Title: Quantum Matter Lecture

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Collection: Quantum Matter 2023/24

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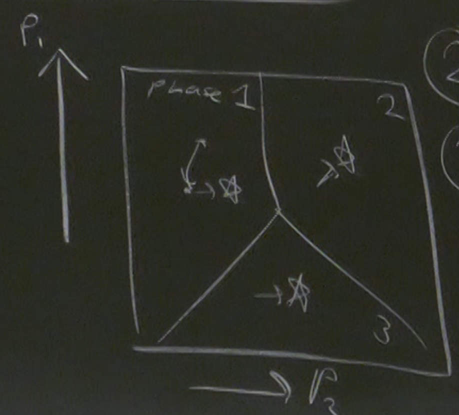
Quantum Phases of Matter

Lattice models (Tim)

Field theory (Yin-Chen)

Motivation:

1. Emergence
2. Practical e.g. platforms for quantum comp
3. Universality



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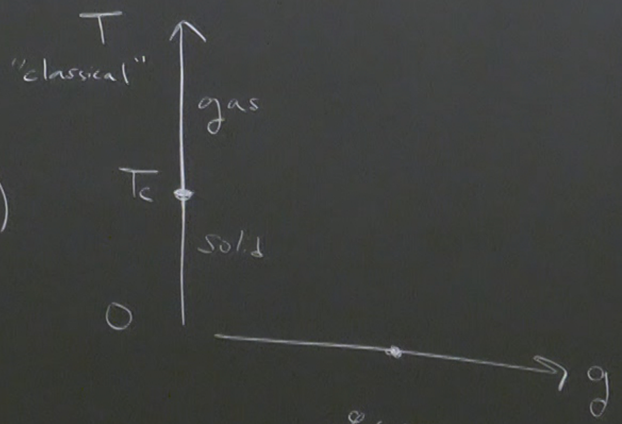


