Title: Gravitational Physics Lecture

Speakers: Ruth Gregory

Collection: Gravitational Physics

Date: February 02, 2024 - 9:00 AM

URL: https://pirsa.org/24020024

# L12 Gravitational Instantons

$$ds^2 = dt^2 - \boxed{\left[ \frac{dr^2}{1-Q/r} + r(r-Q)\,d\Omega_{II}^2 + \left(1-\frac{Q}{r}\right)\left[\boxed{d\psi} + Q(1-\cos\theta)\,d\varphi\right]^2 \right]}$$

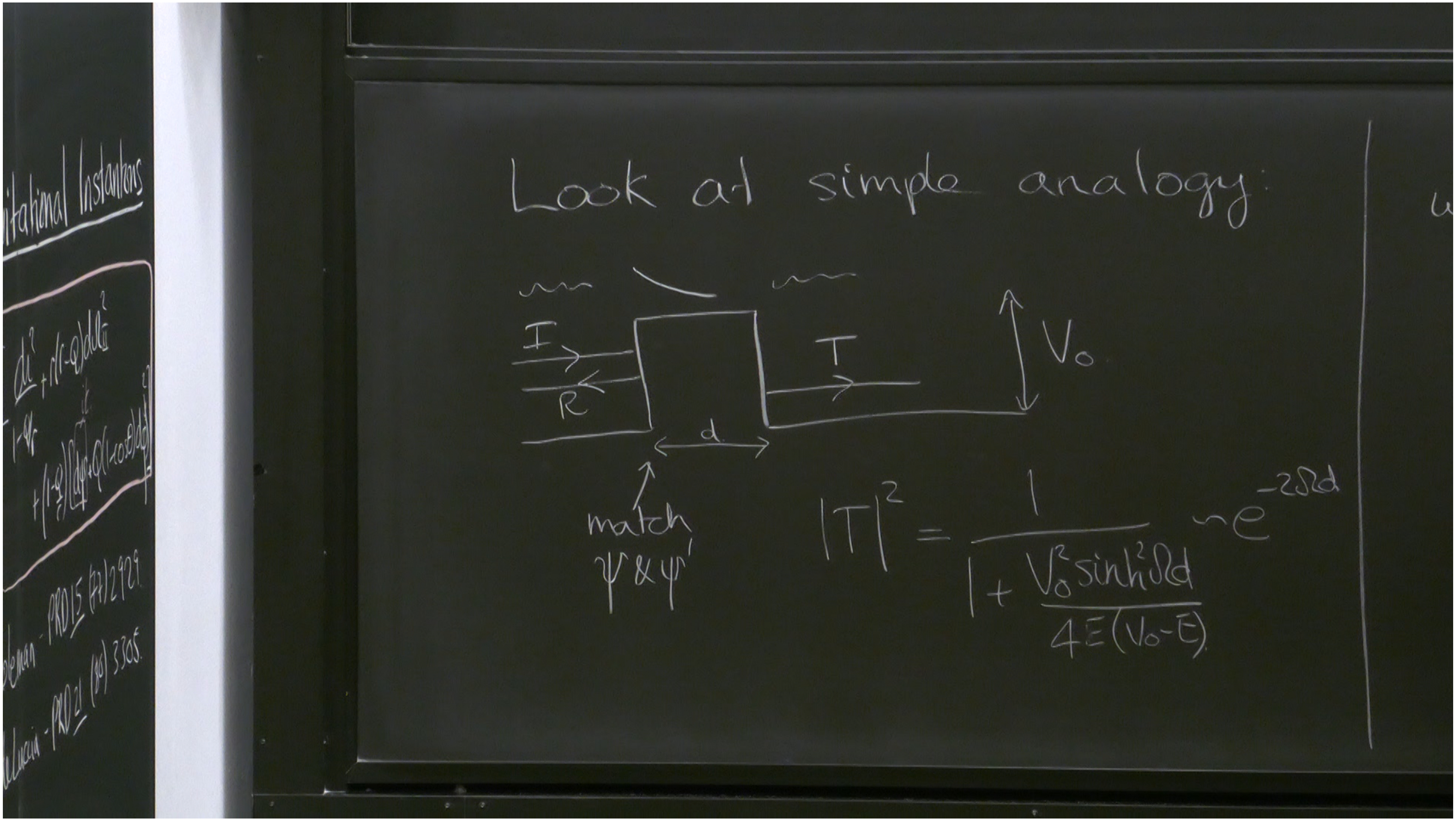
$it$

KK mon.

