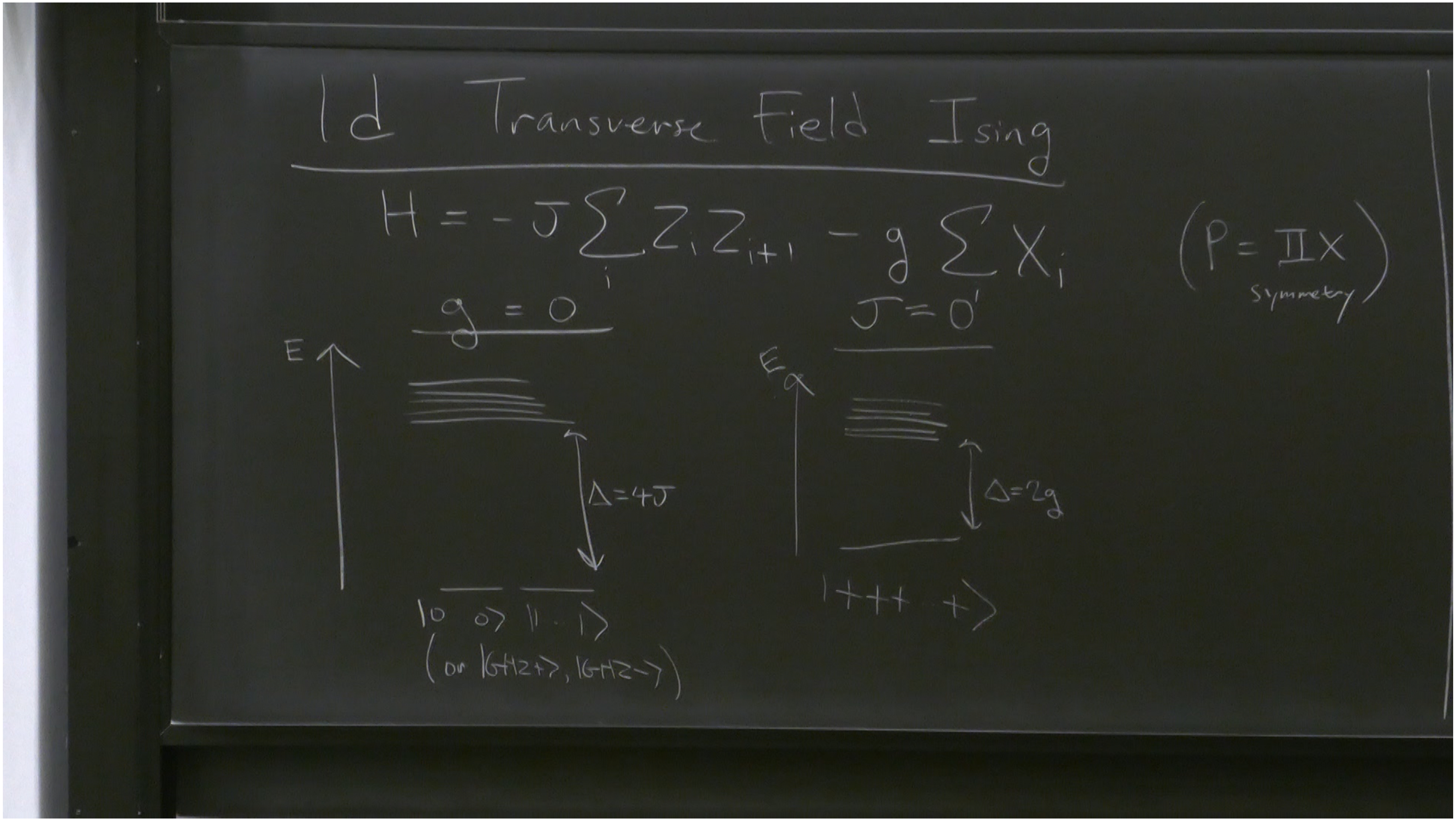
Title: Quantum Matter Lecture

Speakers: Timothy Hsieh

Collection: Quantum Matter 2023/24

Date: February 29, 2024 - 9:00 AM

URL: https://pirsa.org/24020031


1d Transverse Field Ising
H = -J Σ_i Z_i Z_{i+1} - g Σ X_i
(P = ΠX symmetry)
g = 0
J = 0
E
Δ=4J
Δ=2g
|0 ... 0⟩ |1 ... 1⟩
(or |GHZ+⟩, |GHZ-⟩)
|+++ ... +⟩

