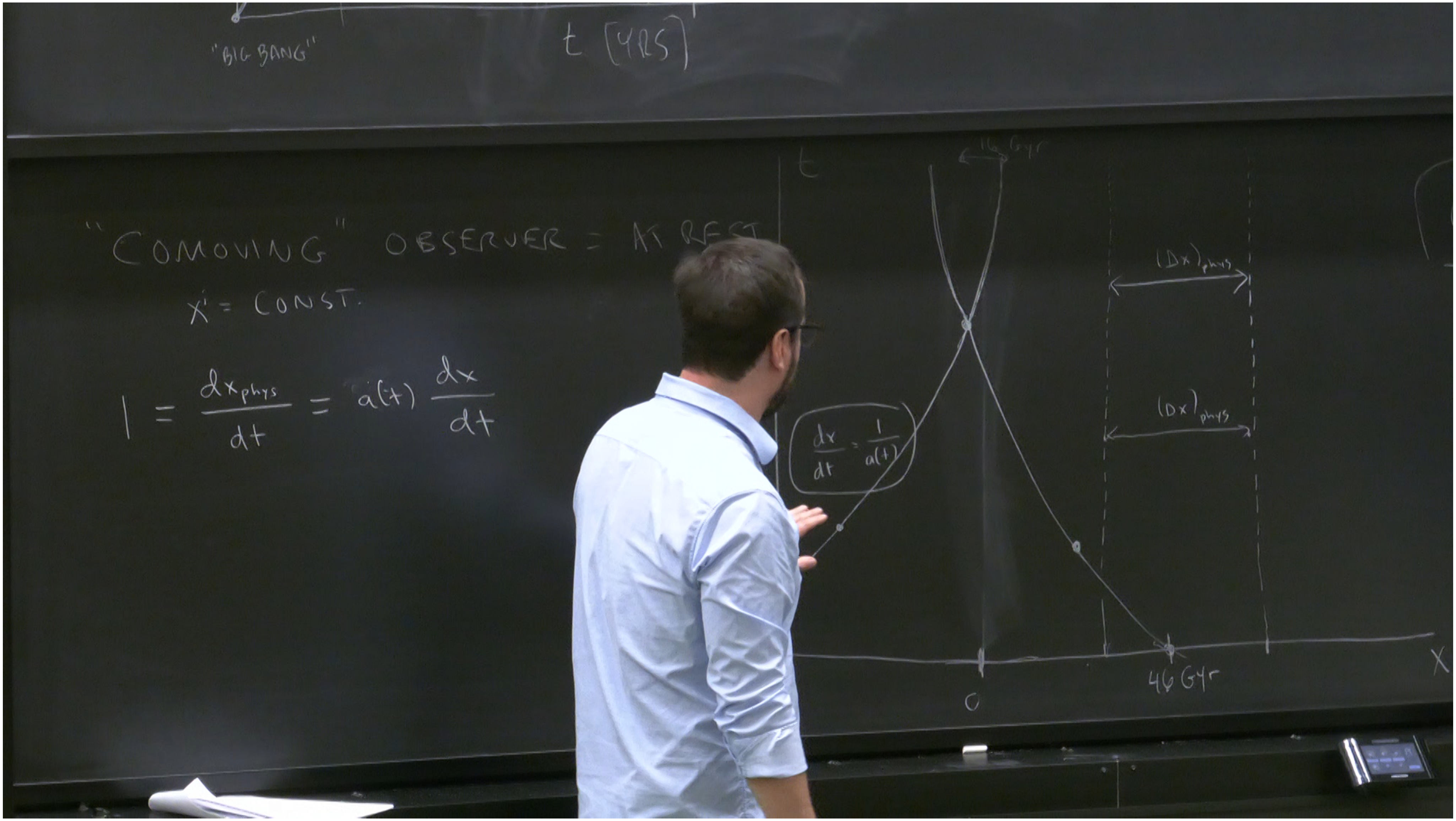
- 3 TRANSLATIONS
- 3 ROTATIONS
- NO BOOSTS ← UNIVERSE HAS PREFERRED REST FRAME
- NO TIME TRANSLATION ← " " " CLOCK

