Title: Effect of non-unital noise on random circuit sampling

Speakers: Kohdai Kuroiwa
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Abstract: In this work, drawing inspiration from the type of noise present in real hardware, we study the output distribution of random quantum circuits under practical non-unital noise sources with constant noise rates. We show that even in the presence of unital sources like the depolarizing channel, the distribution, under the combined noise channel, never resembles a maximally entropic distribution at any depth. To show this, we prove that the output distribution of such circuits never anticoncentrates -- meaning it is never too "flat" -- regardless of the depth of the circuit. This is in stark contrast to the behavior of noiseless random quantum circuits or those with only unital noise, both of which anticoncentrate at sufficiently large depths. As consequences, our results have interesting algorithmic implications on both the hardness and easiness of noisy random circuit sampling, since anticoncentration is a critical property exploited by both state-of-the-art classical hardness and easiness results.

Zoom link

EFFECT OF NON-UNITAL NOISE ON RANDOM CIRCUIT SAMPLING

arXiv:2306.16659

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Joint work with

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PI Grad Student Seminar, 29th January 2024

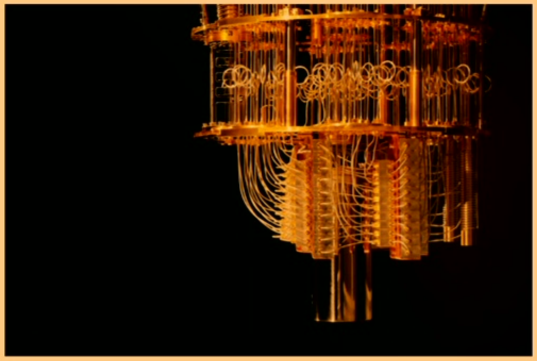
Introduction

CURRENT QUANTUM COMPUTERS

Noisy Intermediate-Scale Quantum (NISQ) devices:

Near term quantum computers that

1. have ~1000 quantum bits (qubits)
2. Do NOT perform error correction.



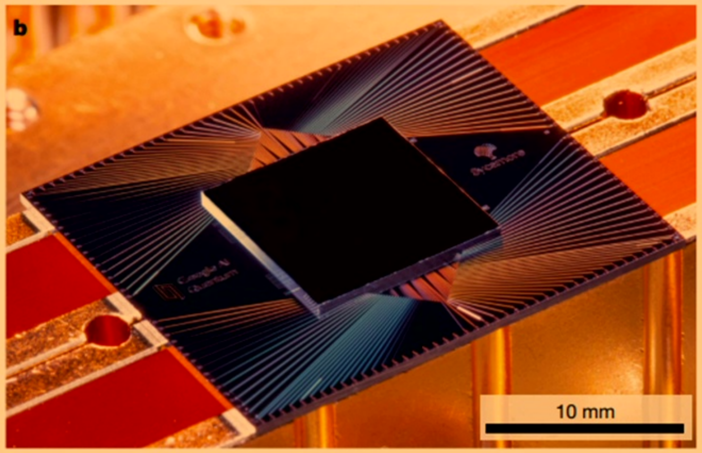
https://newsroom.ibm.com/media-quantum-innovation?keywords=quantum&l=100#gallery_gallery_0:21747

Do quantum computers outperform classical computers?

QUANTUM ADVANTAGE IN NISQ ERA

Quantum Supremacy

Recently, experimentally demonstrated quantum computers can efficiently perform some task, which is expected to be classically hard.

