Title: Dos and Donâ€™ts of Folding Time

Speakers: Sebastian Mizera

Series: Colloquium

Date: January 31, 2024 - 2:00 PM

URL: https://pirsa.org/24010096

Abstract: I will summarize recent progress in uncovering the analytic structure of scattering amplitudes. The overarching theme will be exploiting new intricate ways of analytically continuing time, extending beyond the Wick rotation. I will highlight a broad range of applications: from high-precision calculations in particle physics, through computations of gravitational waves, to formal topics in the scattering of strings.

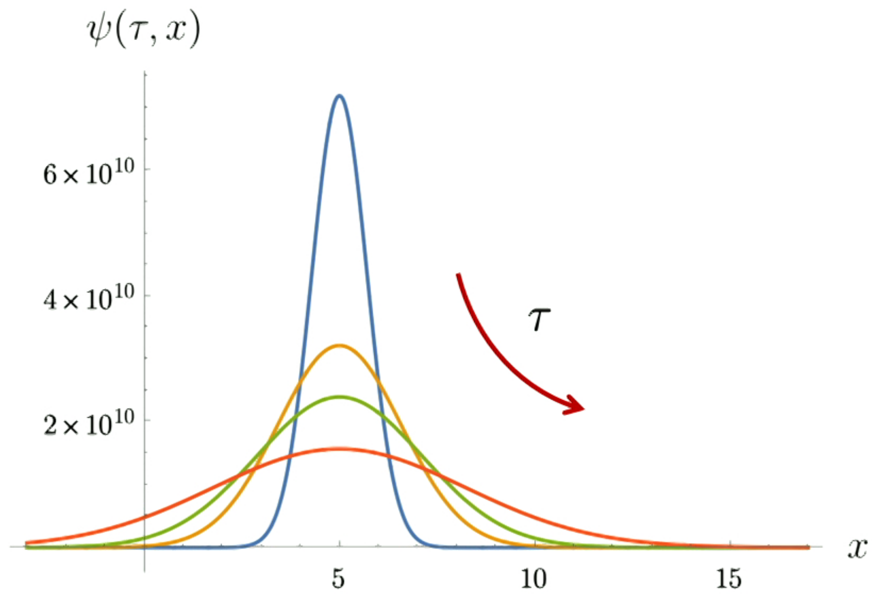
Zoom link

Dos and Don'ts of Folding Time

Sebastian Mizera (IAS Princeton)

Perimeter Institute, 31 January 2024

Heat equation: $\frac{\partial \psi}{\partial \tau} = \alpha \frac{\partial^2 \psi}{\partial x^2}$



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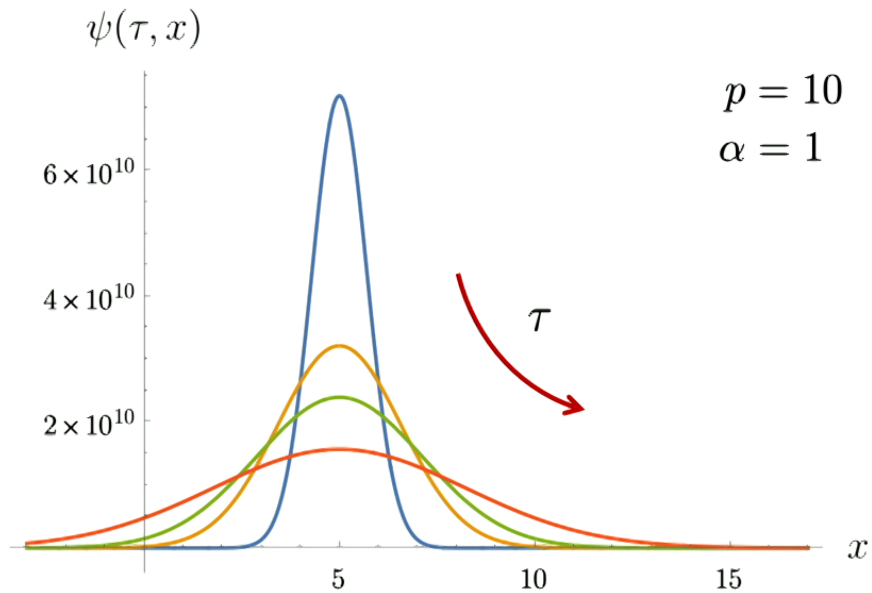

$$\psi(\tau, x) = \frac{1}{\sqrt{1 + 4\alpha\tau}} e^{\frac{p^2 \alpha \tau + p x - x^2}{1 + 4\alpha\tau}}$$

Free Schrödinger equation: $$-i\frac{\partial\psi}{\partial t} = \underbrace{\frac{\hbar}{2m}}_{\alpha}\frac{\partial^2\psi}{\partial x^2}$$

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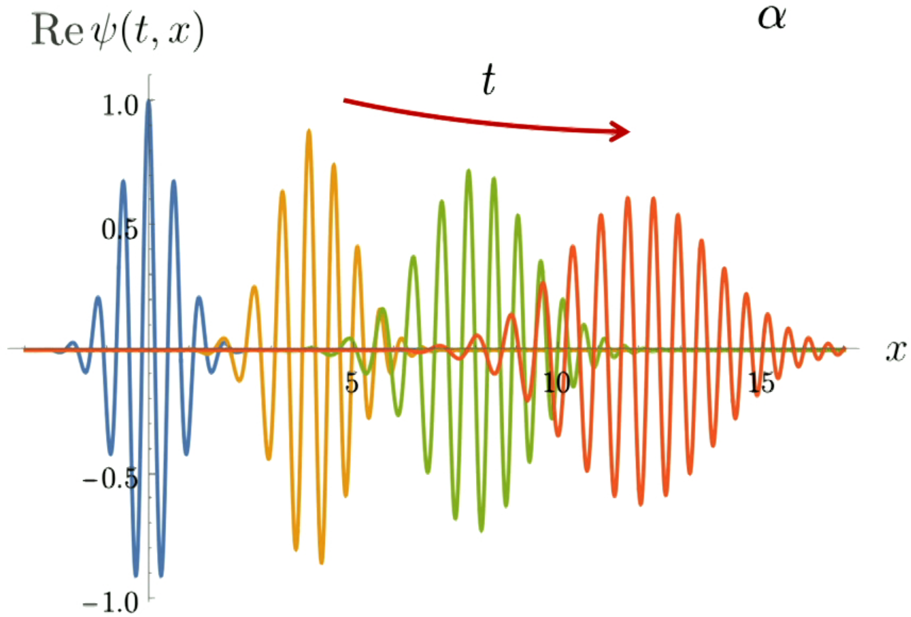
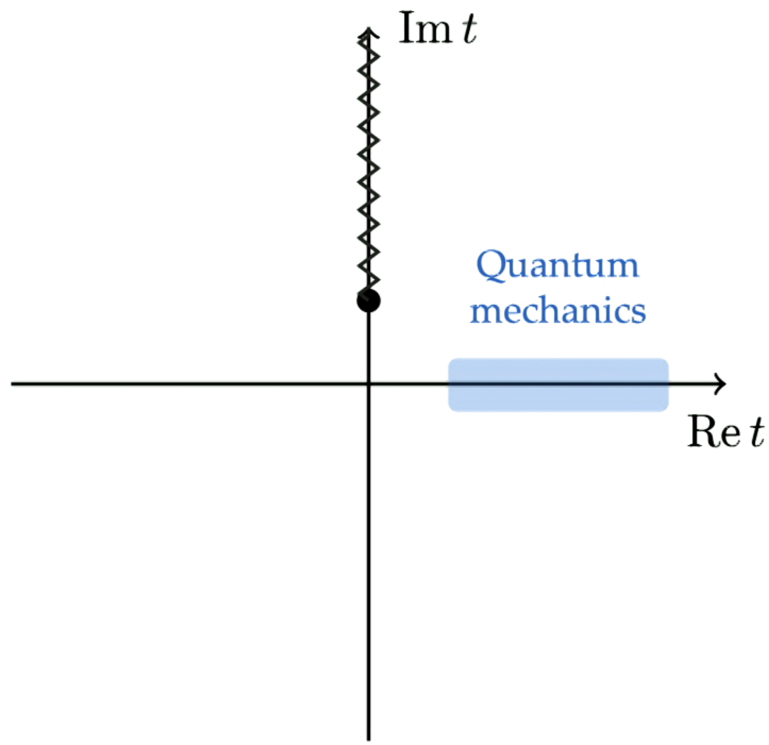


Example of an analytic continuation: Wick rotation



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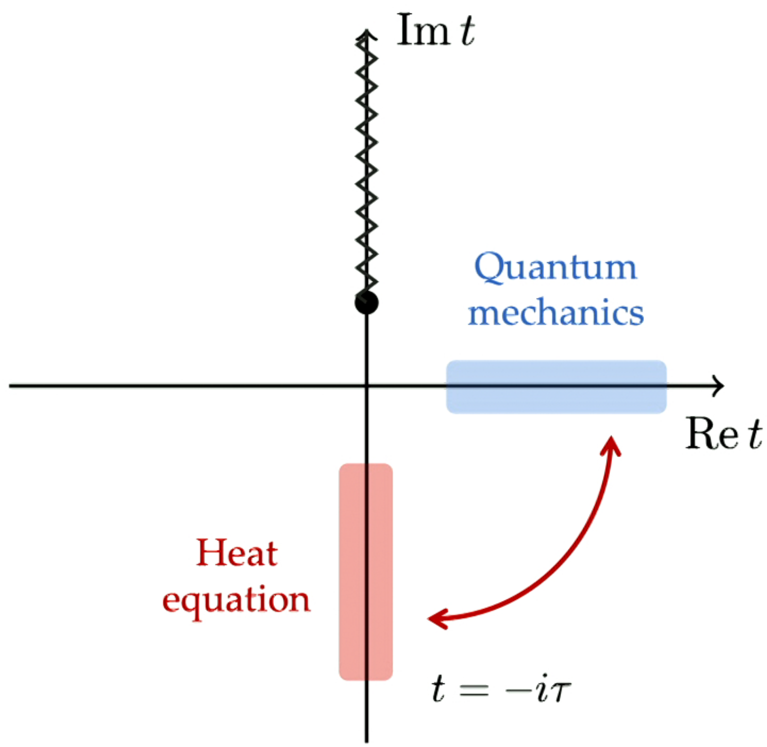
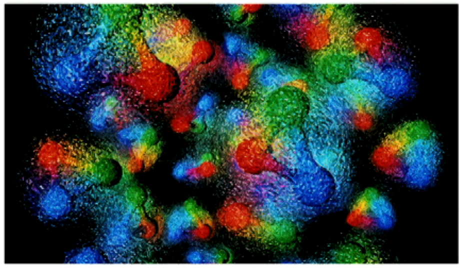
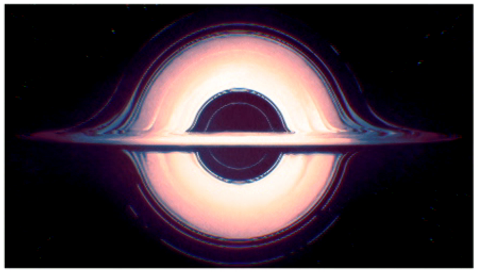