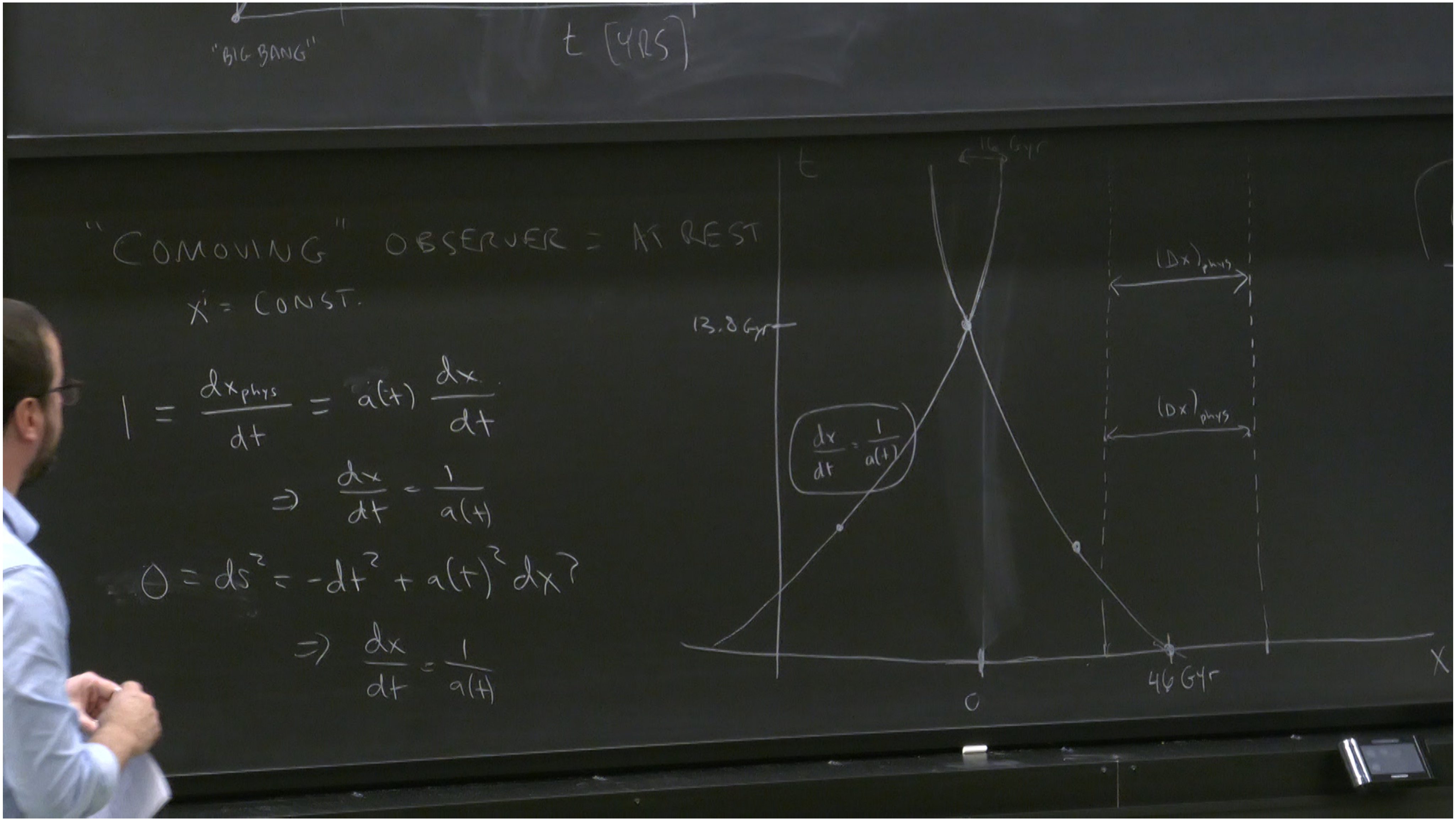
CMB = { MONOPOLE ~ 2.7 K
DIPOLE ~ 3 mK
PRIMORDIAL ~ 100 μK


"BIG BANG"
t [YRS]
"COMOVING" OBSERVER = AT REST
x' = CONST.


PRIMORDIAL
t
(Δx)phys
(Δx)phys
X
(Δx)phys = a(t) (Δx)coord
= ∫ ds
= ∫ a(t) dx


13.8×10
[YRS]
PRIMORDIAL
~100 mK
AT REST
t
16 Gyr
13.8 Gyr
dx/dt = 1/a(t)
(Dx)phys
(Dx)phys
0
46 Gyr
X
(Dx)phys = a(t) (Dx)coord
= ∫ ds
= ∫ a(t) dx


"BIG BANG"
t [YRS]
"COMOVING" OBSERVER = AT REST
x = CONST.
1 = dx_phys / dt = a(t) dx / dt
t
16 Gyr
(Δx)_phys
(Δx)_phys
dx/dt = 1/a(t)
0
46 Gyr
x


"BIG BANG"
t [YRS]
"COMOVING" OBSERVER = AT REST
x = CONST.
1 = dx_phys / dt = a(t) dx/dt
⇒ dx/dt = 1/a(t)
0 = ds² = -dt² + a(t)² dx²
⇒ dx/dt = 1/a(t)
t
13.8 Gyr
dx/dt = 1/a(t)
(Dx)_phys
(Dx)_phys
0
46 Gyr
x

