Title: Quantum Field Theory for Cosmology - Lecture 20240201

Speakers: Achim Kempf

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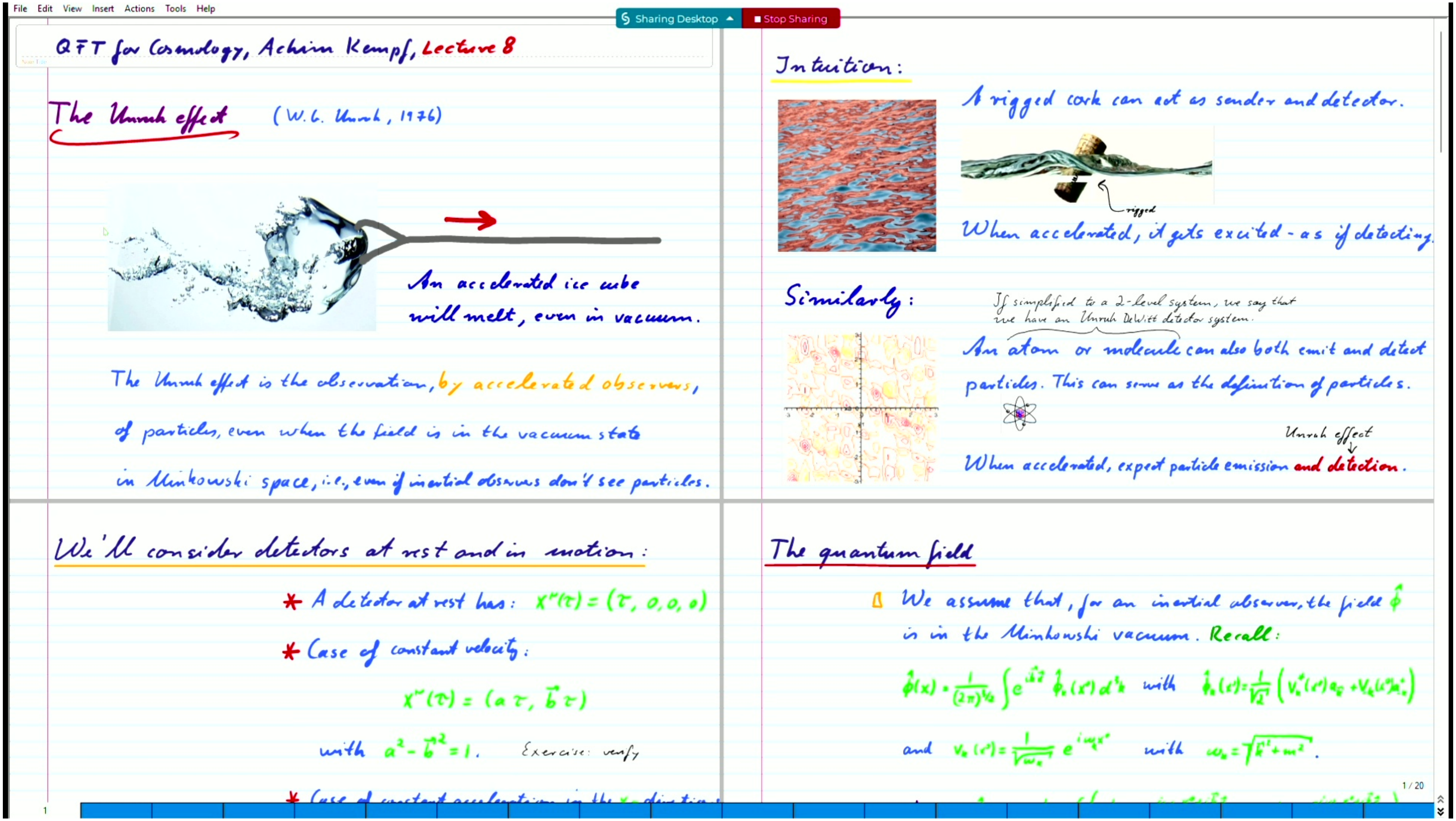


QFT for Cosmology, Achim Kempf, **Lecture 8**

The Unruh effect

(W.G. Unruh, 1976)

An accelerated ice cube will melt, even in vacuum.

The Unruh effect is the observation, by accelerated observers, of particles, even when the field is in the vacuum state in Minkowski space, i.e., even if inertial observers don't see particles.

Intuition:

A rigged cork can act as sender and detector.

When accelerated, it gets excited - as if detecting.

Similarly:

If simplified to a 2-level system, we say that we have an Unruh DeWitt detector system.

An atom or molecule can also both emit and detect particles. This can serve as the definition of particles.

Unruh effect

When accelerated, expect particle emission **and detection**.

We'll consider detectors at rest and in motion:

- A detector at rest has: $x^\mu(\tau) = (\tau, 0, 0, 0)$
- Case of constant velocity:

$$x^\mu(\tau) = (a\tau, \vec{b}\tau)$$

with $a^2 - \vec{b}^2 = 1$. Exercise: verify

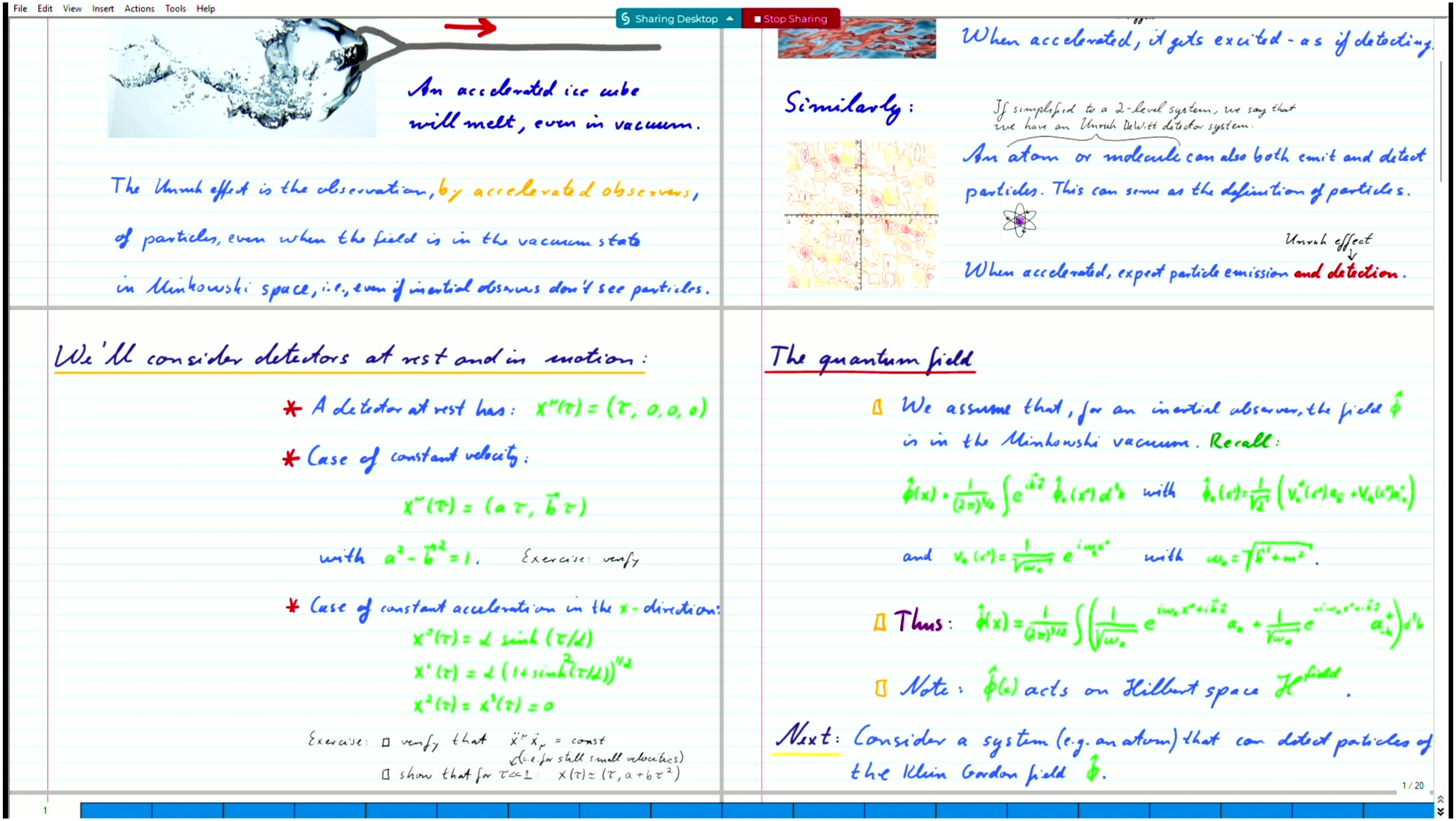
- Case of constant acceleration in the x-direction:

The quantum field

We assume that, for an inertial observer, the field $\hat\phi$ is in the Minkowski vacuum. **Recall:**

$$\hat\phi(x) = \frac{1}{(2\pi)^{3/2}}\int e^{i\vec{k}\vec{x}}\,\hat\phi_k(x^0)\,d^3k \quad \text{with} \quad \hat\phi_k(x^0) = \frac{1}{\sqrt{2}}\left(v_k^*(x^0)a_k + v_k(x^0)a_{-k}^\dagger\right)$$

$$\text{and} \quad v_k(x^0) = \frac{1}{\sqrt{\omega_k}}e^{i\omega_k x^0} \quad \text{with} \quad \omega_k = \sqrt{k^2+m^2}.$$



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$$x^\mu(\tau) = (a\tau, \vec{b}\tau)$$

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* Case of constant acceleration in the x-direction:

$$x^0(\tau) = \alpha \sinh(\tau/\alpha)$$
$$x^1(\tau) = \alpha\left(1+\sinh^2(\tau/\alpha)\right)^{1/2}$$
$$x^2(\tau) = x^3(\tau) = 0$$

Exercise: ☐ verify that $\ddot{x}^\mu \ddot{x}_\mu$ = const

(i.e. for still small velocities)

☐ show that for $\tau \ll 1$: $x(\tau) \simeq (\tau, a + b\tau^2)$

The quantum field

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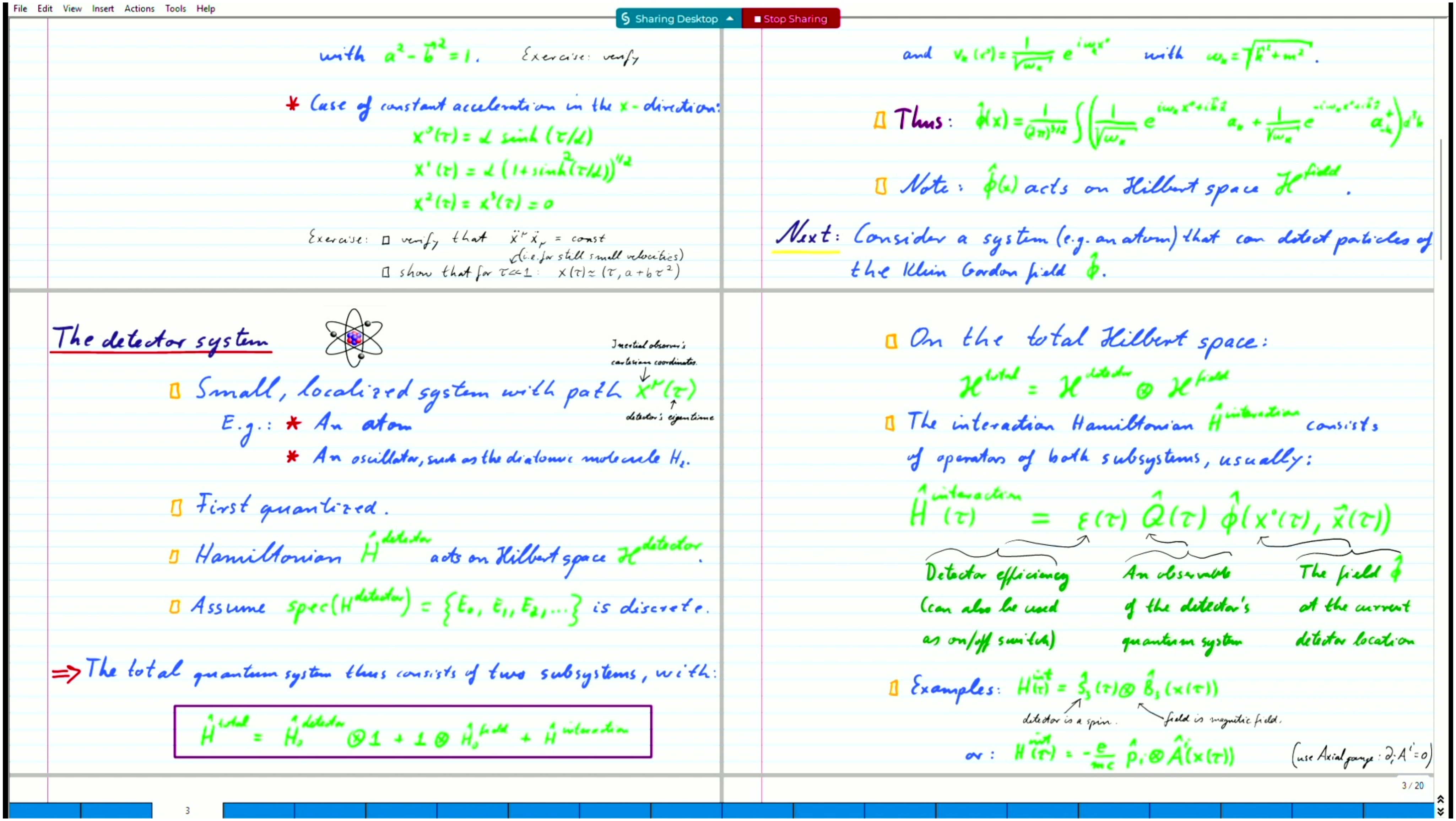
☐ Thus: $\hat\phi(x) = \frac{1}{(2\pi)^{3/2}}\int\left(\frac{1}{\sqrt{\omega_k}}e^{i\omega_k x^0 + i\vec{k}\vec{x}}a_k + \frac{1}{\sqrt{\omega_k}}e^{-i\omega_k x^0 + i\vec{k}\vec{x}}a_{-k}^\dagger\right)d^3k$

☐ Note: $\hat\phi(x)$ acts on Hilbert space $\mathcal{H}^{field}$.

Next: Consider a system (e.g. an atom) that can detect particles of the Klein Gordon field $\hat\phi$.



with $a^2 - \vec{b}^2 = 1$. Exercise: verify

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☐ show that for $\tau \ll 1$: $x(\tau) \approx (\tau, a + b\tau^2)$

and $v_k(x^0) = \frac{1}{\sqrt{\omega_k}} e^{i\omega_k x^0}$ with $\omega_k = \sqrt{\vec{k}^2 + m^2}$.

☐ **Thus:** $\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \left(\frac{1}{\sqrt{\omega_k}} e^{i\omega_k x^0 + i\vec{k}\vec{x}} a_k + \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k x^0 + i\vec{k}\vec{x}} a^+_{-k} \right) d^3k$

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The detector system

☐ Small, localized system with path $x^\mu(\tau)$

Inertial observer's cartesian coordinates.

detector's eigentime

E.g.: * An atom

* An oscillator, such as the diatomic molecule H_2.

☐ First quantized.

☐ Hamiltonian $\hat{H}^{detector}$ acts on Hilbert space $\mathcal{H}^{detector}$.

