Title: Boundary vertex operator algebras of 3d N=4 rank-zero SCFTs.

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Series: Quantum Fields and Strings

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Abstract: I will talk about the boundary vertex operator algebra of topologically twisted 3d N=4 rank-zero SCFTs. The latter is recently introduced family of N=4 SCFTs with zero-dimensional Higgs and Coulomb branches, which are expected to support rational VOAs at the boundary. I will discuss the construction of the simplest class of rank-zero SCFTs T_r, and argue that they admit the simple affine VOAs L_r(osp(1|2)) at their boundary. In the simplest case, this leads to a physical realization of a novel level-rank duality between L_1(osp(1|2)) and the Virasoro minimal model M(2,5).

Zoom link

Boundary VOAs of 3d $\mathcal{N}=4$ rank-zero SCFTs

with D. Gang, S. Stubbs

A. Ferrari, N. Garner

3d $\mathcal{N}=2$ HT

$\mathcal{N}=4$ TFT_A

TFT_B

2d boundary

[Coulomb branch

Higgs branch

$\rightsquigarrow$ non-semisimple log-VOAs

TFTA $\rightsquigarrow$ $Z(S^2 \times S^1) \neq 1$

Q 3d $\mathcal{N}=4$? that supports rational VOAs on boundary?

TFT_B 2d boundary

[Coulomb branch
Higgs branch

$TFT_A \rightsquigarrow Z(S^2 \times S^1) \neq 1$

Q 3d $\mathcal{N}=4$? that supports rational VOAs on boundary?

① $TFT_A \rightsquigarrow$ Coulomb branch operators

— Coulomb branch is a point

② Higgs branch is a point

$\rightsquigarrow$ rank-zero theories

max (dim (Coulomb branch), dim (Higgs branch))

1) T_{min}

TFT_B 3d boundary

[Coulomb branch
Higgs branch

$\mathrm{TFT}_A \rightsquigarrow Z(S^2 \times S^1) \neq 1$

Q 3d $\mathcal{N}=4$? that supports rational VOAs on boundary?

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$\rightsquigarrow$ rank-zero theories

max (dim (Coulomb branch) , dim (Higgs branch))

1) T_{min} [Gang–Yamazaki '18]