Taub-NUT instanton

Coleman - PRD 15 (77) 2929.

+deLuccia - PRD 21 (80) 3305.


Look at simple analogy:
I
R
T
V₀
d
match ψ & ψ'
|T|² = 1 / (1 + V₀² sinh² Ωd / 4E(V₀ − E)) ~ e^(−2Ωd)
itational Instantons
leman - PRD 15 (77) 2929
Lucia - PRD 21 (80) 3305

analogy:

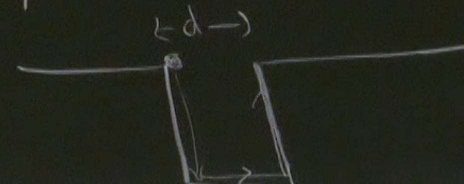
$V_0$

$$\frac{1}{\frac{V_0^2 \sinh^2 \Omega d}{4E(V_0-E)}} \sim e^{-2\Omega d}$$

where $\Omega^2 = \frac{2m}{\hbar^2}(V_0 - E) \sim \frac{2m}{\hbar^2}\Delta V$

$$\Omega d = \int_0^d \frac{\sqrt{2m\Delta V}}{\hbar}\, dx$$

Now consider a particle moving in a well of depth $\Delta V$ & width $d$.




$$\frac{1}{2} m \dot{x}^2 = \Delta V$$

S

$\Delta V$

...ing in

$\dot{x}^2 = \Delta V$

So $\displaystyle\int_0 \sqrt{2m\Delta V}\, dx = \int \sqrt{2m\Delta V}\, \dot{x}\, dt$

$\displaystyle = \int \Delta V + \frac{1}{2} m \dot{x}^2 \; dt$

$= S_E$. Action of ptcle in well

Quantum tunneling process $\longleftrightarrow$ Classical Euclidean trajectory.

in

$\Delta V.$

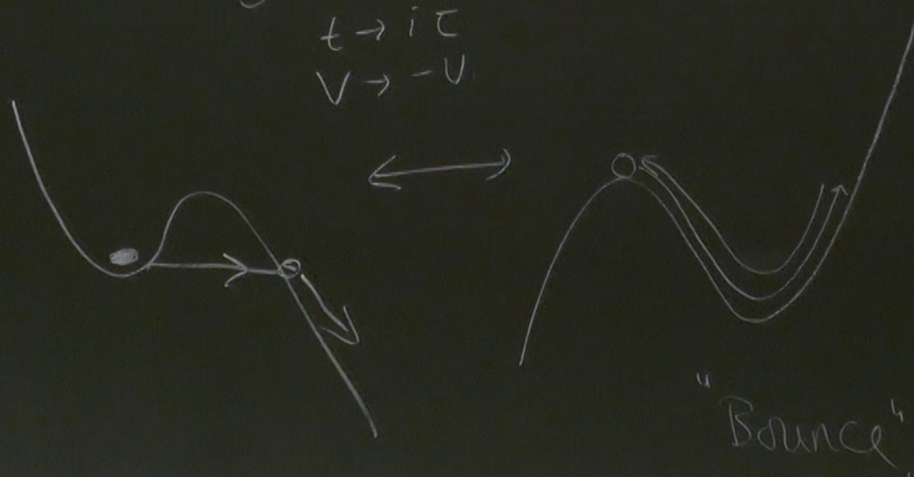
So $\displaystyle\int_0^d \sqrt{2m\Delta V}\, dx = \int \sqrt{2m\Delta V}\, \dot{x}\, dt$

$\displaystyle = \int \Delta V + \frac{1}{2} m \dot{x}^2 \, dt$

$= S_E.$ Action of ptcle in well

Quantum tunneling process $\longleftrightarrow$ Classical Euclidean trajectory.