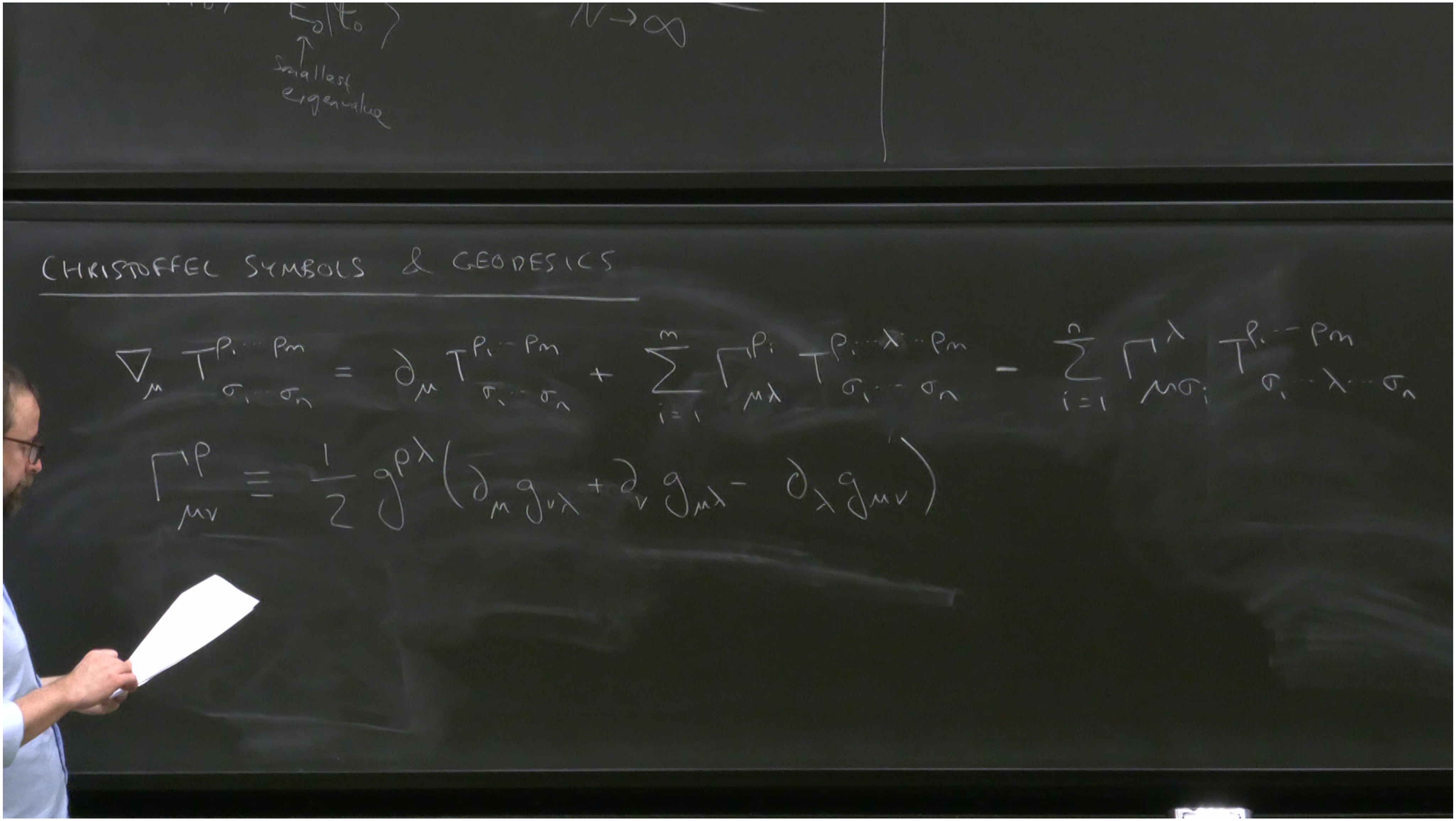
$E_0 | \psi_0 \rangle$

smallest eigenvalue

$N \to \infty$

CHRISTOFFEL SYMBOLS & GEODESICS

$$\nabla_\mu T^{\rho_1 \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} = \partial_\mu T^{\rho_1 \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} + \sum_{i=1}^{m} \Gamma^{\rho_i}{}_{\mu\lambda} T^{\rho_1 \cdots \lambda \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} - \sum_{i=1}^{n} \Gamma^{\lambda}{}_{\mu\sigma_i} T^{\rho_1 \cdots \rho_m}{}_{\sigma_1 \cdots \lambda \cdots \sigma_n}$$


$E_0 | t_0 \rangle$
smallest eigenvalue
$N \to \infty$
CHRISTOFFEL SYMBOLS & GEODESICS
$\nabla_\mu T^{\rho_1 \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} = \partial_\mu T^{\rho_1 \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} + \sum_{i=1}^{m} \Gamma^{\rho_i}_{\mu\lambda} T^{\rho_1 \cdots \lambda \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} - \sum_{i=1}^{n} \Gamma^{\lambda}_{\mu\sigma_i} T^{\rho_1 \cdots \rho_m}{}_{\sigma_1 \cdots \lambda \cdots \sigma_n}$
$\Gamma^{\rho}_{\mu\nu} \equiv \frac{1}{2} g^{\rho\lambda} \left(\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu} \right)$

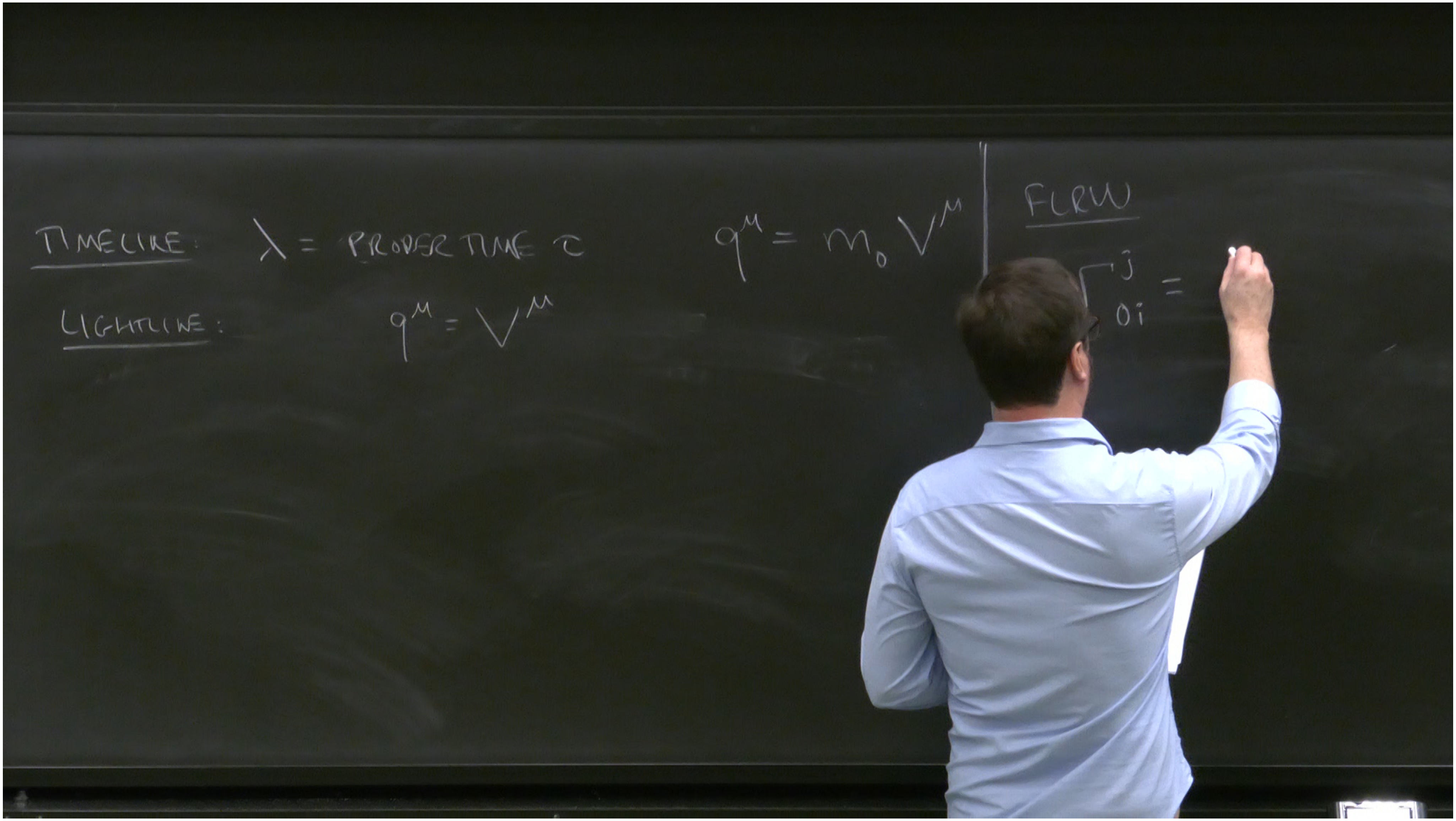
CHRISTOFFEL SYMBOLS & GEODESICS

$$\nabla_\mu T^{\rho_1 \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} = \partial_\mu T^{\rho_1 \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} + \sum_{i=1}^{m} \Gamma^{\rho_i}_{\mu\lambda} T^{\rho_1 \cdots \lambda \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} - \sum_{i=1}^{n} \Gamma^{\lambda}_{\mu\sigma_i} T^{\rho_1 \cdots \rho_m}_{\sigma_1 \cdots \lambda \cdots \sigma_n}$$