Cornerstone of many fields of physics

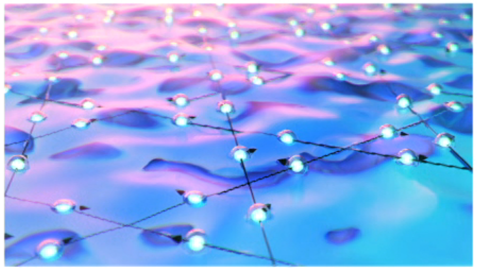
- Lattice field theory



- Black holes



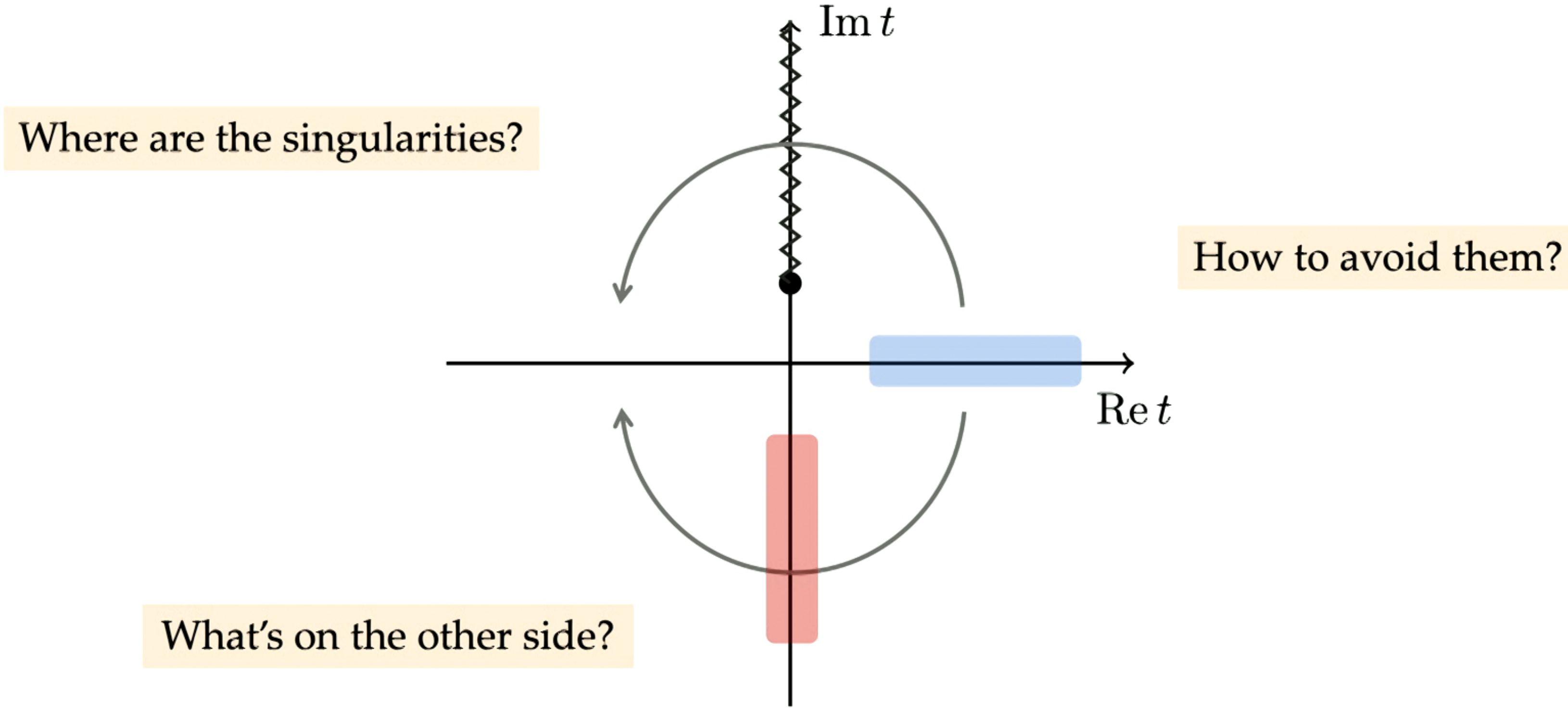
- Condensed matter



[Graphics: Quanta Magazine]

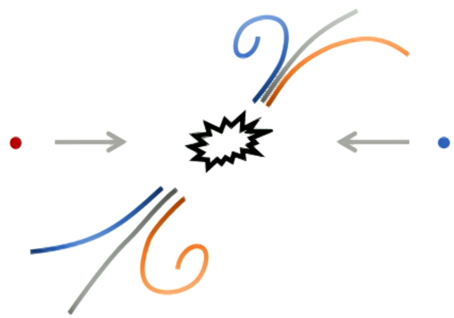
This talk is about even more intricate ways of analytically continuing time

Requires deep understanding of the analytic structure

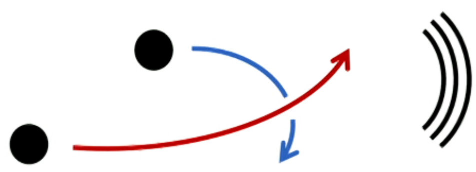


Plan for the talk

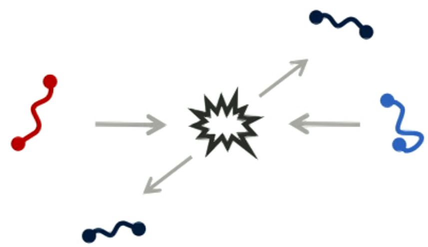
- Particle physics



- Gravitational waves



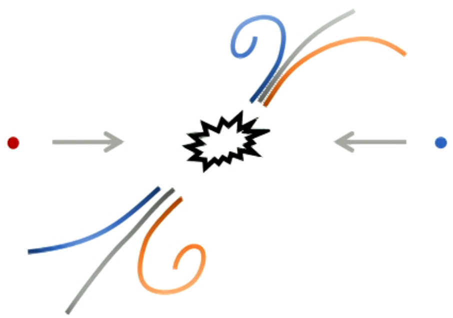
- String theory



Foundations of scattering theory $\Longrightarrow$ **Practical applications**

Plan for the talk

- Particle physics



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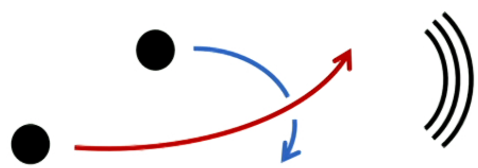

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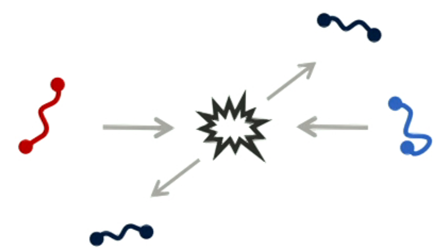
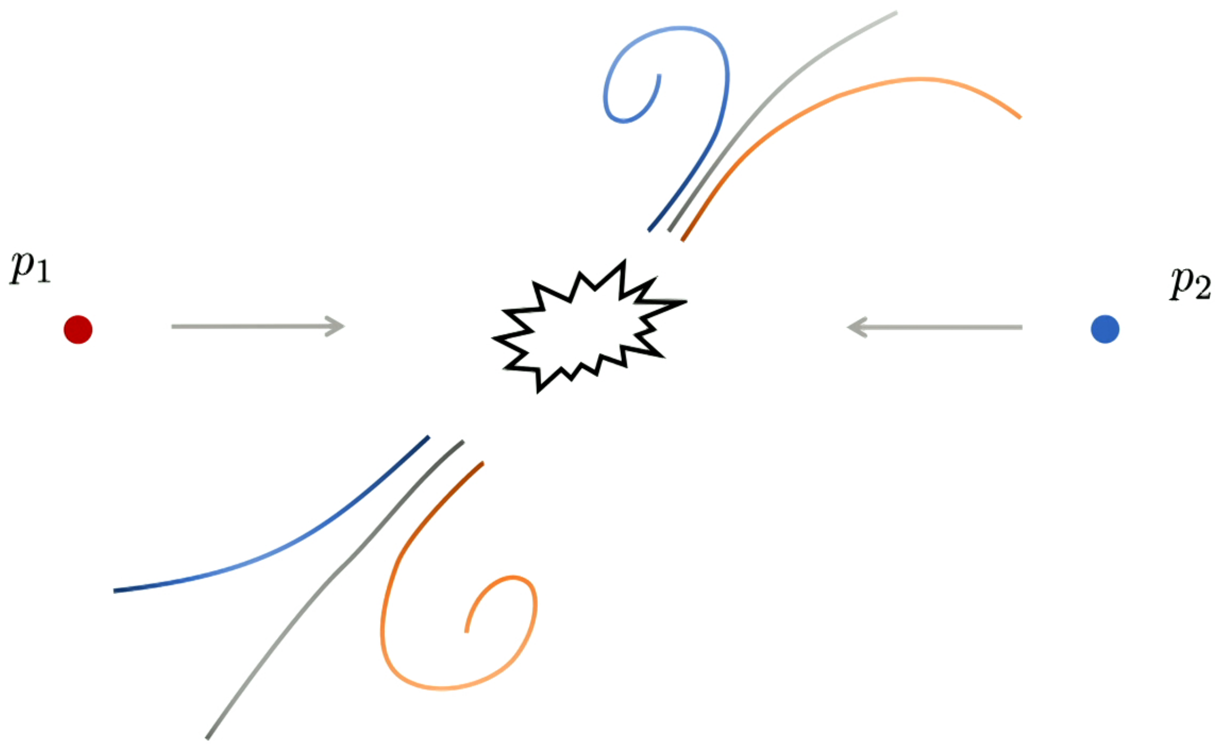
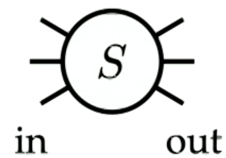


