Title: Numerical Methods Lecture

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Collection: Numerical Methods 2023/24

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URL: https://pirsa.org/24020017

$$\langle f | g \rangle = \int f(x)\, g(x)\, dx$$

$$f(x) = \sum_{i=0}^{n} c^i \, b_i(x)$$

$$\int b_i(x)\, f(x)\, dx = \sum_j c^j \underbrace{\int b_i(x)\, b_j(x)\, dx}_{M_{ij}}$$

"mass matrix"

[diagonal : orthogonal basis"

$$f'(x) = \sum c^i \, b_i'(x)$$

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"mass matrix"

[diagonal : orthogonal basis"

$$f'(x) = \sum c^i\, b_i'(x)$$

$$\int b_i(x) \left(\sum_j c^j b_j'(x) \right) dx = \sum_j c^j \underbrace{\int b_i(x)\, b_j'(x)\, dx}_{D^i_j}$$

ℓ	$P_\ell(x)$	$P_\ell'(x)$	
0	1	0	0
1	x	1	$P_0(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$
2	$\frac{1}{2}(3x^2-1)$	$3x$	$3P_1(x) = \begin{pmatrix} 0 \\ 3 \\ 0 \\ 0 \end{pmatrix}$
3	$\frac{1}{2}\left(5x^3-3x\right)$	$\frac{1}{2}\left(15x^2-3\right)$	$5P_2(x) + P_0(x) = \begin{pmatrix} 1 \\ 0 \\ 5 \\ 0 \end{pmatrix}$

$$f(x) = \begin{pmatrix} c^0 \\ c^1 \\ c^2 \\ c^3 \end{pmatrix}$$

$$f'(x) = \begin{pmatrix} c^1 + 0 + c^3 \\ 0 + 3c^2 + 0 \\ 0 + 0 + 5c^3 \\ 0 + 0 + 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 1 & \\ 0 & 0 & 3 & 0 & \cdots \\ 0 & 0 & 0 & 5 & \\ 0 & 0 & 0 & 0 & \end{pmatrix} \cdot \begin{pmatrix} c^0 \\ c^1 \\ c^2 \\ c^3 \\ \vdots \end{pmatrix}$$

