Title: QFT III Lecture

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Collection: QFT III 2023/24

Date: February 28, 2024 - 10:15 AM

URL: https://pirsa.org/24020029

# QFT III

- Jaume Gomis
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$\boxed{\text{Conformal}}$ Field Theory = CFT

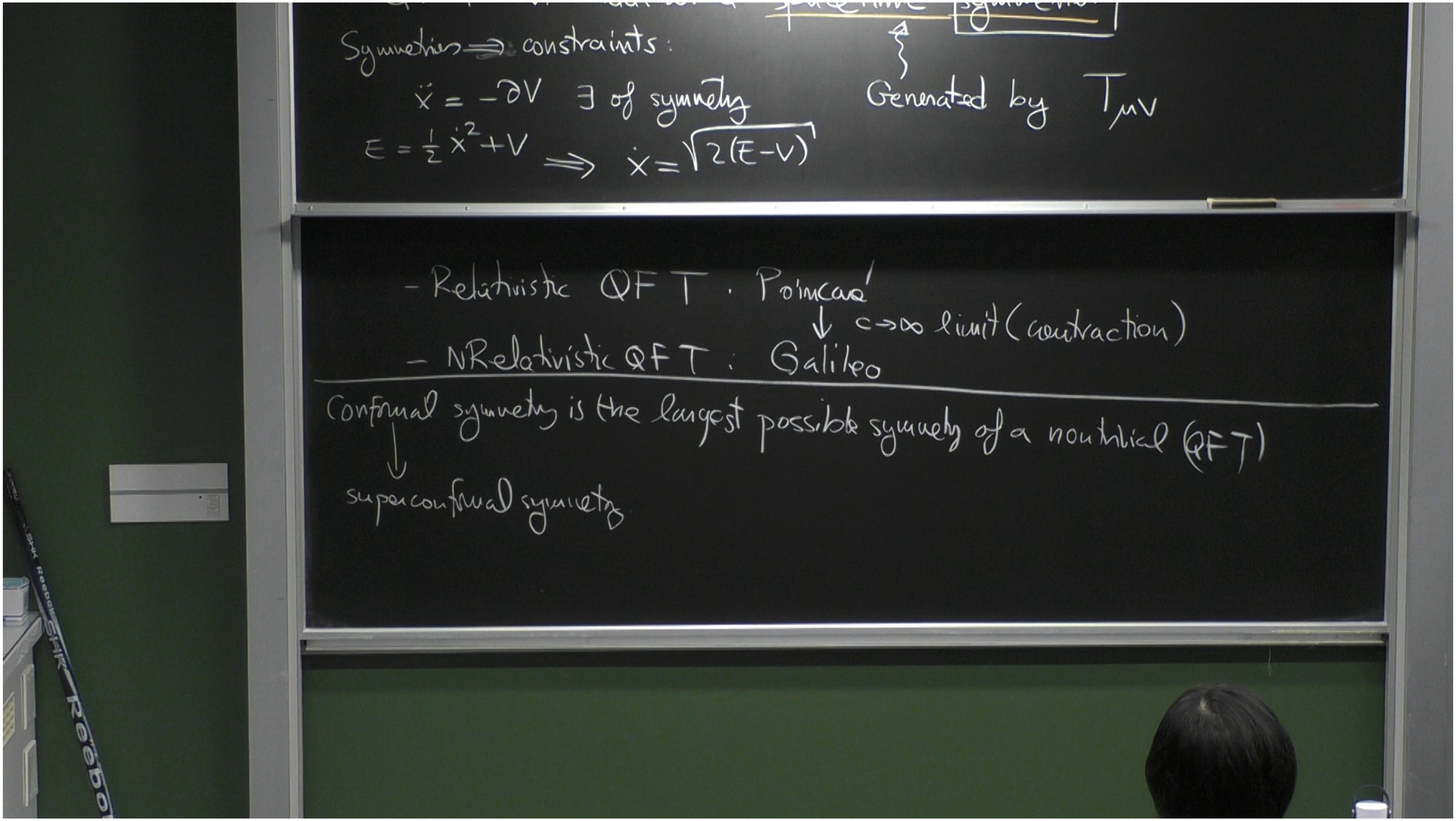
- QFT w/ additional spacetime $\boxed{\text{symmetries}}$