High field limit

$$|i\rangle = Z_i |\psi_0\rangle$$

$$(|++\,+\cdots \underset{i}{-} \cdots ++\rangle)$$

$$|i,j\rangle = Z_i Z_j |\psi_0\rangle$$

(energy $4g$)

$$|\psi_0'\rangle = |\psi_0\rangle + \frac{\bar{v}}{4g} \sum_i |i,i+1\rangle + O(v^2)$$

$$Z_i Z_{i+1} |i\rangle = |i+1\rangle$$

$$Z_{i-1} Z_i |i\rangle = |i-1\rangle$$

$$\rangle$$

$$\frac{J}{4g}\sum_i |i,i+1\rangle + O(J^2)$$

$$Z_i Z_{i+1}|i\rangle = |i+1\rangle$$

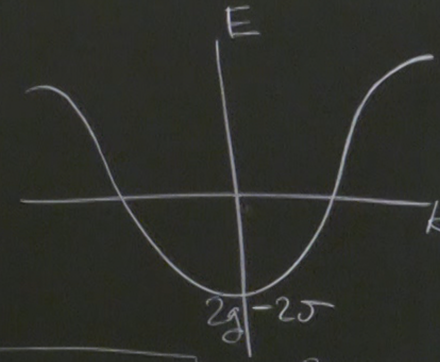
$$Z_{i-1} Z_i|i\rangle = |i-1\rangle$$

$$\langle i|H_{eff}|j\rangle = -J(\delta_{j,i+1}+\delta_{j,i-1})$$

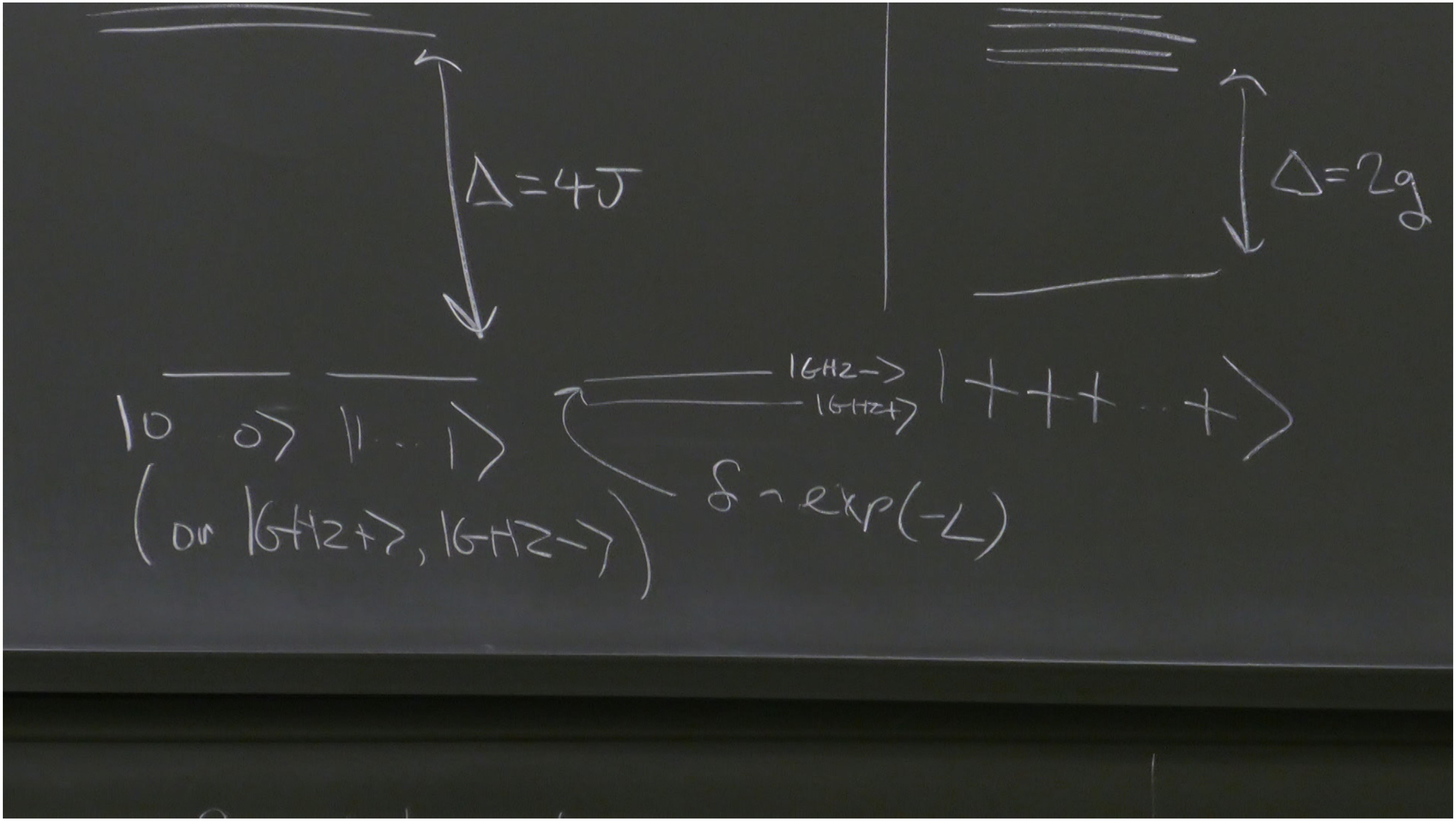
$$|k\rangle = \sum_j e^{ikj}|j\rangle$$

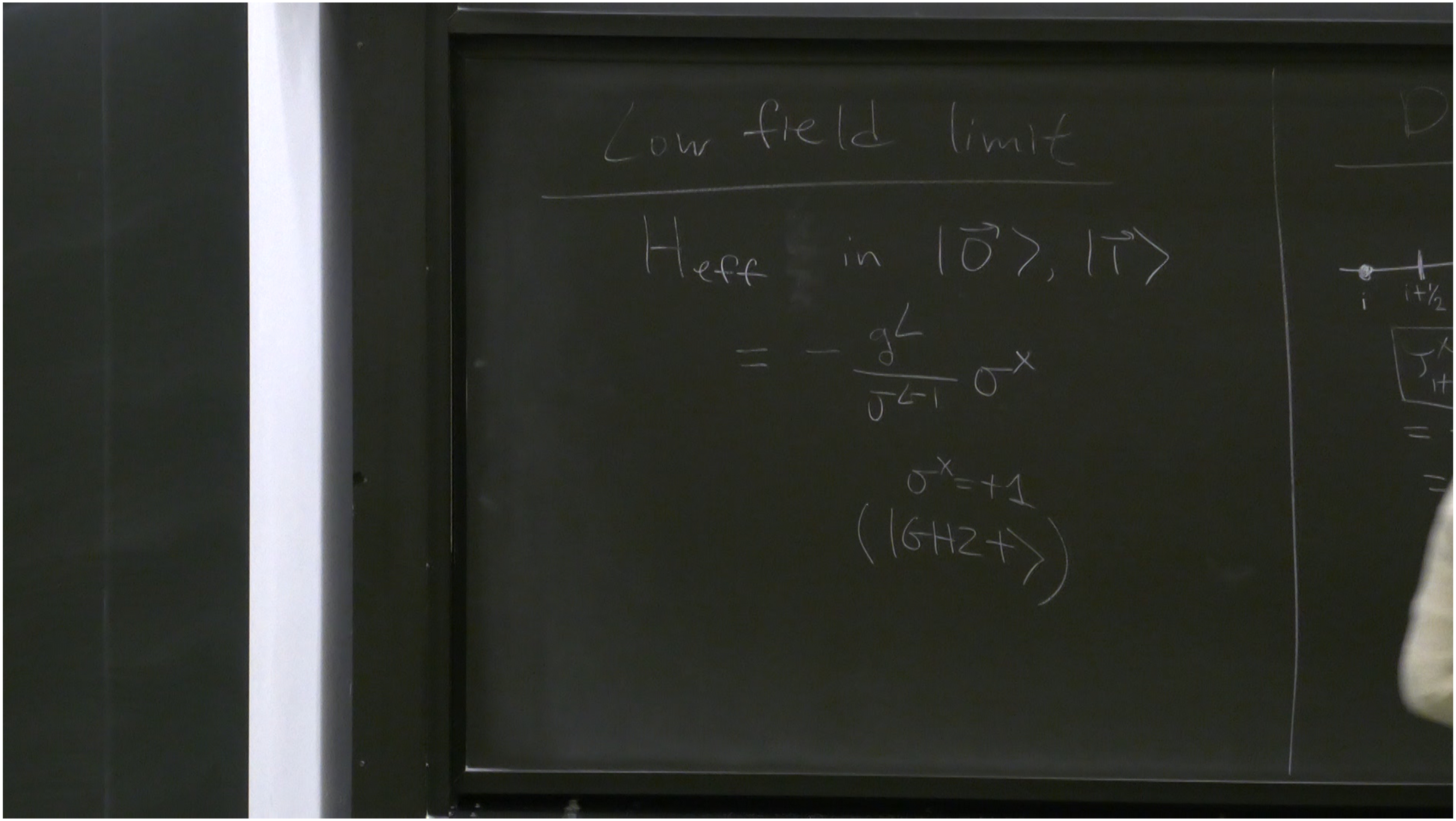
$$H_{eff}|k\rangle = E_k|k\rangle$$

$$E_k = -J(e^{ik}+e^{-ik}) = \boxed{-2J\cos k}$$



energy band


Δ=4J
