

Title: Quantum Information Lecture

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Collection: Quantum Information 2023/24

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URL: <https://pirsa.org/24020032>

- + Classical Information Intro
- + Review of QM with an Info perspective
- + Shannon and Von Neumann Entropies / Thermodynamic entropy
- + Multiparticle quantum systems
- + Entanglement Vs Classical correlations
- + EPR argument and Violations of local realism
- + Peres criterion and Entanglement measures
- + Mutual Information and Quantum Discord
- + Measurements in QM and Quantum information
- + The church of the larger Hilbert space
- Ignorance Vs Entanglement

+ Quantum Channels and Channel capacities

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- + Quantum Channels and Channel capacities
 - + Quantum Teleportation and dense coding
 - + the Measurement problem(s) and Relativity and QFT
 - + Quantum Logic Vs Classical Logic
 - + Quantum algorithms (Grover and Shor)
 - + Quantum Cryptography
 - + An example of RQI protocol: Quantum Energy Teleportation
 - + (Extras)

Consider a Probability distribution $P(X)$ for a random variable
($P_i = P(x_i)$) Shannon Measure of Ignorance: $H(X) = \langle$

What is $S_u(X) \equiv S_u(P(X))$? X sets of outcome

1. $S_u(X) = S_u(P(X))$ is monotonically decreasing with p , the larger the
2. $S_u(X) \geq 0$ (the amount of info learned from an experiment is either zero or p
3. $S_u(X) = 0 \Leftrightarrow P(X) = 1$ if you already knew, you are not surprised!
4. $S_u(X = \{x_1, x_2\}) = S_u(P_1, P_2) = S_u(P_1) + S_u(P_2)$

$$\text{From 4: } S_u(P_1 P_2) = S_u(P_1) + S_u(P_2) \xrightarrow{\partial_{P_1}} P_2 S'_u(P_1 P_2) = S'_u(P_1)$$

$$\rightarrow S'_u(\underbrace{P_1 P_2}_{P_1}) + \underbrace{P_2 P_1}_P S''_u(\underbrace{P_1 P_2}_P) = 0 \rightarrow S'_u(P) + P S''_u(P) = 0$$

$$\Rightarrow (P S'_u(P))' = 0 \Rightarrow P S'_u(P) = \kappa \Rightarrow S'_u(P) = \frac{\kappa}{P} \Rightarrow S_u(P) = \kappa \log P$$

From 3: $C=0$, From 2: $\kappa < 0$ (since $\log(P) \leq 0$)

$$\underline{S_u(P) = -\log_a P} \quad \text{we pick } S_u(x) = -\log_2(P(x))$$

) for a random variable X that can take n values x_1, \dots, x_n with

ence:
$$H(X) = \langle S_u(X) \rangle = \sum_i P(x_i) S_u(x_i) = \sum_i P_i$$

X sets of outcomes of evaluating the random variable Entropy

g with P , the larger the probability the less surprised you will be! (the larger P the
payment is either zero or positive) you are not surprised!

5. $S_u(X) = S_u(P(X))$ should be a smooth end

$$H(X) = - \sum_i P_i \log_2(P_i)$$

x can take n values x_1, \dots, x_n with probabilities P_1, \dots, P_n

$$H = \sum_i P(x_i) S_u(x_i) = \sum_i P_i S_u(x_i) \quad \sum P_i = 1$$

evaluating the random variable / Entropy Axioms

the less surprised you will be! (the larger P the less information you learned)

$S_u(x) = S_u(P(x))$ should be a smooth enough function of P

$P_i \log_2(P_i)$ | Shannon Entropy

From 4: $S_u(P_1 P_2) = S_u(P_1) + S_u(P_2) \xrightarrow{\partial_{P_1}} P_2 S'_u(P_1 P_2)$
 $\rightarrow S'_u(P_1 P_2) + \frac{P_2 P_1}{P} S''_u(P_1 P_2) = 0 \rightarrow S'_u(P) + P S''_u(P) = 0$
 $\Rightarrow (P S'_u(P))' = 0 \Rightarrow P S'_u(P) = \kappa \Rightarrow S'_u(P) = \frac{\kappa}{P} \Rightarrow S_u(P) = \kappa \ln P$

From 3: $C=0$, From 2: $\kappa < 0$ (since $\log(P) \leq 0$)

$|S_u(P) = -\log_a P|$

We pick $S_u(x) = -\log_2(P(x))$

$\hat{H} = \frac{\hat{p}^2}{2m} + m\omega \hat{x}^2$

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$$\frac{P_1 P_2}{P} + \frac{P_2 P_1}{P} S''_u(P_1 P_2) = 0 \longrightarrow S'_u(P) + P S''_u(P) = 0 \implies$$

$$))' = 0 \implies P S'_u(P) = \kappa \implies S'_u(P) = \frac{\kappa}{P} \implies S_u(P) = \kappa \log P$$

From 2: $\kappa < 0$ (since $\log(P) \leq 0$) $\beta = \frac{1}{2} (101)$

$\log_a P$ | We pick $S_u(x) = -\log_2(P(x))$ $147 = \frac{1}{\sqrt{2}} (10$

$$\frac{\hat{p}^2}{2m} + m\omega \hat{x}^2 + \underbrace{i[\hat{x}, \hat{p}]}_{i\hbar} \left(\frac{\hat{p}^2}{2m} + m\omega \hat{x}^2 \right)$$

$$P(x_i) S_u(x_i) = \sum P_i S_u(x_i) \quad \sum P_i = 1$$

the random variable Entropy Axioms
 used you will be! (the larger P the less information you learned)

$S_u(P(x))$ should be a smooth enough function of P $H = \hat{P}^{1/2} \hat{X} \hat{P}^{1/2}$

Shannon Entropy

$$H = X P$$

$$\hat{H} = \frac{1}{2} (\hat{X} \hat{P} + \hat{P} \hat{X})$$