how non-classical? quantum
quantify using entanglement

Lattice model

Hilbert space

• • • • • • • N sites

↑ finite-dim $\mathcal{H}$

spin/qubit

$\sigma^{x,y,z}$

$\sigma^z|0\rangle = |0\rangle$

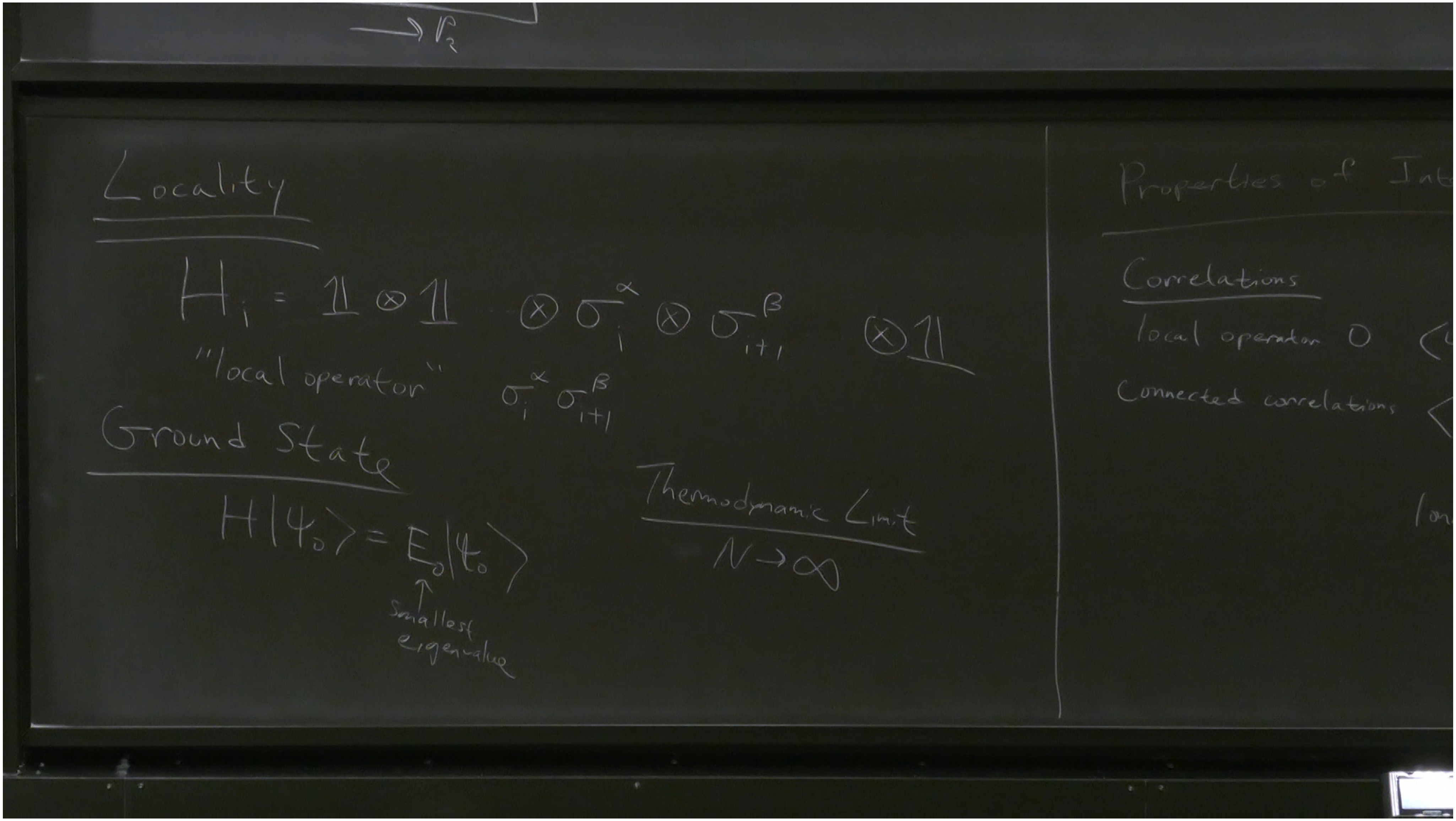
$\sigma^z|1\rangle = -|1\rangle$

$\mathcal{H}_{tot} = \otimes \mathcal{H}$

$\dim \sim 2^N$

Hamiltonian

$$H = \sum_i H_i$$


Locality
$H_i = \mathbb{1} \otimes \mathbb{1} \cdots \otimes \sigma_i^\alpha \otimes \sigma_{i+1}^\beta \otimes \mathbb{1}$
"local operator" $\sigma_i^\alpha \sigma_{i+1}^\beta$
Ground State
$H|\psi_0\rangle = E_0|\psi_0\rangle$
smallest eigenvalue
Thermodynamic Limit
$N \to \infty$
Properties of
Correlations
local operator O
Connected correlations

$\sigma^{\beta}_{i+1} \otimes \mathbb{1}$

...modynamic Limit

$N \to \infty$

Properties of Interest

Correlations

local operator O $\langle \psi_0 | O | \psi_0 \rangle$

Connected correlations $\langle \psi_0 | O_i O_j | \psi_0 \rangle - \langle \psi_0 | O_i | \psi_0 \rangle \langle \psi_0 | O_j | \psi_0 \rangle$

$|i-j| \to \infty$

long ranged?

2 q...

$0|\psi_0\rangle$

$|O_i O_j|\psi_0\rangle - \langle\psi_0|O_i|\psi_0\rangle\langle\psi_0|O_j|\psi_0\rangle$

$|\to\infty$

ranged?

Entanglement

How quantum is ψ_0?

2 qubits: $|00\rangle = |0\rangle\otimes|0\rangle$ product state

$|00\rangle + |11\rangle$ Bell pair entangled

? $(\; + |01\rangle + |10\rangle) = |+\rangle \otimes |+\rangle$

$\sigma^x|+\rangle = |+\rangle$

need measures!

$|\psi_{AB}\rangle$ reduced density matrix $\rho_A \equiv tr_B |\psi_{AB}\rangle\langle\psi_{AB}|$ $\quad tr(\rho_A O_A) = \langle\psi_{AB}|O_A|\psi_{AB}\rangle$

for prod. state $|\phi_A\rangle \otimes |\phi_B\rangle$ $\quad \rho_A = |\phi_A\rangle\langle\phi_A|$ pure state (no uncertainty about A's state)

non-product state ρ_A mixed

e.g. $|00\rangle + |11\rangle$ $\quad \rho_A = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$

von Neumann entanglement entropy

$$S_A = tr(\rho_A \ln \rho_A)$$

$S_A = \ln 2$ $\quad |\psi_0\rangle$:

B (A) B

$S_A \sim |A|$ vol. law

$\sim |\partial A|$ area law

reduced density matrix $\rho_A \equiv \mathrm{tr}_B |\psi_{AB}\rangle\langle\psi_{AB}|$ $\mathrm{tr}(\rho_A O_A) = \langle\psi_{AB}|O_A|\psi_{AB}\rangle$

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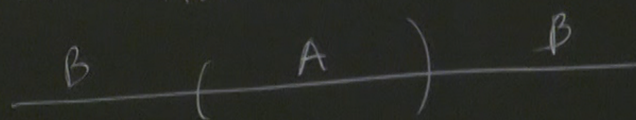

