

Title: Numerical Methods Lecture

Speakers: Dustin Lang

Collection: Numerical Methods 2023/24

Date: February 06, 2024 - 10:15 AM

URL: <https://pirsa.org/24020018>

Solving Hyperbolic PDEs via Finite Differences

```
[1]: using CairoMakie  
      using DifferentialEquations  
      using LinearAlgebra
```

[]:



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Finite Diffe... (4) - JupyterLab

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Representing Functions.ipynk Multiscale.ipynb Finite Differences.ipynb +

Code Julia 1.10.0

Solving Hyperbolic PDEs via Finite Differences

```
[1]: using CairoMakie
      using DifferentialEquations
      using LinearAlgebra
```

```
[ ]: |
```

Simple 0 \$ 5 Julia 1.10.0 | Idle Mode: Edit Ln 1, Col 1 Finite Differences.ipynb 0

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Representing Functions.ipynk Multiscale.ipynb Finite Differences.ipynb +

Code Julia 1.10.0

```
[4]: U = zeros(npoints, npoints, 2)
```

```
[4]: 5x5x2 Array{Float64, 3}:  
[:, :, 1] =  
 0.0  0.0  0.0  0.0  0.0  
 0.0  0.0  0.0  0.0  0.0  
 0.0  0.0  0.0  0.0  0.0  
 0.0  0.0  0.0  0.0  0.0  
 0.0  0.0  0.0  0.0  0.0  
  
[:, :, 2] =  
 0.0  0.0  0.0  0.0  0.0  
 0.0  0.0  0.0  0.0  0.0  
 0.0  0.0  0.0  0.0  0.0  
 0.0  0.0  0.0  0.0  0.0  
 0.0  0.0  0.0  0.0  0.0
```

[]: |

Simple 0 5 Julia 1.10.0 | Idle Mode: Edit Ln 1, Col 1 Finite Differences.ipynb 0

```
h = 2 / (npoints-1)
```

[2]: 0.5

```
•[4]: function init()
      x0 = 0.0
      y0 = -0.5
      W = 0.2
      U = zeros(npoints, npoints, 2)
      for j in 1:npoints, i in 1:npoints
        x = -1 + (i-1) * h
        y = -1 + (j-1) * h
        phi = exp(-((x-x0)^2+(y-y0)^2)/2W)
        psi = 0
        U[i,j,1] = phi
        U[i,j,2] = psi
      end
      return U
    end
```

[4]: 5x5x2 Array{Float64, 3}:
[. . . 1]

```
U[i,j,2] = 0
else
  x = -1 + (i-1) * h
  y = -1 + (j-1) * h
  phi = exp(-((x-x0)^2+(y-y0)^2)/2W)
  psi = 0
  U[i,j,1] = phi
  U[i,j,2] = psi
end
end
return U
end
```

[6]: init (generic function with 1 method)

```
[ ]: function rhs(U)
      Udot = similar(U)
```

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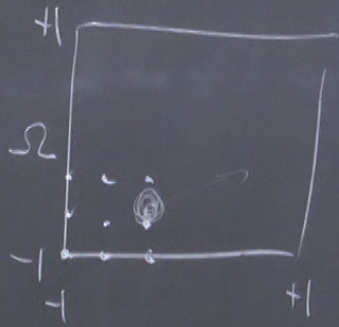
Code Julia 1.10.0

```
        U[i,j,2] = psi
    end
end
return U
end
```

[6]: init (generic function with 1 method)

```
[ ]: function rhs(U)
    Udot = similar(U)
    for j in 1:npoints, i in 1:npoints
        if i==1 || i==npoints || j==1 || j==npoints
            U[i,j,1] = 0
            U[i,j,2] = 0
        else
            end
        end
    end
    return Udot
end
```

Simple 0 5 Julia 1.10.0 | Idle Mode: Edit Ln 8, Col 1 Finite Differences.ipynb 0



$$\partial_t^2 \phi = \partial_x^2 \phi + \partial_y^2 \phi$$

$$\phi|_{\partial\Omega} = 0$$

$$\psi := \partial_t \phi$$

$$\partial_t \phi = \psi$$

$$\partial_t \psi = \partial_x^2 \phi + \partial_y^2 \phi$$

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Finite Diffe... (4) - JupyterLab

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Representing Functions.ipynk Multiscale.ipynb Finite Differences.ipynb +

Code Julia 1.10.0

```
        U[i,j,2] = psi
    end
end
return U
end
```

[6]: init (generic function with 1 method)

```
[ ]: function rhs(U)
    Udot = similar(U)
    for j in 1:npoints, i in 1:npoints
        if i==1 || i==npoints || j==1 || j==npoints
            U[i,j,1] = 0
            U[i,j,2] = 0
        else
            phixx = (U[i-1,j,1] - 2 |
        end
    end
    return Udot
end
```

Simple 0 \$ 5 Julia 1.10.0 | Idle Mode: Edit Ln 8, Col 24 Finite Differences.ipynb 0

```
function rhs(U)
    Udot = similar(U)
    for j in 1:npoints, i in 1:npoints
        if i==1 || i==npoints || j==1 || j==npoints
            U[i,j,1] = 0
            U[i,j,2] = 0
        else
            end
        end
    end
    return Udot
end
```


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eschnetter (4) - JupyterLab x Fix connection errors - Goog x Google

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Representing Functions.ipynb x Multiscale.ipynb x +

Julia 1.10.0

```
[1]: using LegendrePolynomials
[2]: using WGLMakie
[3]: xs = -1.0:0.01:1
      fig = Figure(size=(640, 400))
      axis = Axis(fig[1, 1]; title="Legendre Polynomials")
      for l in 0:4
          lines!(axis, xs, [Pl(x,l) for x in xs])
      end
      fig
Warning: Port in use, using different port. New port: 9386
@ Bonito.HTTPServer ~/.julia/packages/Bonito/ULACo/src/HTTPServer/implementation.jl:274
[3]:
```

```
[4]: using HCubature
[5]: hcubature(x -> sin(x[1]), [0.0], [pi])
[5]: (2.0000000000000004, 1.7896795156957523e-12)
[6]: integrate(f, x0, x1) = hcubature(x -> f(x[1]), [x0], [x1]; atol=sqrt(eps()))[1]
[6]: integrate (generic function with 1 method)
```

Simple 0 0 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Representing Functions.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Goog... x Google x | +

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Representing Functions.ipyn x Finite Differences.ipynb x Multiscale.ipynb x +

Julia 1.10.0

```
[2]: 0.5
```

```
[6]: function init()
      x0 = 0.0
      y0 = -0.5
      W = 0.2
      U = zeros(npoints, npoints, 2)
      for j in 1:npoints, i in 1:npoints
          if i==1 || i==npoints || j==1 || j==npoints
              U[i,j,1] = 0
              U[i,j,2] = 0
          else
              x = -1 + (i-1) * h
              y = -1 + (j-1) * h
              phi = exp(-((x-x0)^2+(y-y0)^2)/2W)
              psi = 0
              U[i,j,1] = phi
              U[i,j,2] = psi
          end
      end
      return U
  end
```

```
[6]: init (generic function with 1 method)
```

```
[ ]: function rhs(U)
      Udot = similar(U)
      for j in 1:npoints, i in 1:npoints
          if i==1 || i==npoints || j==1 || j==npoints
              U[i,j,1] = 0
              U[i,j,2] = 0
          else
              phixx = (U[i-1,j,1] - 2*U[i,j,1] + U[i+1,j,1]) / h^2
          end
      end
      return Udot
  end
```

Simple 0 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Code Julia 1.10.0

```
        U[i,j,2] = psi
    end
end
return U
end
```

[6]: init (generic function with 1 method)

```
[ ]: function rhs(U)
    Udot = similar(U)
    for j in 1:npoints, i in 1:npoints
        if i==1 || i==npoints || j==1 || j==npoints
            Udot[i,j,1] = 0
            Udot[i,j,2] = 0
        else
            phixx = (U[i-1,j,1] - 2*U[i,j,1] + U[i+1,j,1]) / h^2
            phiyy = (U[i,j-1,1] - 2*U[i,j,1] + U[i,j+1,1]) / h^2
        end
    end
    return Udot
end
```