Collider experiments



The S-matrix

Matrix elements: $\langle \text{out}|S|\text{in}\rangle \quad = $ 

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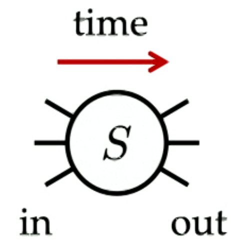
Matrix elements: $\langle \mathrm{out}|S|\mathrm{in}\rangle = $ 

Unitarity: $SS^\dagger = \mathbb{1}$

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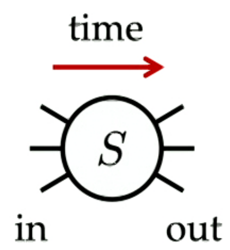
Matrix elements: $\langle \text{out}|S|\text{in}\rangle \quad = $ 

Unitarity: $SS^\dagger = \mathbb{1}$

Separate out the identity: $S = \mathbb{1} + iT$

Optical theorem

$$\frac{1}{2i}(T - T^\dagger) = \frac{1}{2}T^\dagger T$$

Sandwich between $\langle \text{out}|$ and $|\text{in}\rangle$; insert $\mathbb{1} = \sum\!\!\!\!\!\!\int_X |X\rangle\langle X|$

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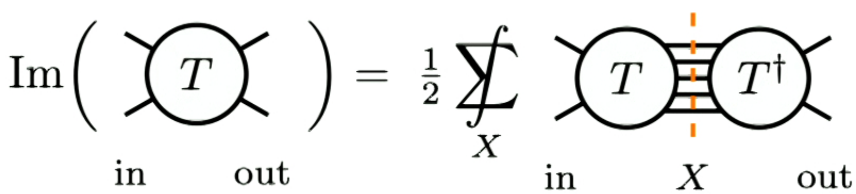

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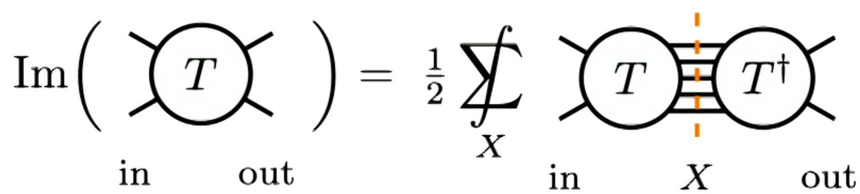

Unitarity cut: Intermediate on-shell states $s \geqslant (m_1 + m_2 + \ldots)^2$

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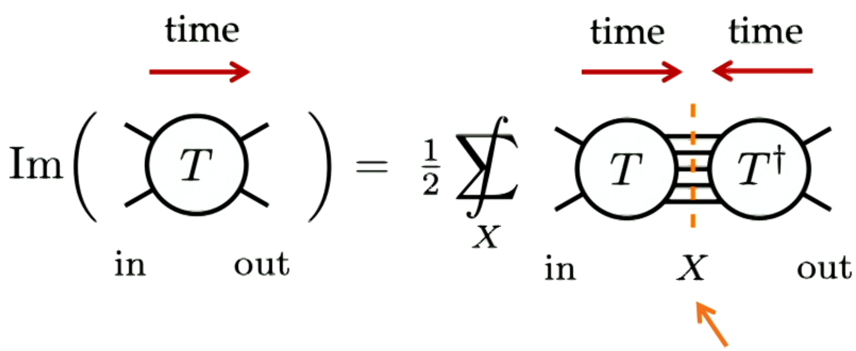


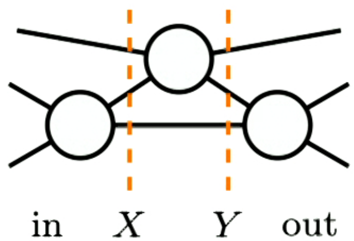
$\propto$ Cross section (in = out)

Unitarity cut: Intermediate on-shell states $s \geqslant (m_1 + m_2 + \ldots)^2$

Unitarity implies an infinite number of discontinuities

$$\frac{1}{2i}(T - T^\dagger) = \frac{1}{2}TT^\dagger$$
$$= \frac{1}{2}(T^2 - iT^2T^\dagger)$$
$$= \frac{1}{2}(T^2 - iT^3 - T^3T^\dagger)$$
$$= \cdots$$

$$T^\dagger = T - iTT^\dagger$$

Example: $T^2T^\dagger \supset \sum\!\!\!\!\!\!\int_{X,Y}$ 

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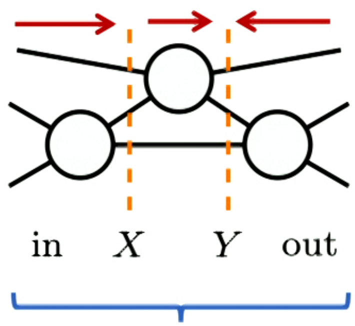

Kinematically allowed for $f \geqslant 0$

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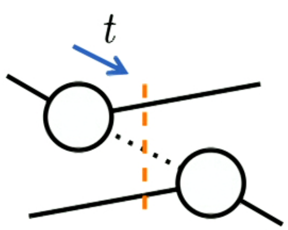
Singularities encode physics

- Analyticity in momentum transfer $t = (p_2 + p_3)^2$ ⇔ **locality**



$$\sim \quad \frac{1}{t-\mu^2} \qquad\qquad V(x) \sim \frac{\mathrm{e}^{-\mu|x|}}{|x|}$$

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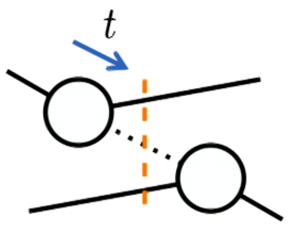
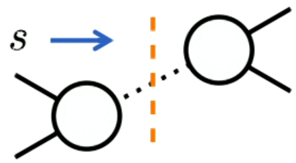
- Analyticity in momentum transfer $t = (p_2 + p_3)^2 \Leftrightarrow$ **locality**



$$\sim \quad \frac{1}{t-\mu^2} \qquad\qquad V(x) \sim \frac{\mathrm{e}^{-\mu|x|}}{|x|}$$

- Analyticity in energy $s = (p_1 + p_2)^2 \Leftrightarrow$ **causality**

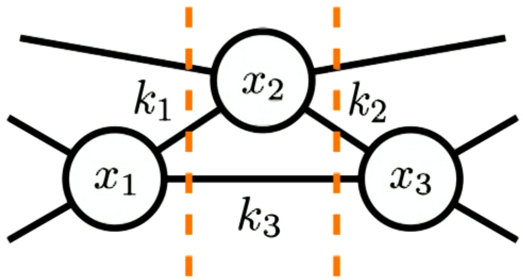


$$\sim \quad \frac{1}{s-\mu^2} \qquad\qquad \psi(t) \sim \mathrm{e}^{-i\mu t}\,\theta(t)$$

[Reviews: Nussenzveig (Elsevier 1972), SM (Physics Reports 2024)]

Anomalous thresholds – most in complex kinematics

(Landau singularities)



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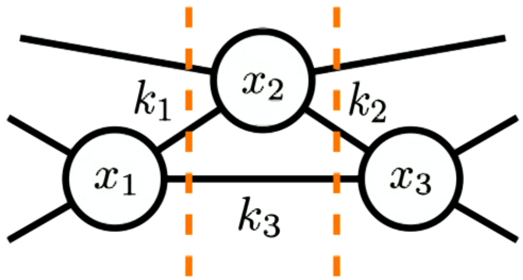

- On-shell particles: $k_i^2 = m_i^2$
- Interacting at space-time points $x_1^\mu, x_2^\mu, x_3^\mu, \ldots$

Anomalous thresholds – most in complex kinematics

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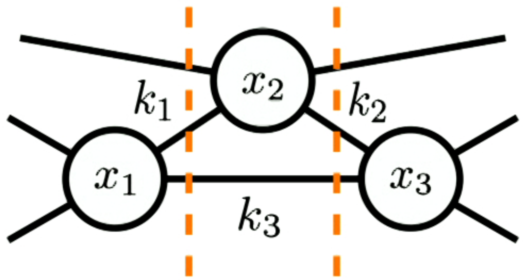

- On-shell particles: $k_i^2 = m_i^2$
- Interacting at space-time points $x_1^\mu, x_2^\mu, x_3^\mu, \ldots$
- In principle **all complex**: $k_i^\mu, x_i^\mu \in \mathbb{C}^4$

Why do we care?