$$\Gamma^{\rho}_{\mu\nu} \equiv \frac{1}{2} g^{\rho\lambda}\left(\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}\right)$$

GEODESIC CURVE $X^\mu(\lambda)$ SATISFYING GEODESIC EQ

$$\frac{dV^\mu}{d\lambda} + \Gamma^{\mu}_{\rho\sigma} V^\rho V^\sigma = 0 \qquad \text{WHERE } V^\mu = \frac{dX^\mu}{d\lambda}$$


TIMELIKE: λ = PROPER TIME τ
q^μ = m₀ V^μ
LIGHTLIKE: q^μ = V^μ
FLRW
Γ^j_0i =

FLRW

$$\Gamma^j_{0i} = \frac{1}{2} g^{j\lambda}\left(\partial_0 g_{i\lambda} + \cancel{\partial_i g_{0\lambda}} - \cancel{\partial_\lambda g_{0i}}\right)$$

$$= \frac{1}{2} g^{jk}\left(\partial_0 g_{ik}\right)$$

$$= \frac{1}{2}\left(a^{-2}\delta^{jk}\right) \partial_t\left(a^2 \delta_{ik}\right)$$

FLRW

$$\Gamma^j_{0i} = \frac{1}{2} g^{j\lambda}\left(\partial_0 g_{i\lambda} + \cancelto{0}{\partial_i g_{0\lambda}} - \cancelto{0}{\partial_\lambda g_{0i}} \right)$$

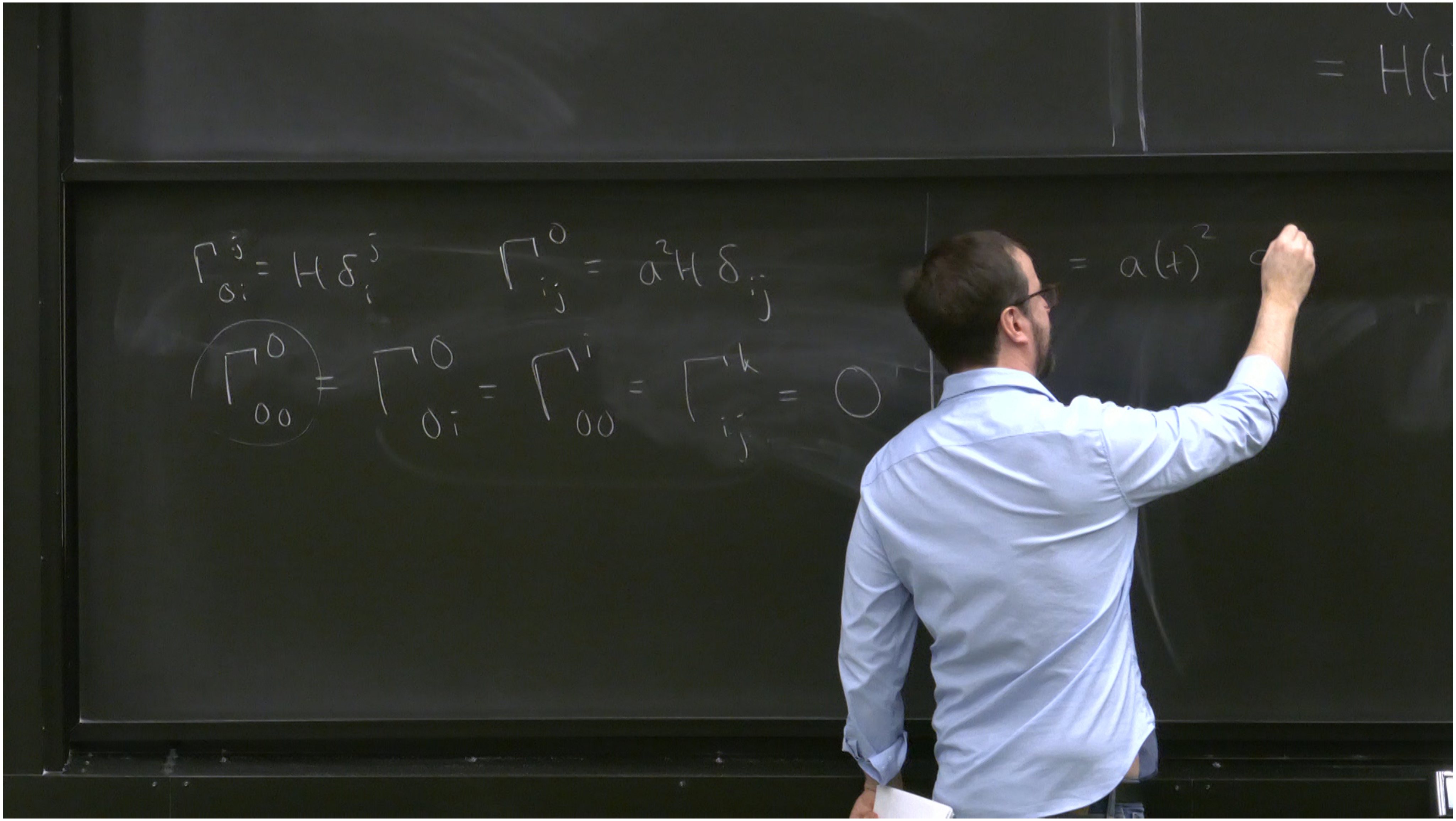
$$= \frac{1}{2} g^{jk}\left(\partial_0 g_{ik} \right)$$

$$= \frac{1}{2}\left(a^{-2}\delta^{jk}\right) \cdot \partial_t\left(a^2 \delta_{ik}\right)$$

$$= \frac{\dot{a}}{a}\delta^j_i$$

$$= H(t)\,\delta^j_i$$

$$\boxed{H(t) \equiv \frac{\dot{a}(t)}{a(t)} \quad \text{HUBBLE EXPANSION RATE}}$$