Simple 0 0 Julia 1.10.0 | Idle Mode: Edit Ln 7, Col 1 Finite Differences.ipynb 0

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Code Julia 1.10.0

```
[7]: function rhs(U)
      Udot = similar(U)
      for j in 1:npoints, i in 1:npoints
          if i==1 || i==npoints || j==1 || j==npoints
              Udot[i,j,1] = 0
              Udot[i,j,2] = 0
          else
              phixx = (U[i-1,j,1] - 2*U[i,j,1] + U[i+1,j,1]) / h^2
              phiyy = (U[i,j-1,1] - 2*U[i,j,1] + U[i,j+1,1]) / h^2
              Udot[i,j,1] = U[i,j,2]
              Udot[i,j,2] = phixx + phiyy
          end
      end
      return Udot
end
```

[7]: rhs (generic function with 1 method)

[]: |

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Edit Ln 1, Col 1 Finite Differences.ipynb 0

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Code Julia 1.10.0

```
end
return U
end

[6]: init (generic function with 1 method)

[7]: function rhs(U)
      Udot = similar(U)
      for j in 1:npoints, i in 1:npoints
          if i==1 || i==npoints || j==1 || j==npoints
              Udot[i,j,1] = 0
              Udot[i,j,2] = 0
          else
              phixx = (U[i-1,j,1] - 2*U[i,j,1] + U[i+1,j,1]) / h^2
              phiyy = (U[i,j-1,1] - 2*U[i,j,1] + U[i,j+1,1]) / h^2
              Udot[i,j,1] = U[i,j,2]
              Udot[i,j,2] = phixx + phiyy
          end
      end
      return Udot
end

[7]: rhs (generic function with 1 method)

[8]: U0 = init()
```

Simple 0 0 Julia 1.10.0 | Idle Mode: Edit Ln 1, Col 15 Finite Differences.ipynb 0

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Code Julia 1.10.0

```
Udot[i,j,2] = phixx + phiyy
    end
end
return Udot
end
```

[9]: rhs (generic function with 2 methods)

```
[11]: U0 = init()
      tspan = (0.0, 4.0)
      prob = ODEProblem(rhs, U0, tspan)
      sol = solve(prob)
```

[11]: retcode: Success
Interpolation: specialized 4th order "free" interpolation, specialized 2nd order "free" stiffness-aware interpolation
t: 26-element Vector{Float64}:
0.0
0.00027008351198874465
0.002970918631876191
0.02147206097588478
0.05960910571175081
0.11329010530910921
0.1834795589660274
0.2749087440943005
0.30124110601158407

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Goog... x Google x | +

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Code Julia 1.10.0

```
Udot[i,j,1] = U[i,j,2]
Udot[i,j,2] = phixx + phiyy
end
end
return Udot
end
```

[9]: rhs (generic function with 2 methods)

```
U0 = init()
tspan = (0.0, 4.0)
prob = ODEProblem(rhs, U0, tspan)
sol = solve(prob);
```

[11]: retcode: Success
Interpolation: specialized 4th order "free" interpolation, specialized 2nd order "free" stiffness-aware interpolation
t: 26-element Vector{Float64}:
0.0
0.00027008351198874465
0.002970918631876191
0.02147206097588478
0.05960910571175081
0.11329010530910921
0.1834795589660274
0.2749087440943005

Simple 0 Julia 1.10.0 | Idle Mode: Edit Ln 4, Col 19 Finite Differences.ipynb

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Goog... x Google x | +

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Representing Functions.ipynb x Finite Differences.ipynb • Multiscale.ipynb x +

Code Julia 1.10.0

```
Udot[i,j,1] = U[i,j,2]
Udot[i,j,2] = phixx + phiyy

end
end
return Udot
end
```

[9]: rhs (generic function with 2 methods)

```
[12]: U0 = init()
tspan = (0.0, 4.0)
prob = ODEProblem(rhs, U0, tspan)
sol = solve(prob);
sol.t
```

[12]: 26-element Vector{Float64}:
0.0
0.00027008351198874465
0.002970918631876191
0.02147206097588478
0.05960910571175081
0.11329010530910921
0.1834795589660274
0.2749087440943005
0.39134119681158497
0.5371764070225316

Simple 0 Julia 1.10.0 | Idle Mode: Edit Ln 5, Col 1 Finite Differences.ipynb

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Finite Diffe... (4) - JupyterLab x Fix connection errora - Goog| x Google x | +

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Code Julia 1.10.0

fig

[14]:

x \ y	-1.0	-0.5	0.0	0.5	1.0
1.0	Dark Purple	Dark Purple	Dark Purple	Dark Purple	Dark Purple
0.5	Dark Purple	Dark Purple	Dark Purple	Dark Purple	Dark Purple
0.0	Dark Purple	Dark Purple	Dark Purple	Dark Purple	Dark Purple
-0.5	Dark Purple	Dark Purple	Dark Purple	Dark Purple	Dark Purple
-1.0	Dark Purple	Dark Purple	Dark Purple	Dark Purple	Dark Purple

Simple 0 0 Julia 1.10.0 | Idle Mode: Edit Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x | +

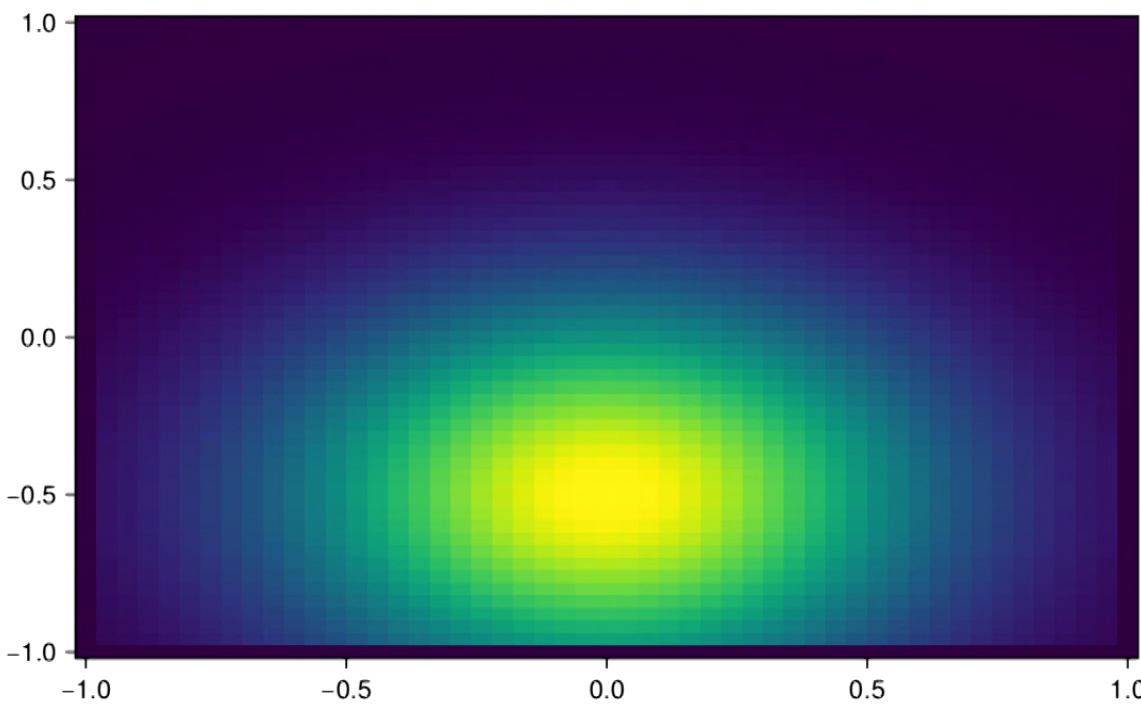
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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

[20]:



1.0
0.5
0.0
-0.5
-1.0

-1.0 -0.5 0.0 0.5 1.0

Simple 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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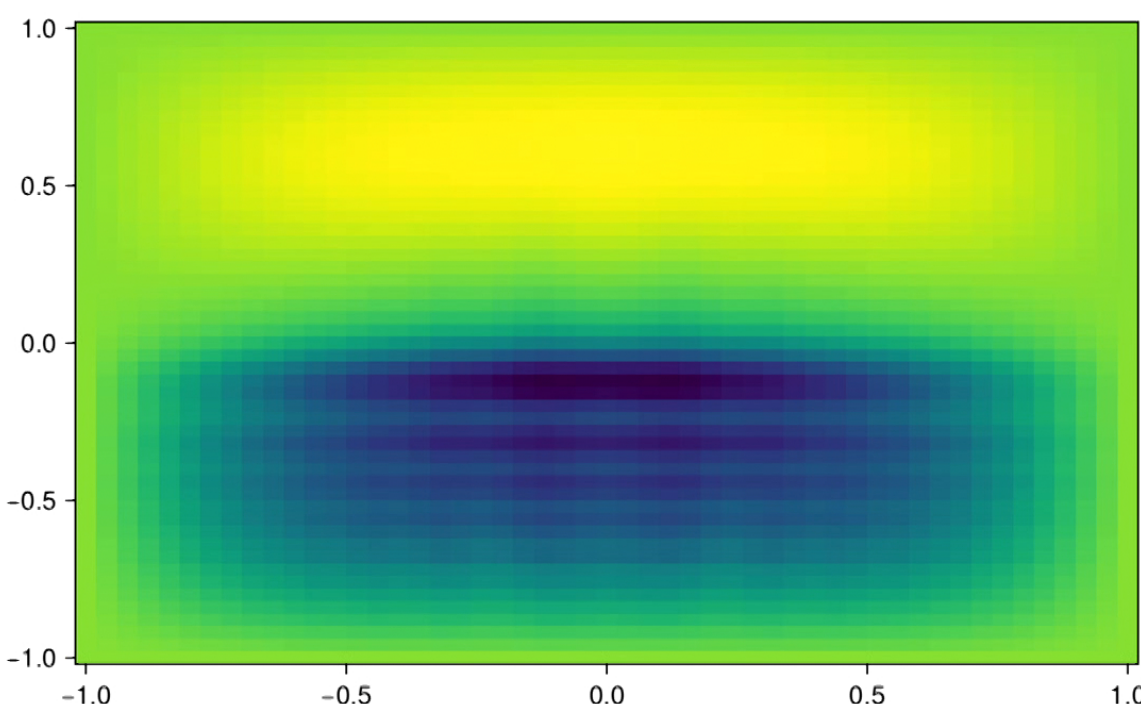
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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
heatmap!(Axis(fig[1,1]), -1:h:+1, -1:h:+1, sol(1.0)[:,:,:1])  
fig
```

[21]:



Simple 0 0 Julia 1.10.0 | Idle Mode: Edit Ln 1, Col 1 Finite Differences.ipynb 0

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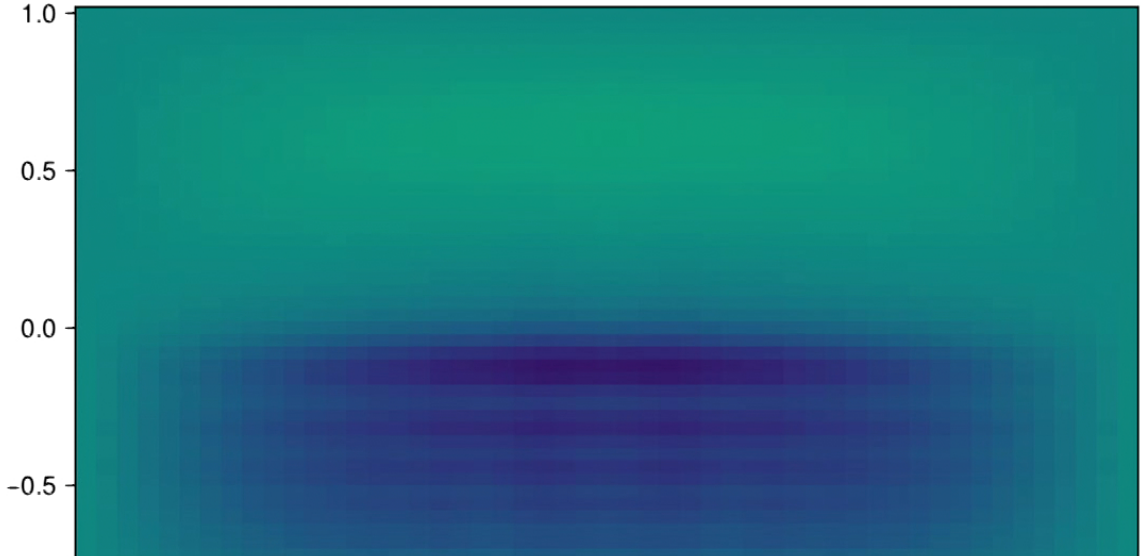
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Representing Functions.ipyn x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
[23]: fig = Figure(size=(640, 400))
heatmap!(Axis(fig[1,1]), -1:h:+1, -1:h:+1, sol(1.0)[:,:,:1]; colorrange=(-1,+1))
fig
```



[23]: 1.0
0.5
0.0
-0.5

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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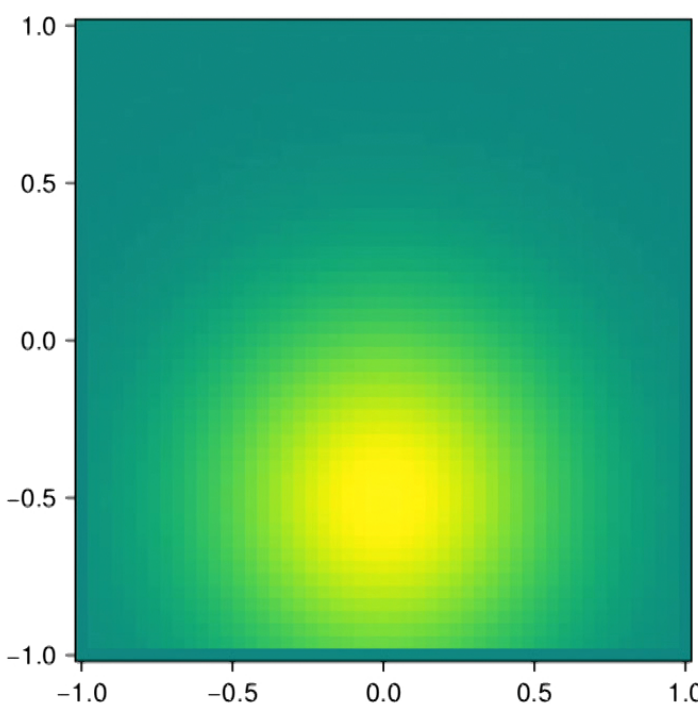
Not Secure <https://symmetry.pi.local/user/eschnetter/lab/tree/home/eschnetter/Finite%20Differences.ipynb>

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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

[24]:



Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Edit Ln 3, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x New Tab x +