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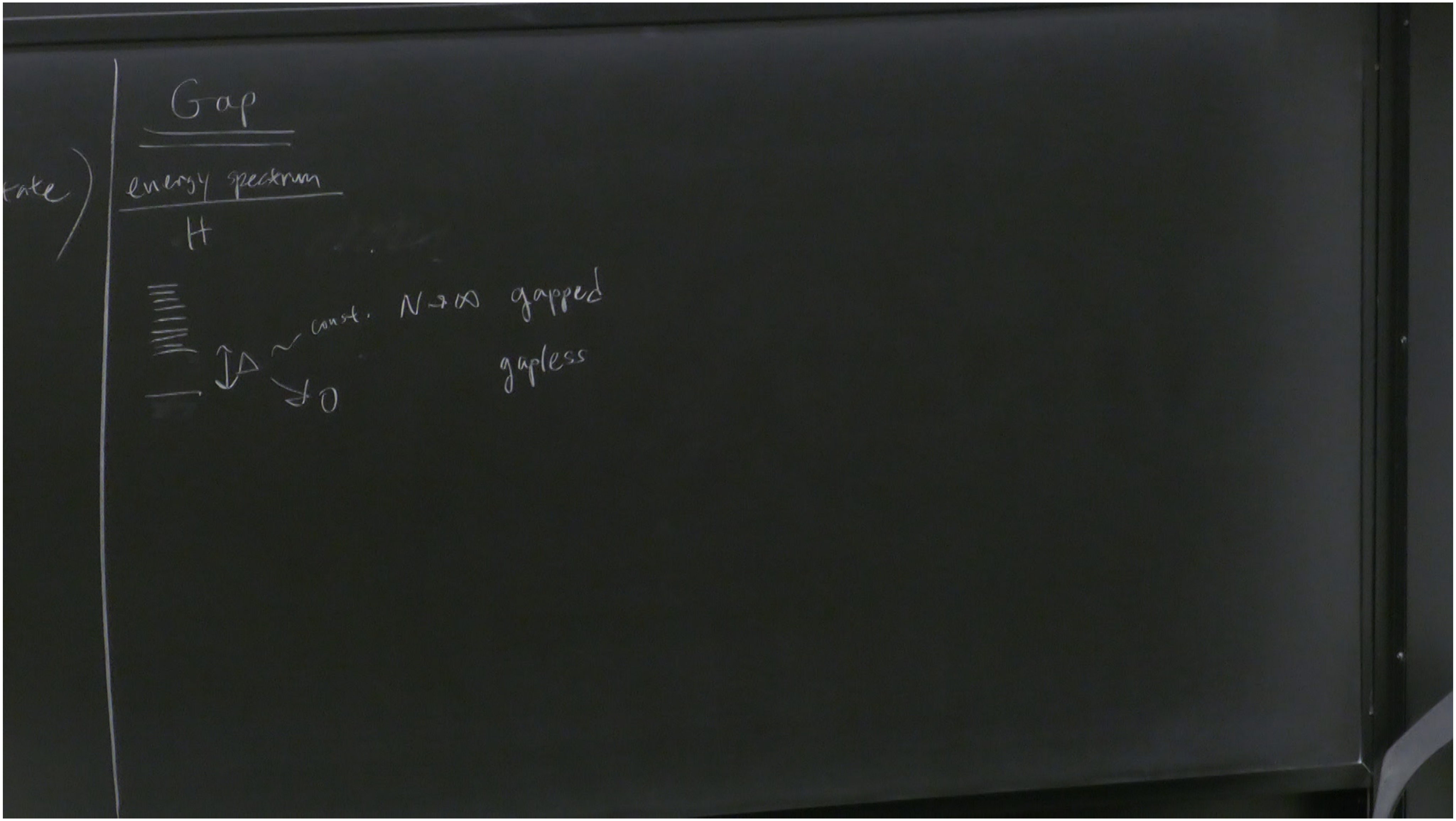
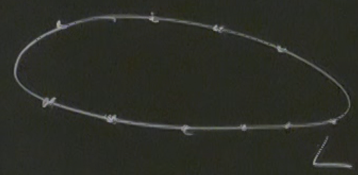