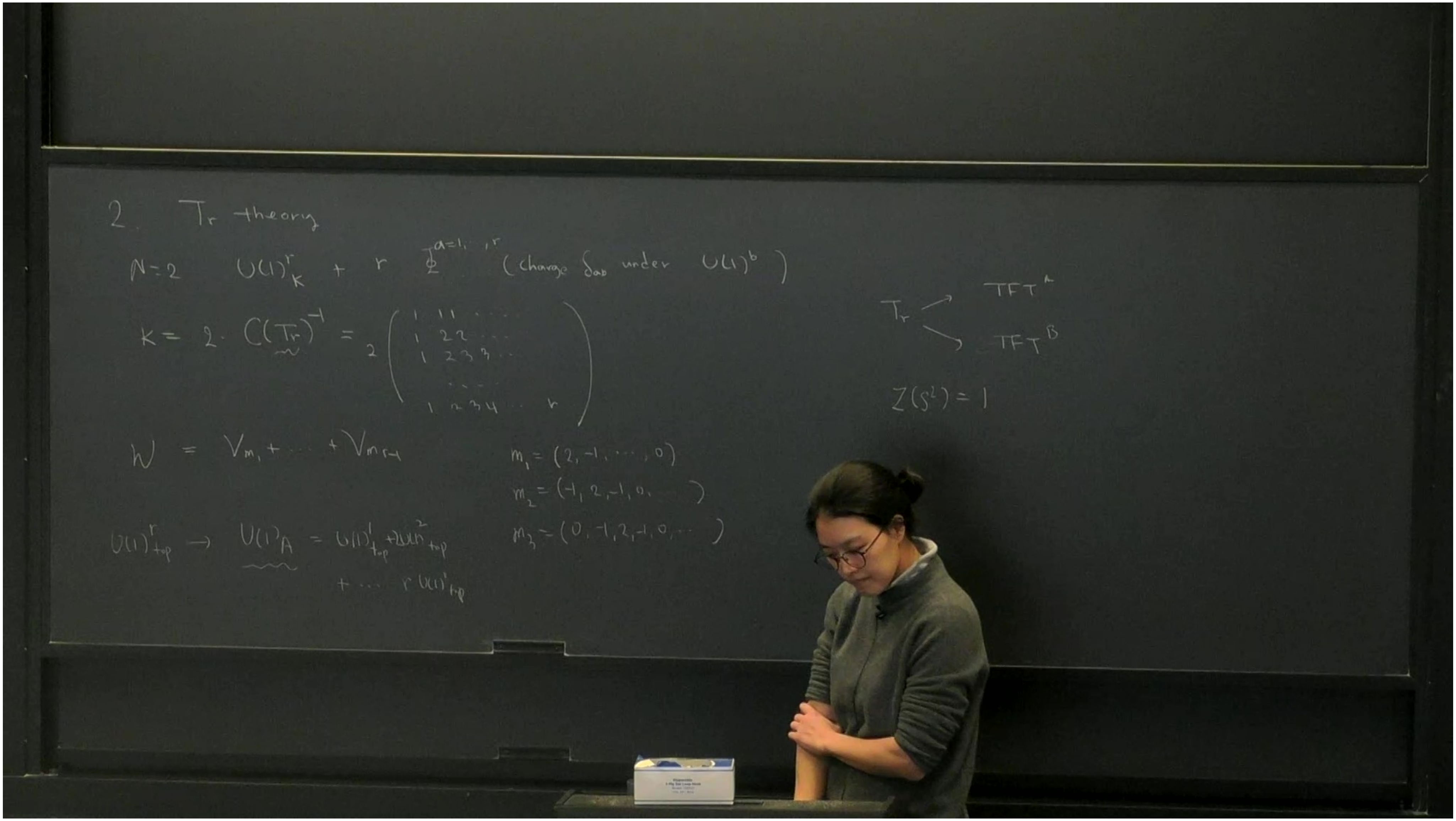
Plan

1) T_r , $T_{r=1} = T_{min}$

2) $\mathrm{TFT}^B_{min} \to \widehat{osp}(1|2)_{k=1}$

$M(2,5)$

level-rank duality


2. T_r theory
N=2 U(1)^r_K + r Φ^(a=1,...,r) (charge δ_ab under U(1)^b)
K = 2·C(T_r)^-1 = 2
1 1 1 ...
1 2 2 ...
1 2 3 3 ...
1 2 3 4 ... r
W = V_m1 + ... + V_m(r-1)
m_1 = (2, -1, ..., 0)
m_2 = (-1, 2, -1, 0, ...)
m_3 = (0, -1, 2, -1, 0, ...)
U(1)^r_top → U(1)_A = U(1)^1_top + 2U(1)^2_top + ... r U(1)^1_top
T_r
TFT^A
TFT^B
Z(S^2) = 1

$$1\ 2\ 3\ 4\ \cdots\ r$$

$$W = V_{m_1} + \cdots + V_{m_{r-1}} \qquad m_1 = (2, -1, \cdots, 0)$$

$$m_2 = (-1, 2, -1, 0, \cdots)$$

$$U(1)^r_{top} \to \underbrace{U(1)_A}_{\sim} = U(1)^1_{top} + 2U(1)^2_{top} + \cdots + r\,U(1)^r_{top} \qquad m_3 = (0, -1, 2, -1, 0, \cdots)$$

3. Half-index

(D, D) $\qquad A_\mu\big|_\partial = 0 \qquad \phi\big|_\partial = 0$

$\underbrace{U(1)^r}_{\sim}$ global sym. on boundary

$$\frac{1}{2\pi}\int_{HS^2} F = m \in \mathbb{Z}^r$$

$$r=1 \qquad I_B = \frac{1}{(q)_\infty}\sum_{m\in\mathbb{Z}} \underbrace{q^{m^2}}\, s^{2m}\,(q^{1-m}s^{-1};q)_\infty \quad \xrightarrow{s\to 1}$$

$$I_A = \qquad q^{m^2-m}$$

$$I_A(s\to 1) = \sum_{m\in\mathbb{Z}} \frac{q^{m^2+m}}{(q)_m}$$

. Vac character of $M(2,5)$

$$I_A[W](s\to 1) =$$

$$\underbrace{I_A(s\to 1) = \sum_{m\in\mathbb{Z}^r} \frac{q^{\frac{1}{2}km^2 + Qm}}{(q)_{m_1}\cdots(q)_{m_r}}}_{\sim} \quad : \text{Vac } M(2, 2r+3)$$