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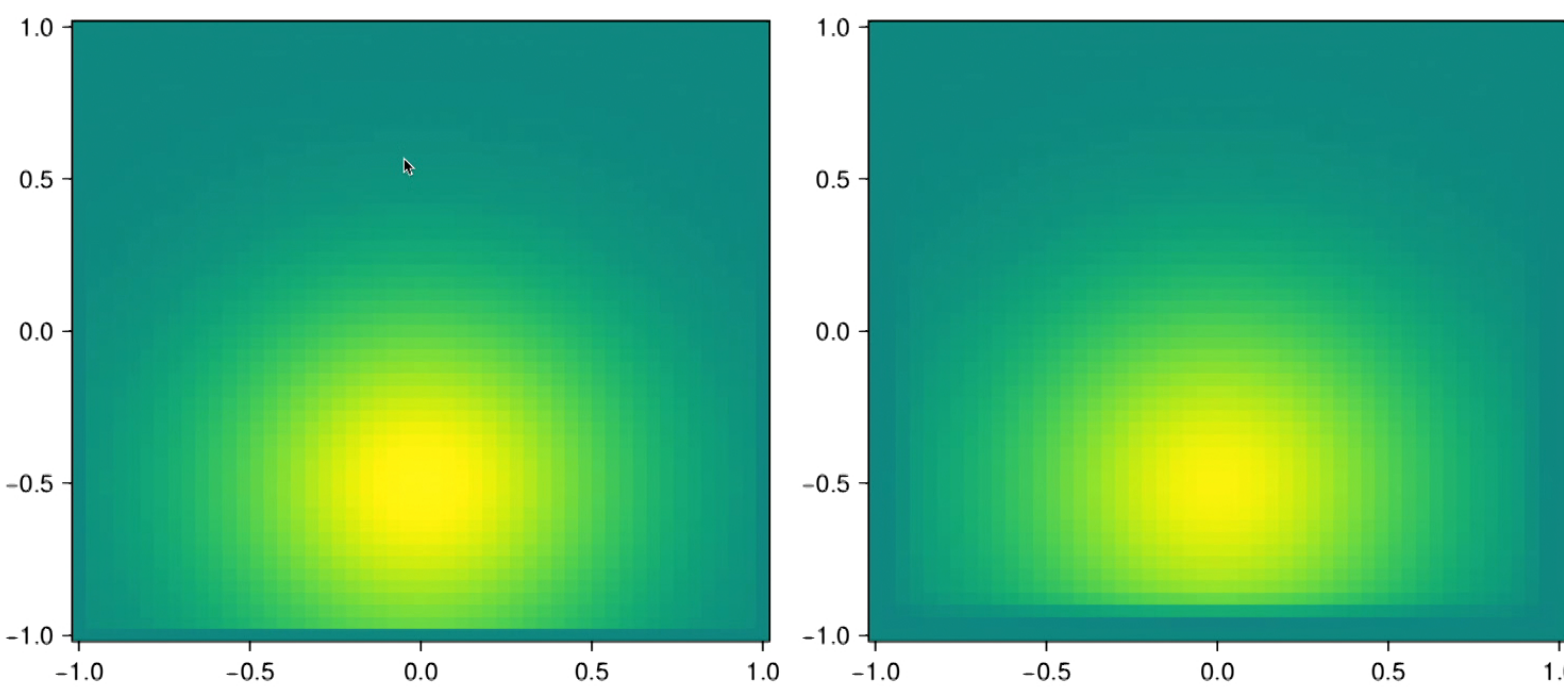
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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
heatmap!(Axis(fig[1,2]), -1:h:+1, -1:h:+1, sol(0.1)[:,:,1]; colorrange=(-1,+1))  
fig
```

[28]:



Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 3, Col 1 Finite Differences.ipynb 0

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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

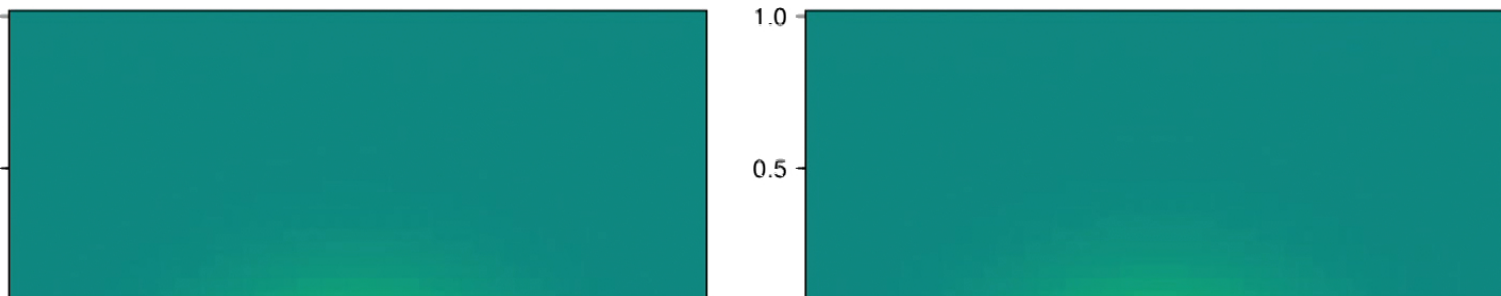
```
[32]: rhs (generic function with 2 methods)
```

```
[33]: U0 = init()
      tspan = (0.0, 4.0)
      prob = ODEProblem(rhs, U0, tspan)
      sol = solve(prob);
      length(sol.t)
```

```
[33]: 184
```

```
[34]: fig = Figure(size=(900, 400))
      heatmap!(Axis(fig[1,1]), -1:h:+1, -1:h:+1, sol(0.0)[:,:,:1]; colrange=(-1,+1))
      heatmap!(Axis(fig[1,2]), -1:h:+1, -1:h:+1, sol(0.1)[:,:,:1]; colrange=(-1,+1))
      fig
```

```
[34]:
```



Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x New Tab x | +

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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
[30]: 0.04
```

```
[31]: function init()
      x0 = 0.0
      y0 = -0.5
      W = 0.1
      U = zeros(npoints, npoints, 2)
      for j in 1:npoints, i in 1:npoints
          if i==1 || i==npoints || j==1 || j==npoints
              U[i,j,1] = 0
              U[i,j,2] = 0
          else
              x = -1 + (i-1) * h
              y = -1 + (j-1) * h
              phi = exp(-((x-x0)^2+(y-y0)^2)/2W)
              psi = 0
              U[i,j,1] = phi
              U[i,j,2] = psi
          end
      end
      return U
    end
```

```
[31]: init (generic function with 1 method)
```

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Edit Ln 3, Col 10 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x New Tab x +

Not Secure https://symmetry.pi.local/user/eschnetter/lab/tree/home/eschnetter/Finite%20Differences.ipynb

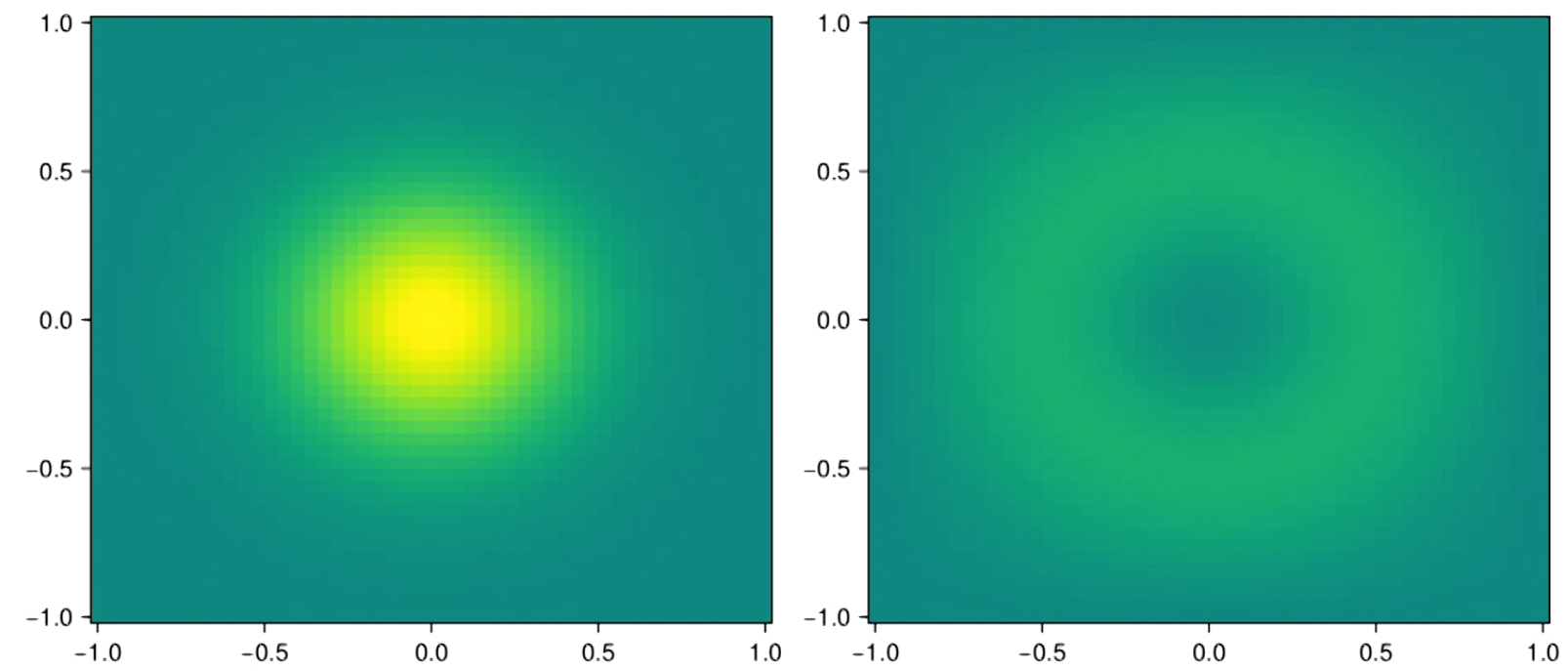
File Edit View Run Kernel Tabs Settings Help

Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

fig

[46]:



Simple 0 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Goog x Google x New Tab x | +

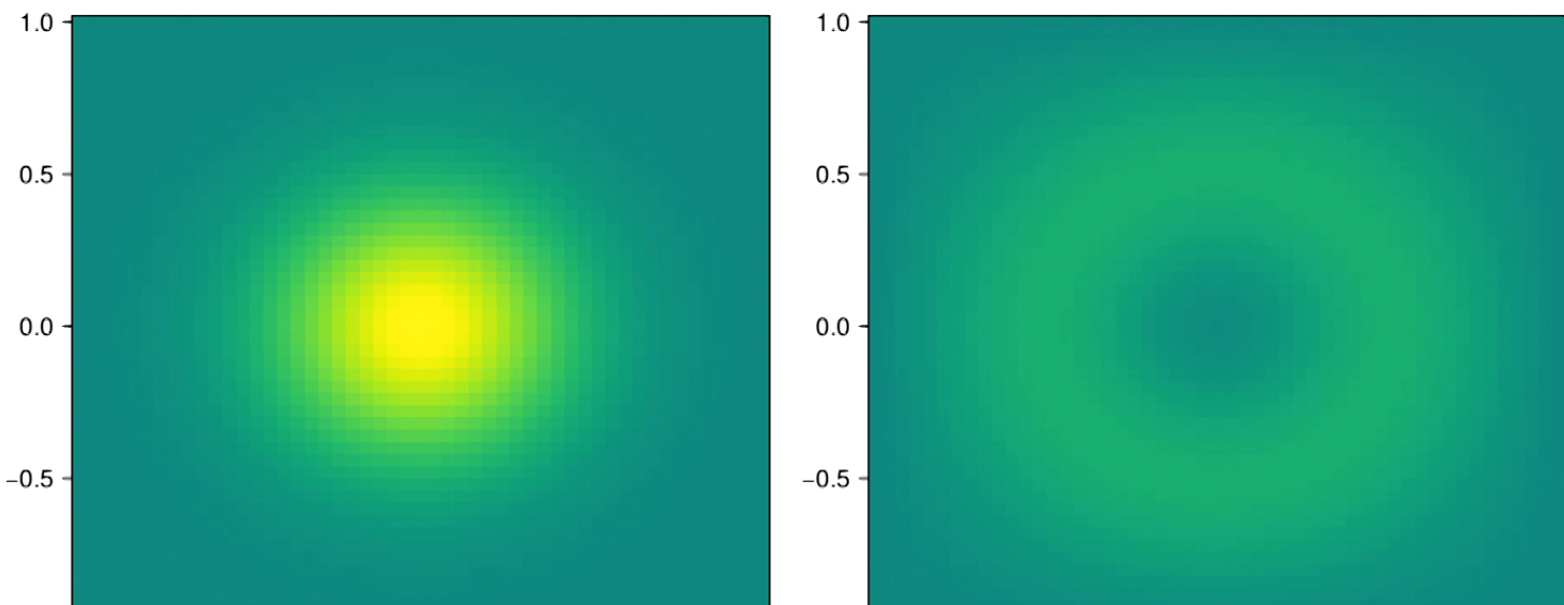
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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
[46]: fig = Figure(size=(900, 400))
      contourf!(Axis(fig[1,1]), -1:h:+1, -1:h:+1, sol(0.0)[:,:,:1]; colrange=(-1,+1))
      c!(Axis(fig[1,2]), -1:h:+1, -1:h:+1, sol(0.4)[:,:,:1]; colrange=(-1,+1))
      fig
```



[46]: 1.0 0.5 0.0 -0.5

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Edit Ln 3, Col 2 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x New Tab x +

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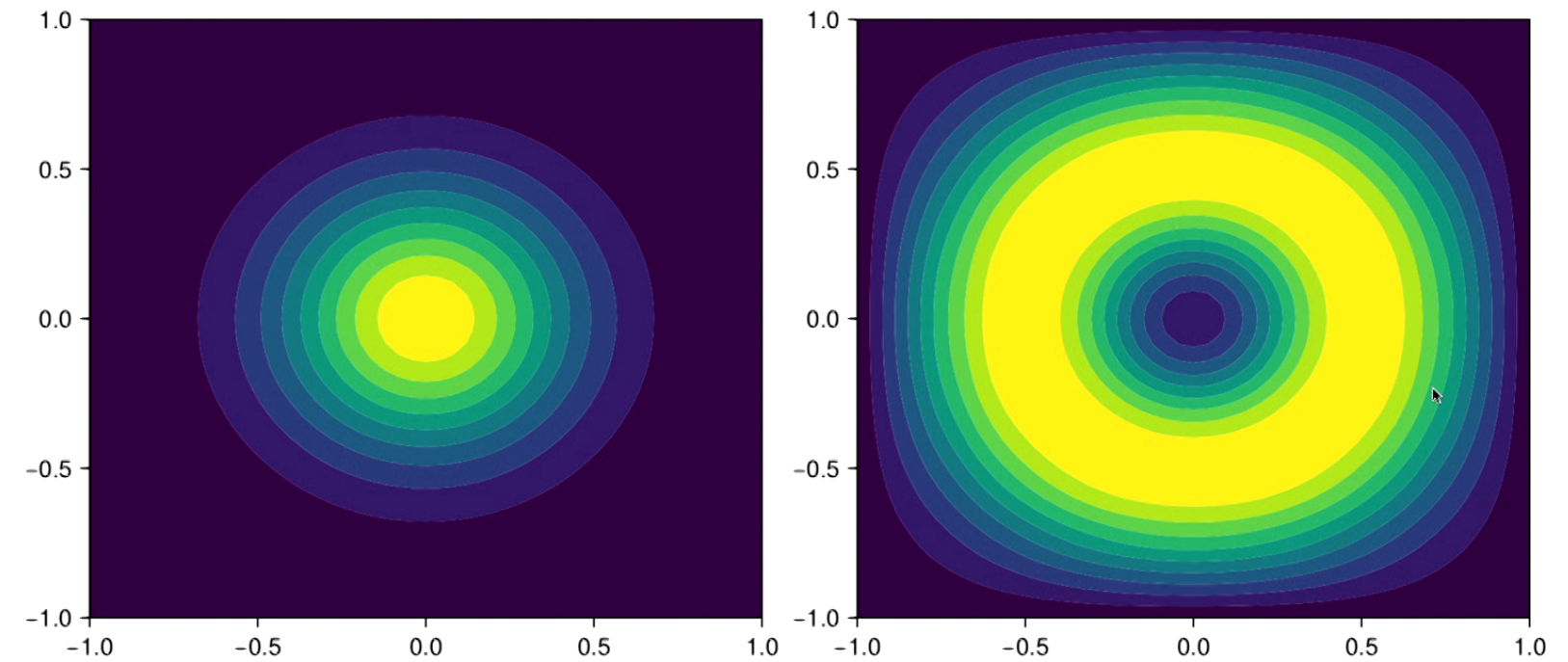
File Edit View Run Kernel Tabs Settings Help

Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

fig

[47]:



Simple 0 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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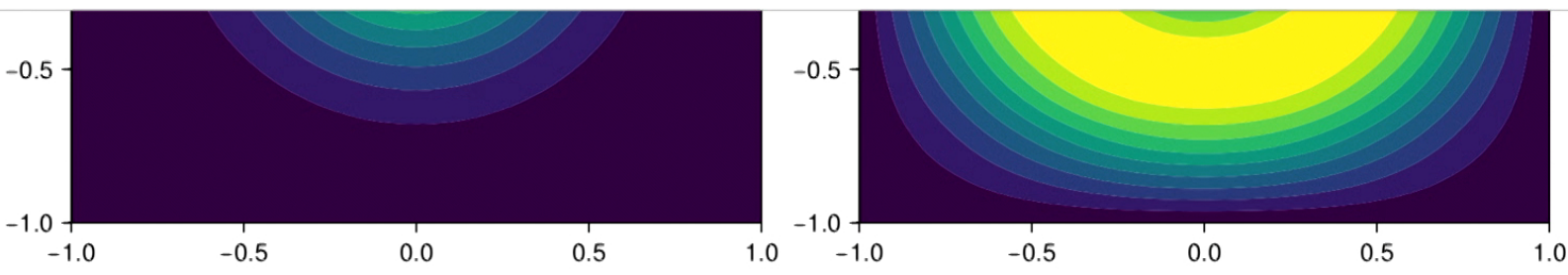
Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x New Tab x | +

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File Edit View Run Kernel Tabs Settings Help

Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0