Δ=2g
|GHz−⟩
|GHz+⟩
|+++ ... +⟩
|0 ... 0⟩ |1 ... 1⟩
(or |GHz+⟩, |GHz−⟩)
δ ~ exp(−L)


Low field limit
H_eff in |0>, |1>
= − g^L / J^{L−1} σ^x
σ^x = +1
(|GHZ+>)

$$\boxed{\tau^x_{i+1/2} = Z_i Z_{i+1}}$$

$= +1$ (no dw) $\uparrow\uparrow$

$= -1$ (dw) $\uparrow\downarrow$

domain wall creation

$$\boxed{\tau^z_{i+1/2} = \prod_{j\leq i} X_j}$$

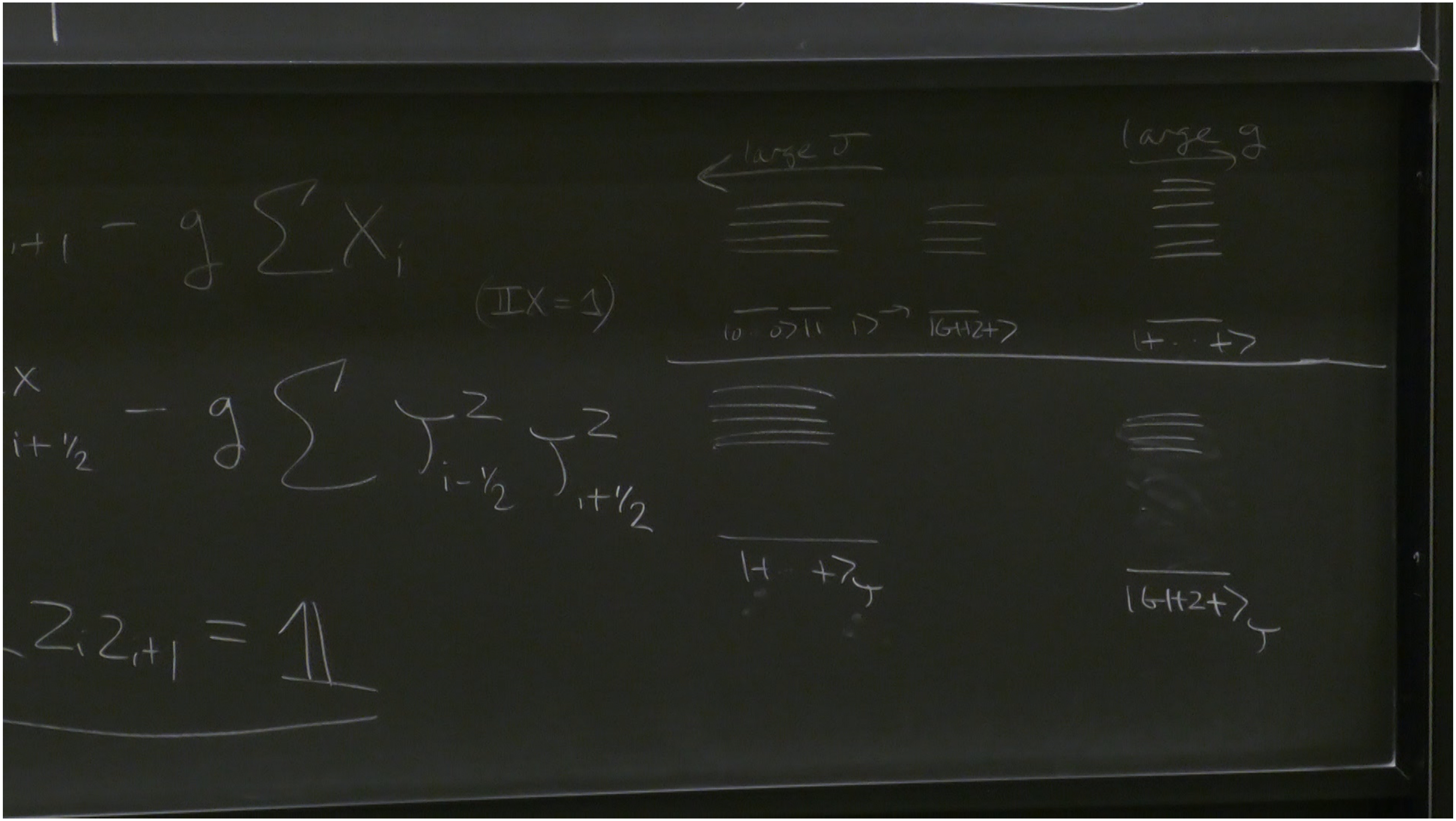
(disorder operator)