☐ Assume $spec(H^{detector}) = \{E_0, E_1, E_2, \ldots\}$ is discrete.

⇒ The total quantum system thus consists of two subsystems, with:

$$\boxed{\hat{H}^{total} = \hat{H}_0^{detector} \otimes 1 + 1 \otimes \hat{H}_0^{field} + \hat{H}^{interaction}}$$

☐ On the total Hilbert space:

$$\mathcal{H}^{total} = \mathcal{H}^{detector} \otimes \mathcal{H}^{field}$$

☐ The interaction Hamiltonian $\hat{H}^{interaction}$ consists of operators of both subsystems, usually:

$$\hat{H}^{interaction}(\tau) = \varepsilon(\tau)\, \hat{Q}(\tau)\, \hat{\phi}(x^0(\tau), \vec{x}(\tau))$$

Detector efficiency (can also be used as on/off switch)

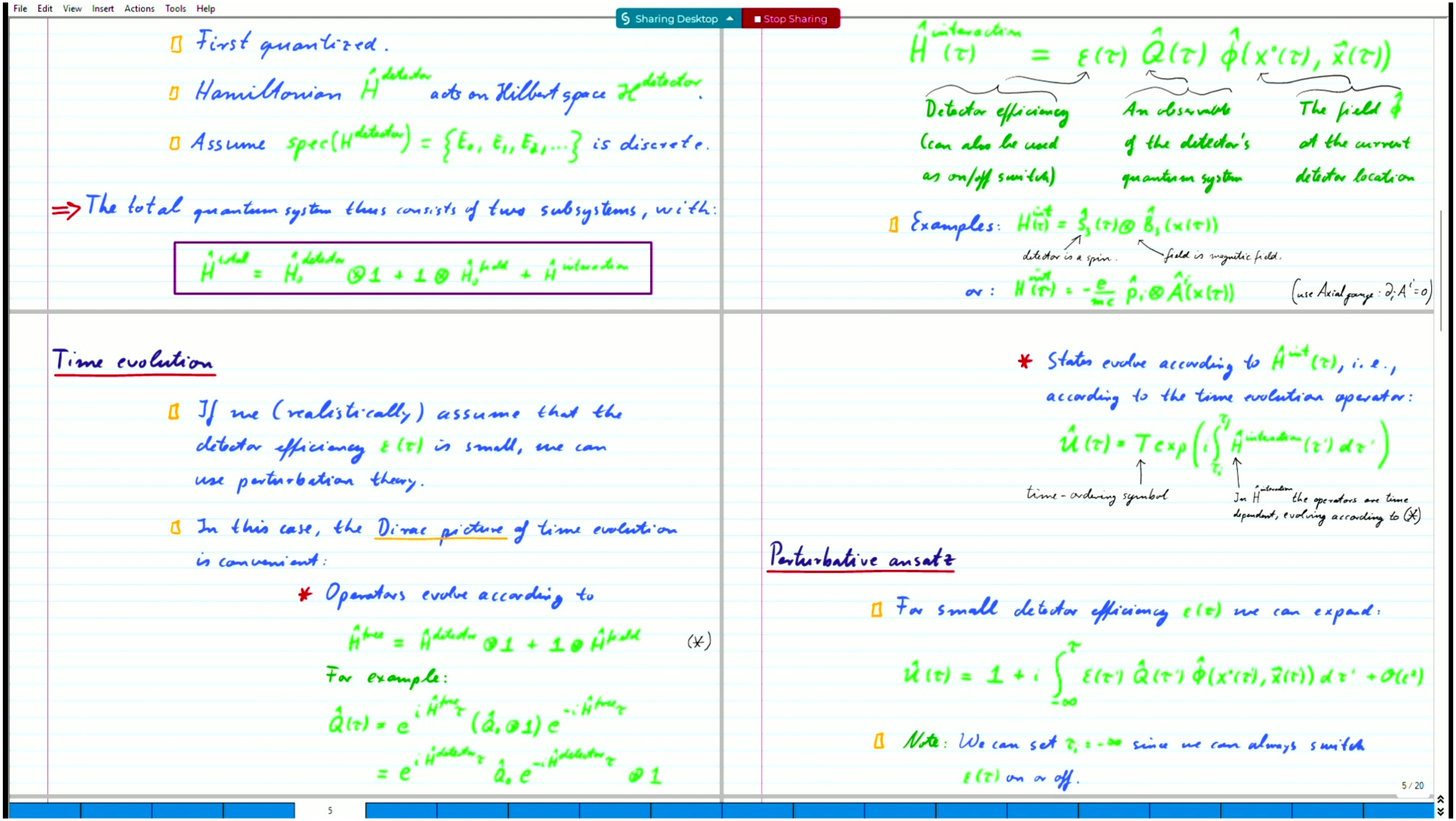
An observable of the detector's quantum system

The field $\hat{\phi}$ at the current detector location

☐ Examples: $H^{int}(\tau) = \hat{S}_3(\tau) \otimes \hat{B}_3(x(\tau))$

detector is a spin. field is magnetic field.

or: $H^{int}(\tau) = -\frac{e}{mc} \hat{p}_i \otimes \hat{A}^i(x(\tau))$ (use Axial gauge: $\partial_i A^i = 0$)



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$$\hat{H}^{interaction}(\tau) = \underbrace{\varepsilon(\tau)}_{\substack{\text{Detector efficiency}\\ \text{(can also be used}\\ \text{as on/off switch)}}} \underbrace{\hat{Q}(\tau)}_{\substack{\text{An observable}\\ \text{of the detector's}\\ \text{quantum system}}} \underbrace{\hat{\phi}(x^0(\tau), \vec{x}(\tau))}_{\substack{\text{The field } \hat{\phi}\\ \text{at the current}\\ \text{detector location}}}$$

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Time evolution

- If we (realistically) assume that the detector efficiency $\varepsilon(\tau)$ is small, we can use perturbation theory.
- In this case, the Dirac picture of time evolution is convenient:

  * Operators evolve according to

  $$\hat{H}^{free} = \hat{H}^{detector} \otimes 1 + 1 \otimes \hat{H}^{field} \qquad (*)$$

  For example:

  $$\hat{Q}(\tau) = e^{i\hat{H}^{free}\tau} (\hat{Q}_0 \otimes 1) e^{-i\hat{H}^{free}\tau}$$

  $$= e^{i\hat{H}^{detector}\tau} \hat{Q}_0 e^{-i\hat{H}^{detector}\tau} \otimes 1$$

  * States evolve according to $\hat{H}^{int}(\tau)$, i.e., according to the time evolution operator:

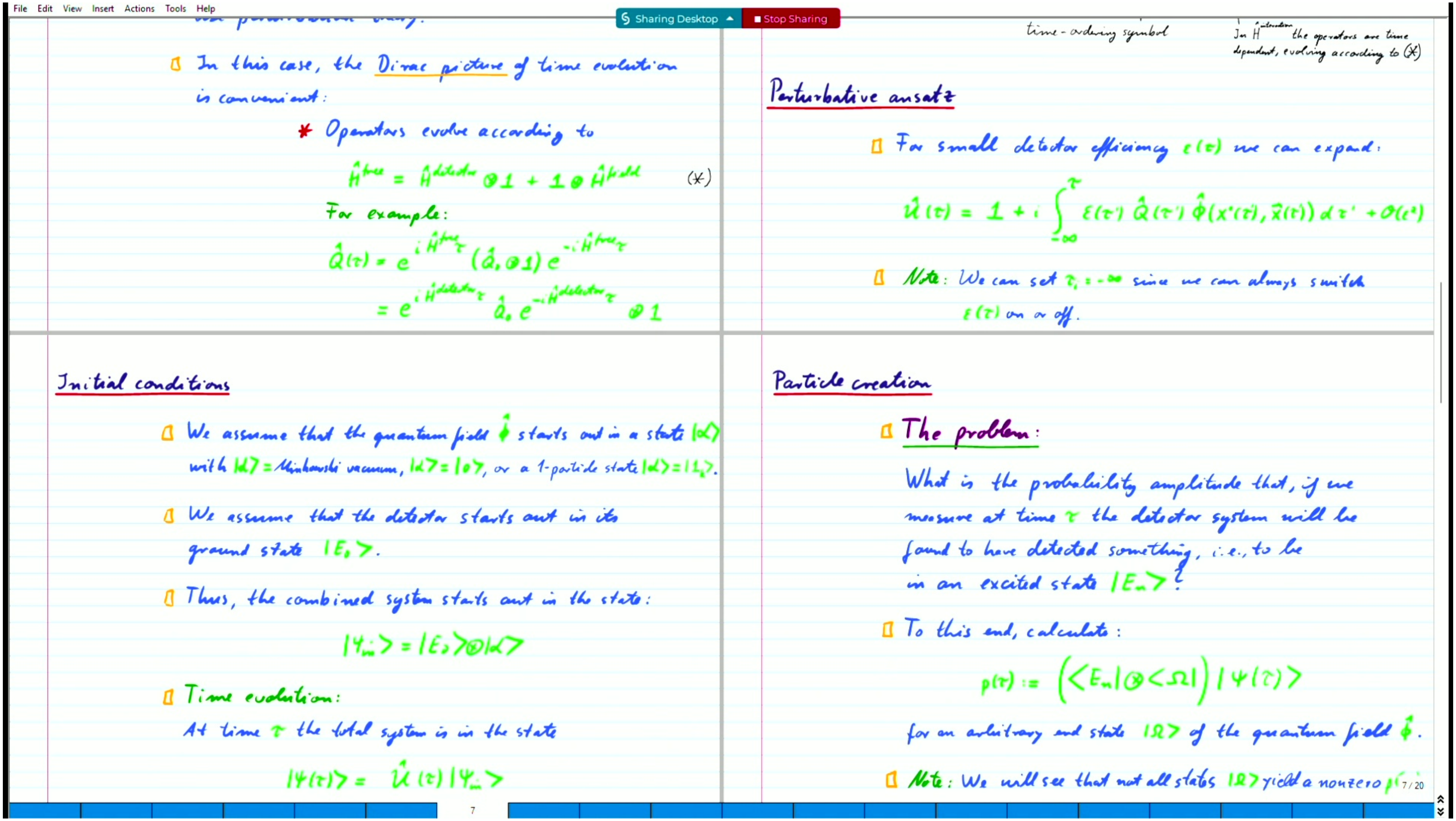
  $$\hat{U}(\tau) = \underset{\substack{\uparrow\\ \text{time-ordering symbol}}}{T} \exp\left(i \int_{\tau_i}^{\tau_f} \underset{\substack{\uparrow\\ \text{In } \hat{H}^{interaction} \text{ the operators are time}\\ \text{dependent, evolving according to } (*)}}{\hat{H}^{interaction}}(\tau')\, d\tau'\right)$$

Perturbative ansatz

- For small detector efficiency $\varepsilon(\tau)$ we can expand:

  $$\hat{U}(\tau) = 1 + i \int_{-\infty}^{\tau} \varepsilon(\tau') \hat{Q}(\tau') \hat{\Phi}(x^0(\tau'), \vec{x}(\tau')) \, d\tau' + O(\varepsilon^2)$$

- Note: We can set $\tau_i = -\infty$ since we can always switch $\varepsilon(\tau)$ on or off.



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time-ordering symbol

In $\hat{H}^{interaction}$ the operators are time dependent, evolving according to $(*)$

Perturbative ansatz

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- Note: We can set $\tau_i = -\infty$ since we can always switch $\varepsilon(\tau)$ on or off.

Initial conditions

- We assume that the quantum field $\hat{\phi}$ starts out in a state $|\alpha\rangle$ with $|\alpha\rangle =$ Minkowski vacuum, $|\alpha\rangle = |0\rangle$, or a 1-particle state $|\alpha\rangle = |1_k\rangle$.

- We assume that the detector starts out in its ground state $|E_0\rangle$.

- Thus, the combined system starts out in the state:

$$|\Psi_{in}\rangle = |E_0\rangle \otimes |\alpha\rangle$$

- Time evolution:

  At time τ the total system is in the state

$$|\Psi(\tau)\rangle = \hat{U}(\tau) |\Psi_{in}\rangle$$

Particle creation

- The problem:

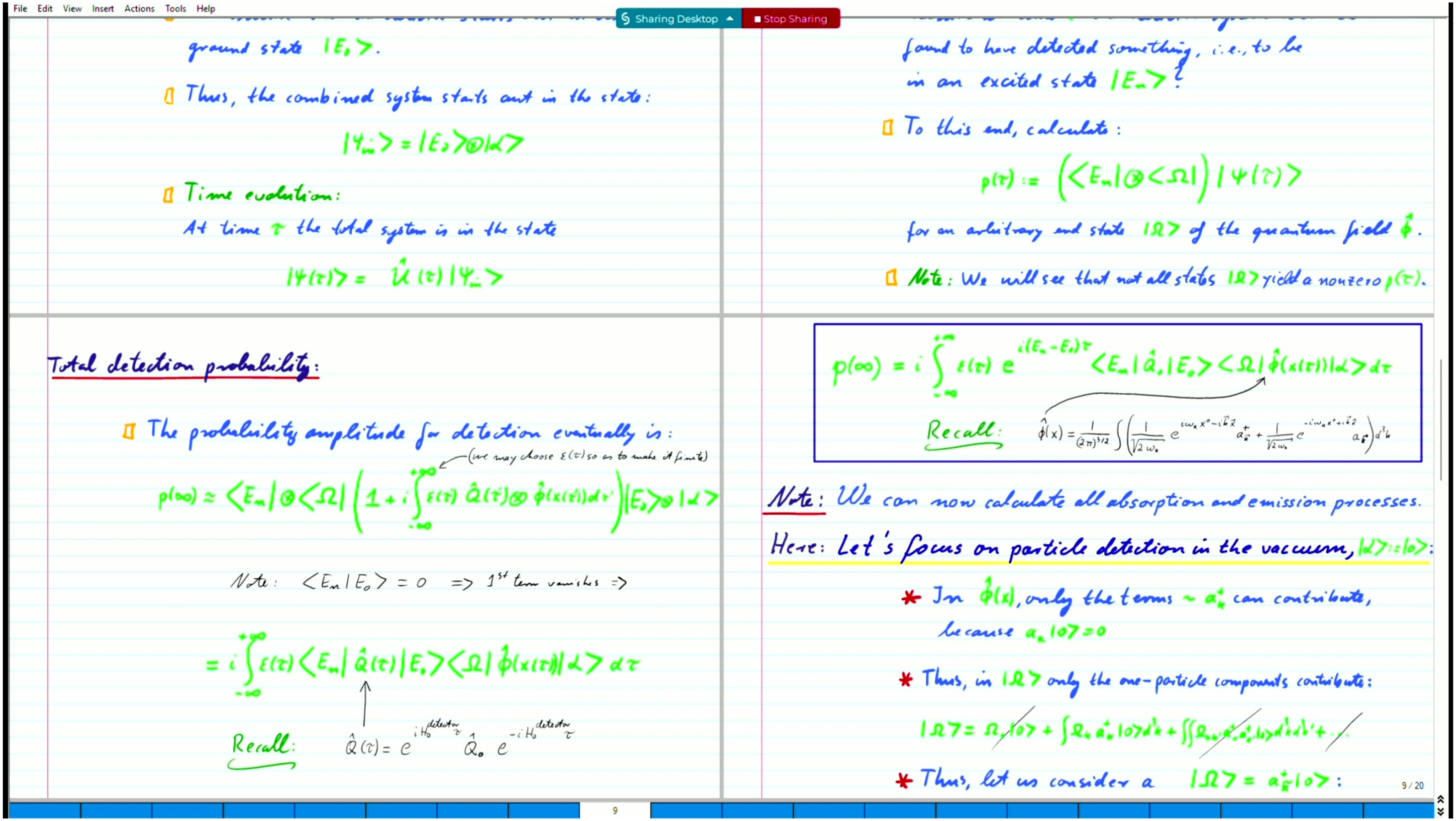
  What is the probability amplitude that, if we measure at time τ the detector system will be found to have detected something, i.e., to be in an excited state $|E_n\rangle$?