```
[ ]: fig = Figure(size=(640, 400))
      scatt!(Axis(fig[1,1]), -1:h:+1, sol(0.0)[:,(npoints-1):,1]; colrange=(-1,+1))
      fig
```

Simple 0 0 Julia 1.10.0 | Idle Mode: Edit Ln 2, Col 55 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x New Tab x +

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File Edit View Run Kernel Tabs Settings Help

Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

fig

[49]:

x	y
-1.0	0.00
-0.9	0.01
-0.8	0.03
-0.7	0.06
-0.6	0.11
-0.5	0.18
-0.4	0.28
-0.3	0.41
-0.2	0.57
-0.1	0.76
0.0	1.00
0.1	0.76
0.2	0.57
0.3	0.41
0.4	0.28
0.5	0.18
0.6	0.11
0.7	0.06
0.8	0.03
0.9	0.01
1.0	0.00

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x New Tab x +

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File Edit View Run Kernel Tabs Settings Help

Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
U[i,j,1] = 0
U[i,j,2] = 0
else
    x = -1 + (i-1) * h
    y = -1 + (j-1) * h
    phi = exp(-((x-x0)^2+(y-y0)^2)/2W)
    psi = 0
    U[i,j,1] = phi
    U[i,j,2] = psi
end
end
return U
end
```

[52]: init (generic function with 1 method)

```
[53]: function rhs(U, p, t)
        Udot = similar(U)
        for j in 1:npoints, i in 1:npoints
            if i==1 || i==npoints || j==1 || j==npoints
                Udot[i,j,1] = 0
                Udot[i,j,2] = 0
            else
                phixx = (U[i-1,j,1] - 2*U[i,j,1] + U[i+1,j,1]) / h^2
                phiyy = (U[i,j-1,1] - 2*U[i,j,1] + U[i,j+1,1]) / h^2
                Udot[i,j,1] = U[i,j,2]
```

Simple 0 \$ 0 Julia 1.10.0 | Busy Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Goog x Google x New Tab x | +

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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
[53]: rhs (generic function with 2 methods)
```

```
[54]: U0 = init()
      tspan = (0.0, 4.0)
      prob = ODEProblem(rhs, U0, tspan)
      sol = solve(prob);
      length(sol.t)
```

```
[54]: 149
```

```
[*]: fig = Figure(size=(900, 400))
      contourf!(Axis(fig[1,1]), -1:h:+1, -1:h:+1, sol(0.0)[:,:1]; colrange=(-1,+1))
      contourf!(Axis(fig[1,2]), -1:h:+1, -1:h:+1, sol(0.4)[:,:1]; colrange=(-1,+1))
      fig
```

```
[*]: fig = Figure(size=(640, 400))
      scatter!(Axis(fig[1,1]), -1:h:+1, sol(0.0)[:(npoints+1)+2,1])
      fig
```

```
[ ]:
```

Simple 0 \$ 0 Julia 1.10.0 | Busy Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Goog x Google x New Tab x +

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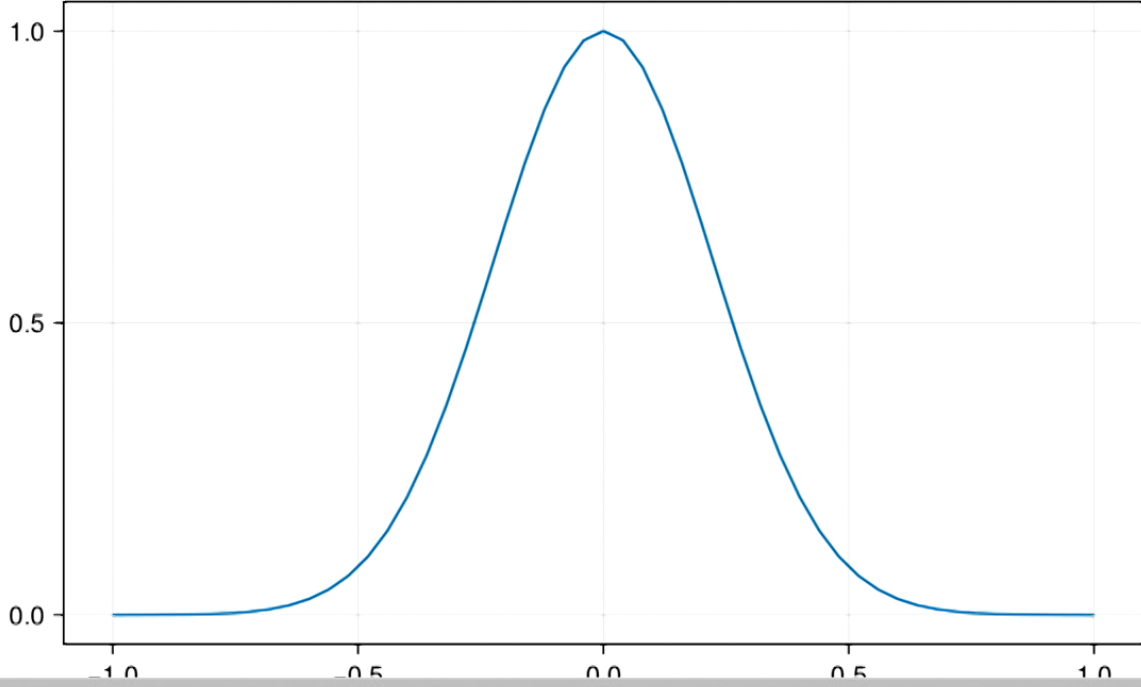
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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
[57]: fig = Figure(size=(640, 400))
      lines!(Axis(fig[1,1]), -1:h:+1, sol(0.0)[:,(npoints+1)+2,1])
      fig
```

[57]:



Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Goog... x Google x New Tab x +

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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
lines = axes(fig[1,1]), ylims = axes(fig[2,1]), xlims = axes(fig[3,1]), (ipoints[1], 1))
```

fig

[60]:

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Goog... x Google x New Tab x +

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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

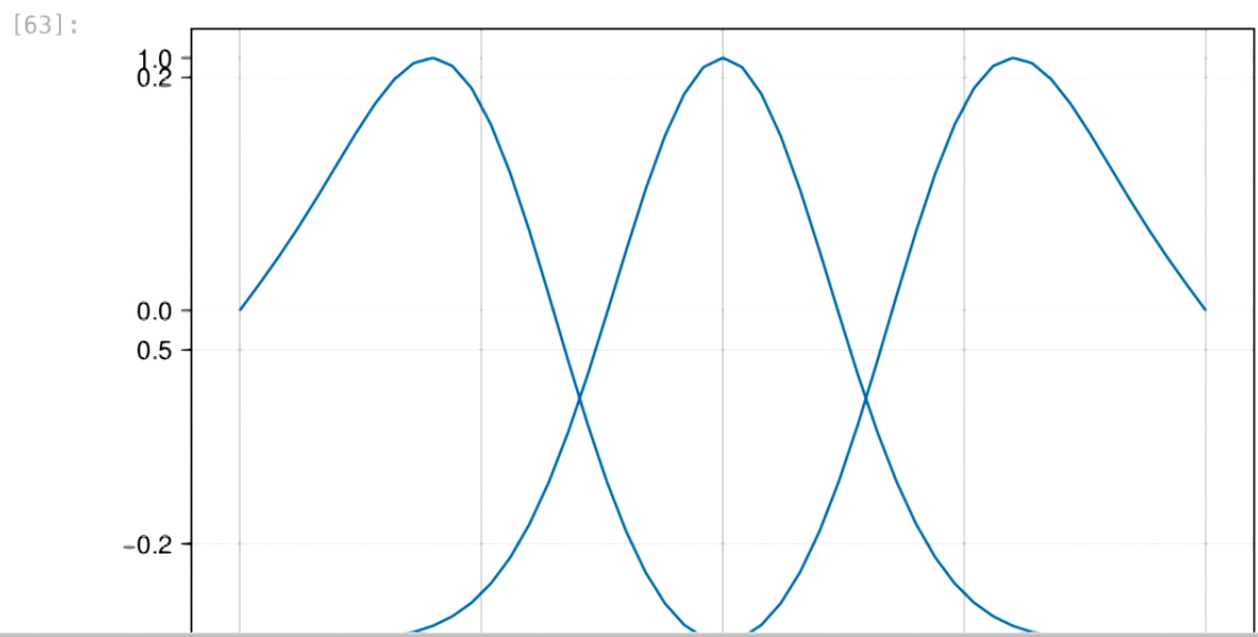
[61]:

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
[63]: fig = Figure(size=(640, 400))  
lines!(Axis(fig[1,1]), -1:h:+1, sol(0.0)[:,(npoints+1)+2,1]; yrange=(-1,1))  
lines!(Axis(fig[1,1]), -1:h:+1, sol(0.5)[:,(npoints+1)+2,1]; yrange=(-1,1))  
# lines!(Axis(fig[1,1]), -1:h:+1, sol(1.0)[:,(npoints+1)+2,1])  
fig
```