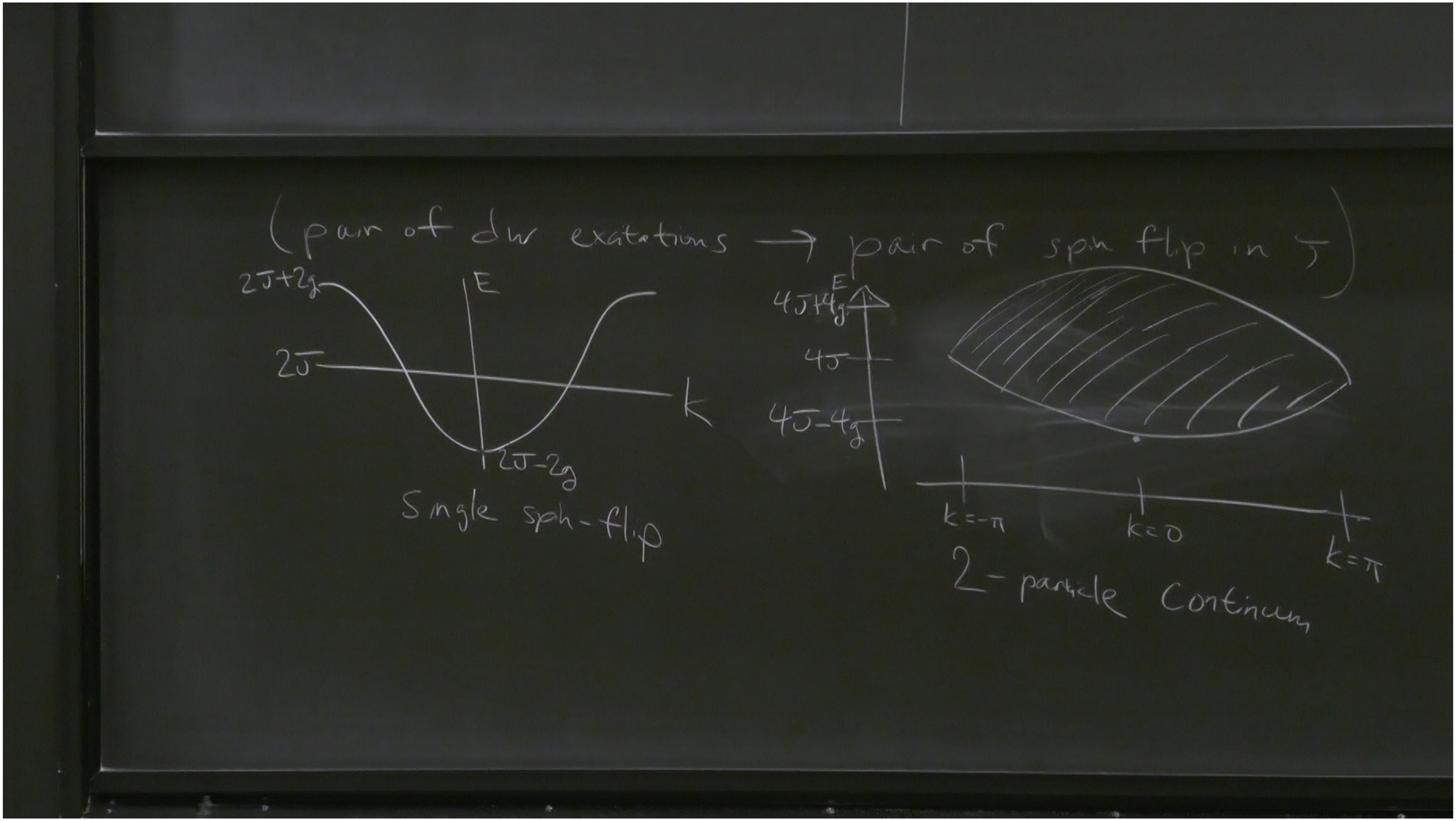
$$\boxed{\tau^z_{i-1/2}\,\tau^z_{i+1/2} = X_i}$$