Generated by $T_{\mu\nu}$

Symmetries $\Rightarrow$ constraints:

$\ddot{x} = -\partial V$ $\exists$ of symmetry

$E = \frac{1}{2}\dot{x}^2 + V \Longrightarrow \dot{x} = \sqrt{2(E-V)}$


Symmetries ⟹ constraints:
$\ddot{x} = -\partial V$ ∃ of symmetry
Generated by $T_{\mu\nu}$
$E = \frac{1}{2}\dot{x}^2 + V \Longrightarrow \dot{x} = \sqrt{2(E-V)}$
- Relativistic QFT · Poincaré
c→∞ limit (contraction)
- NRelativistic QFT : Galileo
Conformal symmetry is the largest possible symmetry of a nontrivial (QFT)
superconformal symmetry
Reebok

Conformal Transformations: scale transformation

$$\vec{x} \to \lambda \vec{x}$$
$$t \to \lambda^{z} t$$

$z$: dynamical critical exponent

- $z=1$ in relativistic QFT
- $z=2$ Free Schrödinger equation.

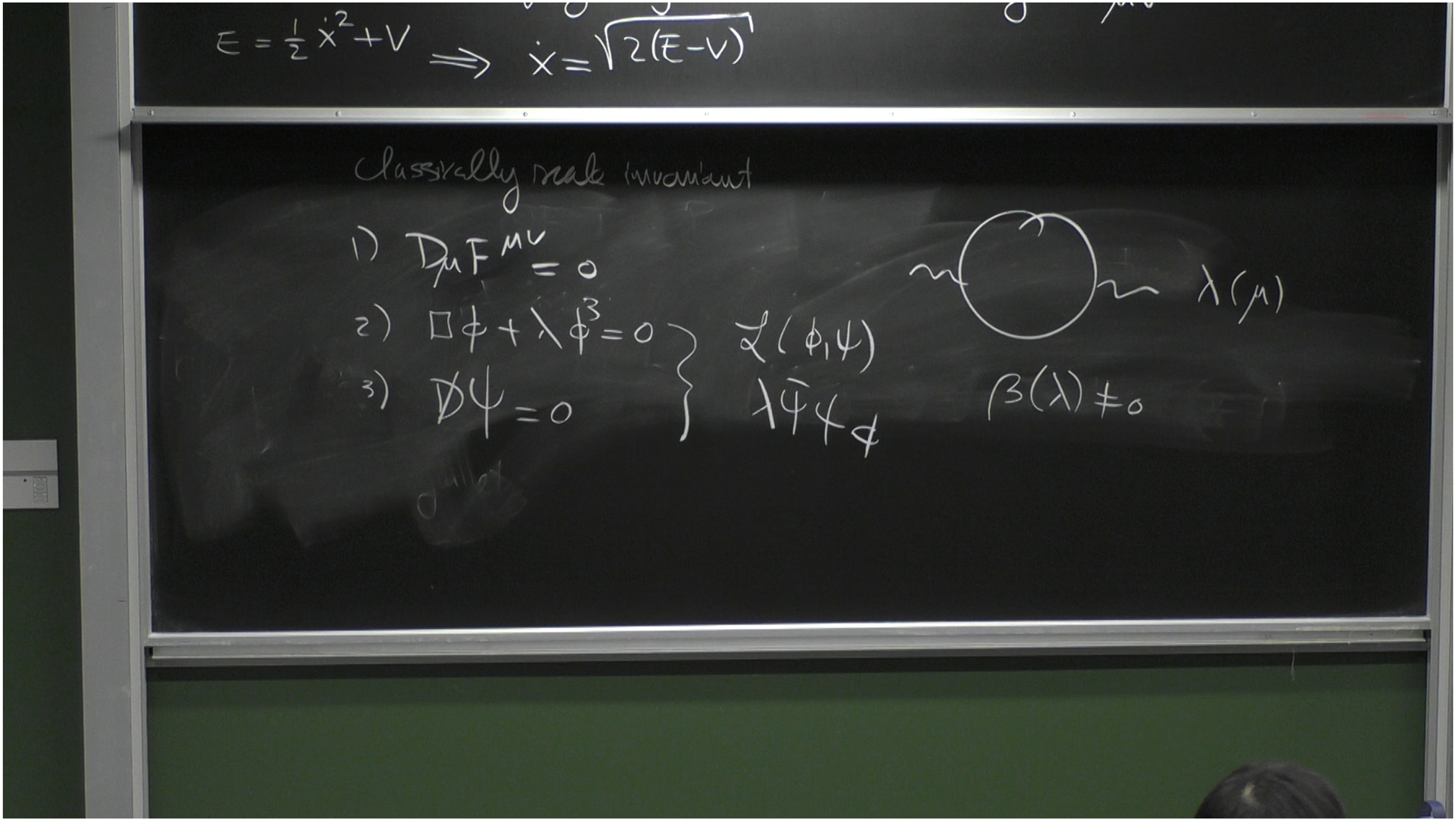
why CFTs?