$= H(t$
$\Gamma^j_{0i} = H\delta^j_i$
$\Gamma^0_{ij} = a^2 H \delta_{ij}$
$= a(t)^2$
$\Gamma^0_{00} = \Gamma^0_{0i} = \Gamma^i_{00} = \Gamma^k_{ij} = 0$

$$= \frac{\dot{a}}{a} \delta^j_i$$

$$= H(t) \delta^j_i$$

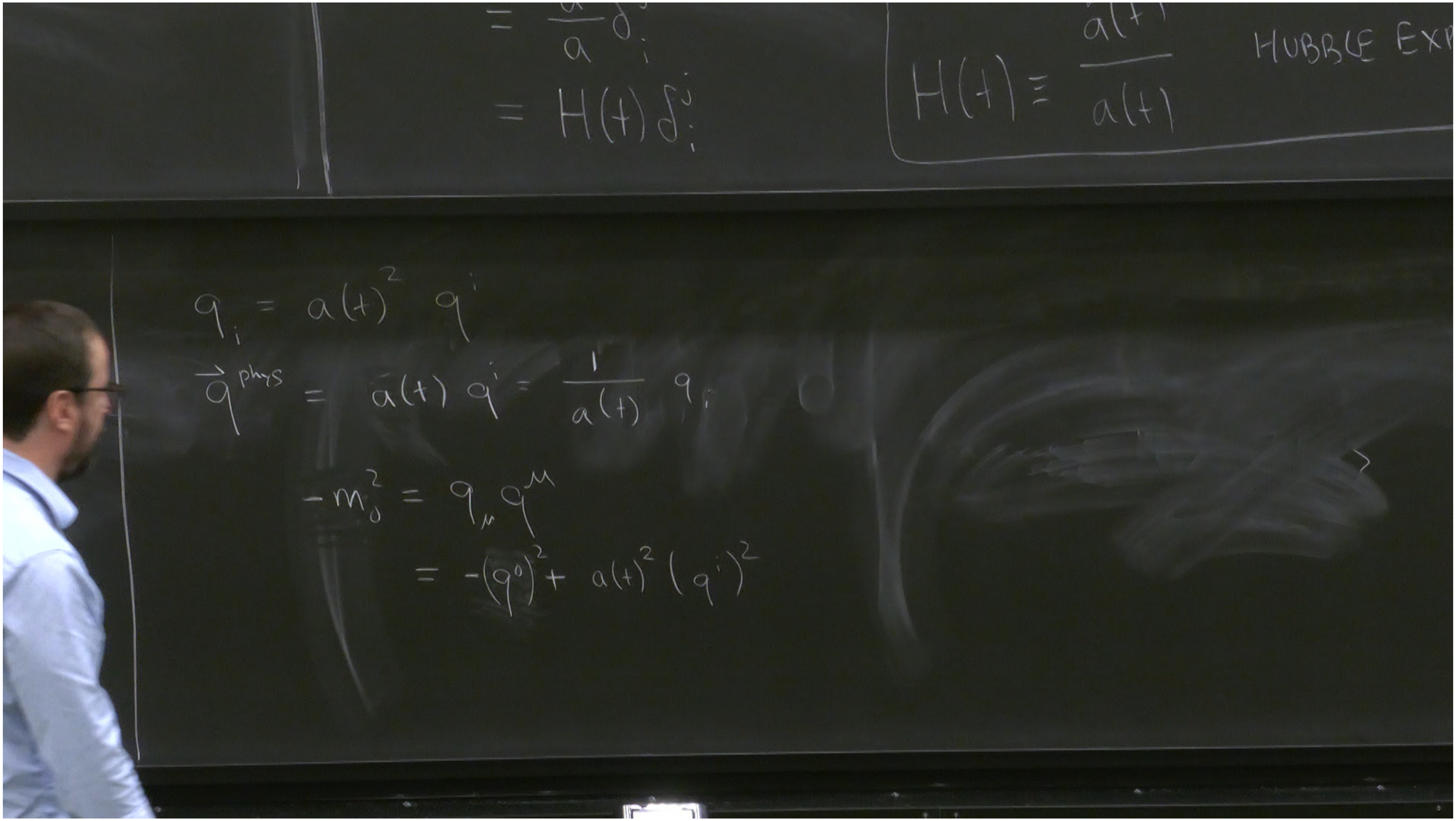
$$\boxed{H(t) \equiv \frac{\dot{a}(t)}{a(t)}}$$

HUBBLE EXP

$$q_i = a(t)^2 q^i$$

$$\vec{q}^{\,phys} = a(t) q^i = \frac{1}{a(t)} q_i$$

$$m_0^2 = q_\mu$$


= H(t) δ_i^j
H(t) ≡ ȧ(t)/a(t)
HUBBLE EXP
q_i = a(t)^2 q^i
q⃗^phys = a(t) q^i = 1/a(t) q_i
−m_j^2 = q_μ q^μ
= −(q^0)^2 + a(t)^2 (q^i)^2

$$= \frac{\dot a}{a}\,\delta^j_i$$

$$= H(t)\,\delta^j_i$$

$$\boxed{H(t) \equiv \frac{\dot a(t)}{a(t)}}$$

HUBBLE EXP

$$q_i = a(t)^2\, q^i$$

$$\vec q^{\,phys} = a(t)\, q^i = \frac{1}{a(t)}\, q_i$$

$$E_{phys}^2 - q_{phys}^2 = m_0^2$$

$$-m_0^2 = q_\mu q^\mu$$

$$= -\underbrace{(q^0)^2}_{q^0 = E_{phys}} + a(t)^2 \underbrace{(q^i)^2}_{q_{phys} = a q^i}$$