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High-precision measurements in QFT

Simple example: Complex singularities in experiments

Example: $J/\psi \to \gamma\pi^0\pi^0$ and $\gamma K_S^0 K_S^0$ decays

18

Simple example: Complex singularities in experiments

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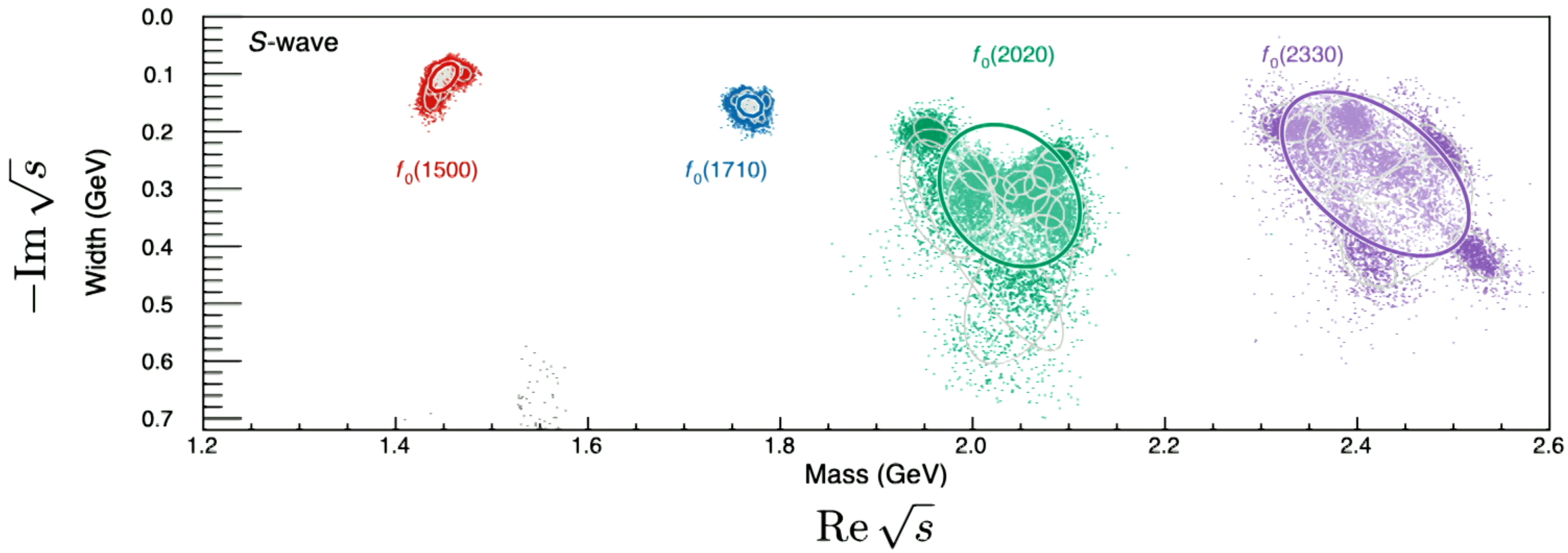
Complex energy plane



18

Simple example: Complex singularities in experiments

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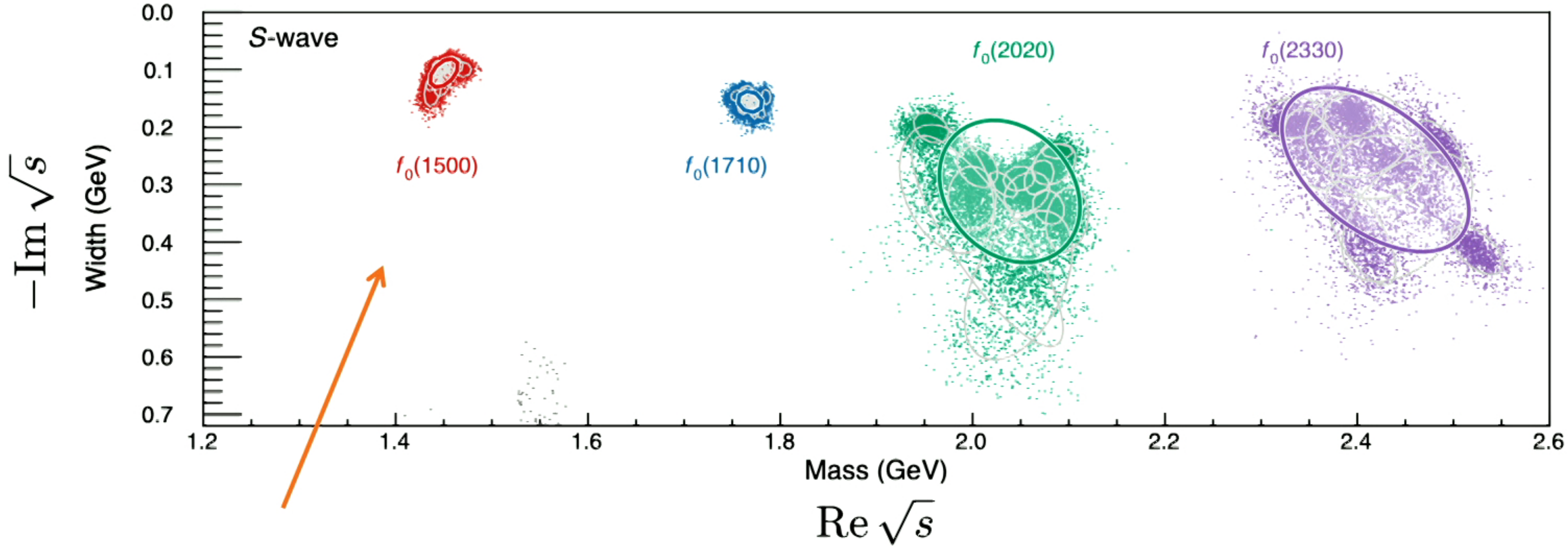
Complex energy plane



Complex pole = QCD bound state

[JPAC collaboration (Eur. Phys. J. C 82, 2022)]

High Energy Physics - Phenomenology

[Submitted on 15 Mar 2022]

Snowmass white paper: Need for amplitude analysis in the discovery of new hadrons

Miguel Albaladejo, Marco Battaglieri, Lukasz Bibrzycki, Andrea Celentano, Igor V. Danilkin, Sebastian M. Dawid, Michael Doring, Cristiano Fanelli, Cesar Fernandez-Ramirez, Sergi Gonzalez-Solis, Astrid N. Hiller Blin, Andrew W. Jackura, Vincent Mathieu, Mikhail Mikhasenko, Victor I. Mokeev, Emilie Passemar, Robert J. Perry, Alessandro Pilloni, Arkaitz Rodas, Matthew R. Shepherd, Nathaniel Sherrill, Jorge A. Silva-Castro, Tomasz Skwarnicki, Adam P. Szczepaniak, Daniel Winney (for the JPAC Collaboration)

We highlight the need for the development of comprehensive amplitude analysis methods to further our understanding of hadron spectroscopy. Reaction amplitudes constrained by first principles of S-matrix theory and by QCD phenomenology are needed to extract robust interpretations of the data from experiments and from lattice calculations.

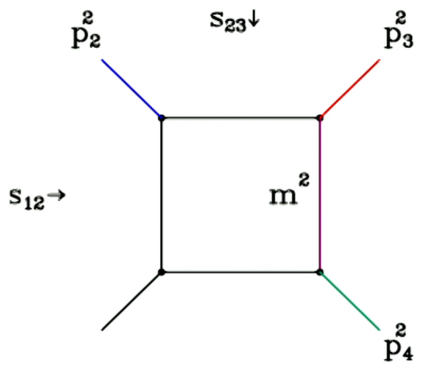
Simple example: Complex singularities in perturbative computations

$$I_4^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2,p_4^2;s_{12},s_{23};0,0,0,m^2) = \frac{1}{(s_{12}s_{23}-m^2s_{12}-p_2^2p_4^2+m^2p_2^2)}$$

$$\times \left[\frac{1}{\epsilon}\ln\left(\frac{(m^2-p_4^2)p_2^2}{(m^2-s_{23})s_{12}}\right)+\mathrm{Li}_2\left(1+\frac{(m^2-p_3^2)(m^2-s_{23})}{p_2^2m^2}\right)-\mathrm{Li}_2\left(1+\frac{(m^2-p_3^2)(m^2-p_4^2)}{s_{12}m^2}\right)\right.$$

$$+\ 2\,\mathrm{Li}_2\left(1-\frac{m^2-s_{23}}{m^2-p_4^2}\right)-2\,\mathrm{Li}_2\left(1-\frac{p_2^2}{s_{12}}\right)+2\,\mathrm{Li}_2\left(1-\frac{p_2^2(m^2-p_4^2)}{s_{12}(m^2-s_{23})}\right)$$

$$\left.+\ 2\ln\left(\frac{\mu m}{m^2-s_{23}}\right)\ln\left(\frac{(m^2-p_4^2)p_2^2}{(m^2-s_{23})s_{12}}\right)\right]+\mathcal{O}(\epsilon)$$



[QCDloop repository]

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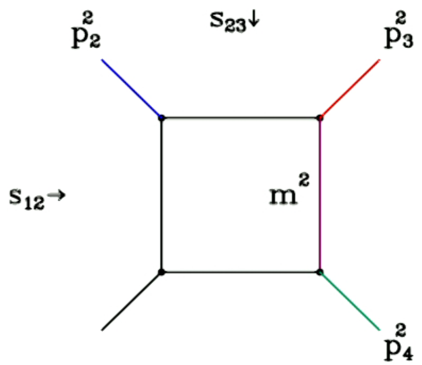
$$\times \left[\frac{1}{\epsilon}\ln\left(\frac{(m^2-p_4^2)p_2^2}{(m^2-s_{23})s_{12}}\right) + \mathrm{Li}_2\left(1+\frac{(m^2-p_3^2)(m^2-s_{23})}{p_2^2m^2}\right) - \mathrm{Li}_2\left(1+\frac{(m^2-p_3^2)(m^2-p_4^2)}{s_{12}m^2}\right)\right.$$
$$+ 2\,\mathrm{Li}_2\left(1-\frac{m^2-s_{23}}{m^2-p_4^2}\right) - 2\,\mathrm{Li}_2\left(1-\frac{p_2^2}{s_{12}}\right) + 2\,\mathrm{Li}_2\left(1-\frac{p_2^2(m^2-p_4^2)}{s_{12}(m^2-s_{23})}\right)$$
$$\left. + 2\ln\left(\frac{\mu m}{m^2-s_{23}}\right)\ln\left(\frac{(m^2-p_4^2)p_2^2}{(m^2-s_{23})s_{12}}\right)\right] + \mathcal{O}(\epsilon)$$



[QCDloop repository]

17 distinct singularities: Need to know them all!