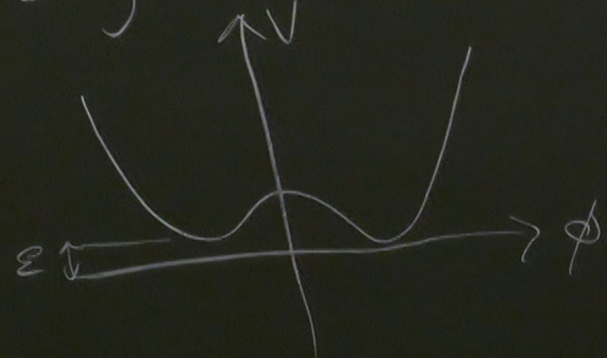
The exponent in amplitude corresponds to a "bounce" - particle moving to exit pt & back again.



...nds

...t pt

Suppose we have a $V$ with nearly degenerate minima:




$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V = \frac{\lambda}{2}(\phi^2-\eta^2)^2 - \epsilon\frac{(\phi-\eta)}{2\eta}$$

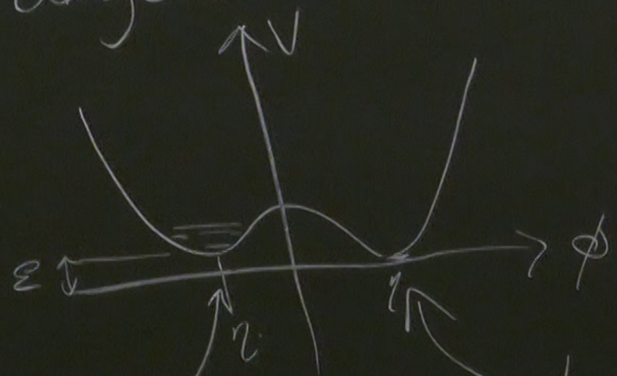
nds

pt

Suppose we have a $V$ with nearly degenerate minima:

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V = \frac{\lambda}{2}(\phi^2-\eta^2)^2 - \epsilon\frac{(\phi-\eta)}{2\eta}$$




$\phi = -\eta$

$V = \varepsilon$

FALSE VACUUM.

$\phi = \eta$

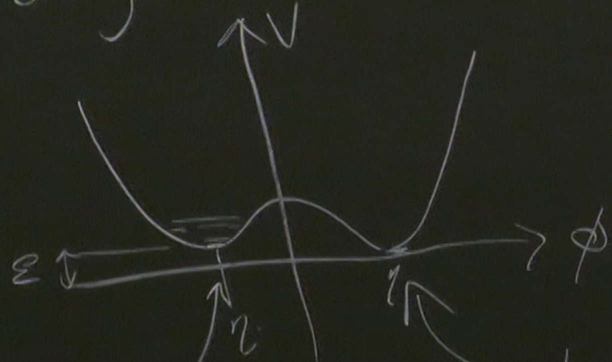
$V = 0$

TRUE VAC.

nds

pt

Suppose we have a $V$ with nearly degenerate minima:




$\phi = -\eta$

$V = \varepsilon$

FALSE VACUUM.

$\phi = \eta$

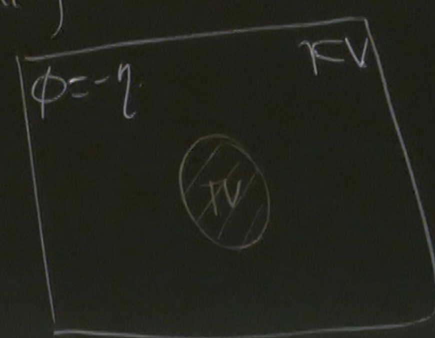
$V = 0$

TRUE VAC.