tate)
Gap
energy spectrum
H
Δ
~ const.
→ 0
N→∞
gapped
gapless

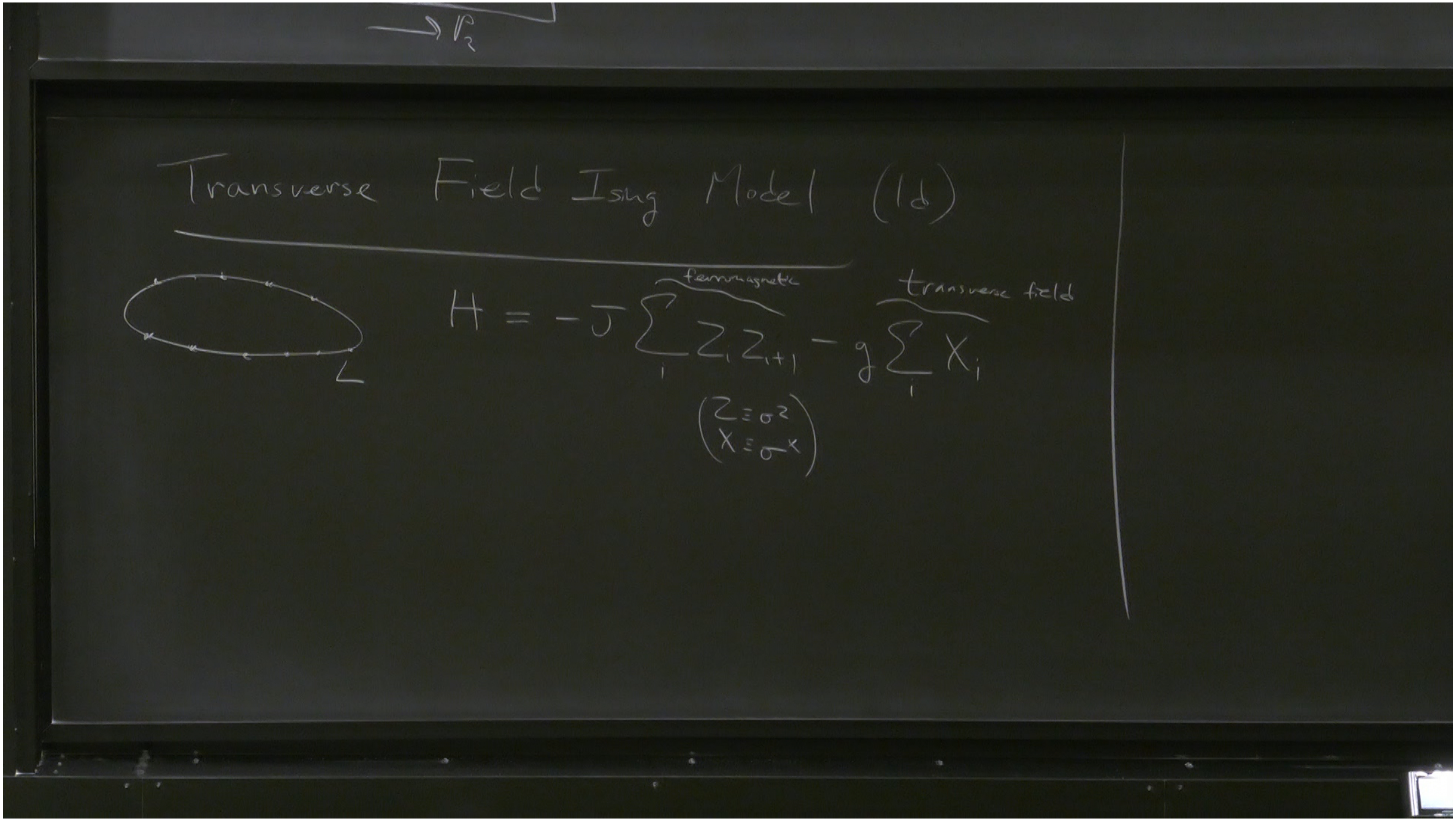
$\rightarrow P_2$

Transverse Field Ising Model (1d)



$$H = -J \overbrace{\sum_i Z_i Z_{i+1}}^{\text{ferromagnetic}} - g \overbrace{\sum_i X_i}^{\text{transverse field}}$$

$$\begin{pmatrix} Z \equiv \sigma^z \\ X \equiv \sigma^x \end{pmatrix}$$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ferromagnetic
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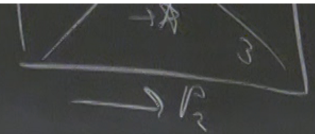
P_2