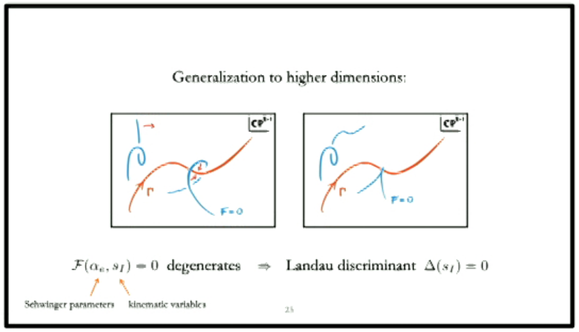
(one of the bottlenecks)

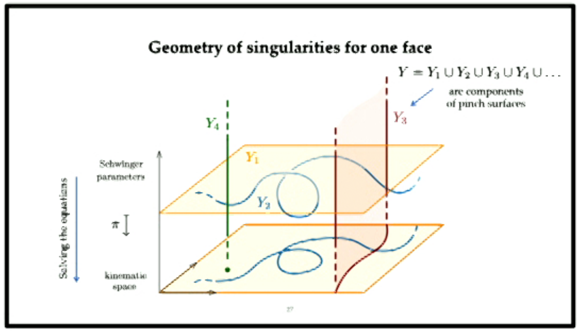
Analyticity enters at all stages of the modern pipeline for Standard Model computations

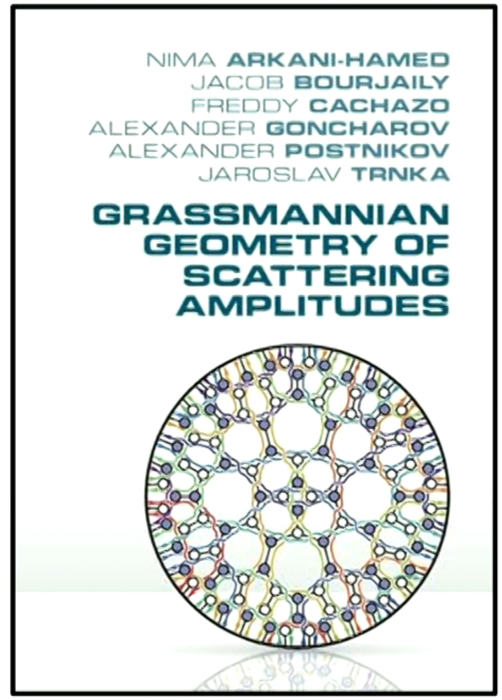
- Generalized unitarity
- Integration-by-parts reduction
- Differential equations
- Symbol alphabet
- Singularity analysis
- Elliptic functions
- Finite-field methods
- Numerical integration
- …

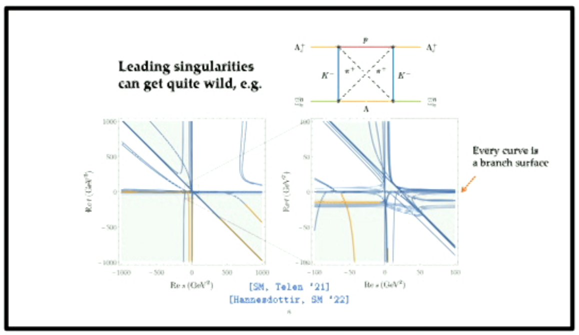
[Reviews: Weinzierl (Springer 2022), Badger, Henn, Plefka, Zoia (Springer 2024)]

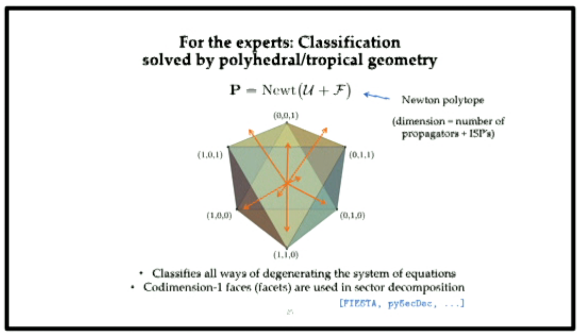
Start of an adventure into analytic properties of the S-matrix











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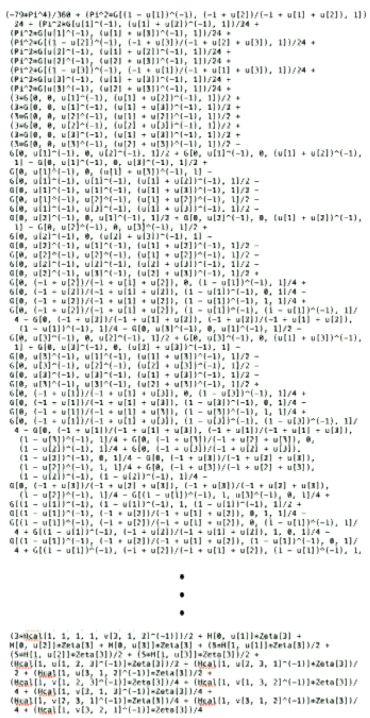
Highlight: 2-loop 6-pt MHV remainder function

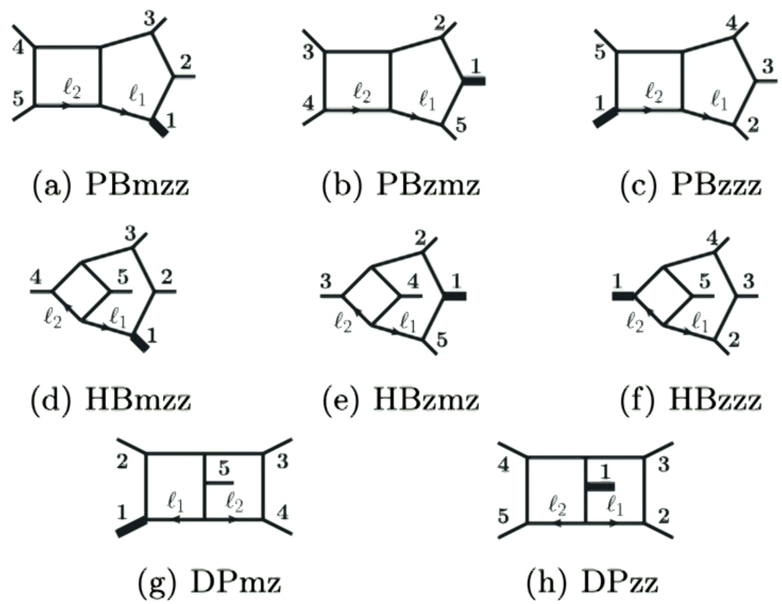
17 pages



[Del Duca, Duhr, Smirnov (JHEP 2010)]

Highlight: State-of-the-art QCD computations

(for example, $pp \to H + \text{jets}$)



(a) PBmzz (b) PBzmz (c) PBzzz

(d) HBmzz (e) HBzmz (f) HBzzz

(g) DPmz (h) DPzz

[Abreu, Chicherin, Ita, Page, Sotnikov, Tschernow, Zoia (2023)]

Summary of the 1st part

- Scattering amplitudes have an intricate **analytic structure**

- Understanding it is central for **precision measurements** in QFT


ias.edu/amplitudes2024
Amplitudes 2024
Conference
Institute for Advanced Study, Princeton, NJ, United States
10 - 14 June 2024
Organizers:
Nima Arkani-Hamed, Jacob Bourjaily,
Hofie Hannesdottir, Sebastian Mizera
Image credits: Gaia Fontana
Carl P. Feinberg Program in Cross-Disciplinary Innovation
IAS INSTITUTE FOR ADVANCED STUDY


ias.edu/amplitudes2024
Amplitudes 2024
Summer School
Institute for Advanced Study, Princeton, NJ, United States
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Organizers:
Nima Arkani-Hamed, Jacob Bourjaily,
Hofie Hannesdottir, Sebastian Mizera
Lecturers:
Michael Borinsky
Fabrizio Caola
Simon Caron-Huot
Lucía Córdova
Henriette Elvang
Donal O'Connell
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Carl P. Feinberg Program in Cross-Disciplinary Innovation
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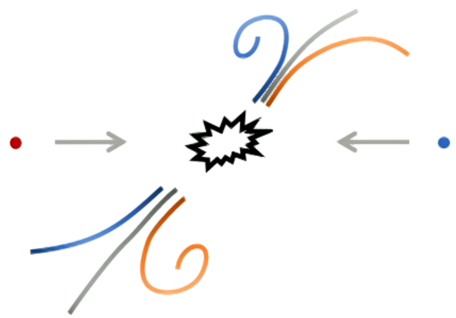
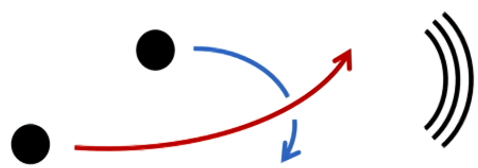
Plan for the talk