- To this end, calculate:

$$p(\tau) := \left(\langle E_n| \otimes \langle \Omega|\right) |\Psi(\tau)\rangle$$

  for an arbitrary end state $|\Omega\rangle$ of the quantum field $\hat{\phi}$.

- Note: We will see that not all states $|\Omega\rangle$ yield a nonzero $p($



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Total detection probability:

- The probability amplitude for detection eventually is:

(we may choose $\varepsilon(\tau)$ so as to make it finite)

$$p(\infty) \approx \langle E_m| \otimes \langle\Omega| \left(1 + i\int_{-\infty}^{+\infty} \varepsilon(\tau)\, \hat{Q}(\tau') \otimes \hat{\phi}(x(\tau'))\,d\tau'\right) |E_0\rangle \otimes |\alpha\rangle$$

Note: $\langle E_m|E_0\rangle = 0$ => 1st term vanishes =>

$$= i\int_{-\infty}^{+\infty} \varepsilon(\tau) \langle E_m|\hat{Q}(\tau)|E_0\rangle \langle\Omega|\hat{\phi}(x(\tau))|\alpha\rangle\, d\tau$$

Recall: $\hat{Q}(\tau) = e^{iH_0^{detector}\tau}\, \hat{Q}_0\, e^{-iH_0^{detector}\tau}$

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$$\boxed{\begin{gathered} p(\infty) = i\int_{-\infty}^{+\infty} \varepsilon(\tau)\, e^{i(E_m - E_0)\tau} \langle E_m|\hat{Q}_0|E_0\rangle \langle\Omega|\hat{\phi}(x(\tau))|\alpha\rangle\, d\tau \\ \text{Recall: } \hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \left(\frac{1}{\sqrt{2\omega_k}} e^{i\omega_k x^0 - i\vec{k}\vec{x}} a_{\vec{k}}^{+} + \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k x^0 + i\vec{k}\vec{x}} a_{\vec{k}}\right) d^3k \end{gathered}}$$

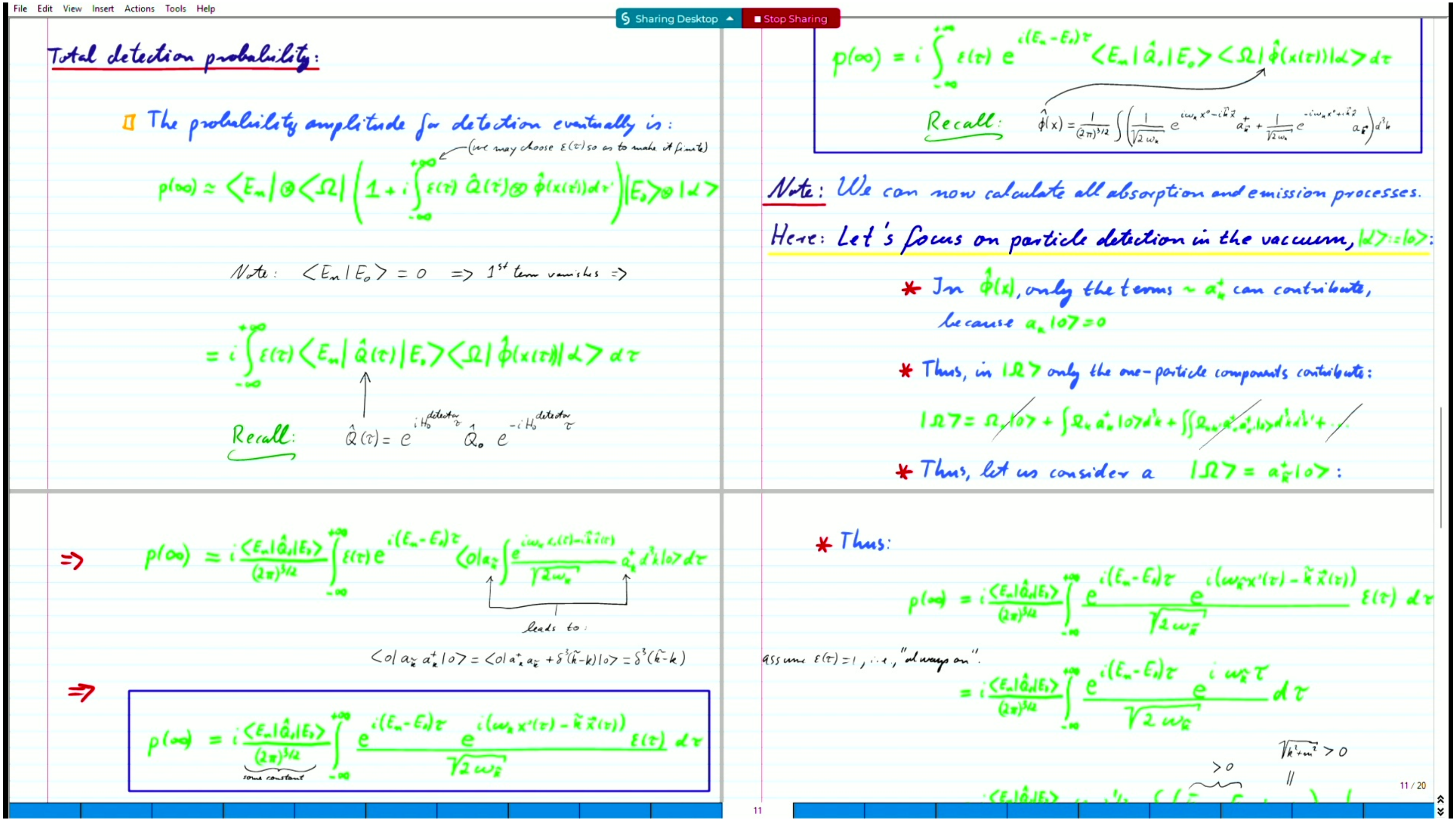
Note: We can now calculate all absorption and emission processes.

Here: Let's focus on particle detection in the vacuum, $|\alpha\rangle := |0\rangle$:

- ✱ In $\hat{\phi}(x)$, only the terms $\sim a_k^{+}$ can contribute, because $a_k|0\rangle = 0$
- ✱ Thus, in $|\Omega\rangle$ only the one-particle components contribute:

$$|\Omega\rangle = \cancel{\Omega_0|0\rangle} + \int \Omega_k a_k^{+}|0\rangle d^3k + \cancel{\iint \Omega_{kk'} a_k^{+} a_{k'}^{+}|0\rangle d^3k\, d^3k'} + \cancel{\ldots}$$

- ✱ Thus, let us consider a $|\Omega\rangle = a_{\vec{k}}^{+}|0\rangle$:



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$$= i\int_{-\infty}^{+\infty} \varepsilon(\tau) \langle E_m | \hat{Q}(\tau) | E_0\rangle \langle \Omega | \hat{\phi}(x(\tau)) | \alpha \rangle \, d\tau$$

Recall: $\hat{Q}(\tau) = e^{iH_0^{detector}\tau}\, \hat{Q}_0\, e^{-iH_0^{detector}\tau}$

$$\Rightarrow \quad p(\infty) = i\frac{\langle E_m|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} \varepsilon(\tau)\, e^{i(E_m-E_0)\tau} \langle 0 | a_{\tilde{k}} \int \frac{e^{i\omega_k x_0(\tau) - i\vec{k}\vec{x}(\tau)}}{\sqrt{2\omega_k}}\, a_k^+ \, d^3k |0\rangle \, d\tau$$

leads to:

$$\langle 0 | a_{\tilde{k}}\, a_k^+ | 0 \rangle = \langle 0 | a_k^+ a_{\tilde{k}} + \delta^3(\tilde{k}-k) | 0 \rangle = \delta^3(\tilde{k}-k)$$

$$\Rightarrow \quad \boxed{p(\infty) = i\underbrace{\frac{\langle E_m|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}}_{\text{some constant}} \int_{-\infty}^{+\infty} \frac{e^{i(E_m-E_0)\tau}\; e^{i(\omega_{\tilde{k}} x^0(\tau) - \tilde{k}\vec{x}(\tau))}\, \varepsilon(\tau)}{\sqrt{2\omega_{\tilde{k}}}} \, d\tau}$$

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Note: We can now calculate all absorption and emission processes.

