Title: Numerical Methods Lecture

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Collection: Numerical Methods 2023/24

Date: February 01, 2024 - 10:15 AM

URL: https://pirsa.org/24020017

$$f(x) = \sum_{i=0}^{n} c^i \, b_i(x)$$

$$\langle f | g \rangle = \int f(x) \, g(x) \, dx$$

$$\int b_i(x) \, f(x) \, dx = \sum_j c^j \underbrace{\int b_i(x) \, b_j(x) \, dx}_{M_{ij}}$$

"mass matrix"

[diagonal: orthogonal basis"

$$f'(x) = \sum c^i \, b_i'(x)$$

$$f(x) = \sum_{i=0}^{n} c^i \, b_i(x)$$

$$\langle f | g \rangle = \int f(x) \, g(x) \, dx$$

$$\int b_i(x) \, f(x) \, dx = \sum_j c^j \underbrace{\int b_i(x) \, b_j(x) \, dx}_{M_{ij}}$$

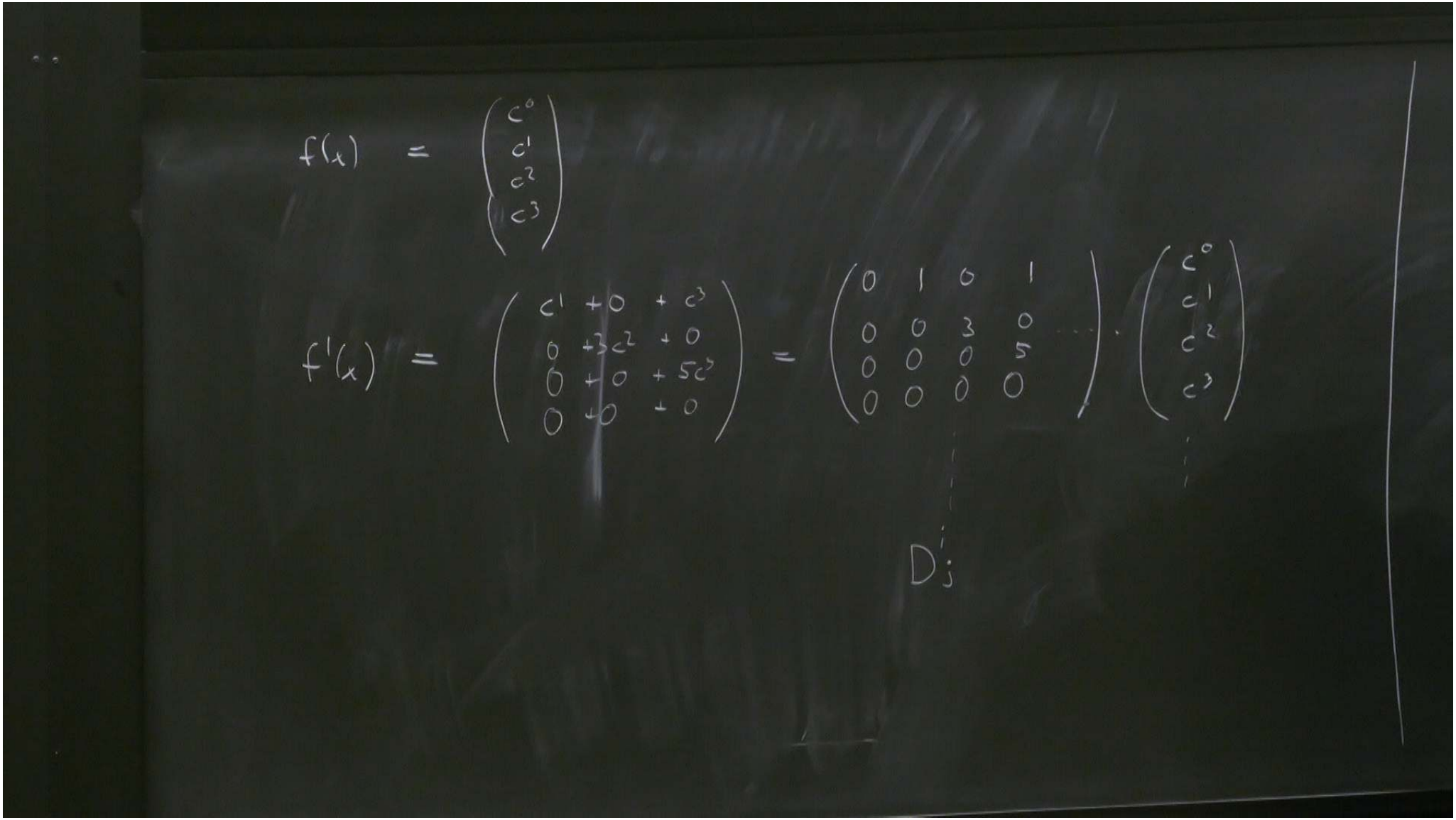
"mass matrix"