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V = \frac{\lambda}{2}(\phi^2-\eta^2)^2 - \frac{\epsilon(\phi-\eta)}{2\eta}$$

Initially

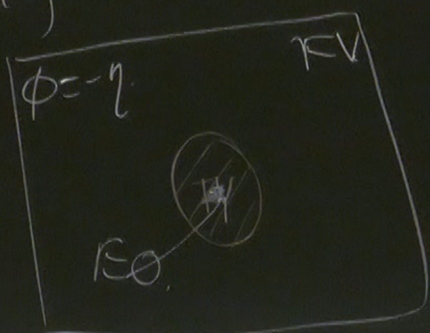


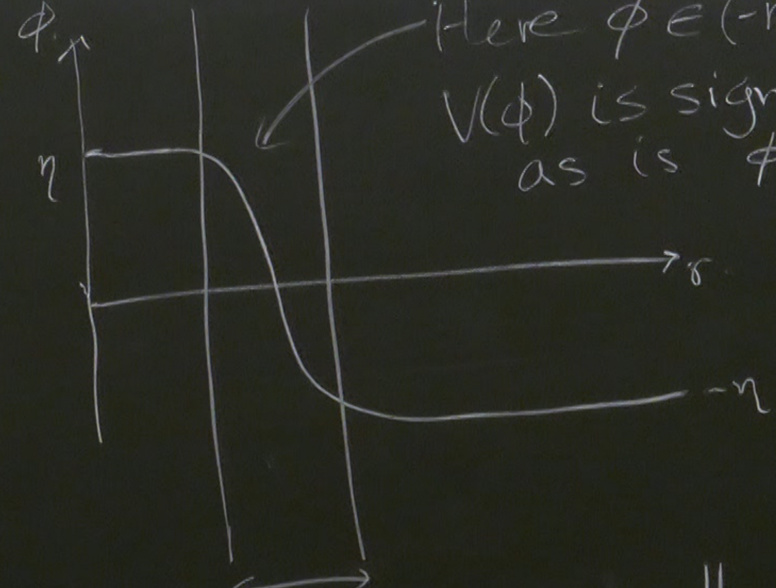
V with nearly

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V = \frac{\lambda}{2}(\phi^2-\eta^2)^2 - \epsilon\frac{(\phi-\eta)}{2\eta}$$

Initially




Here $\phi \in (-\eta, \eta)$ so $V(\phi)$ is significant as is $\phi'$.

higher barrier $\leftrightarrow$ smaller width

Local energy density.

"Bounce"

FALSE VACUUM

$$\Box\phi + \frac{\partial V}{\partial \phi} = 0$$

$$-\phi'' + V'(\phi) = 0$$

$\cdot \phi'$ & $\int$. $\quad -\frac{1}{2}\phi'^2 + V = \text{const}$

to $O(\epsilon^0)$ const $= 0$

$$T_{ab} = \phi_{,a}\phi_{,b} - g_{ab}\left(\tfrac{1}{2}(\partial\phi)^2 - V\right)$$

$$T^0{}_0 = V + \tfrac{1}{2}\phi'^2 = 2V. \ (= T_{\text{parallel}})$$

$$T^\perp{}_\perp = 0$$

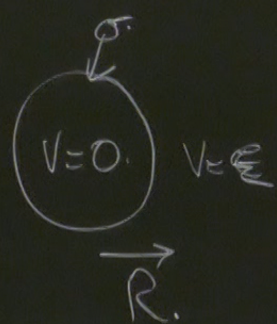
Bubble wall has $T_{ab} \simeq \sigma(r)\, h_{ab}$

VAC.

"Energy budget": By quantum uncertainty, a bubble nucleates. If too small, energy of wall causes it to recollapse. Too big & has a much lower probability to form.