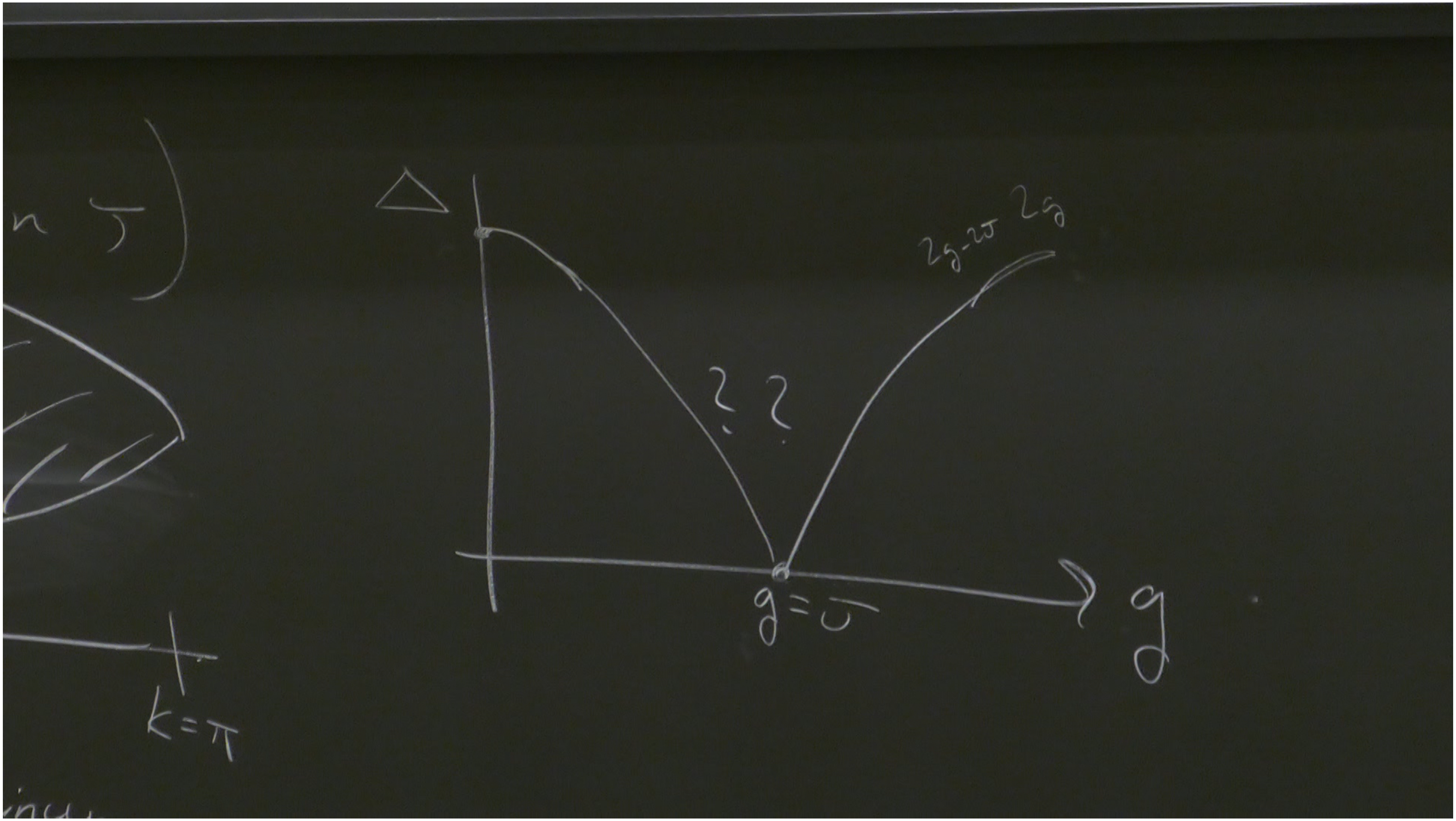
$$H = -J \sum Z_i Z_{i+1} - g \sum X_i$$