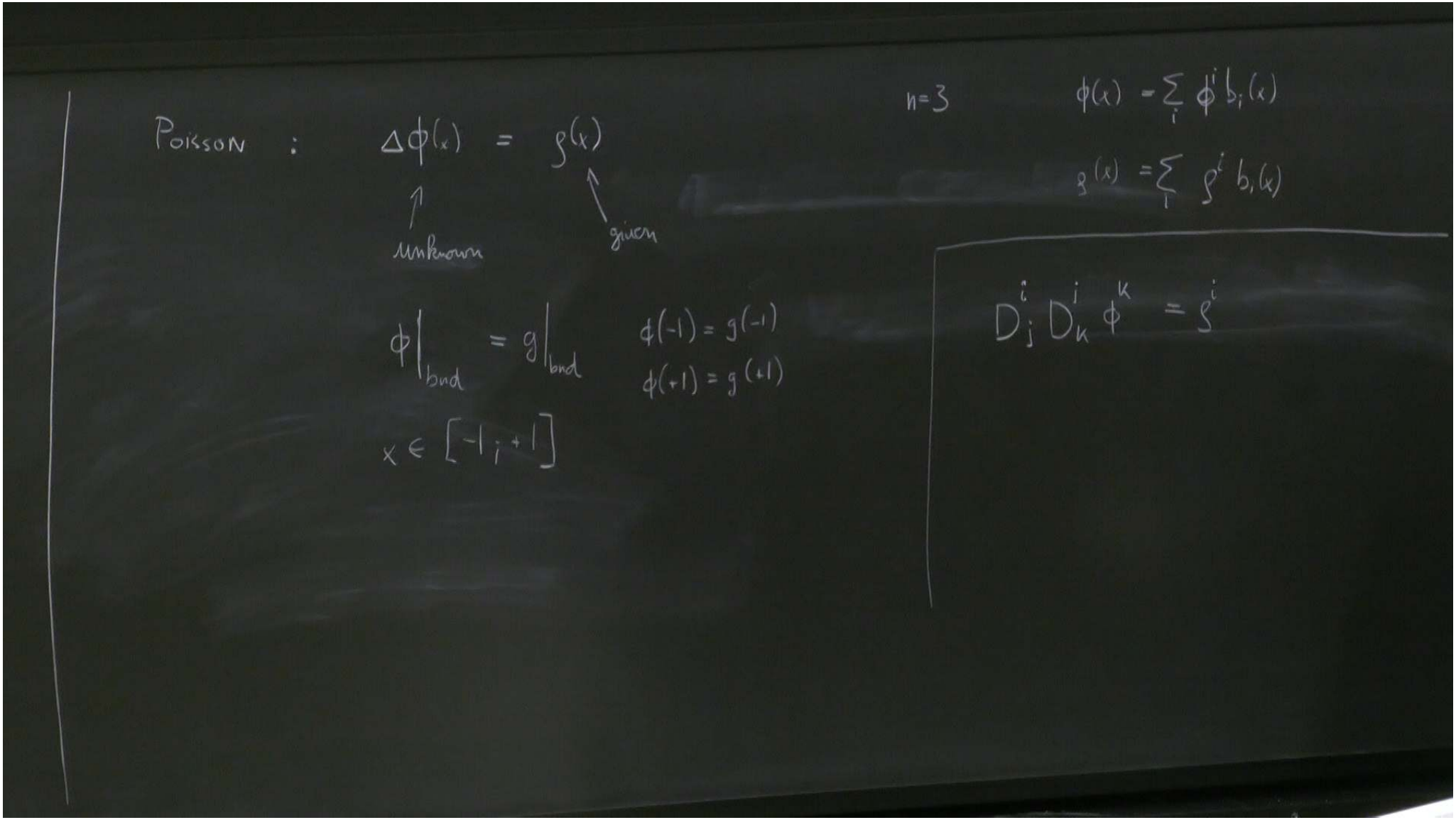
[diagonal : orthogonal basis"