Energy in wall $= 2\pi^2 R^3 \sigma$. (4D)

Energy gain from TV $= \frac{\pi^2}{2} R^4 \varepsilon$.



$$\frac{d}{dR}\left(\delta E = 2\pi^2 R^3 \sigma - \frac{\pi^2}{2} R^4 \varepsilon\right)$$

$$= 6\pi^2 R^2 \sigma - 2\pi^2 R^3 \varepsilon$$

$$= 0 \text{ for } R_0 = 3\sigma/\varepsilon.$$

Action (or cost) $$B = \frac{\pi^2}{2} R_0^{\,3} (4\sigma - \varepsilon R_0)$$

$$= \frac{27\pi^2}{2} \sigma^4/\varepsilon^3.$$

What happens next?

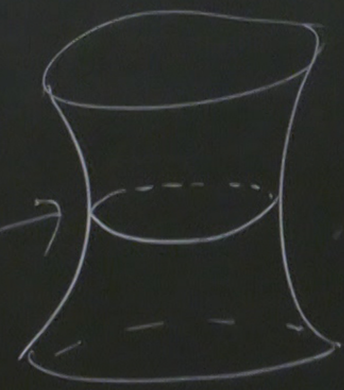
Spherical bubble in $\mathbb{R}^4$

$$\rho^2+\tau^2=R_0^2$$

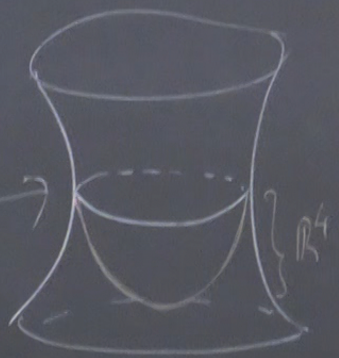
$x^2+y^2+z^2$

$\rightarrow$ Lorentzian time $\tau \to it$.