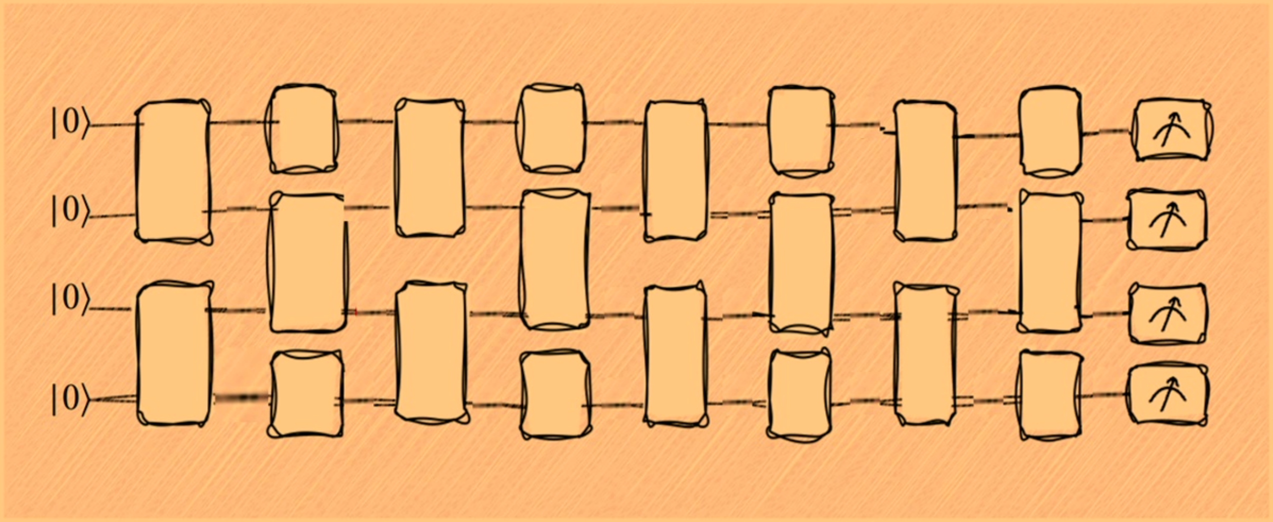


Arute *et al.*, Nature **574**, 505–510 (2019)

THE TASK FOR QUANTUM ADVANTAGE

Random circuit sampling

1. Construct a "random" quantum state
2. Sample from the resulting state via quantum measurement in $\{|0\rangle, |1\rangle\}$



White rectangles: random gates

OUR QUESTION

Have we really achieved quantum advantage?

1. Is it classically hard? ⇨ Yes, if noiseless
   - We have **classical hardness evidence** for "noiseless" case (Aaronson and Chen, 2016) (Bouland *et al.* 2018)
   - Current classical algorithms fail to simulate the latest noisy quantum advantage demonstration in a reasonable amount of human time (Morvan et al, 2023)

2. Doesn't noise matter? ⇨ Unfortunately, yes…
   - The fidelity is low due to the noise…
   - Noise causes the decay of "quantum signal" as the system size grows

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The effects of noise must be taken into account

TAKEAWAY MESSAGE

We show that in case of realistic noise models, all current techniques to analyze quantum advantage experiments with random circuits fail.

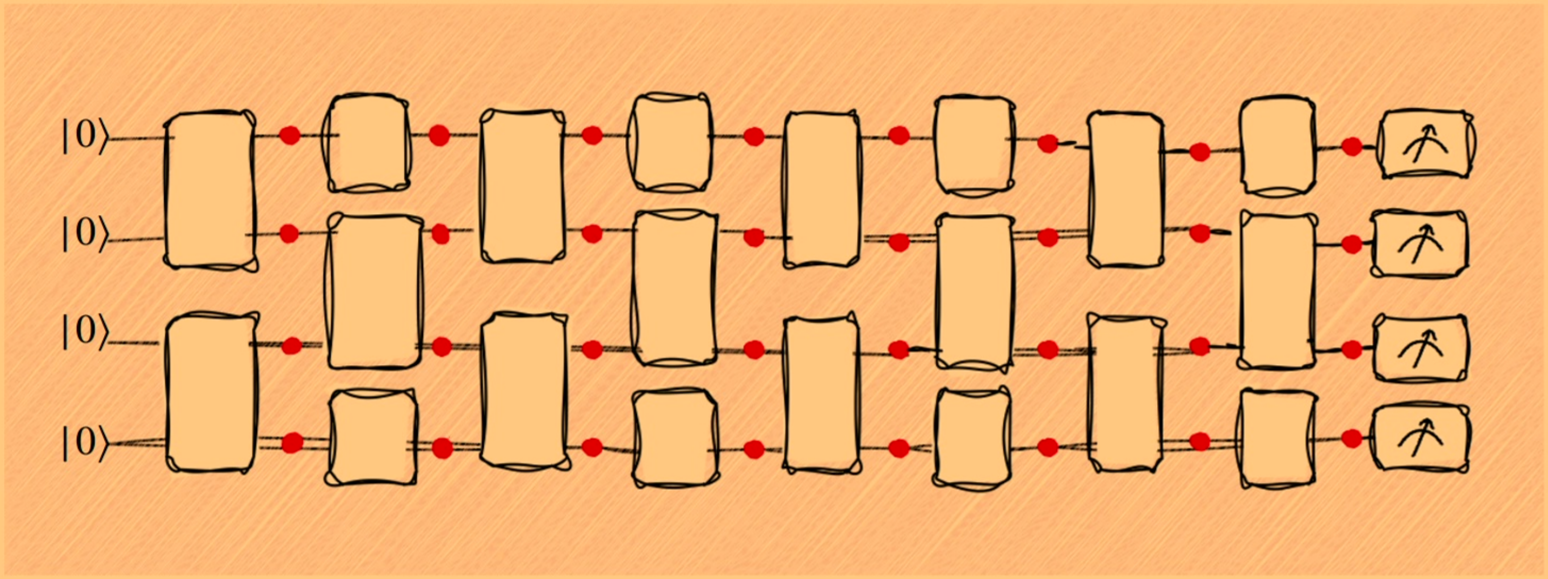
We need new techniques to discuss quantum advantage in this noisy regime!