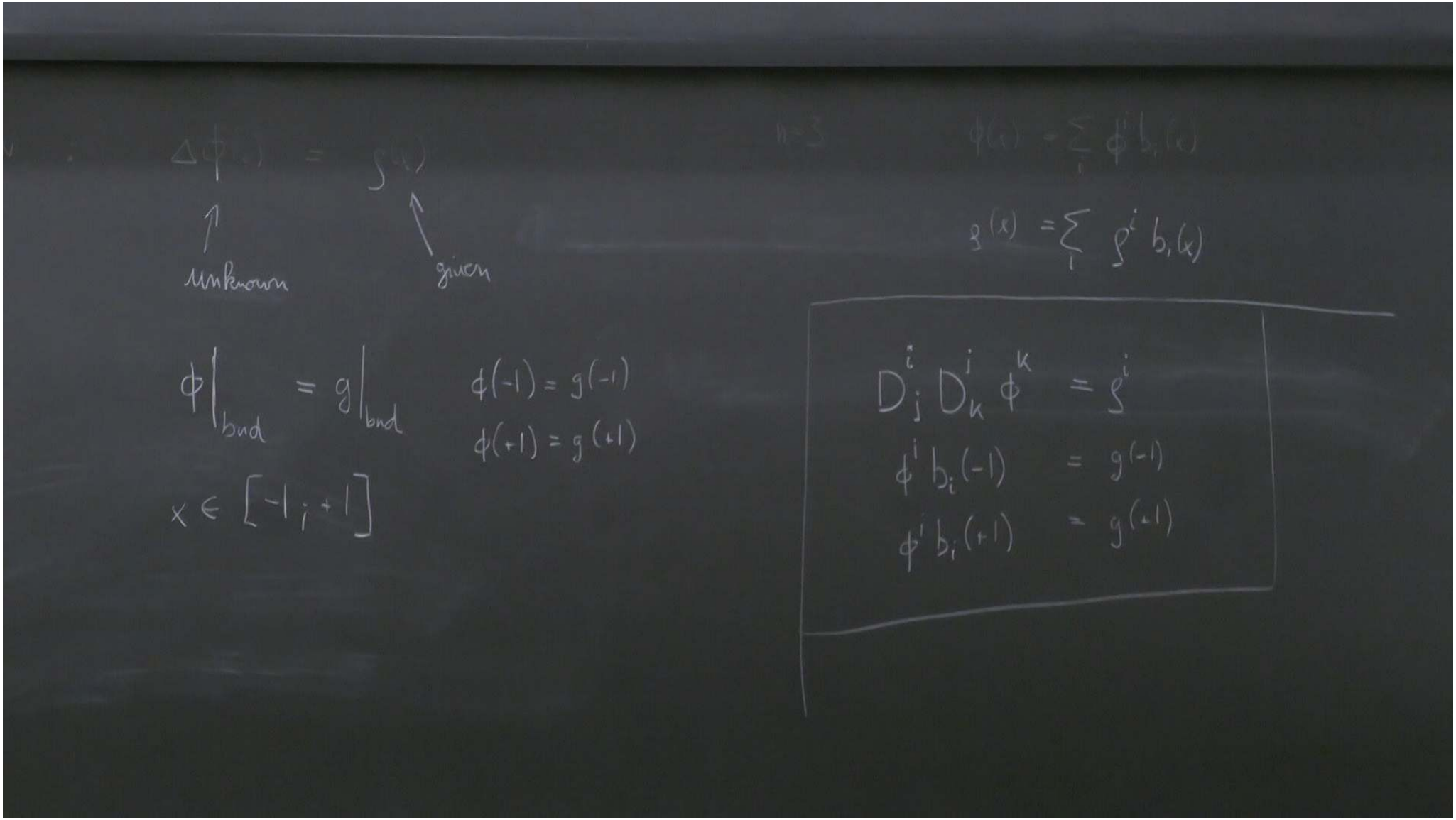
$$f'(x) = \sum c^i \, b_i'(x)$$

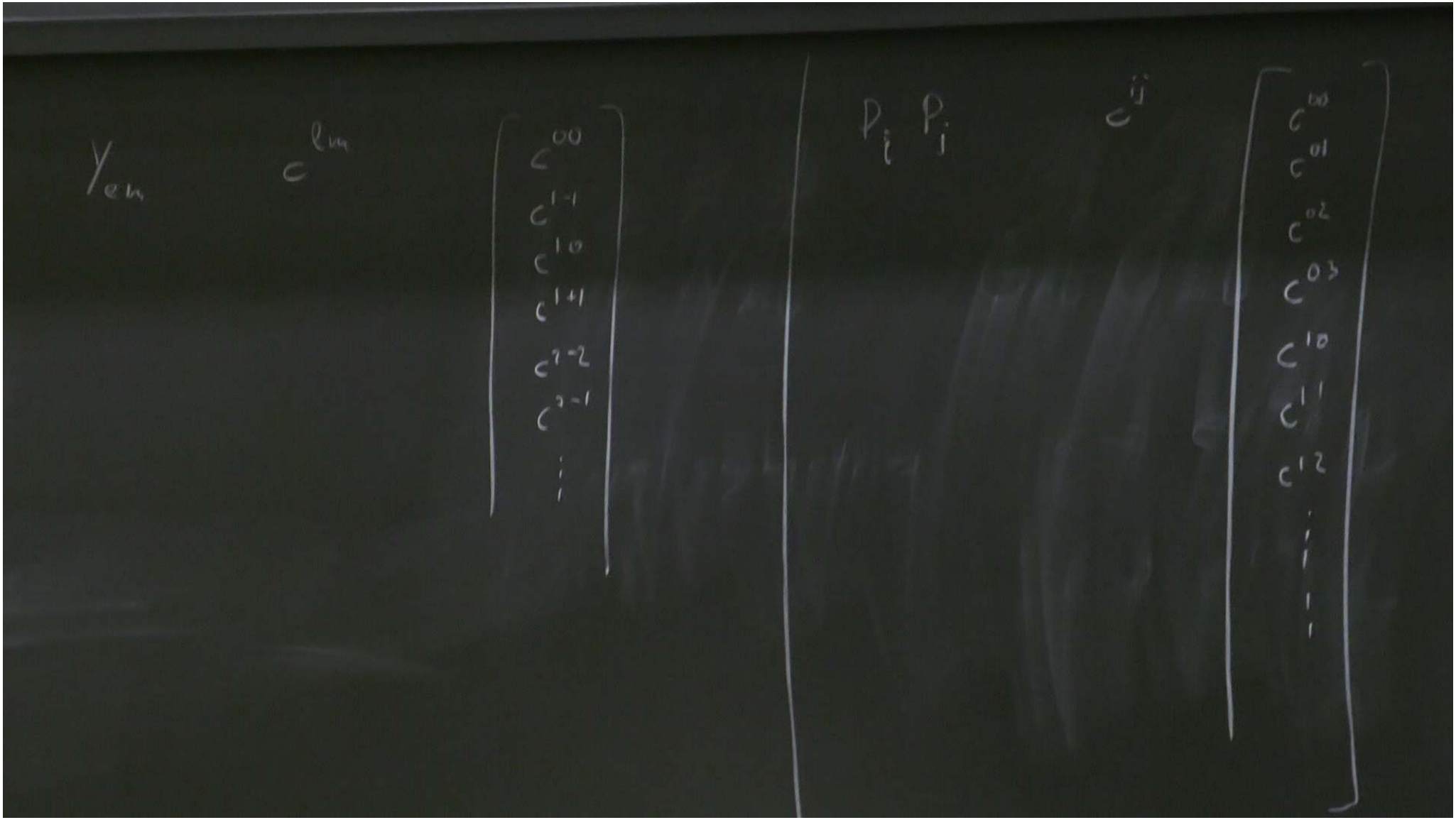
$$\int b_i(x) \left(\sum_j c^j b_j'(x) \right) dx = \sum_j c^j \underbrace{\int b_i(x) \, b_j'(x) \, dx}_{D^i_j}$$