1. Asymptotic low energy (IR) behavior of any QFT is scale invariant:

a. trivial: $\nexists$ local operators. $\langle \mathcal{O}(x)\, \mathcal{O}(0) \rangle \xrightarrow{|x|\to\infty} 0$ $\quad \mathcal{O}(x) \to 0$

Topological QFT (TQFT)

b. non trivial CFT


$E = \frac{1}{2}\dot{x}^2 + V \Longrightarrow \dot{x} = \sqrt{2(E-V)}$
classically scale invariant
1) $D_\mu F^{\mu\nu} = 0$
2) $\Box\phi + \lambda\phi^3 = 0$
3) $\not{D}\psi = 0$
$\mathcal{L}(\phi, \psi)$
$\lambda\bar{\psi}\psi\phi$
$\lambda(\mu)$
$\beta(\lambda) \neq 0$

3. CFT's induce an ordering in space of QFTs.

UV $\quad CFT_1 + \int \lambda \mathcal{O}$ ← relevant

$\Big\downarrow$ deep IR

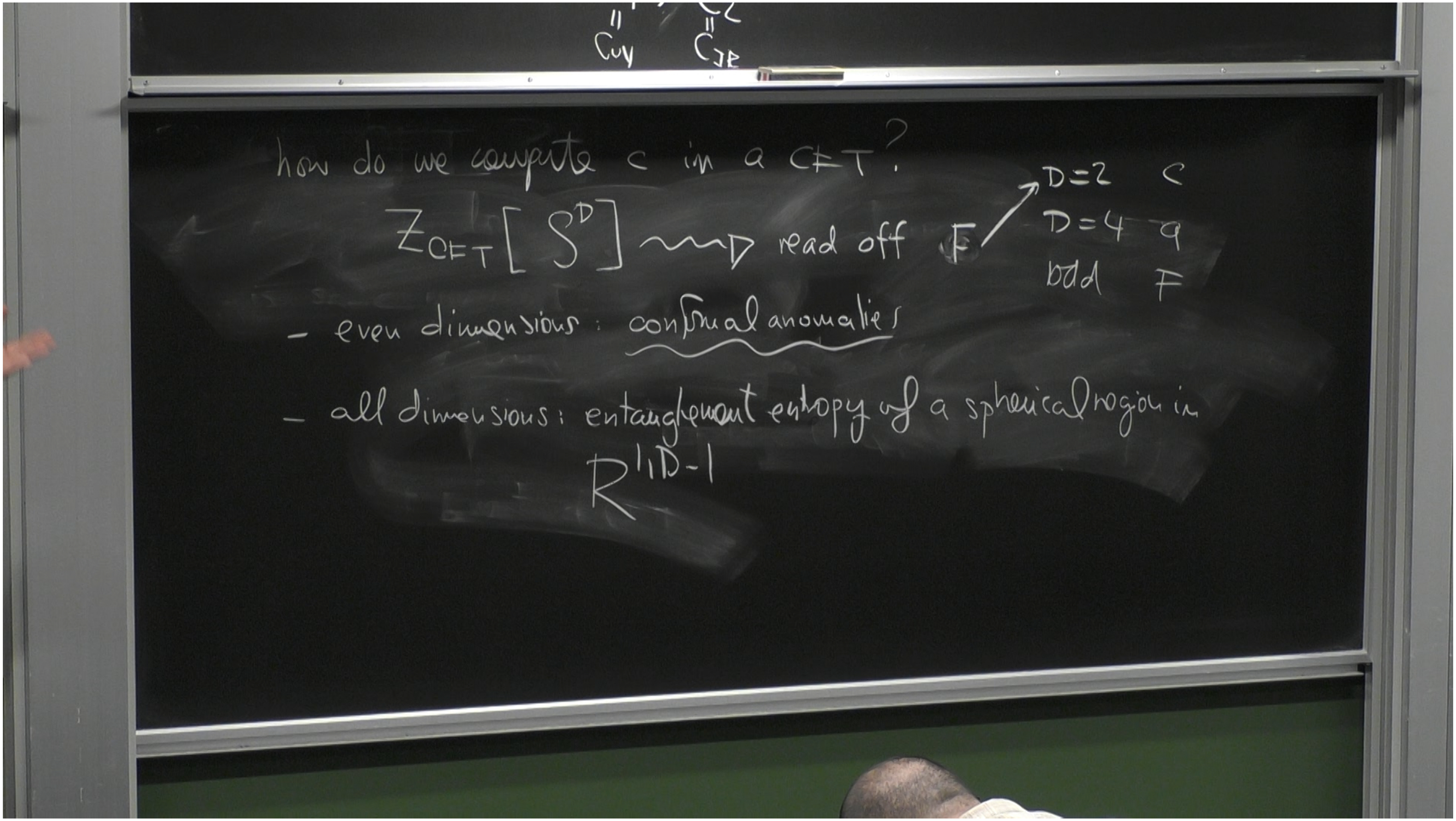
IR. $\quad CFT_2$

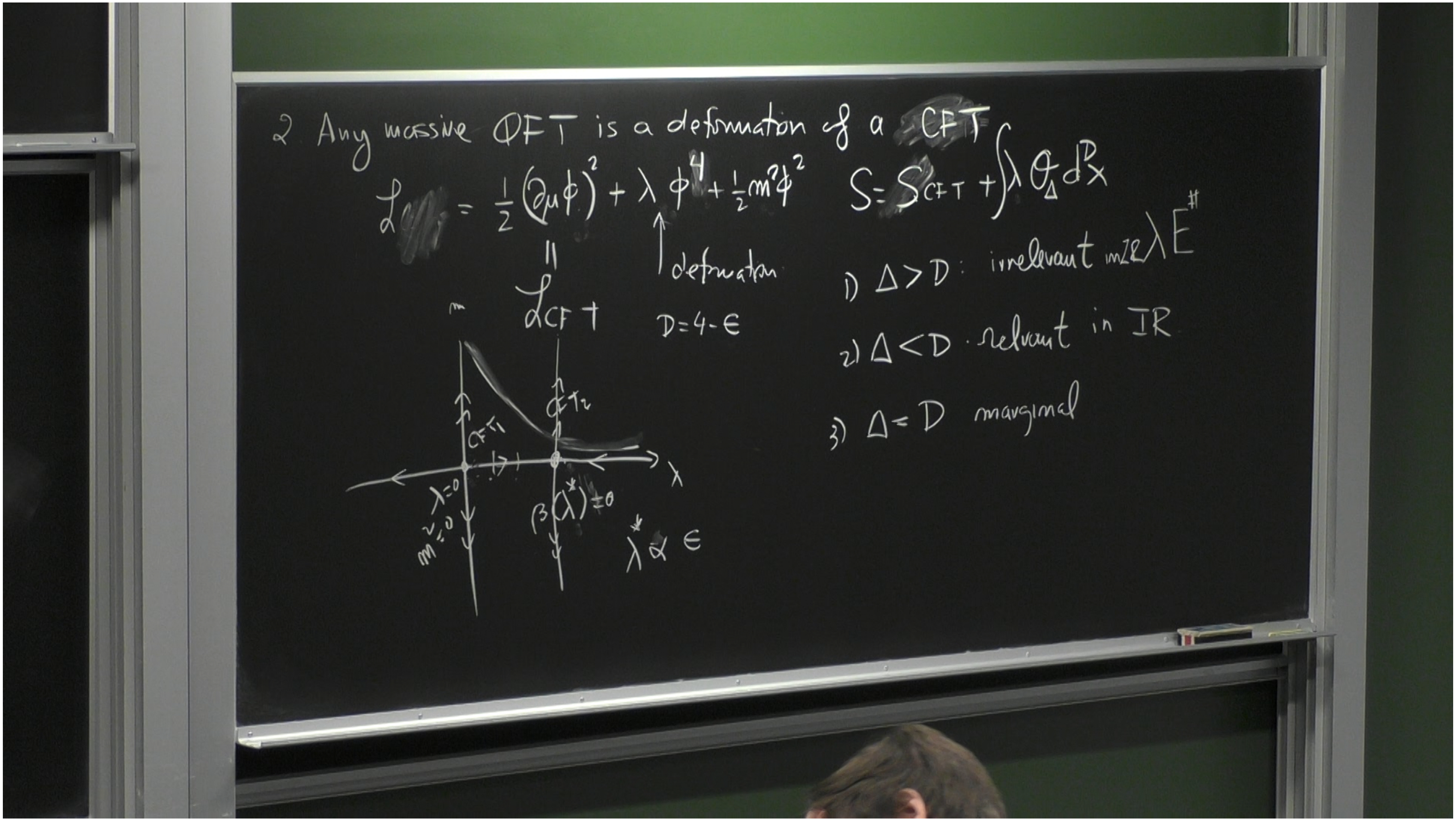
$\Longrightarrow$ ~~$CFT_2 \to CFT_1$~~

irreversibility of RG flow

Assign a "height" function $\boxed{c}$ that has monotonicity property

$$\begin{array}{ccc} c_1 & > & c_2 \\ \| & & \| \\ c_{UV} & & c_{IR} \end{array}$$

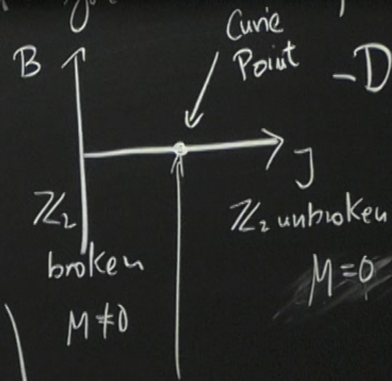

how do we compute c in a CFT?
Z_CFT[S^D] ⇝ read off F
D=2 c
D=4 a
odd F
- even dimensions: conformal anomalies
- all dimensions: entanglement entropy of a spherical region in
R^{1,D-1}


2 Any massive QFT is a deformation of a CFT
$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \lambda \phi^4 + \frac{1}{2} m^2 \phi^2$
$S = S_{CFT} + \int \lambda\, \mathcal{O}_\Delta\, d^D x$
deformation
$\mathcal{L}_{CFT}$
$D = 4 - \epsilon$
1) $\Delta > D$ : irrelevant in IR $\lambda E^{\#}$
2) $\Delta < D$ : relevant in IR
3) $\Delta = D$ marginal
CFT1
CFT2
$\lambda = 0$
$m^2 = 0$
$\beta(\lambda^*) = 0$
$\lambda$
$\lambda^* \propto \epsilon$