$$\rho^2-t^2=R_0^2$$



nucleation




nucleation

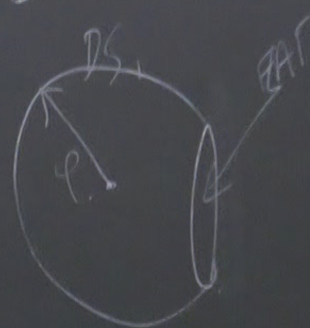
INSTANTON

Energy gravitates, so need to include GR

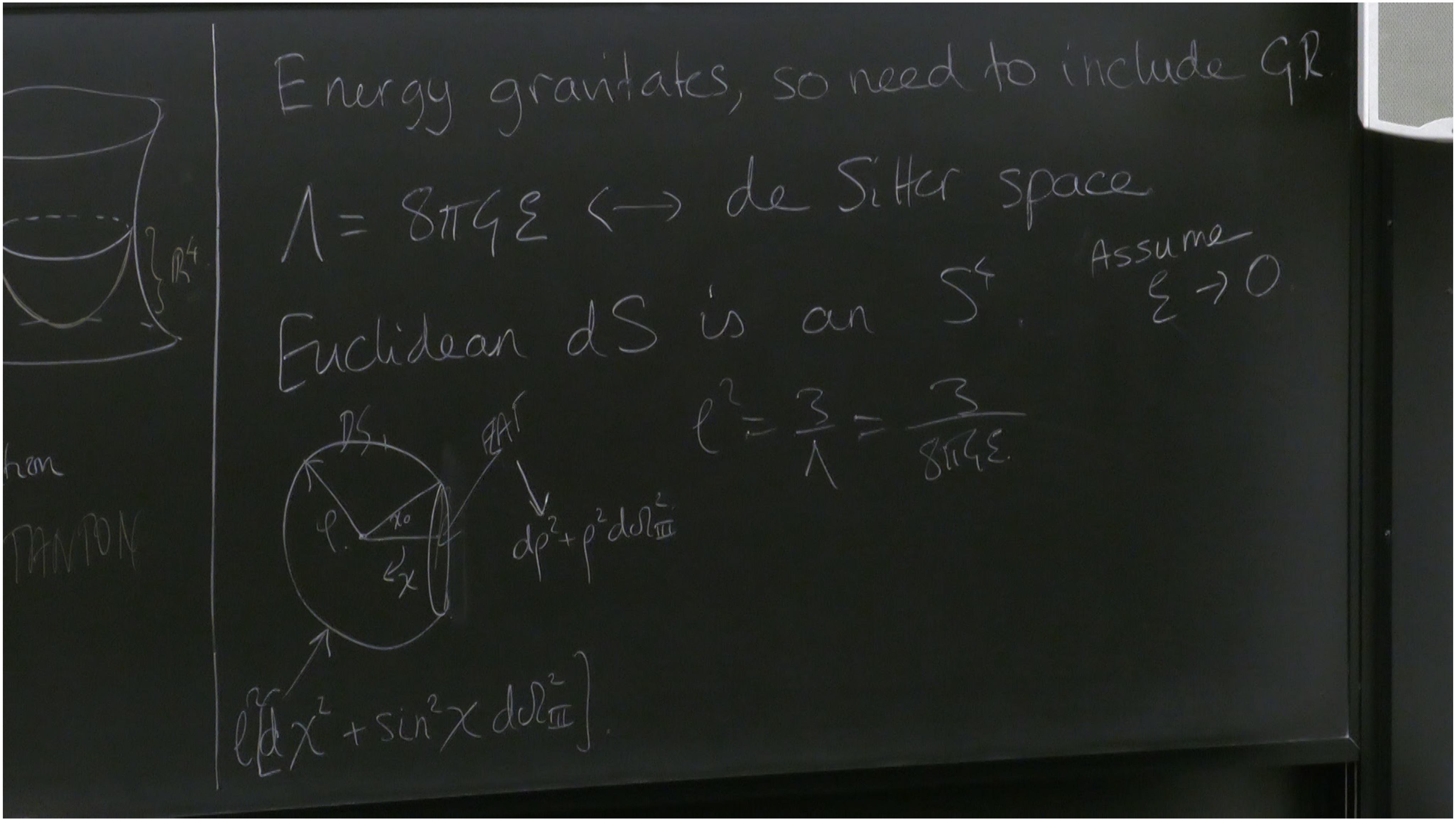
$\Lambda = 8\pi G \mathcal{E} \longleftrightarrow$ de Sitter space

Euclidean dS is an $S^4$

Assume $\mathcal{E} \to 0$




$$\ell^2 = \frac{3}{\Lambda} = \frac{3}{8\pi G \mathcal{E}}$$


Energy gravitates, so need to include G.R.
Λ = 8πGε ⟷ de Sitter space
Assume
ε → 0
Euclidean dS is an S⁴.
ℓ² = 3/Λ = 3/(8πGε).
DS
FLAT
dρ² + ρ² dΩ²_III
ρ
χ₀
χ
ℓ²[dχ² + sin²χ dΩ²_III].
ℝ⁴
TANTON

Instanton comprises $\mathcal{M}_-, W, \mathcal{M}_+$

$$\mathcal{M}_- : \quad ds_-^2 = d\rho^2 + \rho^2 d\Omega_{\mathrm{III}}^2 \qquad \rho < R_0 = l \sin\chi_0 .$$

$$\mathcal{M}_+ : \quad ds_+^2 = l^2 [d\chi^2 + \sin^2\chi \, d\Omega_{\mathrm{III}}^2] \qquad \chi > \chi_0 .$$

$$W : \quad ds_3^2 = R_0^2 \, d\Omega_{\mathrm{III}}^2 \;=\; \partial\mathcal{M}_- = \partial\mathcal{M}_+$$

Recalling the GH boundary term, $(K_{ab} - K h_{ab})\delta h^{ab}$

$\mathcal{M}_+$

$< R_0 = l \sin \chi_0 .$

$\chi > \chi_0 .$

$\partial \mathcal{M}_- = \partial \mathcal{M}_+$

$(K_{ab} - K h_{ab}) \delta h^{ab}$

Dernie Israel eqns $\quad \int_-^+ T_{ab}\, d\text{"}x\text{"}$

$$\Delta K_{ab} - \Delta K h_{ab} = 8\pi G S_{ab}$$
$$= 8\pi G \sigma h_{ab}$$

$$n_+ = \frac{1}{l}\frac{\partial}{\partial \chi} \qquad n_- = \frac{\partial}{\partial \rho}$$