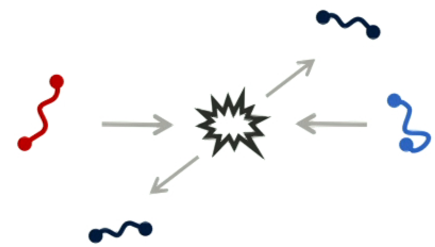
- Particle physics



- Gravitational waves



- String theory



Beyond the S-matrix

Example: Gravitational Bremsstrahlung

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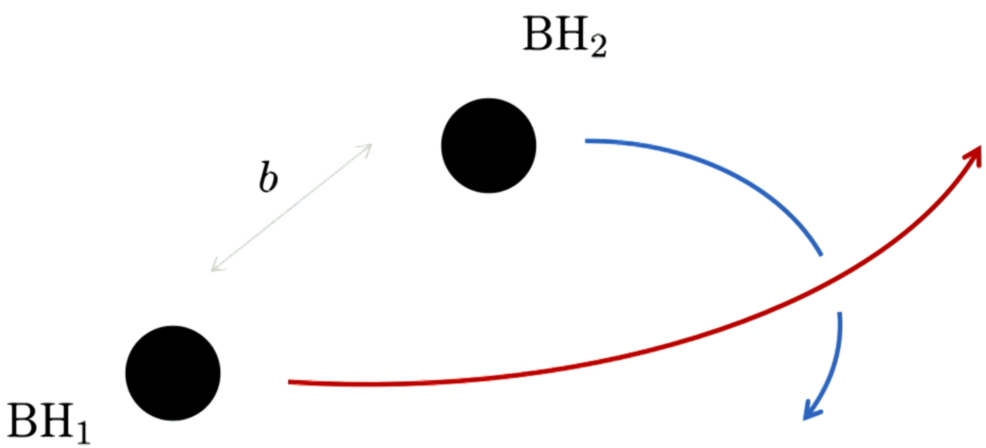
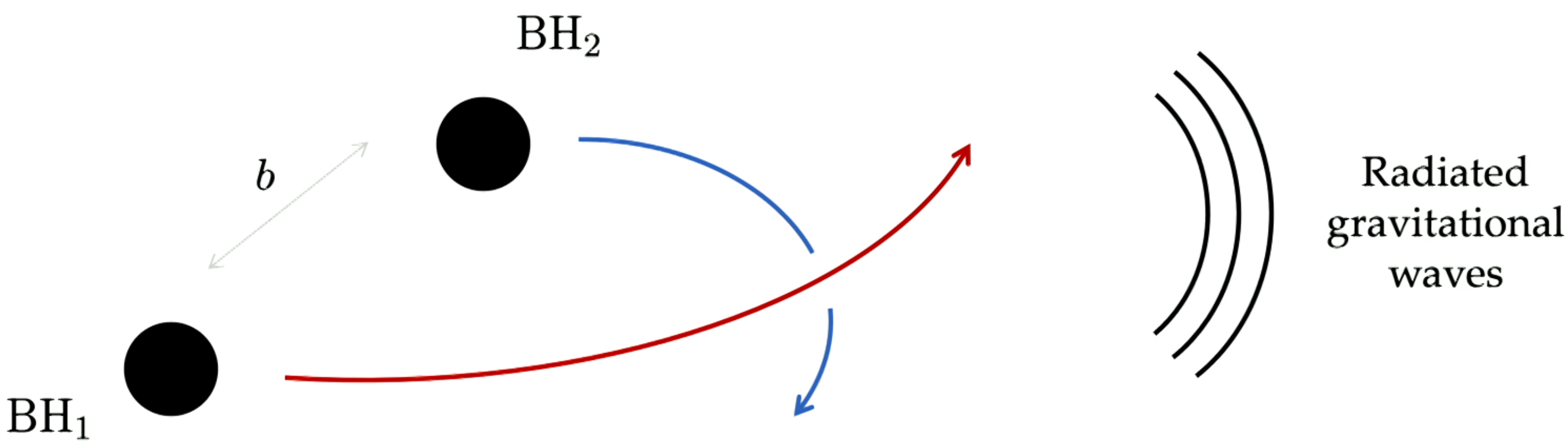


Beyond the S-matrix

Example: Gravitational Bremsstrahlung



Leading order in $\frac{GM}{b}$ computed in [Kovacs, Thorne (Astrophys. J. 224, 1978)]

Asymptotic observables beyond the S-matrix

Expectation value of the metric $h^{\mu\nu}$:

$$\langle \mathrm{BH}_1' \mathrm{BH}_2' | a_h^{\dagger \mathrm{out}} | \mathrm{BH}_1 \mathrm{BH}_2 \rangle$$

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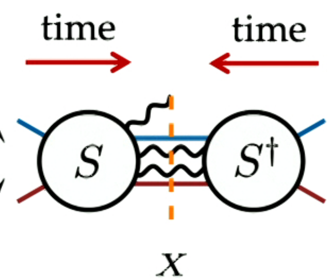
$$S^\dagger a_h^{\dagger \mathrm{in}} S$$

$$\mathbb{1} = \sum\!\!\!\!\!\!\int_X |X\rangle\langle X|$$

Asymptotic observables beyond the S-matrix

Expectation value of the metric $h^{\mu\nu}$:

$$\langle \mathrm{BH}_1' \mathrm{BH}_2' | \underbrace{a_h^{\dagger\mathrm{out}}}_{S^\dagger a_h^{\dagger\mathrm{in}} S} | \mathrm{BH}_1 \mathrm{BH}_2 \rangle \quad = \quad \sum\!\!\!\!\!\!\int_X$$

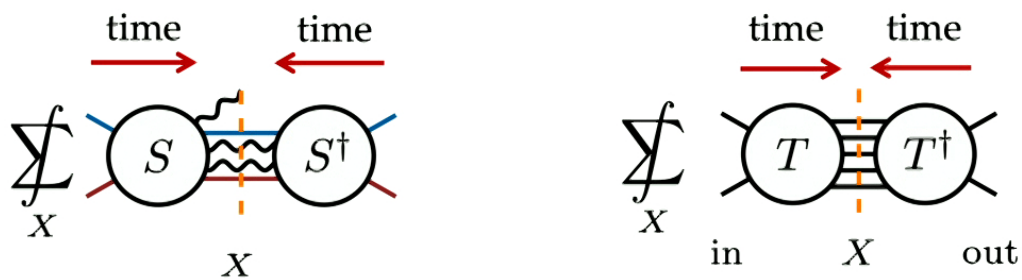


↑

$$\mathbb{1} = \sum\!\!\!\!\!\!\int_X |X\rangle\langle X|$$

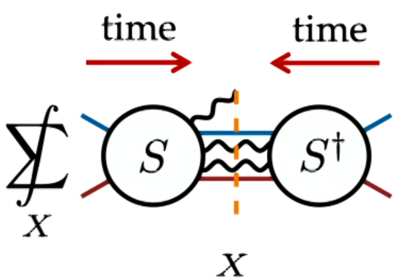
[Kosower, Maybee, O'Connell (JHEP 2019)]

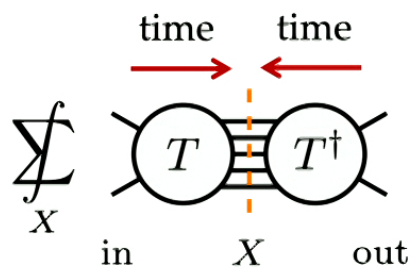
Looks familiar…



Could it be related to the scattering amplitude?

Looks familiar…





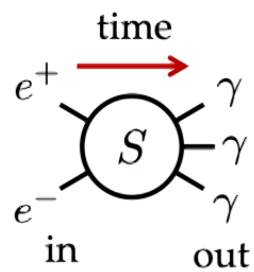
Could it be related to the scattering amplitude?

Yes, through crossing!

Interlude: CPT theorem

Interlude: CPT theorem

$$\langle \text{out}|S|\text{in}\rangle \quad =$$



Interlude: CPT theorem

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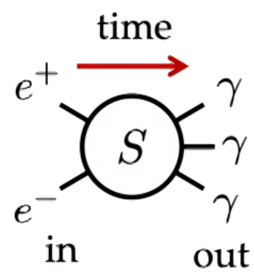
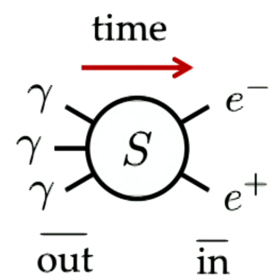


$$\|$$

$$\langle \overline{\text{in}}|S|\overline{\text{out}}\rangle =$$

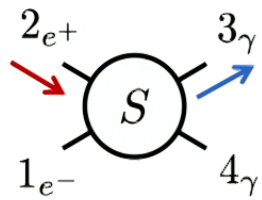


CP = exchange particles and anti-particles

T = exchange incoming and outgoing states

Crossing: Analog of CPT for individual particles

(consider $2 \to 2$ case first)

$\langle 34|S|12\rangle \quad =$ 

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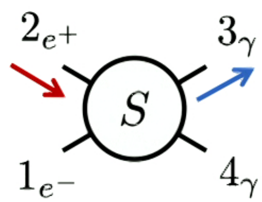
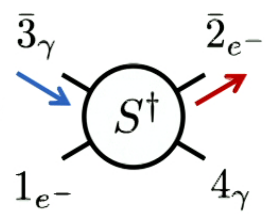


$\updownarrow$

$$\langle \bar{2}4|S^\dagger|1\bar{3}\rangle =$$



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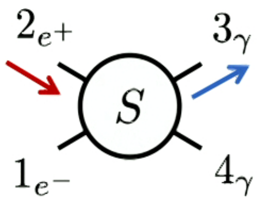
$\langle 34|S|12\rangle \quad =$