$$\tilde{H} = -J \sum \tau^x_{i+1/2} - g \sum \tau^z_{i-1/2} \tau^z_{i+1/2}$$

$$\prod_i \tau^x_{i+1/2} = \prod_i Z_i Z_{i+1} = \mathbb{1}$$


large J
large g
$-g\sum X_i$
$(\Pi X = 1)$
$|+\cdots+\rangle$
$-g\sum \tau^z_{i-1/2}\,\tau^z_{i+1/2}$
$|+\cdots+\rangle_\tau$
$Z_i Z_{i+1} = 1$


(pair of dw excitations → pair of spin flip in J)
2J+2g
E
2J
k
2J−2g
Single spin-flip
E
4J+4g
4J
4J−4g
k=−π
k=0
k=π
2-particle Continuum



Δ
2g-2J
2g
? ?
g=J
g
k=π

Jordan-Wigner transformation

FM $\langle Z \rangle \neq 0$

PM $\langle \tau^z \rangle \neq 0$

γ_{2i} γ_{2i+1}

i $\quad$ $i+1$

$$\gamma_{2i} = \Big(\prod_{j\leq i} X_j\Big) Z_i$$

$$\tau^z_{i-1/2} Z_i = \Big(\prod_{j\leq i} X_j\Big) Z_i$$

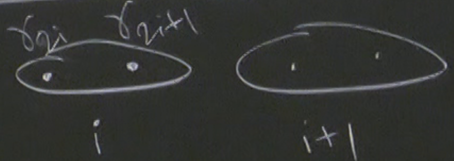
$$\gamma_{2i+1} = \Big(\prod_{j\leq i} X_j\Big) Y_i$$

$$\{\gamma_i, \gamma_j\} = \delta_{ij}$$

x

z

...mation



$$c_i^\dagger = \frac{\gamma_{2i} + i\gamma_{2i+1}}{2}$$

$$c_i = \frac{\gamma_{2i} - i\gamma_{2i+1}}{2}$$

$$\underline{i\gamma_{2i}\gamma_{2i+1}} = 1 - 2c_i^\dagger c_i = (-1)^{n_i}$$

$$\gamma_{2i} = \Big(\prod_{j\leq i} X_j\Big) Z_i$$

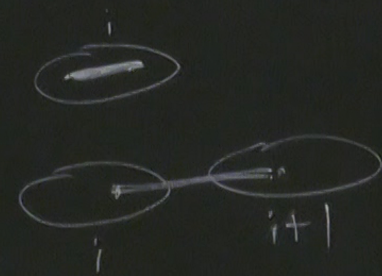
$$\gamma_{2i+1} = \Big(\prod_{j\leq i} X_j\Big) Y_i$$

$$\{\gamma_i, \gamma_j\} = \delta_{ij}$$

$$X_i = i\gamma_{2i}\gamma_{2i+1}$$

$$Z_i Z_{i+1} = i\gamma_{2i+1}\gamma_{2i+2}$$

$H = -$



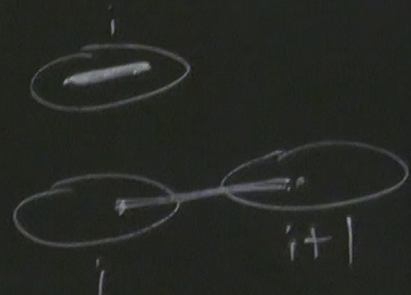
$$H = -J \sum_i i \gamma_{2i+1} \gamma_{2i+2} - g \sum_i i \gamma_{2i} \gamma_{2i+1}$$

$$\text{Ising sym} = \Pi \gamma = (-1)^N$$

$$c_i^+ c_i = (-1)^{n_i}$$

$$\gamma_{2i} \gamma_{2i+1}$$

$$\gamma_{2i+1} \gamma_{2i+2}$$



large g

large J