$$\vdots$$

$$D_j$$

Poisson : $\Delta \phi(x) = \rho(x)$

unknown given

$\phi \big|_{bnd} = g \big|_{bnd}$

$\phi(-1) = g(-1)$

$\phi(+1) = g(+1)$

$x \in [-1 ; +1]$

$n = 3$

$\phi(x) = \sum_i \phi^i b_i(x)$

$\rho(x) = \sum_i \rho^i b_i(x)$

$D^i_j D^j_k \phi^k = \rho^i$

$$\Delta \phi(x) = \rho(x)$$

unknown — given

$n=3$

$$\phi(x) = \sum_i \phi^i b_i(x)$$

$$\rho(x) = \sum_i \rho^i b_i(x)$$

$$\phi\big|_{bnd} = g\big|_{bnd}$$

$$\phi(-1) = g(-1)$$

$$\phi(+1) = g(+1)$$

$$x \in [-1; +1]$$

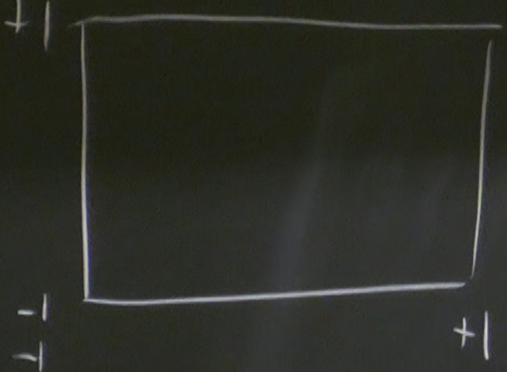
$$D^i_j D^j_k \phi^k = \rho^i$$

$$\phi^i b_i(-1) = g(-1)$$

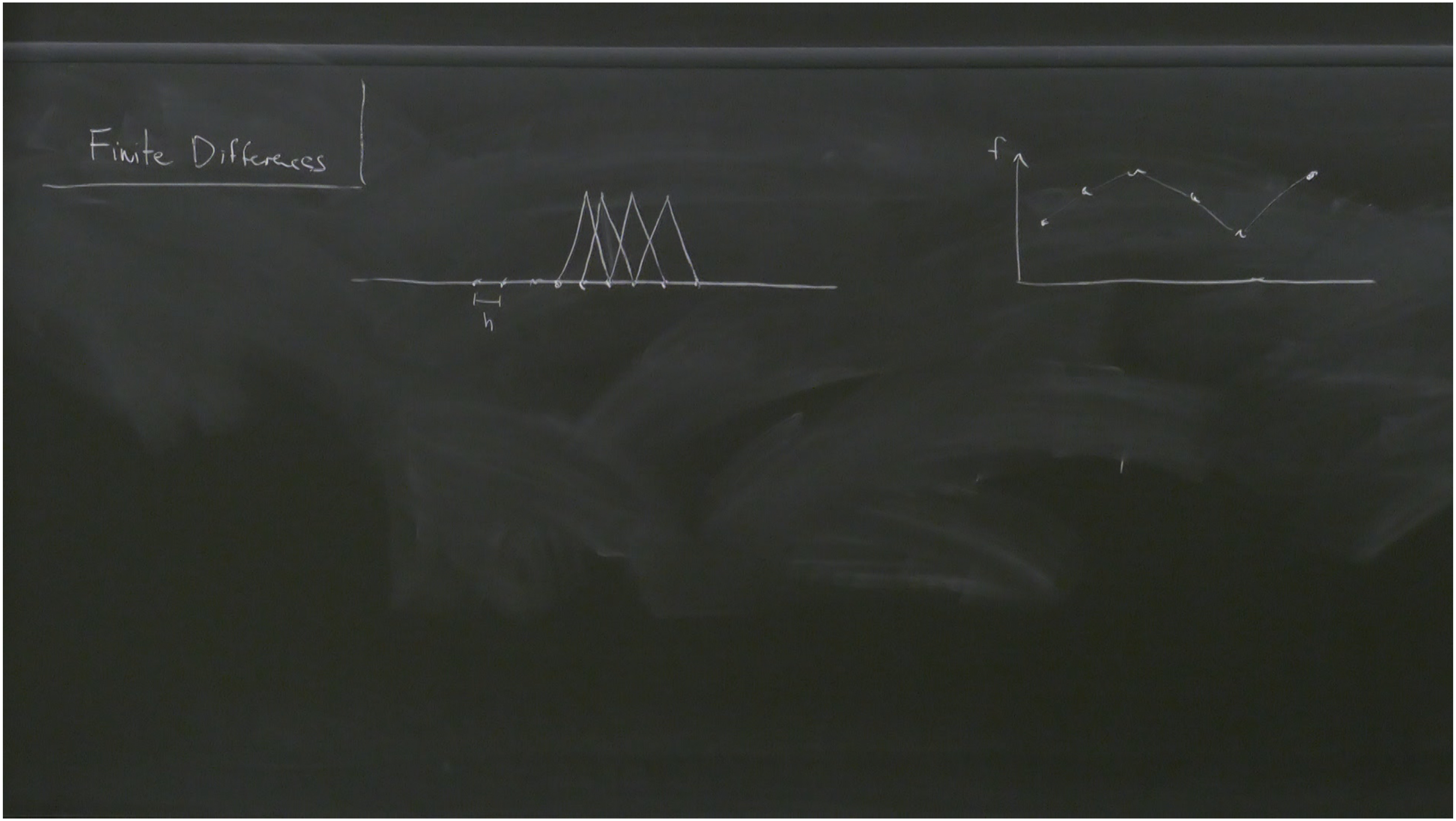
$$\phi^i b_i(+1) = g(+1)$$

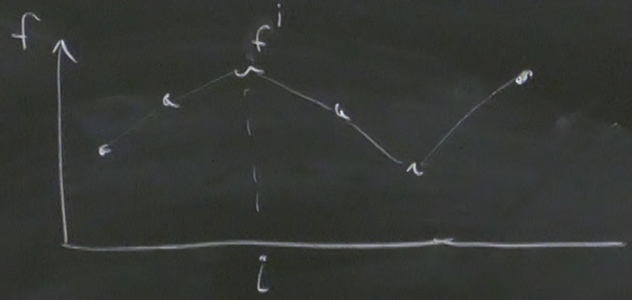
$Y_{\ell m}$ $\qquad c^{\ell m}$ $\qquad \begin{bmatrix} c^{00} \\ c^{1-1} \\ c^{10} \\ c^{1+1} \\ c^{2-2} \\ c^{2-1} \\ \vdots \end{bmatrix}$

$P_i \ P_j$ $\qquad c^{ij}$ $\qquad \begin{bmatrix} c^{00} \\ c^{01} \\ c^{02} \\ c^{03} \\ c^{10} \\ c^{11} \\ c^{12} \\ \vdots \end{bmatrix}$



$$f(x,y) = \sum_{ij}^{n} c^{ij} \, P_i(x) \, P_j(y)$$


Finite Differences
h
f



$$(f')^i = \frac{f^{i+1} - f^{i-1}}{2h}$$

$$\partial_x f \qquad (f')^i = \frac{f^{i+1} - f^{i-1}}{2h}$$

$$D = \frac{1}{2h}\begin{pmatrix} \ddots & & & & \\ \cdots & -1 & 0 & +1 & \cdots \\ & & & & \ddots \end{pmatrix}$$

$$(f')^i = \frac{f^{i+1} - f^{i-1}}{2h}$$

$$D = \frac{1}{2h}$$

$$(f'')^i = \frac{-f^{i-1} + 2f^i - f^{i+1}}{h^2}$$