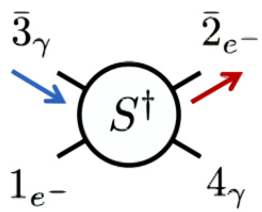


$s = (p_1 + p_2)^2 > 0$

Energy

analytic continuation in s upper half-plane

$\langle \bar{2}4|S^\dagger|1\bar{3}\rangle \quad =$

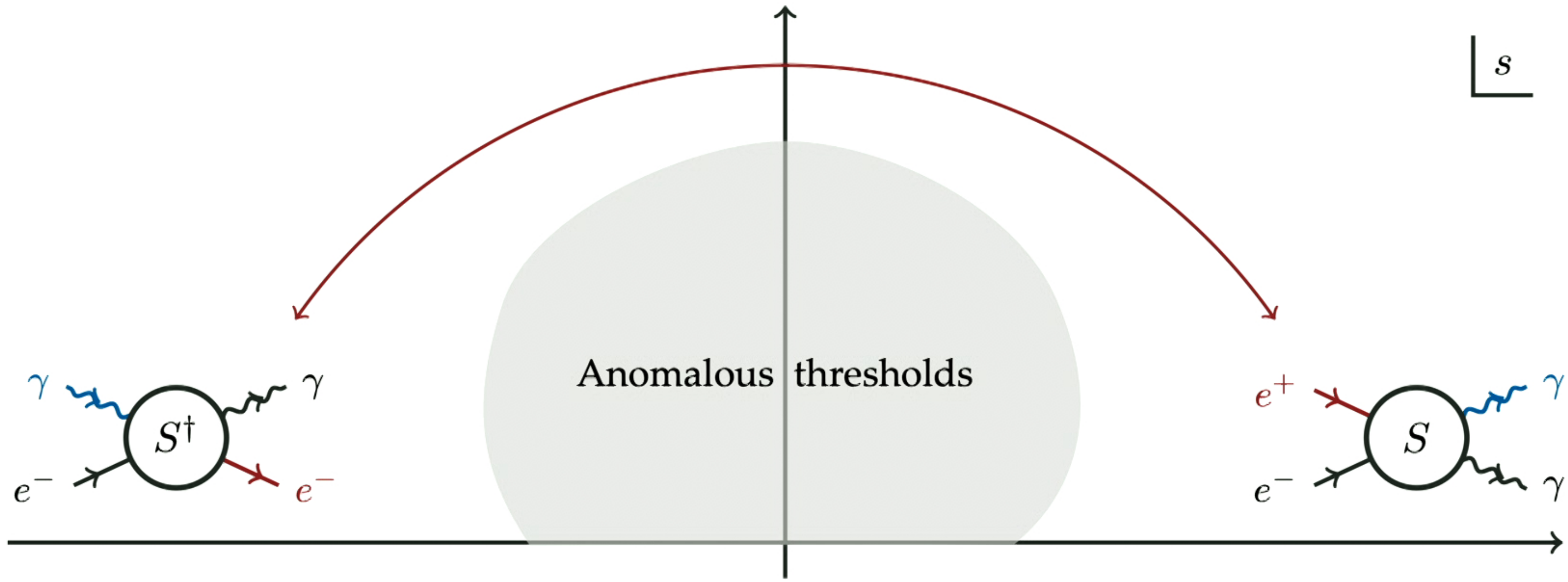


$s = (p_1 + p_2)^2 < 0$

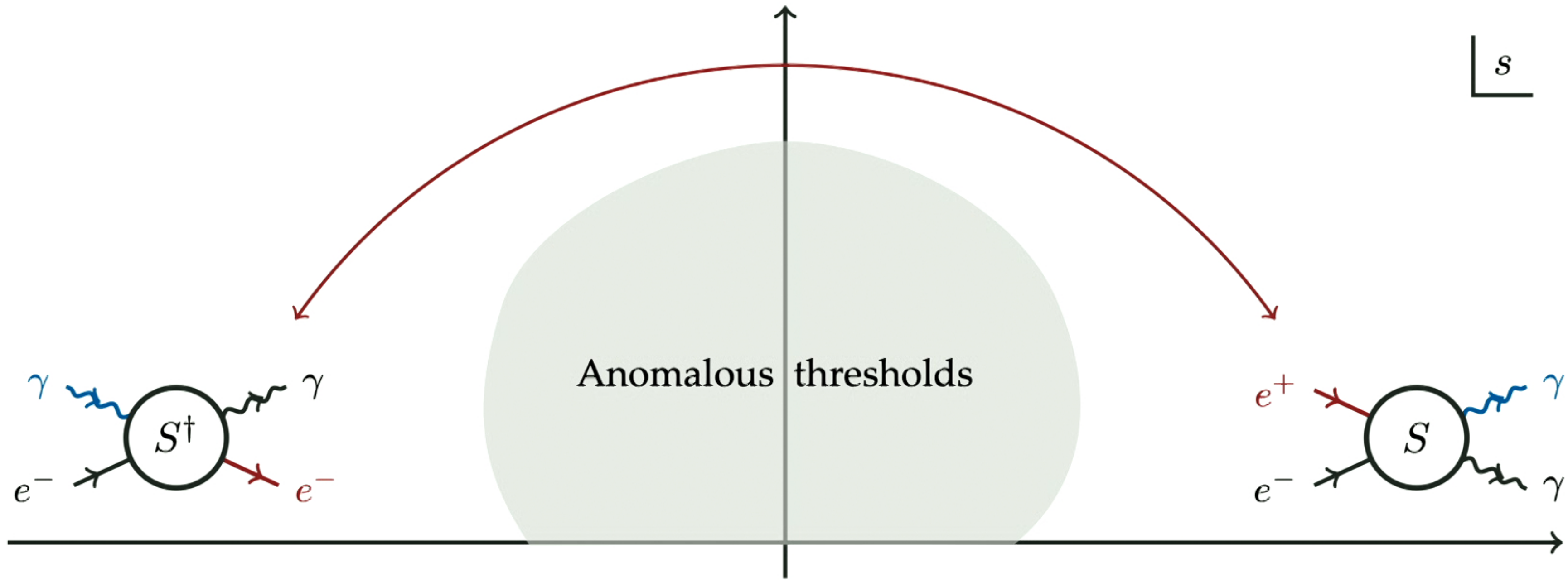
Mom. transfer

[Gell-Mann, Goldberger, Thirring (Phys. Rev. 1954)]

[Bros, Epstein, Glaser (Helv. Phys. Acta 1964)]



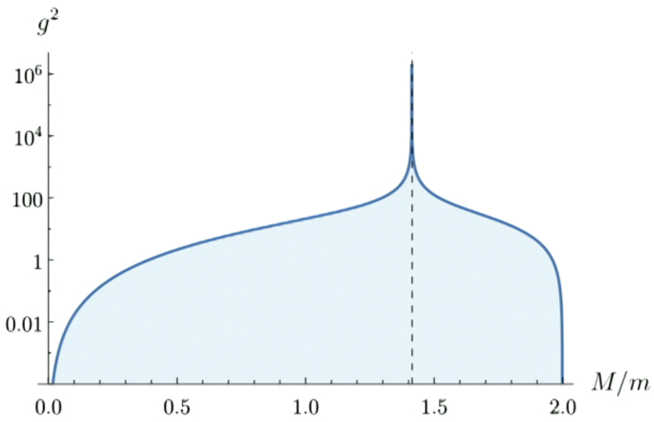
35



s
Anomalous thresholds
γ
γ
S†
e−
e−
e+
γ
S
e−
γ

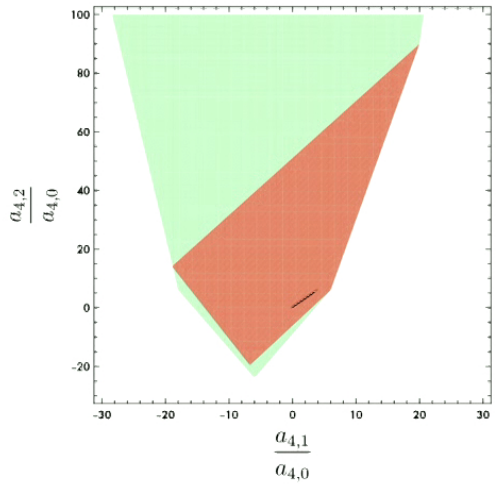
Turns out to be incredibly useful for $2 \to 2$ scattering

- S-matrix bootstrap



[Paulos, Penedones, Toledo, van Rees, Vieira (JHEP 2017), …]

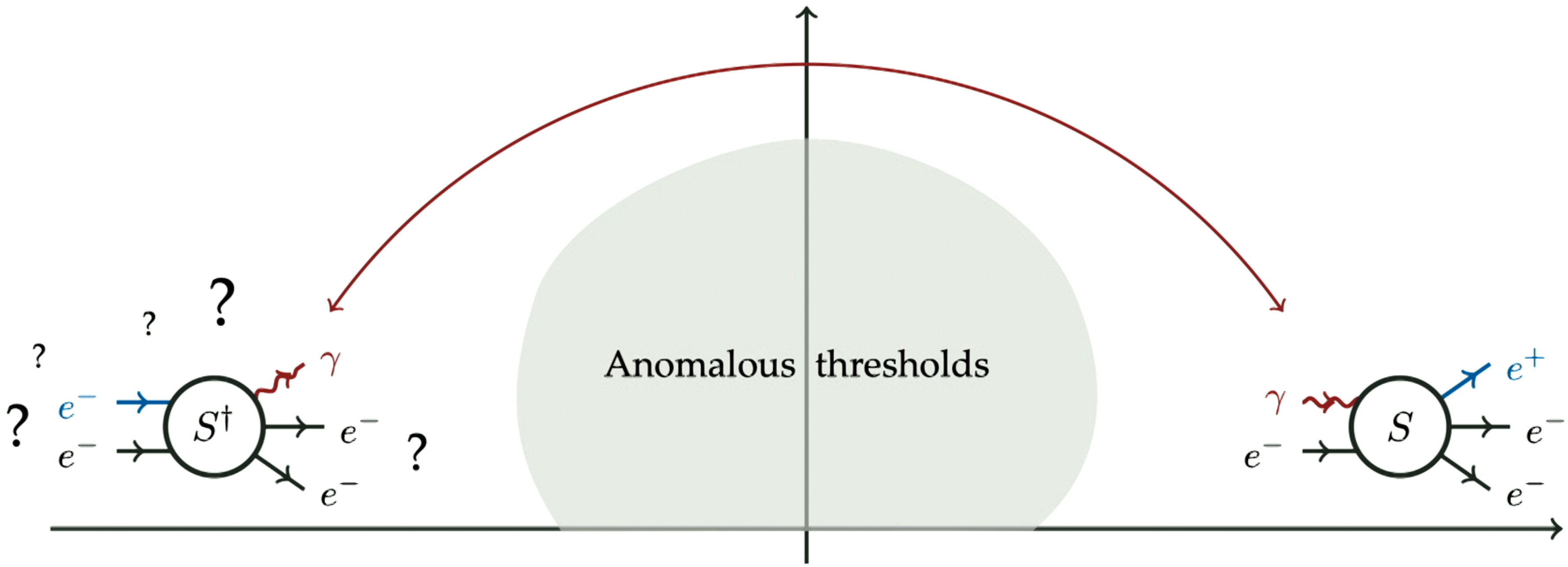
- Bounds on effective field theories



[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi (JHEP 2006), …]

[Plot from Bern, Kosmopoulos, Zhiboedov (J. Phys. A 2021)]

Open question since 1960's



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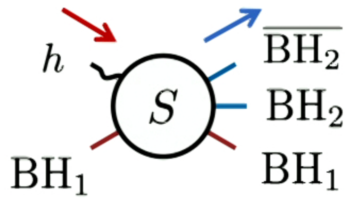
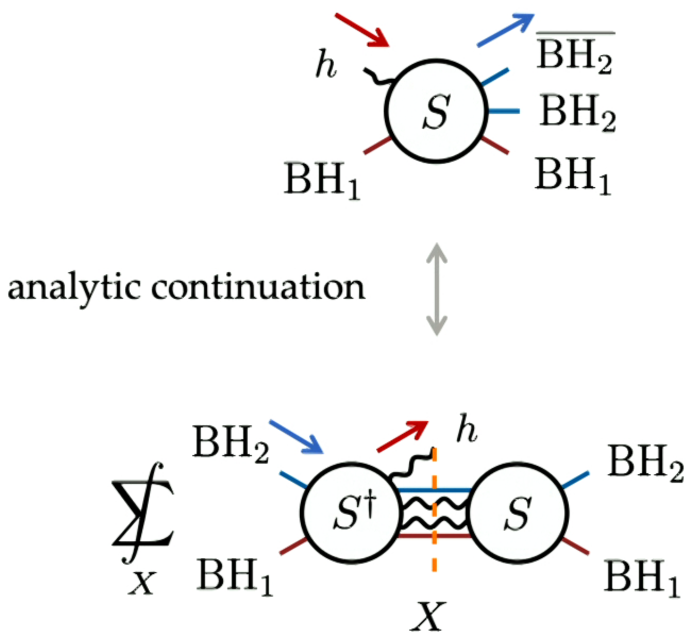
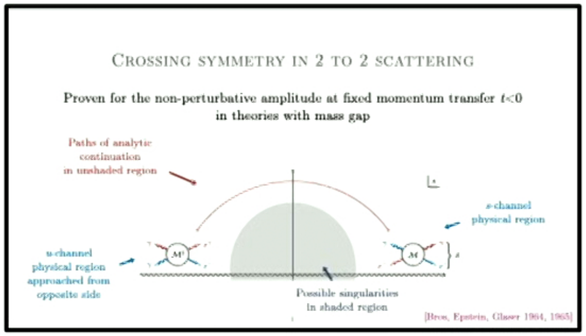
Surprises in crossing beyond $2 \to 2$ scattering