(angles)

$$K_{+\alpha\beta} = \frac{1}{l}\Gamma^{\chi}_{\alpha\beta} = -\frac{\cot \chi_0}{l} g_{\alpha\beta}$$

$$K_{-\alpha\beta} = \Gamma^{\rho}_{\alpha\beta} = -\frac{1}{\rho_0} g_{\alpha\beta}$$

$\Delta K_{\alpha\beta} - \Delta$

eqns $\int_{-}^{+} T_{ab}\, d\text{"}x\text{"}$ $\downarrow$

$= 8\pi G\, S_{ab}$

$= 8\pi G \sigma\, h_{ab}$

$n_{-} = \frac{\partial}{\partial \rho}$

$-\frac{\cot \chi_0}{\ell} g_{\alpha\beta}$

$-\frac{1}{\rho_0} g_{\alpha\beta}$

$$\Delta K_{\alpha\beta} - \Delta K\, h_{\alpha\beta} = -\frac{2 h_{\alpha\beta}}{\ell \sin\chi_0}(1-\cos\chi_0)$$

$$= -8\pi G \sigma\, h_{\alpha\beta}$$

i.e. $$\frac{(1-\cos\chi_0)}{\ell \sin\chi_0} = \frac{\left(1-\sqrt{1-R^2/\ell^2}\right)}{R} = 4\pi G\sigma.$$

b

b

$$\ell \sin\chi_0$$

$$= -8\pi G \sigma \, h_{\alpha\beta}$$

i.e. $$\frac{(1-\cos\chi_0)}{\ell \sin\chi_0} = \frac{\left(1-\sqrt{1-R^2/\ell^2}\right)}{R} = 4\pi G \sigma .$$

$$B = I_{INST} - I_{BACK}$$

$$= \frac{1}{16\pi G}\int_{\chi<\chi_0} \sqrt{g}\,\underbrace{(R-2\Lambda)}_{2\Lambda} - \frac{1}{8\pi G}\int_{\partial M} \overset{12\pi G\sigma}{\Delta K}\sqrt{h} + \int_{W} \sigma\sqrt{h}$$

Recalling the GH boundary term, ... | $K_{\alpha\beta} = T_{\alpha\beta}$ ...

$$= \frac{3}{8\pi G \ell^2} \cdot 2\pi^2 \ell^4 \int_0^{\chi_0} \sin^3\chi \, d\chi - \sigma \pi^2 R^3 .$$

$$\vdots$$

$$= \frac{\pi \ell^2}{4G} \left(1 - \cos\chi_0\right)^2 = \frac{\pi \ell^2}{4G} \left(4\pi G \sigma R\right)^2 .$$

$$= \frac{3}{8\pi G \ell^2} \cdot 2\pi^2 \ell^4 \int_0^{\chi_0} \sin^3\chi \, d\chi - \sigma \pi^2 R^3$$

to

$B =$

$$\vdots$$

$$= \frac{\pi \ell^2}{4G}\left(1-\cos\chi_0\right)^2 = \frac{\pi \ell^2}{4G}\left(4\pi G \sigma R\right)^2.$$

Finally, solve for $R(\sigma) = \dfrac{2\ell^2 (4\pi G\sigma)}{(1+(4\pi G\sigma \ell)^2)}$

$R^3$.

to get

$$B = \frac{\pi \ell^2}{G} \frac{(4\pi G\sigma\ell)^4}{[1+(4\pi G\sigma\ell)^2]^2}$$

$$\ell^2 = \frac{3}{8\pi G\xi}$$

$G\ell \to 0$ as $G \to 0$

$$\frac{\ldots\pi G\sigma)}{\ldots\pi G\sigma\ell)^2)}$$