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Symmetries

translation sym

global Ising sym $P \equiv \prod_i X_i$

$$[H, P] \stackrel{!}{=} 0.$$

$\prod X_i$

$= 0$

Extreme Limits

$\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$

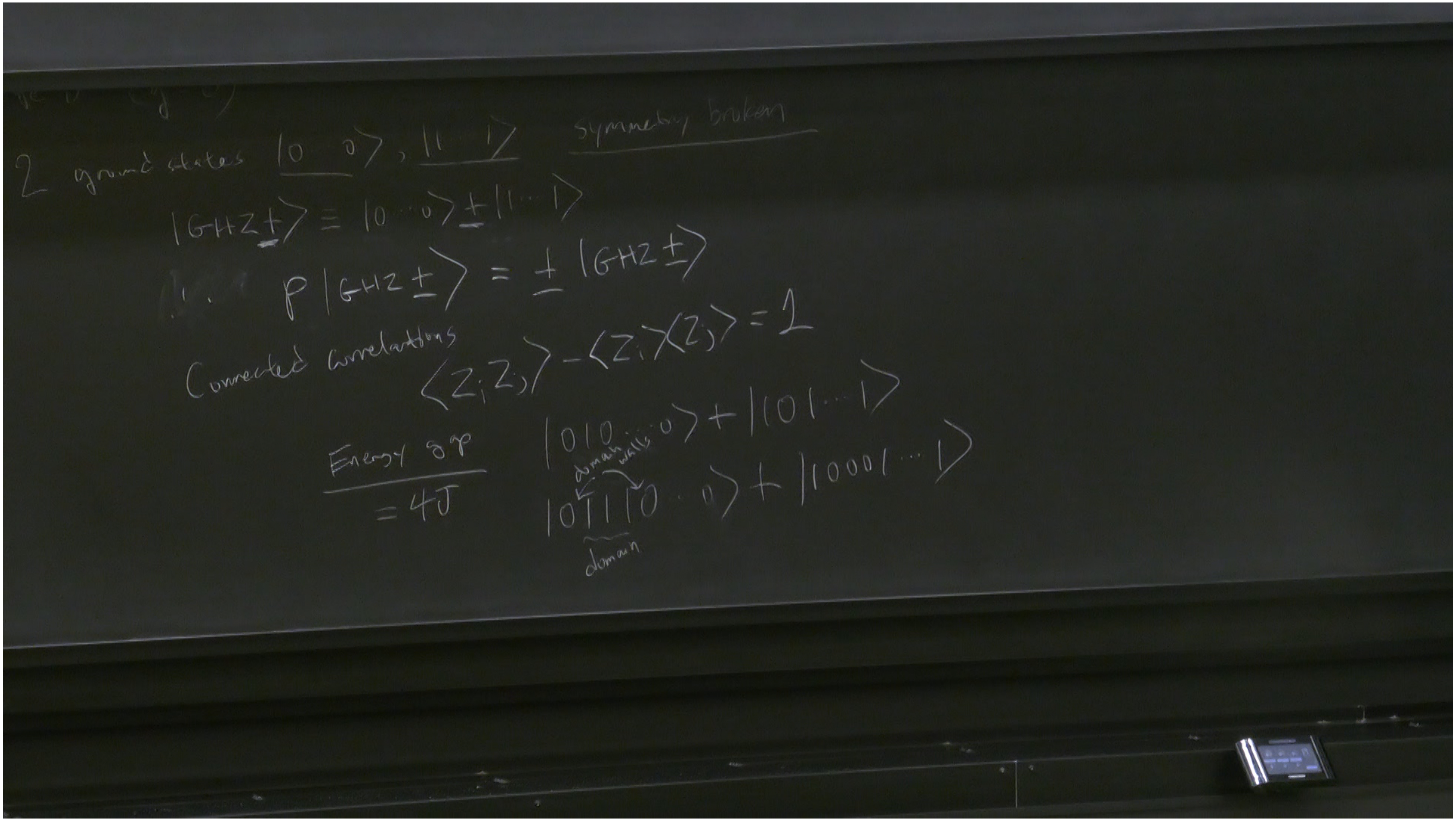
large g $(J=0)$ $|\psi_0\rangle = \bigotimes_i |+\rangle_i$

"paramagnet"

energy gap $\Delta = 2g$

first excited states $|\psi_i\rangle = Z_i |\psi_0\rangle$

(L of them)


2 ground states |0 … 0⟩, |1 … 1⟩
symmetry broken
|GHZ±⟩ ≡ |0 … 0⟩ ± |1 … 1⟩
P |GHZ±⟩ = ± |GHZ±⟩
Connected correlations
⟨Z_i Z_j⟩ − ⟨Z_i⟩⟨Z_j⟩ = 1
Energy gap = 4J
|010 … 0⟩ + |101 … 1⟩
domain walls
|01110 … 0⟩ + |10001 … 1⟩
domain