$$\chi \sim \sum_{m} \frac{q^{Am^2/2 + Bm + C}}{(q)_{m_1} \cdots (q)_{m_r}}$$

$r = 1, 2, 3$

$$Q_B = Q_{HT} + \delta_B$$

$Q^{+\dot{+}}_{+}$ $\qquad$ $Q^{+\dot{-}}_{-}$

b. VOA for $T^B_{min} = T^B_{r=1}$

$k=2$ $\qquad$ $U(1) + \mathbb{Z}_2$ $\qquad$ $k = 3/2$

$$Q(\phi V_{+}) \quad Q(\bar{\phi} V_{-1}) \sim G^{-\dot{+}}$$

$$\sim G^{+\dot{-}}$$

$$\chi \sim \sum_{m} \frac{q^{Am^2/2 + Bm + C}}{(q)_{m_1} \cdots (q)_{m_r}}$$

$r = 1, 2, 3$

b. VOA for $T^B_{min} = T^B_{r=1}$

$N=2$ $\quad U(1) + T_{\frac{1}{2}}$ $\quad k = 3/2$

$$Q(\phi V_+) \quad Q(\bar{\psi} V_{-1}) \sim G^{-\dot{+}}$$

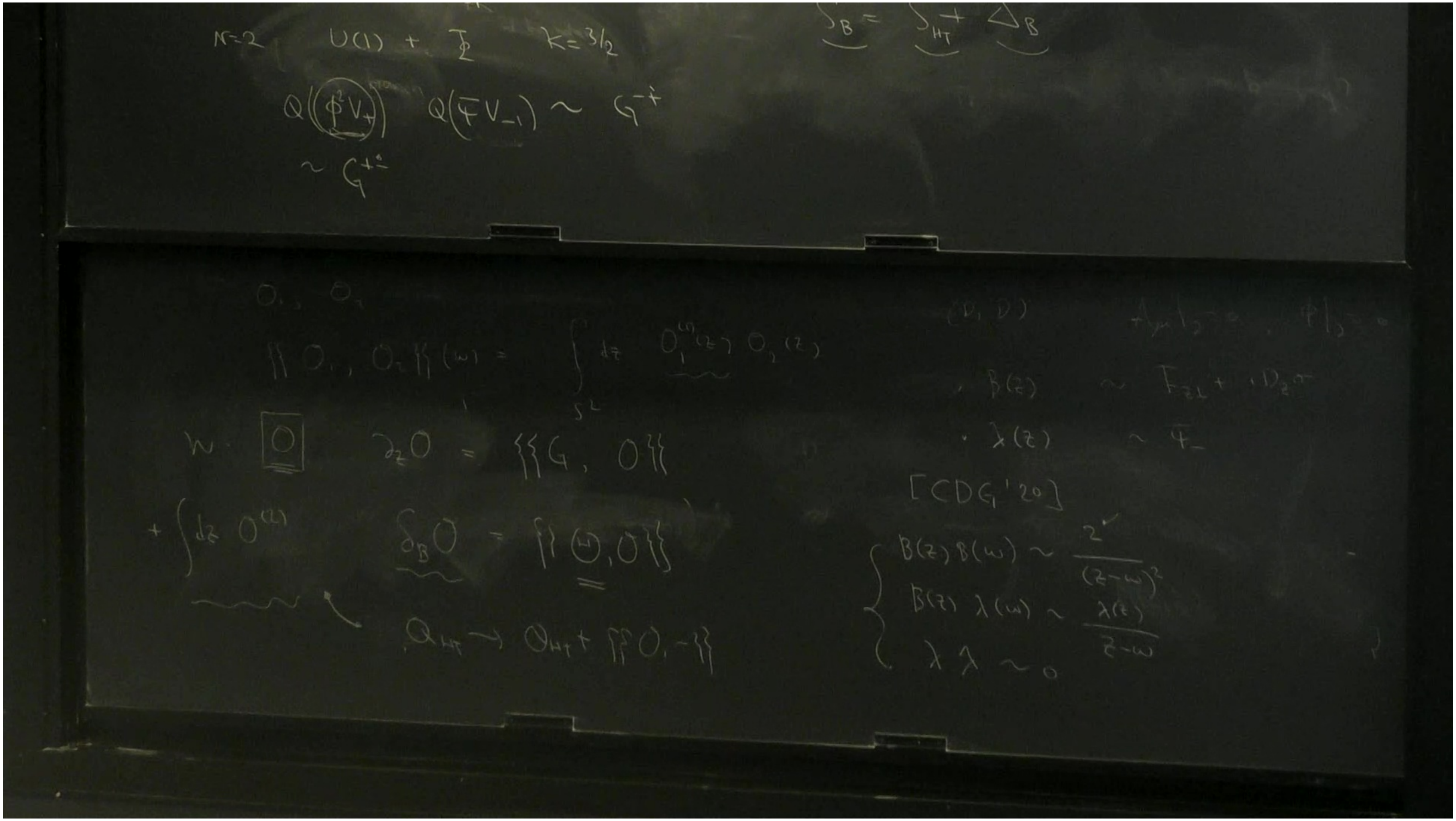
$$\sim G^{+\dot{-}}$$

$$Q_B = Q_{HT} + \delta_B$$

$Q^{+\dot{+}}_{+}$ $\quad Q^{+\dot{-}}_{-}$

$$J_B = J - \frac{R_c}{2}$$

$$S_B = S_{HT} + \Delta_B$$


N=2
U(1) + Φ
k=3/2
[CDG'20]
B(z) B(w) ~ 2/(z-w)^2
B(z) λ(w) ~ λ(z)/(z-w)
λ λ ~ 0

$$1\ 2\ 3\ 4\ \cdots\ r$$

$$W = V_{m_1} + \cdots + V_{m_{r-1}}$$

$$m_1 = (2, -1, \cdots, 0)$$
$$m_2 = (-1, 2, -1, 0, \cdots)$$
$$m_3 = (0, -1, 2, -1, 0, \cdots)$$