4. CFTs are directly relevant for nature $\Longrightarrow$ phase transitions

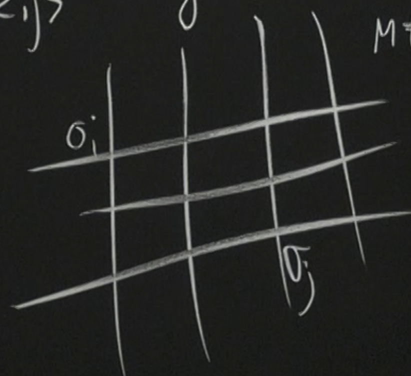
3d Ising Model

$$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j$$

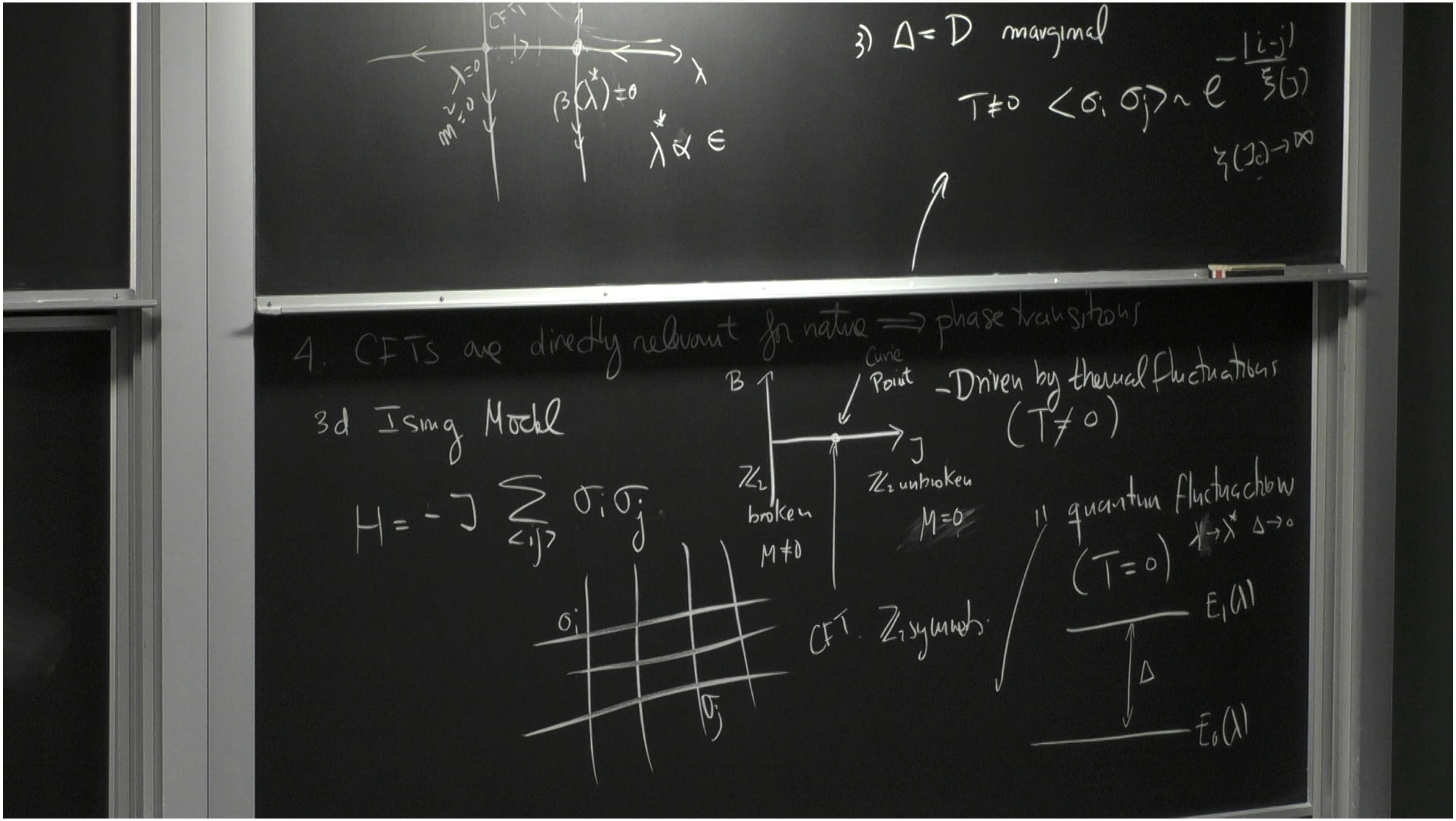


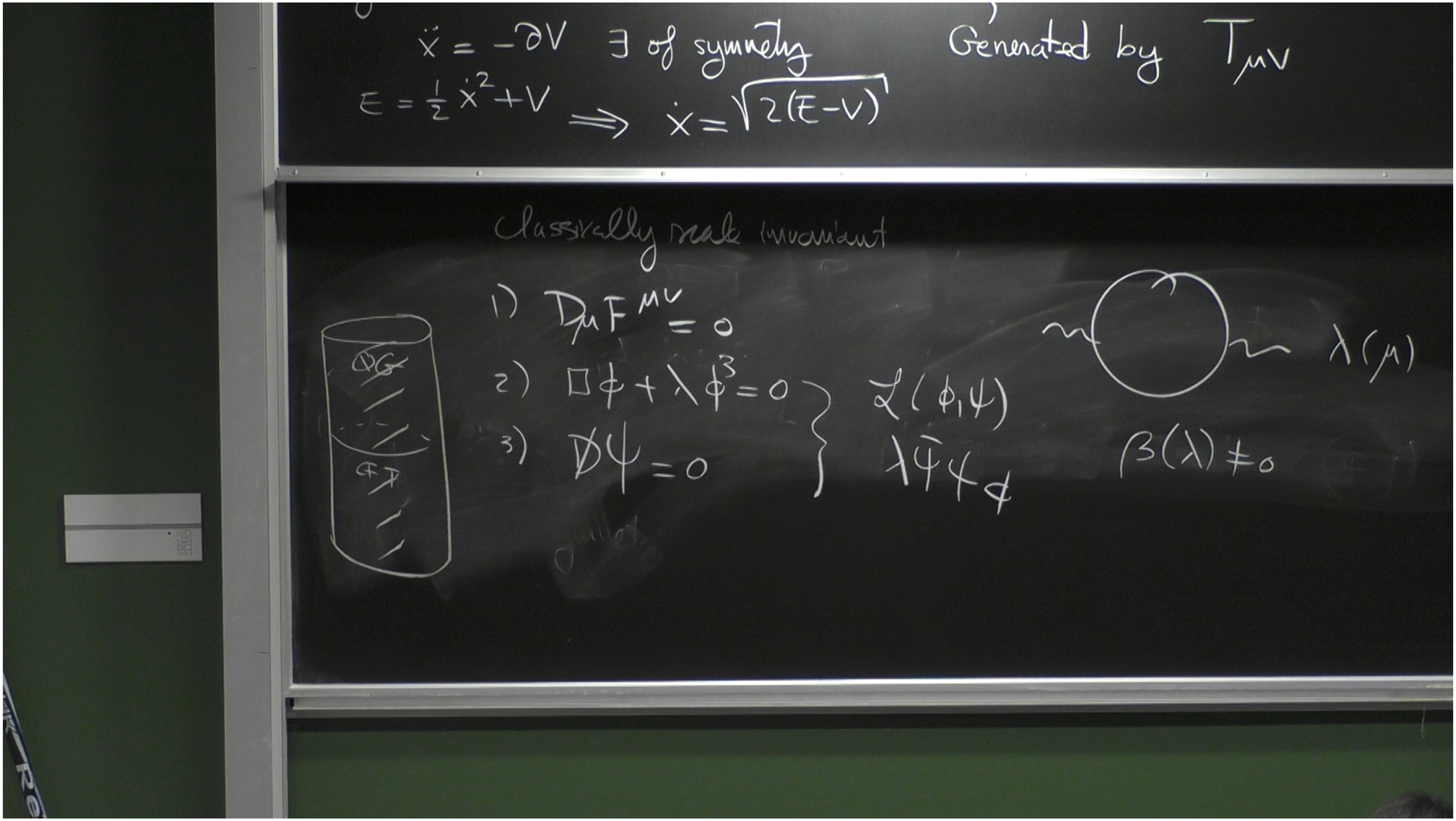

- Driven by thermal fluctuations $(T \neq 0)$

  " quantum fluctuations $(T = 0)$




CFT. $\mathbb{Z}_2$ symmetry.