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assume $\varepsilon(\tau) = 1$, i.e., "always on".

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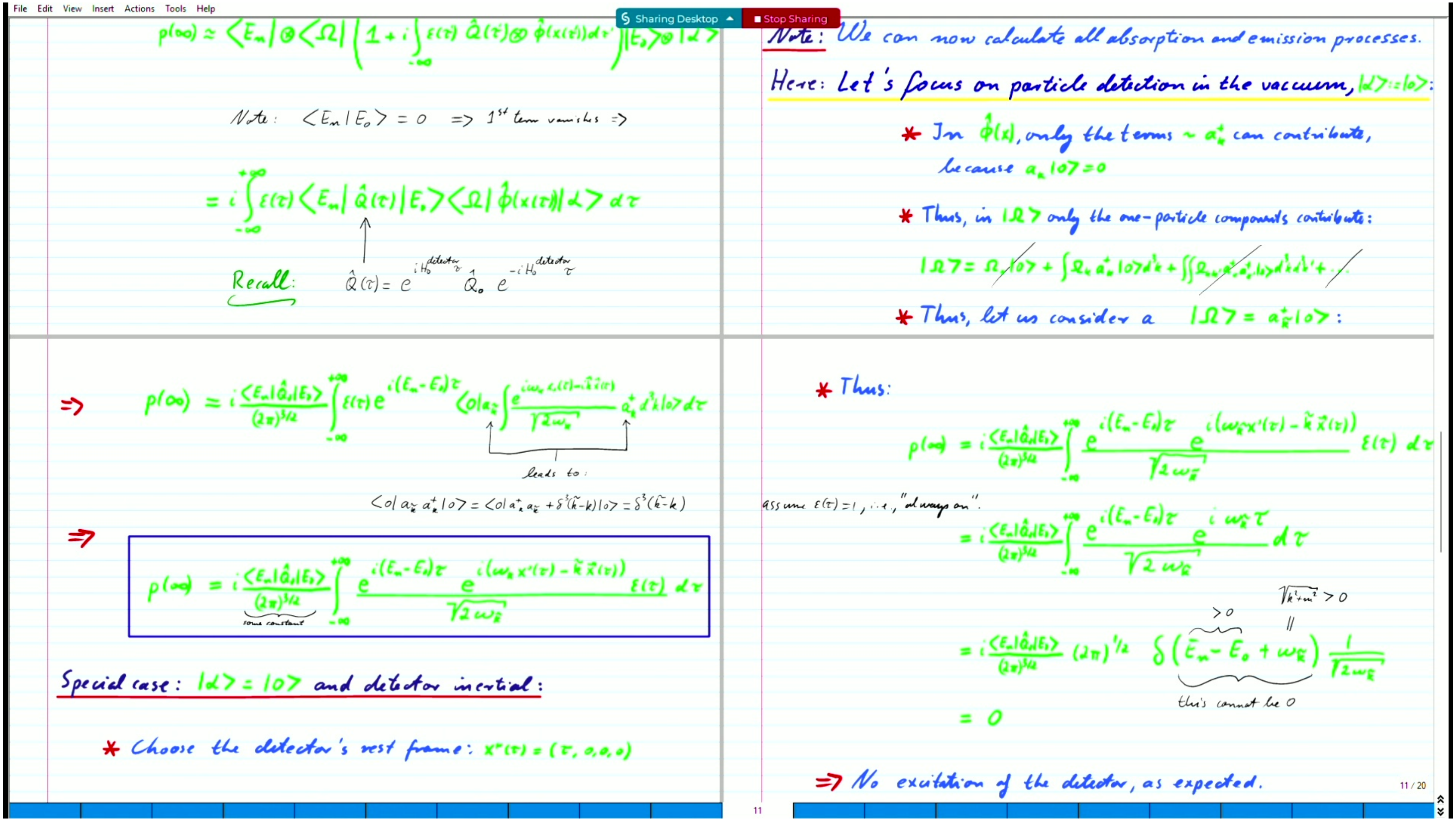
$\sqrt{k^2+m^2} > 0$

$||$

> 0



$$p(\infty) \simeq \langle E_n| \otimes \langle \Omega| \left(1 + i\int_{-\infty} \varepsilon(\tau')\, \hat{Q}(\tau') \otimes \hat{\phi}(x(\tau'))\, d\tau'\right) |E_0\rangle \otimes |\alpha\rangle$$

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Special case: $|\alpha\rangle = |0\rangle$ and detector inertial:

- $*$ Choose the detector's rest frame: $x^\mu(\tau) = (\tau, 0, 0, 0)$

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assume $\varepsilon(\tau) = 1$, i.e., "always on".

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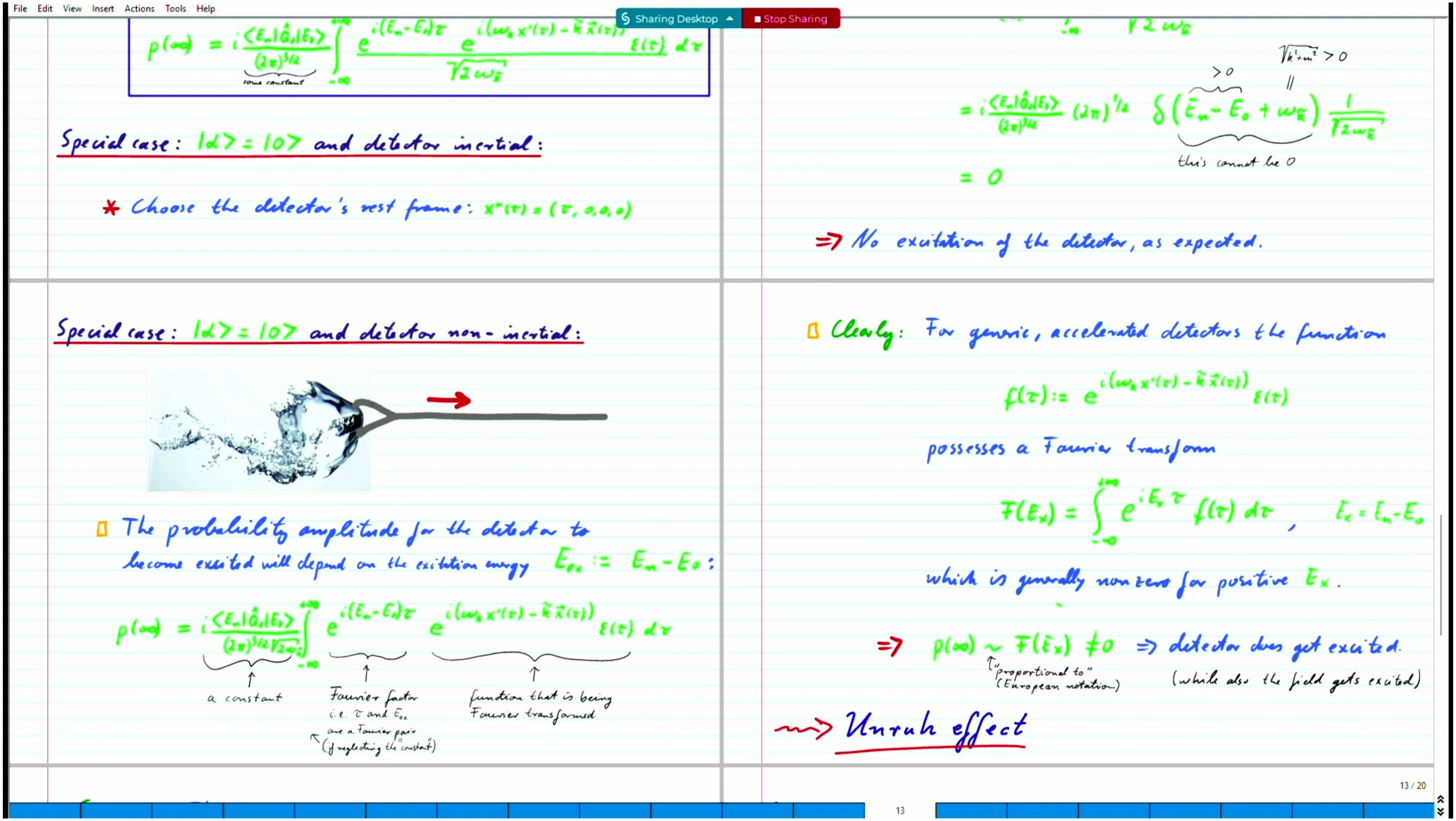
$$= i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}} (2\pi)^{1/2}\, \delta\Big(\underbrace{\overbrace{E_n - E_0}^{>0} + \overset{\sqrt{k^2+m^2} > 0 \\ \|}{\omega_{\tilde{k}}}}_{\text{this cannot be } 0}\Big) \frac{1}{\sqrt{2\omega_{\tilde{k}}}}$$