FLRW GEODESIC $X^\mu(\lambda)$ $\quad V^\mu = \frac{dX^\mu}{d\lambda}$

$$\frac{dV^0}{d\lambda} = -a^2 H \,\delta_{ij} V^i V^j$$

$$\frac{dV^i}{d\lambda} = -2H V^0 V^i$$

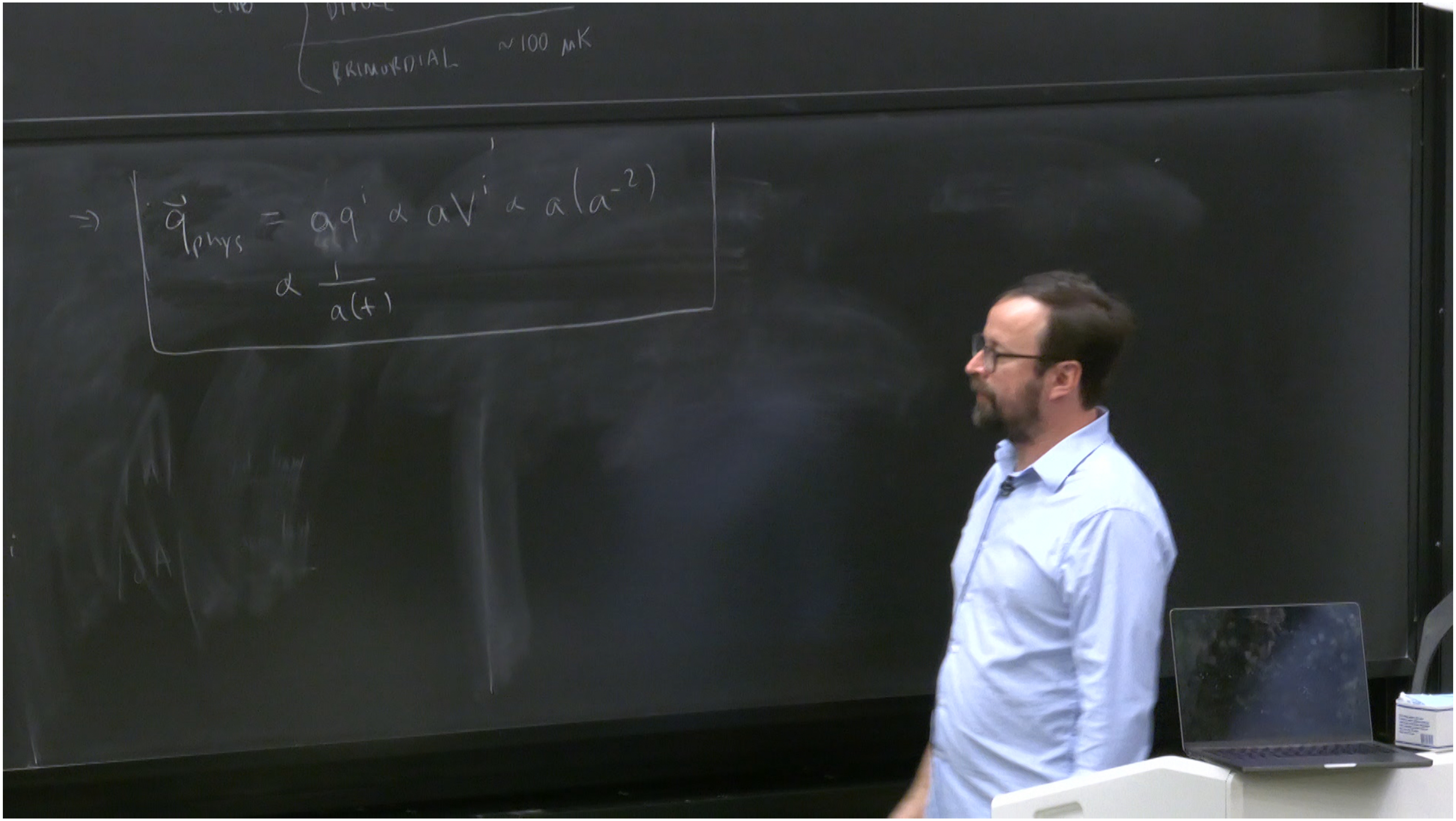
FLRW GEODESIC $x^\mu(\lambda)$ $V^\mu = \frac{dx^\mu}{d\lambda}$