3) Δ = D marginal
T ≠ 0 ⟨σ_i σ_j⟩ ~ e^{-|i-j|/ξ(J)}
ξ(J_c) → ∞
λ = 0
m^2 = 0
β(λ*) = 0
λ* ∝ ε
4. CFTs are directly relevant for nature ⟹ phase transitions
3d Ising Model
H = -J Σ_{⟨ij⟩} σ_i σ_j
σ_i
σ_j
B
J
Curie Point
Z_2 broken
M ≠ 0
Z_2 unbroken
M = 0
CFT. Z_2 symmetry
- Driven by thermal fluctuations (T ≠ 0)
quantum fluctuations (T = 0)
λ → λ*  Δ → 0
E_1(λ)
Δ
E_0(λ)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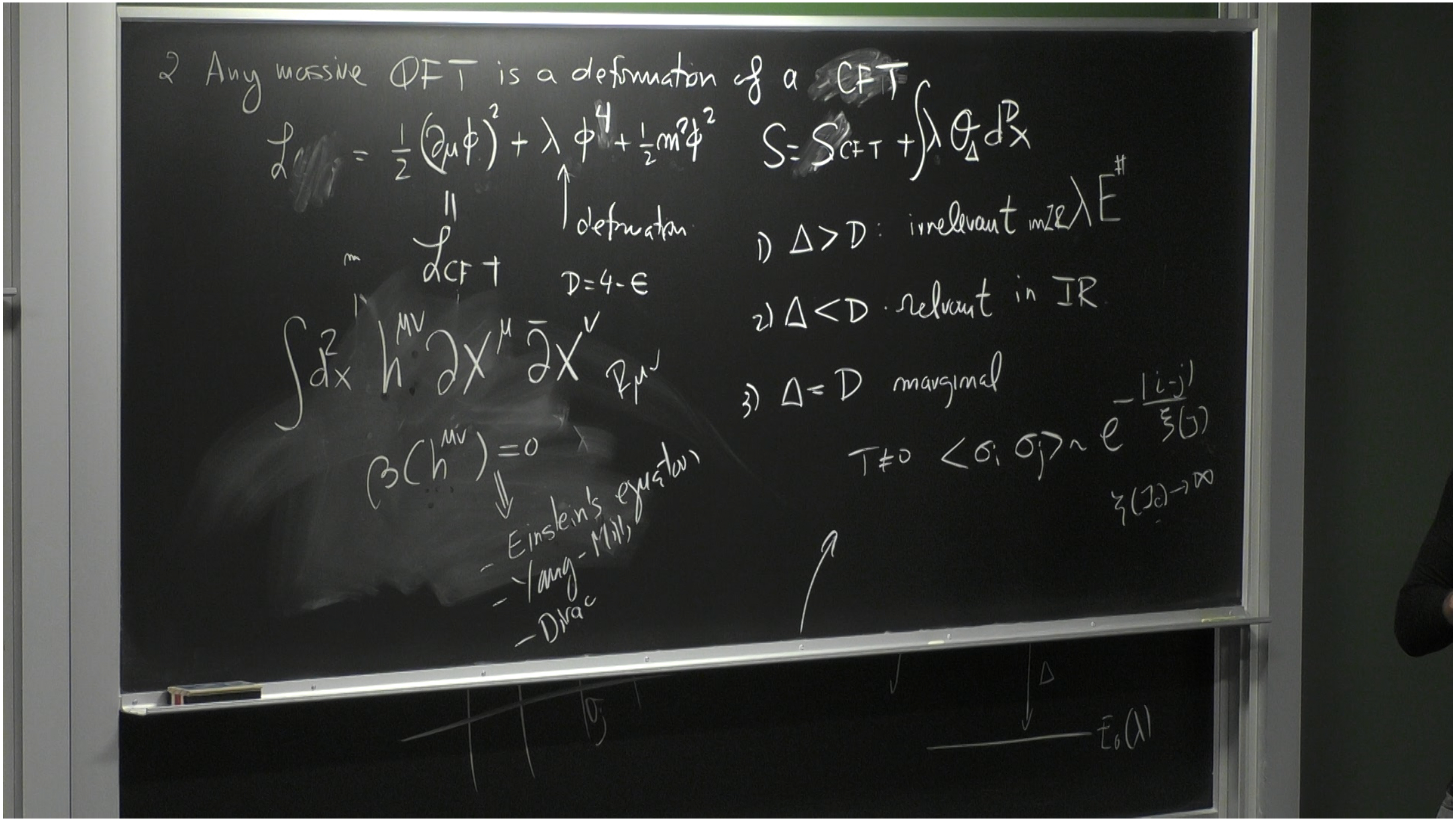
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deformation
$\mathcal{L}_{CFT} +$
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2) $\Delta < D$. relevant in IR
3) $\Delta = D$ marginal
$\int d^2x\, h^{\mu\nu}\partial X^\mu \bar\partial X^\nu$
$R_{\mu\nu}$
$\beta(h^{\mu\nu}) = 0$
$\Downarrow$
- Einstein's equation
- Yang-Mills
- Dirac
$T \neq 0 \quad \langle \sigma_i \sigma_j \rangle \sim e^{-\frac{|i-j|}{\xi(J)}}$
$\xi(J_c) \to \infty$
$\sigma_j$
$\Delta$
$E_0(\lambda)$