Noise model in quantum advantage experiments

HOW DO WE MODEL NOISE?

Conventional approach

Depolarizing noise: $\mathcal{N}_p^{(\mathrm{dep})}(\rho) = (1-p)\cdot\rho + p\cdot\frac{\mathbb{I}}{2}$



White rectangles: random gates; Red dots: depolarizing noise

WHAT HAPPENS UNDER DEPOLARIZING NOISE?

The effects of depolarizing noise

Increasing entropy and the output distribution eventually converges to the flat distribution which is easy to sample from

↓

If the circuit is deep enough, classical sampling should be easy

- At super-logarithmic depth, classically sampling from the uniform distribution suffices (Aharonov et al, 1996) (Deshpande et al, 2022)
- Classical easiness persists at logarithmic depth too. (Aharonov et al, 2022)

IS THE NOISE MODEL REALISTIC?

- Depolarizing noise describes a noise component increasing entropy ("more flatness", unital noise)
- However, other noise sources (T_1 decay, readout error etc) that can decrease entropy also exists("less flatness", non-unital noise)
- In realistic noise model, there is a fight between the two components

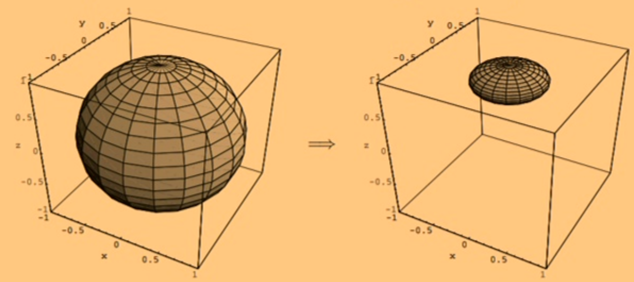
Who wins this fight?

⇨ We show that non-unital noise appears to be winning the fight!

Main results

EXAMPLE OF NON-UNITAL NOISE

(Nielsen and Chuang, 2010)



Amplitude damping noise

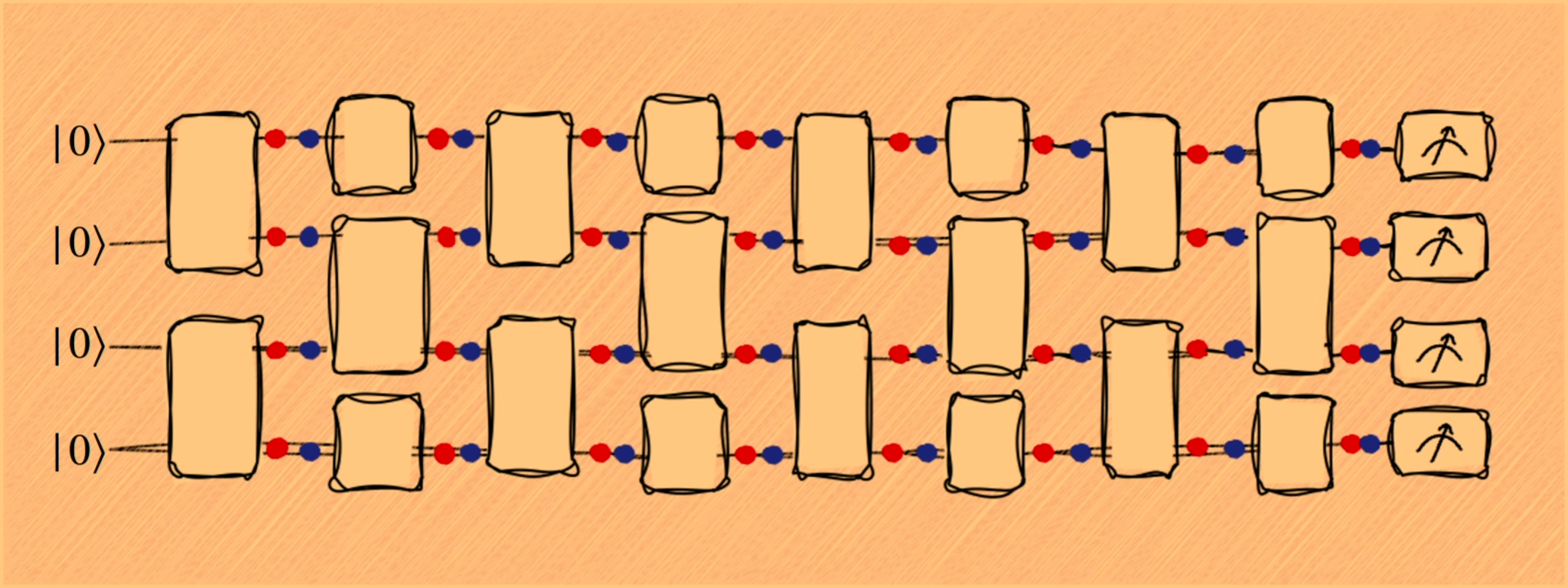
$$U_q^{(\mathrm{amp})}|0\rangle_S|0\rangle_E = |0\rangle_S|0\rangle_E$$
$$U_q^{(\mathrm{amp})}|1\rangle_S|0\rangle_E = \sqrt{q}|0\rangle_S|1\rangle_E + \sqrt{1-q}|1\rangle_S|0\rangle_E$$

(Stinespring representation)

Considered to model T1 decay etc.