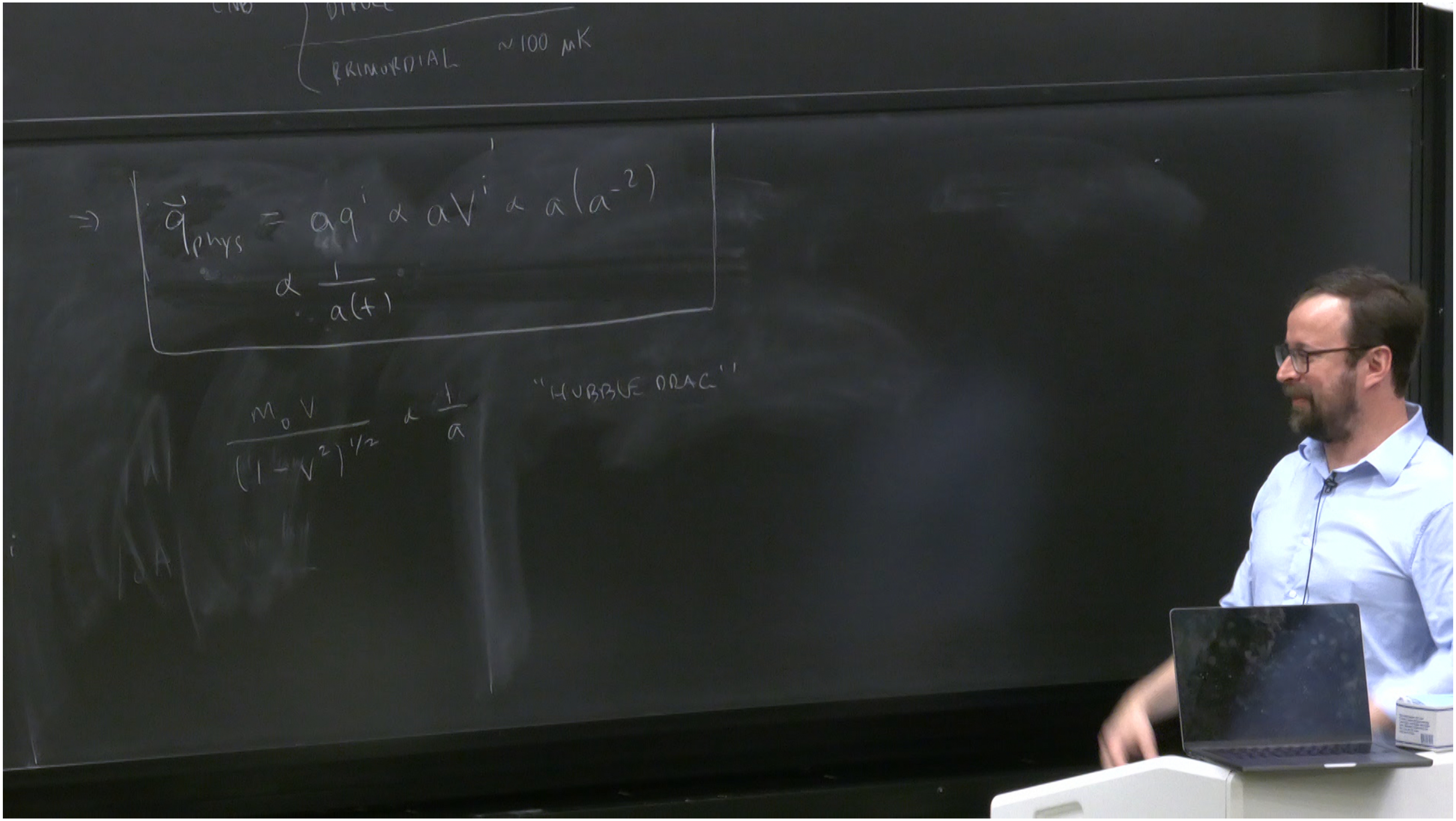
$$\frac{dV^0}{d\lambda} = -a^2 H \delta_{ij} V^i V^j$$