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Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x New Tab x +

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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
[65]: fig = Figure(size=(640, 400))
lines!(Axis(fig[1,1]), -1:h:+1, sol(0.0)[:,(npoints+1)+2,1]; ylims=(-1,1))
lines!(Axis(fig[1,1]), -1:h:+1, sol(0.5)[:,(npoints+1)+2,1]; ylims=(-1,1))
# lines!(Axis(fig[1,1]), -1:h:+1, sol(1.0)[:,(npoints+1)+2,1])
fig
```

[65]:

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Goog... x Google x New Tab x lines - Makie x +

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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
[66]: fig = Figure(size=(640, 400))
      lines(Axis(fig[1,1]), -1:h:+1, sol(0.0)[:,(npoints+1)+2,1])
      lines!(Axis(fig[1,1]), -1:h:+1, sol(0.5)[:,(npoints+1)+2,1])
      # lines!(Axis(fig[1,1]), -1:h:+1, sol(1.0)[:,(npoints+1)+2,1])
      fig
```

Plotting into an axis without !

Stacktrace:

```
[1] error(s::String)
    @ Base ./error.jl:35
[2] create_axis_like(::Lines{Tuple{Vector{Point{2, Float32}}}}, ::Dict{Symbol, Any}, ::Axis)
    @ Makie ~/.julia/packages/Makie/z2T2o/src/figureplotting.jl:205
[3] _create_plot(::Function, ::Dict{Symbol, Any}, ::Axis, ::Vararg{Any})
    @ Makie ~/.julia/packages/Makie/z2T2o/src/figureplotting.jl:249
[4] lines(::Axis, ::Vararg{Any}; kw::@Kwargs{})
    @ MakieCore ~/.julia/packages/MakieCore/UAwps/src/recipes.jl:175
[5] top-level scope
    @ In[66]:2
```

[]:

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

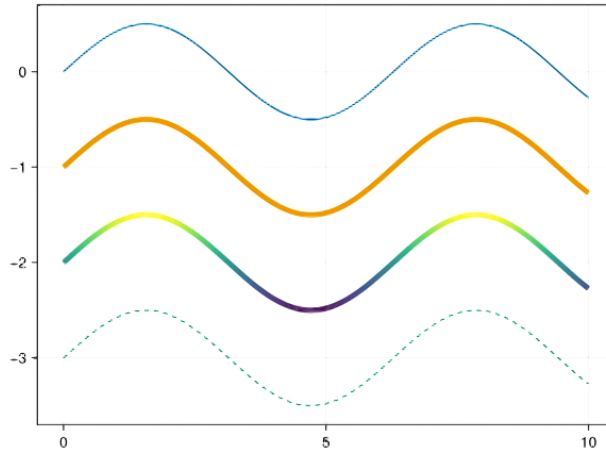


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 - errorbars
 - heatmap
 - hexbin
 - hist
 - hlines and vlines
 - hspan and vspan
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 - lines**
 - linesegments
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 - meshscatter
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 - spy

```
f = Figure()
Axis(f[1, 1])

xs = 0:0.01:10
ys = 0.5 .* sin.(xs)

lines!(xs, ys)
lines!(xs, ys .- 1, linewidth = 5)
lines!(xs, ys .- 2, linewidth = 5, color = ys)
lines!(xs, ys .- 3, linestyle = :dash)
```



Linestyles

```
using CairoMakie
```

```
f = Figure()
Axis(f[1, 1])
```

Examples
Linestyles
Dealing with outline artifacts in GLMakie

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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
[67]: fig = Figure(size=(640, 400))
      axis = Axis(fig[1,1])
      lines!(axis, -1:h:+1, sol(0.0)[:,(npoints+1)÷2,1])
      lines!(axis, -1:h:+1, sol(0.5)[:,(npoints+1)÷2,1])
      # lines!(Axis(fig[1,1]), -1:h:+1, sol(1.0)[:,(npoints+1)÷2,1])
      fig
```

UndefVarError: `axis` not defined

Stacktrace:

- [1] top-level scope @ In[67]:3

[]:

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Representing Functions.ipyn x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

fig

[69]:

x	Blue Curve (y)	Orange Curve (y)	Green Curve (y)
-1.0	0.0	0.0	0.0
-0.5	0.1	0.2	-0.1
0.0	1.0	0.0	-0.1
0.5	0.1	0.2	-0.1
1.0	0.0	0.0	0.0

Simple 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x New Tab x lines - Makie x +

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Representing Functions.ipyn x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

[84]:

Simple 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x New Tab x lines - Makie x +

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Representing Functions.ipynb x Finite Differences.ipynb • Multiscale.ipynb x +

Code Julia 1.10.0

```
using LinearAlgebra

[79]: npoints = 51
      h = 2 / (npoints-1)

[79]: 0.04

•[80]: function init(npoints)
        x0 = 0.0
        y0 = 0.0
        W = 0.01
        U = zeros(npoints, npoints, 2)
        for j in 1:npoints, i in 1:npoints
            if i==1 || i==npoints || j==1 || j==npoints
                U[i,j,1] = 0
                U[i,j,2] = 0
            else
                x = -1 + (i-1) * h
                y = -1 + (j-1) * h
                phi = exp(-((x-x0)^2+(y-y0)^2)/2W)
                psi = 0
                U[i,j,1] = phi
                U[i,j,2] = psi
            end
        end
    end
```

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Edit Ln 1, Col 22 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Goog x Google x New Tab x lines - Makie x +

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Representing Functions.ipynb x Finite Differences.ipynb x Multiscale.ipynb x +

Code Julia 1.10.0

```

    Udot[i,j,2] = 0
  else
    phixx = (U[i-1,j,1] - 2*U[i,j,1] + U[i+1,j,1]) / h^2
    phiyy = (U[i,j-1,1] - 2*U[i,j,1] + U[i,j+1,1]) / h^2
    Udot[i,j,1] = U[i,j,2]
    Udot[i,j,2] = phixx + phiyy
  end
end
return Udot
end

```

```

[*]: U0 = init(51)
      tspan = (0.0, 4.0)
      prob = ODEProblem(rhs, U0, tspan)
      sol = solve(prob);
      length(sol.t)

```

```

[*]: fig = Figure(size=(900, 400))
      contourf!(Axis(fig[1,1]), -1:h:+1, -1:h:+1, sol(0.0)[:,:,1]; colrange=(-1,+1))
      contourf!(Axis(fig[1,2]), -1:h:+1, -1:h:+1, sol(0.4)[:,:,1]; colrange=(-1,+1))
      fig

```

```

[*]: fig = Figure(size=(640, 400))
      axis = Axis(fig[1,1])
      lines!(axis, -1:h:+1, sol(0.0)[:,(npoints+1)+2,1])

```

Simple 0 0 Julia 1.10.0 | Busy Saving completed Mode: Edit Ln 4, Col 4 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Goog... x Google x New Tab x lines - Makie x +

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File Edit View Run Kernel Tabs Settings Help

Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
contourf!(Axis(fig[1,2]), -1:h:+1, -1:h:+1, sol(0.4)[:,:1]; colrange=(-1,+1))
fig
```