$$= 0$$

$\Rightarrow$ No excitation of the detector, as expected.



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$$= 0$$

$\Rightarrow$ No excitation of the detector, as expected.

Special case: $|\alpha\rangle = |0\rangle$ and detector non-inertial:

☐ The probability amplitude for the detector to become excited will depend on the excitation energy $E_{ex} := E_n - E_0$:

$$p(\infty) = \underbrace{i\frac{\langle E_n|\hat{Q}_d|E_0\rangle}{(2\pi)^{3/2}\sqrt{2\omega_{\vec k}}}}_{\substack{\uparrow\\ \text{a constant}}}\int_{-\infty}^{+\infty} \underbrace{e^{i(E_n-E_0)\tau}}_{\substack{\uparrow\\ \text{Fourier factor}\\ \text{i.e. } \tau \text{ and } E_{ex}\\ \text{are a Fourier pair}\\ \text{(if neglecting the "constant")}}}\ \underbrace{e^{i(\omega_k x^0(\tau)-\vec{k}\vec{x}(\tau))}\varepsilon(\tau)}_{\substack{\uparrow\\ \text{function that is being}\\ \text{Fourier transformed}}}\, d\tau$$

☐ Clearly: For generic, accelerated detectors the function

$$f(\tau) := e^{i(\omega_k x^0(\tau)-\vec{k}\vec{x}(\tau))}\varepsilon(\tau)$$

possesses a Fourier transform

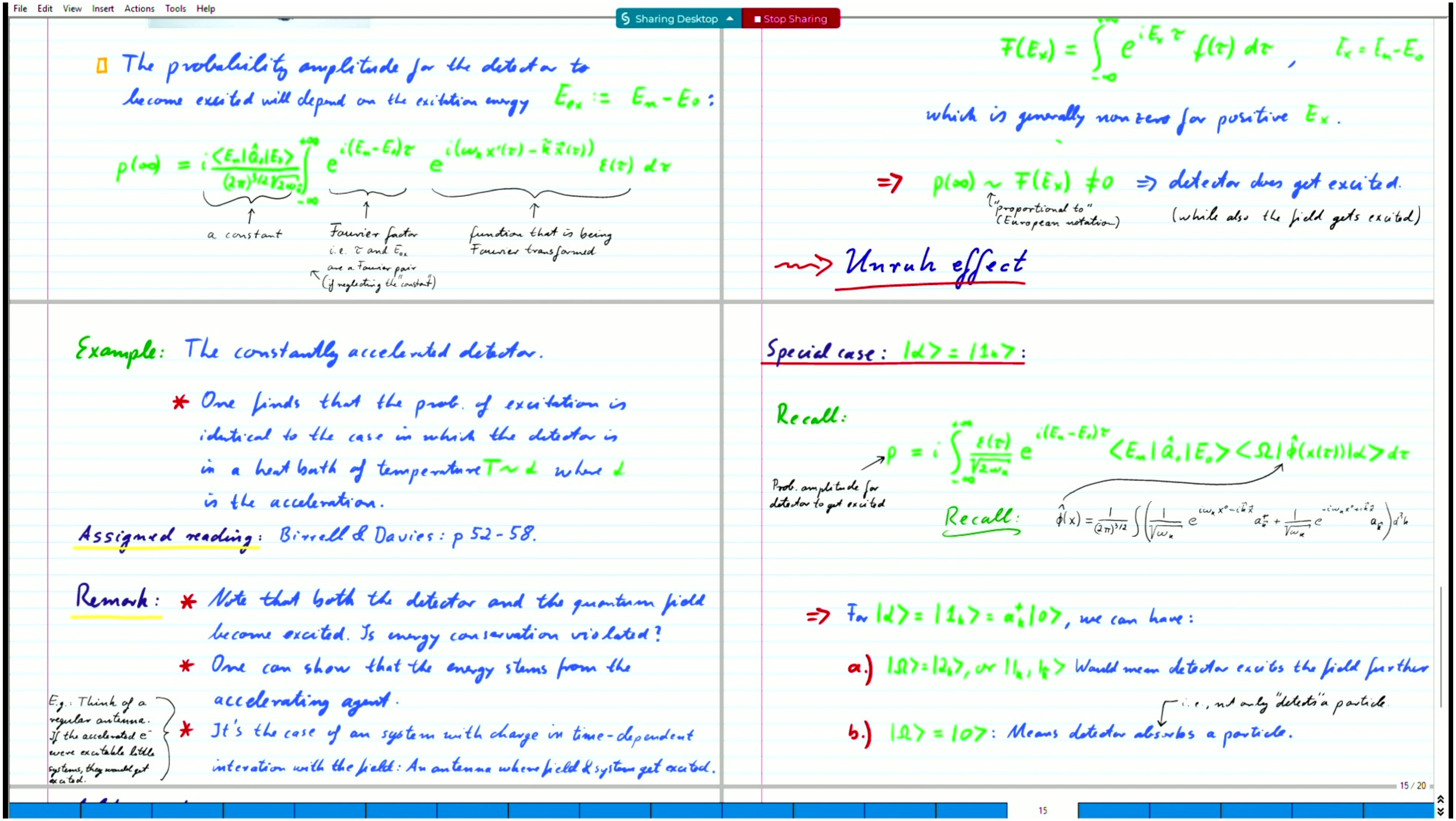
$$F(E_x) = \int_{-\infty}^{+\infty} e^{iE_x\tau} f(\tau)\, d\tau, \quad E_x = E_n - E_0$$

which is generally nonzero for positive E_x.

$\Rightarrow$ $p(\infty) \underset{\substack{\uparrow\\ \text{"proportional to"}\\ \text{(European notation)}}}{\sim} F(E_x) \neq 0 \Rightarrow$ detector does get excited.