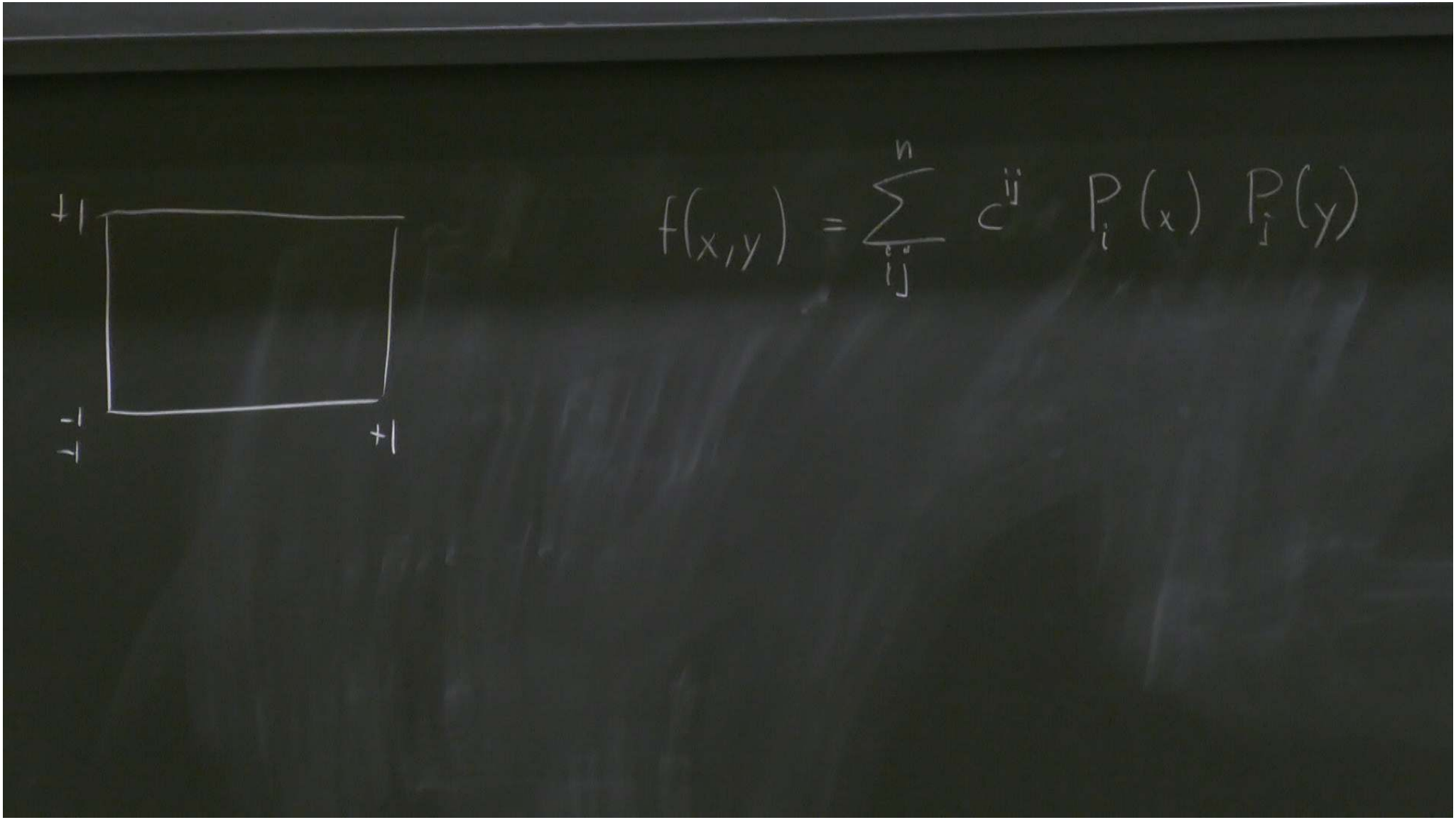
ℓ	$P_\ell(x)$	$P_\ell'(x)$	
0	1	0	0
		1	$P_0(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$
1	x		
		$3x$	$3P_1(x) = \begin{pmatrix} 0 \\ 3 \\ 0 \\ 0 \end{pmatrix}$
2	$\frac{1}{2}(3x^2-1)$		
		$\frac{1}{2}(15x^2-3)$	$5P_2(x) + P_0(x) = \begin{pmatrix} 1 \\ 0 \\ 5 \\ 0 \end{pmatrix}$
3	$\frac{1}{2}(5x^3-3x)$		