UndefVarError: `h` not defined

Stacktrace:

```
[1] top-level scope
 @ In[5]:2
```

```
• [9]: fig = Figure(size=(640, 400))
axis = Axis(fig[1,1])
h0 = 2 / 50
lines!(axis, -1:h0:+1, sol(0.0)[:,(e+1)+2,1])
lines!(axis, -1:h0:+1, sol(0.25)[:,(npoints+1)+2,1])
lines!(axis, -1:h0:+1, sol(0.5)[:,(npoints+1)+2,1])
lines!(axis, -1:h0:+1, sol(0.75)[:,(npoints+1)+2,1])
lines!(axis, -1:h0:+1, sol(1.0)[:,(npoints+1)+2,1])
fig
```

UndefVarError: `npoints` not defined

Stacktrace:

```
[1] top-level scope
 @ In[9]:4
```

[]:

Simple 0 0 Julia 1.10.0 | Idle Mode: Edit Ln 4, Col 37 Finite Differences.ipynb 0

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Representing Functions.ipyn x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
lines!(axis, -1:10:1, sqrt(1.0)[:, (end+1)+2, 1])  
fig
```

[10]:

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x New Tab x lines - Makie x +

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File Edit View Run Kernel Tabs Settings Help

Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```

else
    phixx = (U[i-1,j,1] - 2*U[i,j,1] + U[i+1,j,1]) / h^2
    phiyy = (U[i,j-1,1] - 2*U[i,j,1] + U[i,j+1,1]) / h^2
    Udot[i,j,1] = U[i,j,2]
    Udot[i,j,2] = phixx + phiyy
end
end
return Udot
end

```

[7]: rhs (generic function with 1 method)

```

• [8]: tspan = (0.0, 4.0)
      U0 = init(51)
      prob0 = ODEProblem(rhs, U0, tspan)
      sol0 = solve(prob);
      U0 = init(51)
      prob0 = ODEProblem(rhs, U0, tspan)
      sol0 = solve(prob);

```

[8]: 109

```

[5]: fig = Figure(size=(900, 400))
      contourf!(Axis(fig[1,1]). -1:h+1. -1:h+1. sol(0.0)[...:1]: colorrange=(-1.+1))

```

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Edit Ln 7, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Googl x Google x New Tab x lines - Makie x +

Not Secure https://symmetry.pi.local/user/eschnetter/lab/tree/home/eschnetter/Finite%20Differences.ipynb

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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Julia 1.10.0

```
prob0 = ODEProblem(rhs, U0, tspan)
sol0 = solve(prob0);

U1 = init(101)
prob1 = ODEProblem(rhs, U1, tspan)
sol1 = solve(prob1);
```

[5]:

```
fig = Figure(size=(900, 400))
contourf!(Axis(fig[1,1]), -1:h0+1, -1:h0+1, sol(0.0)[:,:1]; colrange=(-1,+1))
contourf!(Axis(fig[1,2]), -1:h0+1, -1:h0+1, sol(0.4)[:,:1]; colrange=(-1,+1))
fig
```

UndefVarError: `h` not defined

Stacktrace:

```
[1] top-level scope
@ In[5]:2
```

[10]:

```
fig = Figure(size=(640, 400))
axis = Axis(fig[1,1])
h0 = 2 / 50
lines!(axis, -1:h0+1, sol(0.0)[:,(end+1)+2,1])
lines!(axis, -1:h0+1, sol(0.25)[:,(end+1)+2,1])
lines!(axis, -1:h0+1, sol(0.5)[:,(end+1)+2,1])
lines!(axis, -1:h0+1, sol(0.75)[:,(end+1)+2,1])
lines!(axis, -1:h0+1, sol(1.0)[:,(end+1)+2,1])
```

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 2, Col 30 Finite Differences.ipynb 0

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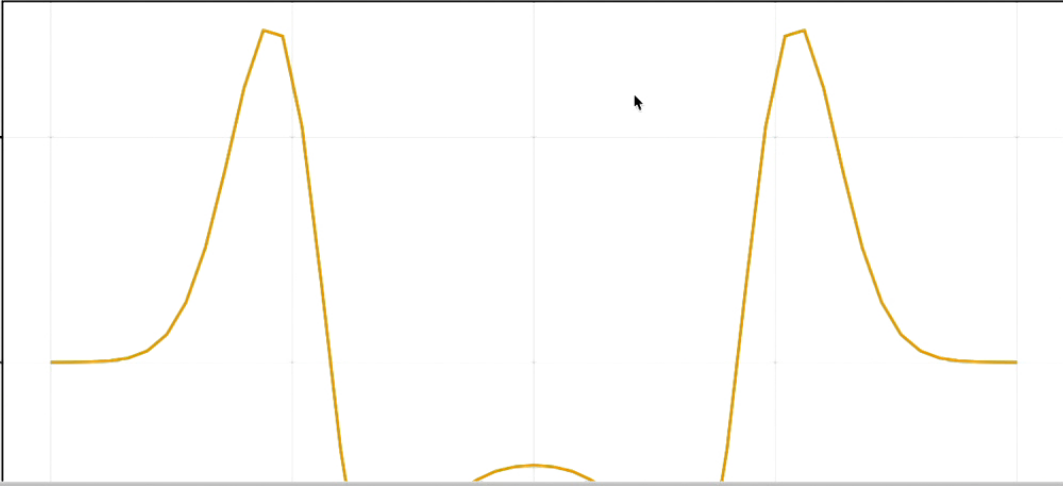
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Representing Functions.ipyn x Finite Differences.ipynb • Multiscale.ipynb x +

Code Julia 1.10.0

```
[13]: fig = Figure(size=(640, 400))
axis = Axis(fig[1,1])
h0 = 2 / 50
h1 = 2 / 100
lines!(axis, -1:h0:+1, sol0(0.5)[:(end+1)+2,1])
lines!(axis, -1:h0:+1, sol1(0.5)[:(end+1)+2,1])
fig
```

[13]:



Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Finite Diffe... (4) - JupyterLab x Fix connection errors - Goog... x Google x New Tab x lines - Makie x +

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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

```
end

[*]: tspan = (0.0, 4.0)

U0 = init(51)
prob0 = ODEProblem(rhs, U0, tspan)
sol0 = solve(prob0);

U1 = init(101)
prob1 = ODEProblem(rhs, U1, tspan)
sol1 = solve(prob1);

[*]: fig = Figure(size=(900, 400))
contourf!(Axis(fig[1,1]), -1:h:+1, -1:h:+1, sol(0.0)[:,: ,1]; colrange=(-1,+1))
contourf!(Axis(fig[1,2]), -1:h:+1, -1:h:+1, sol(0.4)[:,: ,1]; colrange=(-1,+1))
fig

[*]: fig = Figure(size=(640, 400))
axis = Axis(fig[1,1])
h0 = 2 / 50
lines!(axis, -1:h0:+1, sol(0.0)[:,(end+1)+2,1])
lines!(axis, -1:h0:+1, sol(0.25)[:,(end+1)+2,1])
lines!(axis, -1:h0:+1, sol(0.5)[:,(end+1)+2,1])
lines!(axis, -1:h0:+1, sol(0.75)[:,(end+1)+2,1])
lines!(axis, -1:h0:+1, sol(1.0)[:,(end+1)+2,1])
```

Simple 0 0 Julia 1.10.0 | Busy Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

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Representing Functions.ipynb x Finite Differences.ipynb ● Multiscale.ipynb x +

Code Julia 1.10.0

fig

[7]:

The plot displays two curves, one blue and one yellow, which are nearly identical. The x-axis ranges from -1.0 to 1.0 with major ticks at -1.0, -0.5, 0.0, 0.5, and 1.0. The y-axis ranges from -0.1 to 0.1 with major ticks at -0.1, 0.0, and 0.1. The curves are zero at x = -1.0 and x = 1.0. They rise to a peak of approximately 0.15 at x ≈ -0.5 and x ≈ 0.5, dip to a minimum of approximately -0.11 at x ≈ -0.3 and x ≈ 0.3, and then rise to a smaller peak of approximately -0.05 at x = 0.0.

Simple 0 \$ 0 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Finite Differences.ipynb 0

$$\left(\int_{\Omega} [f_h(t,x) - f^*(t,x)]^2 dx \right)^{1/2} = O(h^2)$$