In the paper (arXiv:2306.16659), we analyzed a general non-unital noise!

OUR NOISE MODEL



The red dots are unital sources (depolarizing), the blue ones are non-unital sources (amplitude damping). The relative order of the sources does not matter.

OUR RESULTS (INFORMAL)

We show that such noisy random circuits NEVER become too "flat"!

In more detail…

We show lack of the "flatness" is exhibited at any depth, any relative strength of the amplitude damping and the depolarizing channel, as long as both are constants.

- So, all known results for easiness and hardness of random circuit sampling are no longer valid in this noise model because they all rely on this "flatness" property…

WHAT IS THE CONSEQUENCE?

We do NOT understand the power of noisy random circuit sampling and we need new techniques to understand realistic quantum advantage.

Technical details

FORMAL STATEMENT FOR "FLATNESS" PROPERTY

Mathematically, this concept of "flatness" is called anti-concentration

(Aaronson and Arkhipov, 2010)

For all $x \in \{0,1\}^n$,

$$\Pr_{\mathcal{B}}\left[p_x \geq \frac{\alpha}{2^n}\right] \geq \beta$$

where $\alpha, \beta \in (0,1]$ are constants.

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Indicating the output probabilities of most bit strings in the computational basis are sufficiently large

FORMAL STATEMENT

We show that noisy random circuits with amplitude damping noise and depolarizing noise will NEVER be anti-concentrated

Mathematically, what we want to prove is:

Let $\mathcal{B}$ be an ensemble of noisy random quantum circuits with amplitude damping noise and depolarizing noise. For any $x \in \{0,1\}^n$ with at least $\frac{n}{2}$ 1's and any $\alpha > 0$,

$$\lim_{n\to\infty} \Pr_{\mathcal{B}}\left[p_x < \frac{\alpha}{2^n}\right] = 1$$

PROOF TECHNIQUES

Key ingredient: Chebyshev's inequality

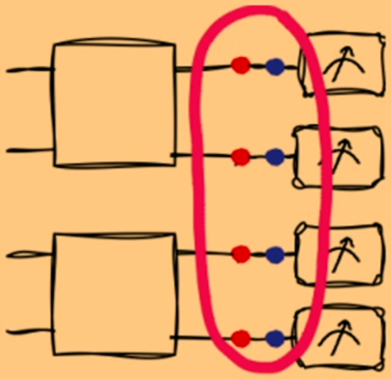
$$\Pr\left(|X - \mathbb{E}[X]| < k\sqrt{\mathrm{Var}[X]}\right) \geq \frac{1}{k^2}$$

To use this bound, we compute the first moment $\mathbb{E}[p_x]$ and the second moments $\mathbb{E}[p_x^2]$ of output probabilities.

See our paper (arXiv:2306.16659) for details!

KEY IDEAS IN OUR PROOF TECHNIQUE

For evaluation of the first and second moments, we "separate out" the last layer of noise.



- A noisy random circuit $\mathcal{C}$ can be written as $\mathcal{C} = \mathcal{N}^{\otimes n} \circ \mathcal{C}'$
- Then, $p_x = \mathrm{Tr}[|x\rangle\langle x|\mathcal{C}(|0^n\rangle\langle 0^n|)] = \mathrm{Tr}[(\mathcal{N}^{\dagger})^{\otimes n}(|x\rangle\langle x|)\mathcal{C}'((|0^n\rangle\langle 0^n|)]$

CONCLUSION

- We showed that under a realistic noise model, noisy random circuits **never anti-concentrate**

CONCLUSION

- We showed that under a realistic noise model, noisy random circuits never anti-concentrate

- Then, how powerful is noisy random circuit sampling?

  We don't know! This is a very important open problem.
  Since anti-concentration fails, we need new techniques.

- The question of classically sampling still remains open.

WHAT'S NEXT?

The first goal is asymptotic hardness
How hard are random quantum circuits with non-unital noise as the system size scales?

The second goal is easiness for finite system sizes
Have finite sized noisy ramdom circuit sampling experiments achieved a Goldilocks' zone that puts them just beyond the reach of existing classical simulators running in reasonable human time?

Thank you!