$f(x) = \begin{pmatrix} c^0 \\ c^1 \\ c^2 \\ c^3 \end{pmatrix}$
$f'(x) = \begin{pmatrix} c^1 + 0 + c^3 \\ 0 + 3c^2 + 0 \\ 0 + 0 + 5c^3 \\ 0 + 0 + 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} c^0 \\ c^1 \\ c^2 \\ c^3 \end{pmatrix}$
D_3'

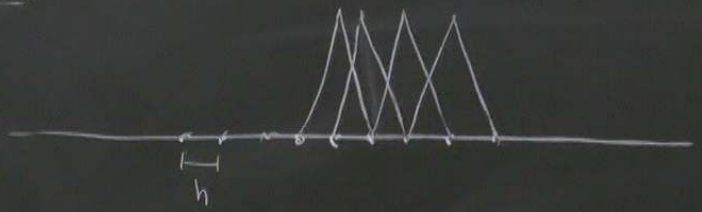

Poisson : $\Delta\phi(x) = \rho(x)$
unknown
given
$n=3$
$\phi(x) = \sum_i \phi^i b_i(x)$
$\rho(x) = \sum_i \rho^i b_i(x)$
$\phi|_{bnd} = g|_{bnd}$
$\phi(-1) = g(-1)$
$\phi(+1) = g(+1)$
$x \in [-1, +1]$
$D^i_j D^j_k \phi^k = \rho^i$

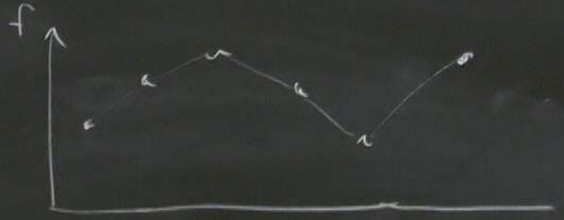

$\Delta\phi(x) = \rho(x)$
unknown
given
$\phi|_{bnd} = g|_{bnd}$
$\phi(-1) = g(-1)$
$\phi(+1) = g(+1)$
$x \in [-1, +1]$
$n=3$
$\phi(x) = \sum_i \phi^i b_i(x)$
$\rho(x) = \sum_i \rho^i b_i(x)$
$D^i_j D^j_k \phi^k = \rho^i$
$\phi^i b_i(-1) = g(-1)$
$\phi^i b_i(+1) = g(+1)$

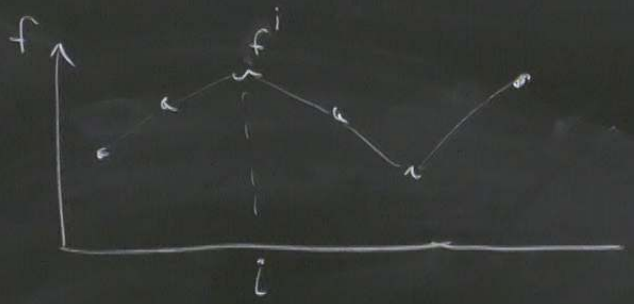



+1
-1
-1
+1
$f(x,y) = \sum_{ij}^{n} c^{ij} P_i(x) P_j(y)$

Finite Differences







$$(f')^i = \frac{f^{i+1} - f^{i-1}}{2h}$$

$$\partial_x f \qquad (f')^i = \frac{f^{i+1} - f^{i-1}}{2h}$$

$$D = \frac{1}{2h} \begin{pmatrix} \ddots & & & \\ & -1 & 0 & +1 \\ & & & \ddots \end{pmatrix}$$

$$(f')^i = \frac{f^{i+1} - f^{i-1}}{2h} \qquad D = \frac{1}{2h}$$

$$(f'')^i = \frac{-f^{i-1} + 2f^i - f^{i+1}}{h^2}$$