(while also the field gets excited)

$\rightsquigarrow$ **Unruh effect**



☐ The probability amplitude for the detector to become excited will depend on the excitation energy $E_{ex} := E_n - E_0$:

$$p(\infty) = \underbrace{i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}\sqrt{2\omega_k}}}_{\text{a constant}}\int_{-\infty}^{+\infty} \underbrace{e^{i(E_n-E_0)\tau}}_{\substack{\text{Fourier factor}\\ \text{i.e. } \tau \text{ and } E_{ex}\\ \text{are a Fourier pair}}} \underbrace{e^{i(\omega_k x^0(\tau) - \vec{k}\vec{x}(\tau))} \varepsilon(\tau)}_{\substack{\text{function that is being}\\ \text{Fourier transformed}}} \, d\tau$$

(if neglecting the "constant")

$$\mathcal{F}(E_x) = \int_{-\infty}^{+\infty} e^{iE_x\tau} f(\tau)\, d\tau \,, \qquad E_x = E_n - E_0$$

which is generally non zero for positive E_x.

$\Rightarrow$ $p(\infty) \sim \mathcal{F}(E_x) \neq 0$ $\Rightarrow$ detector does get excited.

"proportional to" (European notation)

(while also the field gets excited)

$\rightsquigarrow$ **Unruh effect**

Example: The constantly accelerated detector.

* One finds that the prob. of excitation is identical to the case in which the detector is in a heat bath of temperature $T \sim a$ where a is the acceleration.

Assigned reading: Birrell & Davies: p 52 - 58.

Remark: * Note that both the detector and the quantum field become excited. Is energy conservation violated?

* One can show that the energy stems from the accelerating agent.

* It's the case of an system with charge in time-dependent interation with the field: An antenna where field & system get excited.

E.g.: Think of a regular antenna. If the accelerated e^- were excitable little systems, they would get excited.

Special case: $|\alpha\rangle = |1_k\rangle$:

Recall:

$$p = i\int_{-\infty}^{+\infty} \frac{\varepsilon(\tau)}{\sqrt{2\omega_k}} e^{i(E_n-E_0)\tau} \langle E_n|\hat{Q}_0|E_0\rangle\langle\Omega|\hat{\phi}(x(\tau))|\alpha\rangle\, d\tau$$

Prob. amplitude for detector to get excited

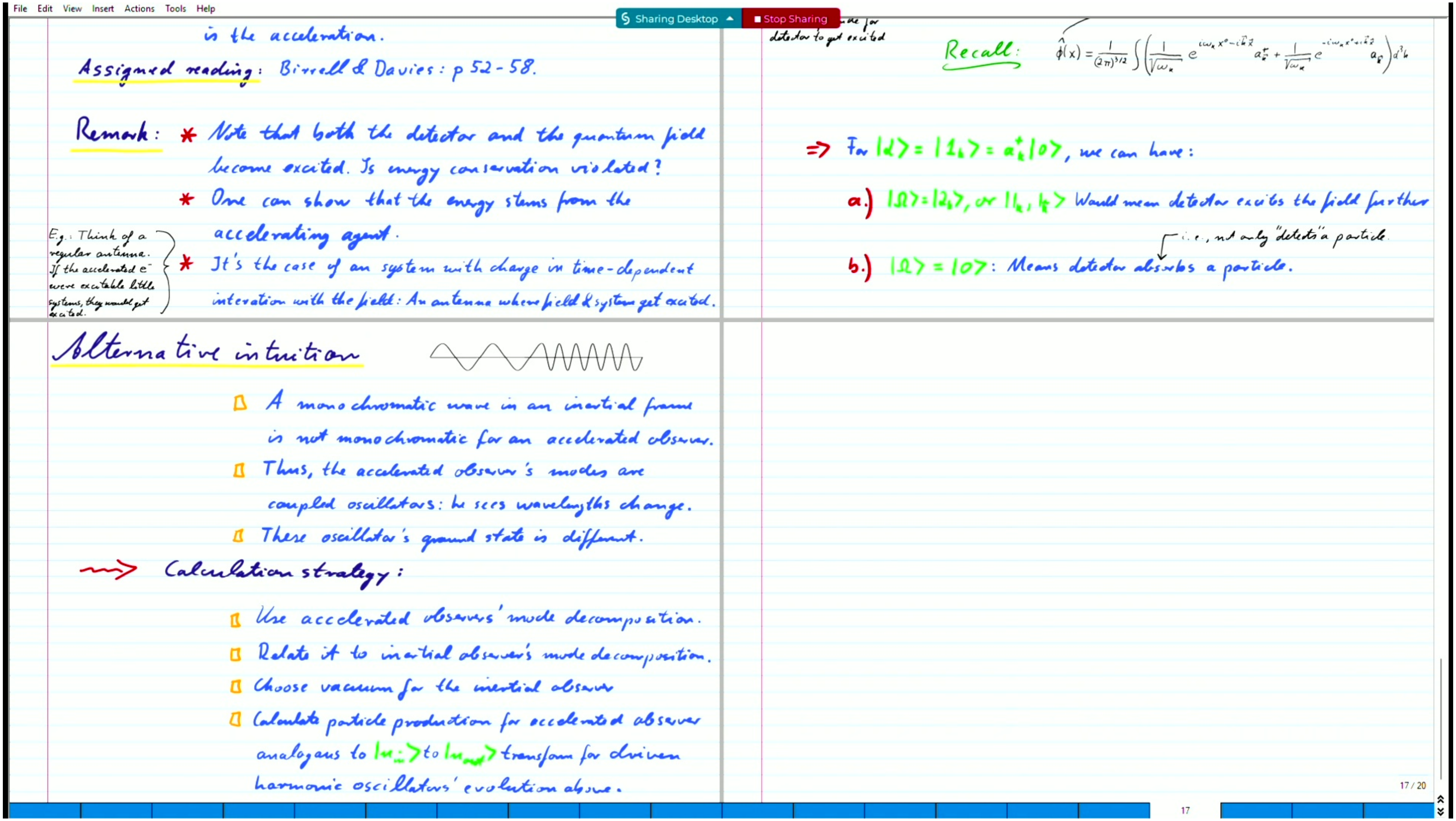
Recall: $$\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}}\int\left(\frac{1}{\sqrt{\omega_k}} e^{i\omega_k x^0 - i\vec{k}\vec{x}} a^+_{\vec{k}} + \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k x^0 + i\vec{k}\vec{x}} a_{\vec{k}}\right) d^3k$$

$\Rightarrow$ For $|\alpha\rangle = |1_k\rangle = a^+_k|0\rangle$, we can have:

a.) $|\Omega\rangle = |2_k\rangle$, or $|1_k, 1_{k'}\rangle$ Would mean detector excites the field further

i.e., not only "detects" a particle

b.) $|\Omega\rangle = |0\rangle$: Means detector absorbs a particle.



is the acceleration.

Assigned reading: Birrell & Davies: p 52-58.

Remark: * Note that both the detector and the quantum field become excited. Is energy conservation violated?

* One can show that the energy stems from the accelerating agent.

* It's the case of an system with charge in time-dependent interation with the field: An antenna where field & system get excited.

E.g.: Think of a regular antenna. If the accelerated e^- were excitable little systems, they would get excited.

Alternative intuition

- A monochromatic wave in an inertial frame is not monochromatic for an accelerated observer.
- Thus, the accelerated observer's modes are coupled oscillators: he sees wavelengths change.
- These oscillator's ground state is different.

→ Calculation strategy:

- Use accelerated observers' mode decomposition.
- Relate it to inertial observer's mode decomposition.
- Choose vacuum for the inertial observer
- Calculate particle production for accelerated observer analogaus to $|u_{in}\rangle$ to $|u_{out}\rangle$ transform for driven harmonic oscillators' evolution above.

for detector to get excited

Recall: $\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \left(\frac{1}{\sqrt{\omega_k}} e^{i\omega_k x^0 - i\vec{k}\vec{x}} a_k^+ + \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k x^0 + i\vec{k}\vec{x}} a_k \right) d^3k$

⇒ For $|\alpha\rangle = |1_k\rangle = a_k^+|0\rangle$, we can have:

a.) $|\Omega\rangle = |2_k\rangle$, or $|1_k, 1_{k'}\rangle$ Would mean detector excites the field further

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