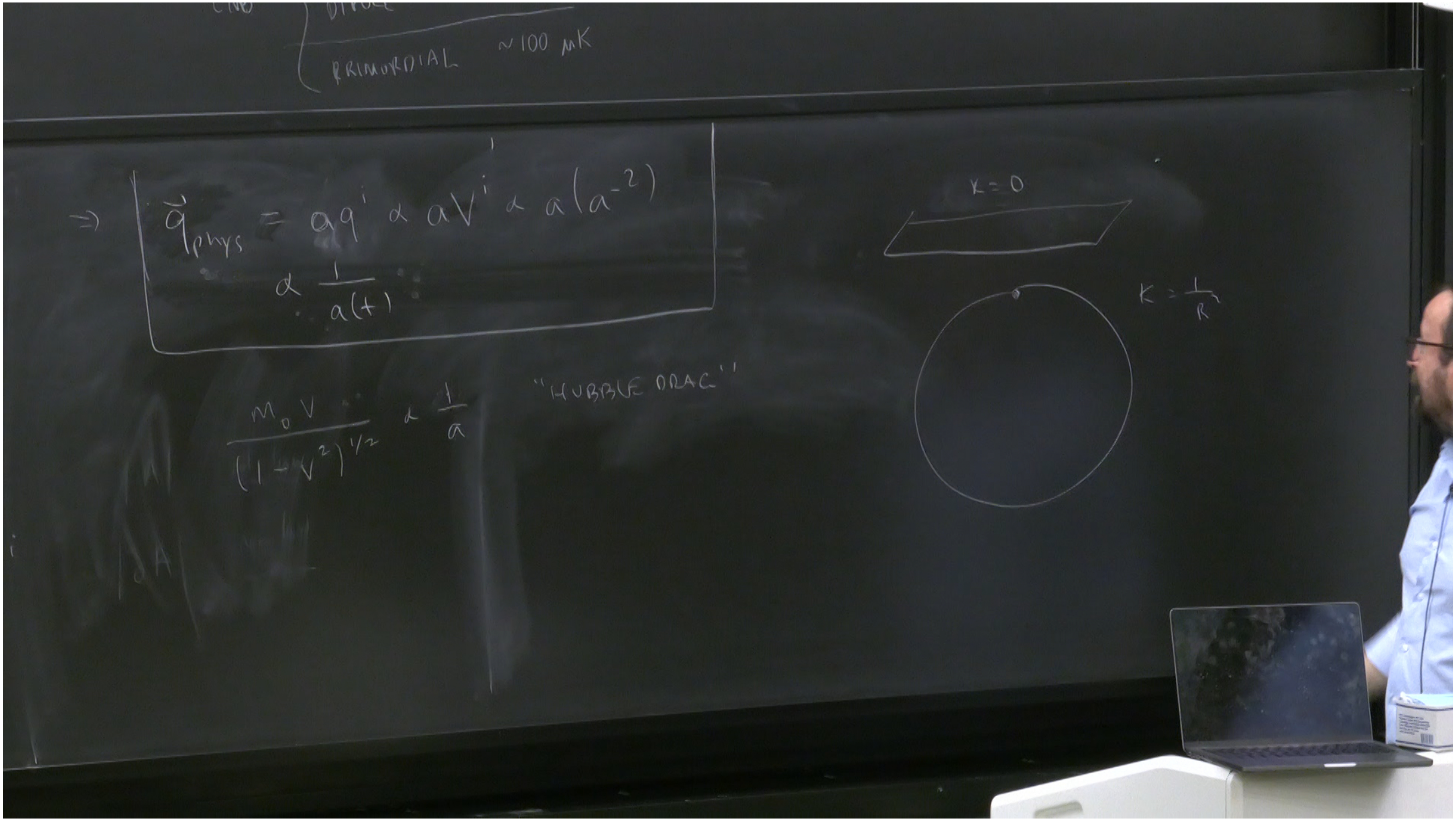
$$\frac{dV^i}{d\lambda} = -2H V^0 V^i$$

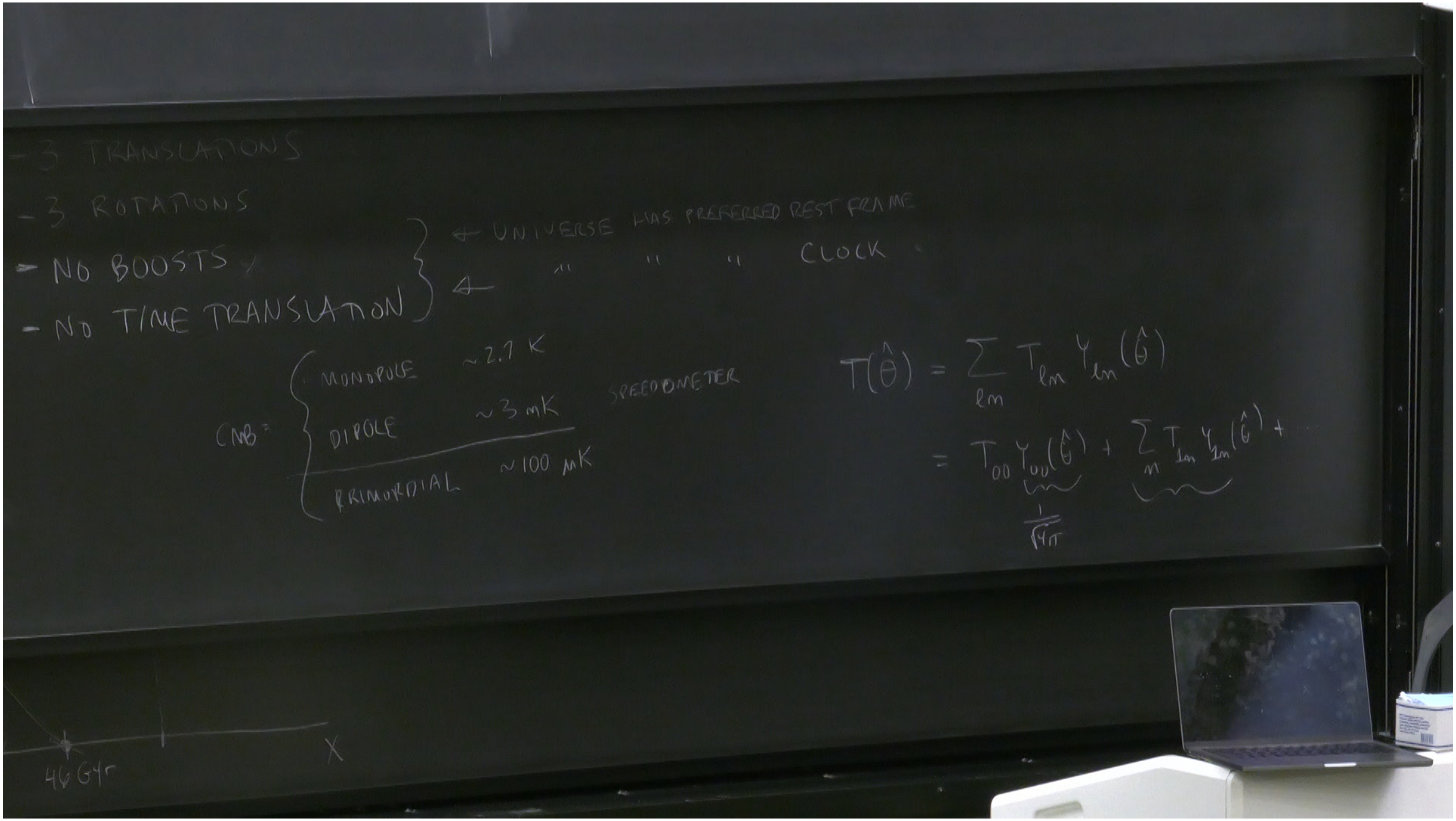
$$= -2\left(\frac{1}{a}\frac{da}{dt}\right)\left(\frac{dt}{d\lambda}\right) V^i \Rightarrow \frac{dV^i}{da} = -\frac{2}{a} V^i$$

$$\Rightarrow V^i \propto a^{-2}$$


PRIMORDIAL ~100 mK
⇒ q_phys = a q^i ∝ a V^i ∝ a(a^-2)
∝ 1/a(t)


PRIMORDIAL ~100 mK
⇒ $\vec{q}_{phys} = a q^i \propto a V^i \propto a(a^{-2}) \propto \frac{1}{a(t)}$
$\frac{m_0 V}{(1 - V^2)^{1/2}} \propto \frac{1}{a}$
"HUBBLE DRAG"


PRIMORDIAL ~100 μK
$\vec{q}_{phys} = a q^i \propto a V^i \propto a(a^{-2}) \propto \frac{1}{a(t)}$
$\frac{m_0 V}{(1-V^2)^{1/2}} \propto \frac{1}{a}$
"HUBBLE DRAG"
$K = 0$
$K = \frac{1}{R^2}$



- 3 TRANSLATIONS
- 3 ROTATIONS
- NO BOOSTS
- NO TIME TRANSLATION
← UNIVERSE HAS PREFERRED REST FRAME
← " " " CLOCK
CMB =
MONOPOLE ~2.7 K
DIPOLE ~3 mK
PRIMORDIAL ~100 μK
SPEEDOMETER
$T(\hat{\theta}) = \sum_{lm} T_{lm} Y_{lm}(\hat{\theta})$
$= T_{00} \underbrace{Y_{00}(\hat{\theta})}_{\frac{1}{\sqrt{4\pi}}} + \underbrace{\sum_m T_{1m} Y_{1m}(\hat{\theta})} + \dots$
46 Gyr
X