$$U(1)^r_{top} \to \underset{\sim}{U(1)_A} = U(1)^1_{top} + 2U(1)^2_{top} + \cdots r\, U(1)^r_{top}$$

$$U(1)_2 \longrightarrow SU(2)_2$$

$$B(z) \qquad B(z),\ V_+,\ V_-$$

$$h \quad e \quad f$$

$$\theta_+ \sim \lambda$$
$$\theta_-$$

$$f(z)\,\theta_+(w) \sim \frac{\theta_-(w)}{z-w}$$

$$e\ \theta_- \sim \frac{\theta_+}{z-w}$$

$$h\ \theta_\pm \sim \frac{\theta_\pm}{z-w}$$

$$W \sim \phi^2 V_+$$

$$\psi\psi \sim \frac{1}{z}\partial^2 W$$

$$\rightsquigarrow \underbrace{\widehat{osp}(1|2)_{k=1}}$$

$$\theta_+\theta_+ \sim \frac{e(w)}{z-w}$$

$\mathcal{N}=2$ $\quad U(1) + \Psi_2 \quad k = 3/2$

$$S_B = S_{HT} + \Delta_B$$

$$Q(\phi^\dagger V_+) \quad Q(\bar{\Psi} V_{-1}) \sim G^{-+}$$

$$\sim G^{+-}$$

$$\frac{\widehat{su}(2)_{k=1}}{\widehat{sl}(2)_{k=1}} \simeq M(3,5)$$

$\beta(z) \sim F_{zz} + \ldots$

$\lambda(z) \sim \bar{\Psi}$

$$\widehat{osp}(1|2)_{k=1} = [M(3,5) \otimes \widehat{sl}(2)_{k=1}] / \mathbb{Z}_2$$

[CDG '20]

Gaiotto - Komargodski - Wu

$$[(U(1)_0 \text{-charge } 2) \otimes U(1)_2] / \mathbb{Z}_2^{[?]} \simeq T_{min}$$

$$\begin{cases} B(z)B(w) \sim \dfrac{2}{(z-w)^2} \\ B(z)\lambda(w) \sim \dfrac{\lambda(z)}{z-w} \\ \lambda\lambda \sim 0 \end{cases}$$

$$I(q,s) = \chi[L_1]\,\chi[V_1^{(3,5)}](q) - \chi[L_2](q,s)\,\chi[V_4^{(3,5)}](q)$$

$$W = V_{m_1} + \cdots + V_{m_{r-1}}$$

$m_1 = (2, -1, \cdots, 0)$

$m_2 = (-1, 2, -1, 0, \cdots)$

$m_3 = (0, -1, 2, -1, 0, \cdots)$

$$U(1)^r_{top} \to \underset{\sim\sim}{U(1)_A} = U(1)^1_{top} + 2U(1)^2_{top} + \cdots r\, U(1)^r_{top}$$

(N, M) Vacuum ch of $M(2,5)$

$$\frac{bc^{\otimes 2}}{M(2,5)} \simeq \widehat{osp}(1|2)_1 \qquad \frac{bc^{\otimes 2}}{\widehat{osp}(1|2)_1} = M(2,5)$$

$\widehat{gl}(k)_n \quad | \quad SU(n)_k \quad | \leftarrow \widehat{sl}(n)_k$

Bounded VOAs

$$\chi \sim \sum_{m} \frac{q^{Am^2/2 + Bm + C}}{(q)_{m_1} \cdots (q)_{m_r}}$$

$r = 1,2,3$

$Q_B = Q_{HT} + \delta_B$

$Q^{+\dot{+}}_{+}$ $\quad Q^{+\dot{-}}_{-}$

$J_B = J - \frac{k_c}{2}$

$S_B = S_{HT} + \Delta_B$

b. VOA for $T^{B}_{min} = T^{B}_{r=1}$

$N=2$ $\quad U(1) + T_2 \quad k = 3/2$

$Q(\bar{\phi} V_+) \quad Q(\bar{\psi} V_{-1}) \sim G^{-\dot{+}}$

$\sim G^{+\dot{-}}$

$A = B \otimes C^{-1}$

$B, C : \; C(A_r) \; C(B_r) \; C(C_r) \; C(T_r)$