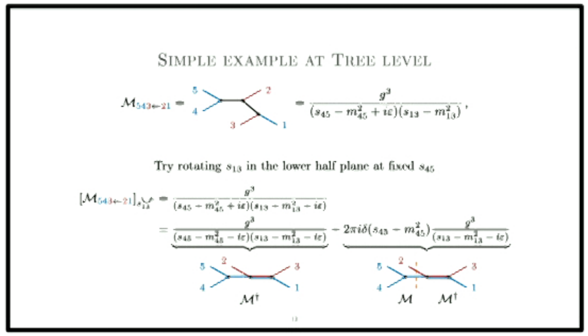


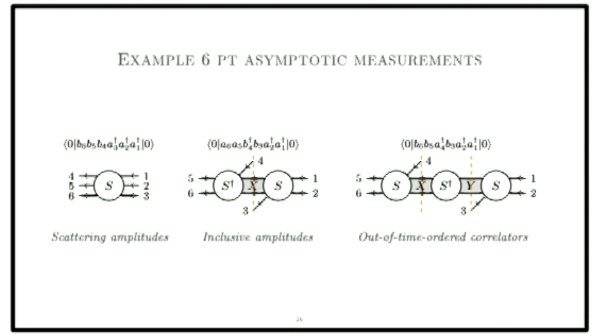
Surprises in crossing beyond $2 \to 2$ scattering

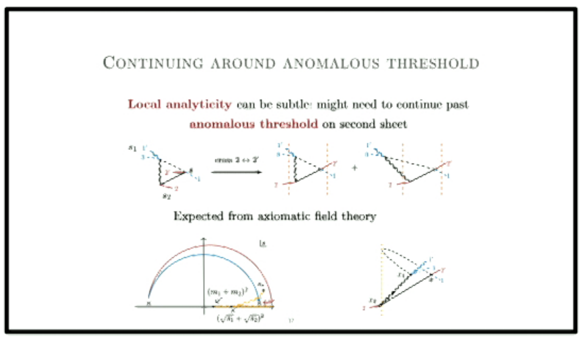


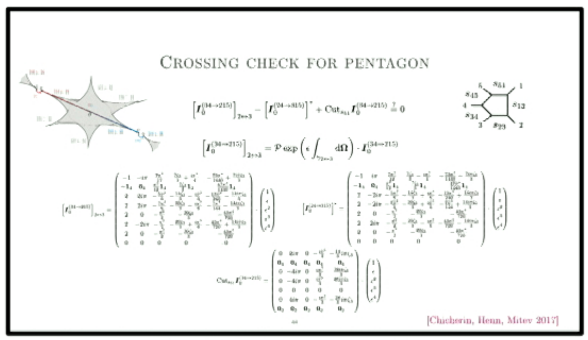
Start of an adventure into analytic continuation between observables

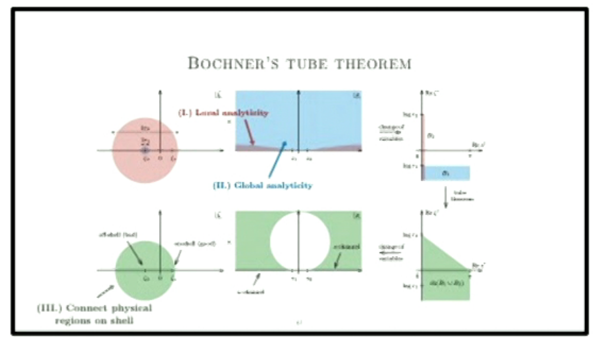












Highlight: First computation Gravitational Bremsstrahlung at NLO

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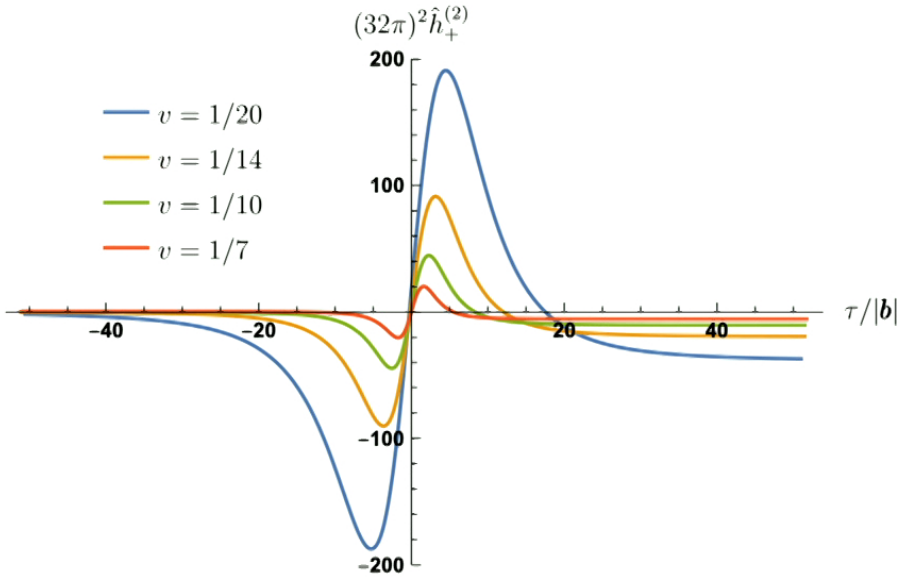

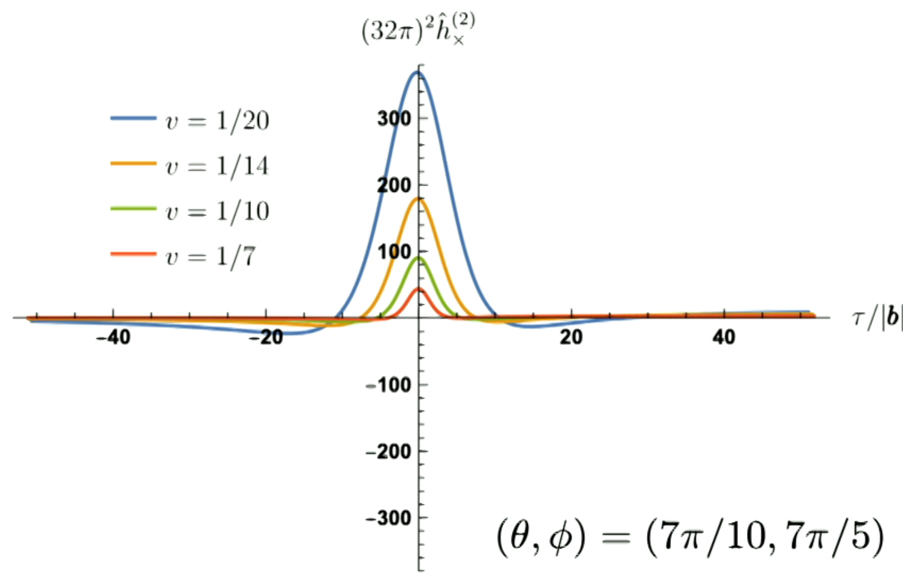

[Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini (JHEP 2023), Herderschee, Roiban, Teng (JHEP 2023), Elkhidir, O'Connell, Sergola, Vazquez-Holm (JHEP 2023), Caron-Huot, Giroux, Hannesdottir, SM (JHEP 2023a)]

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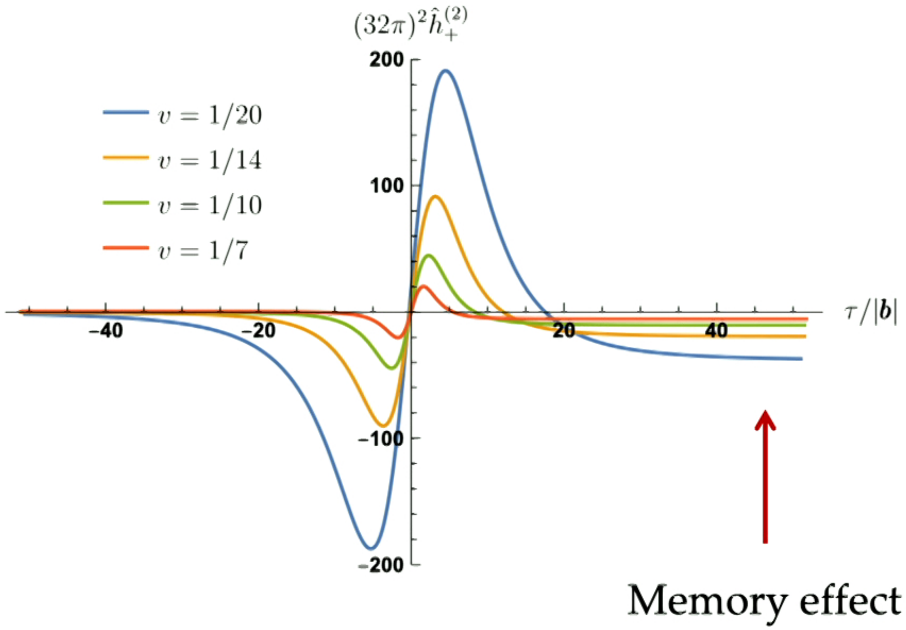

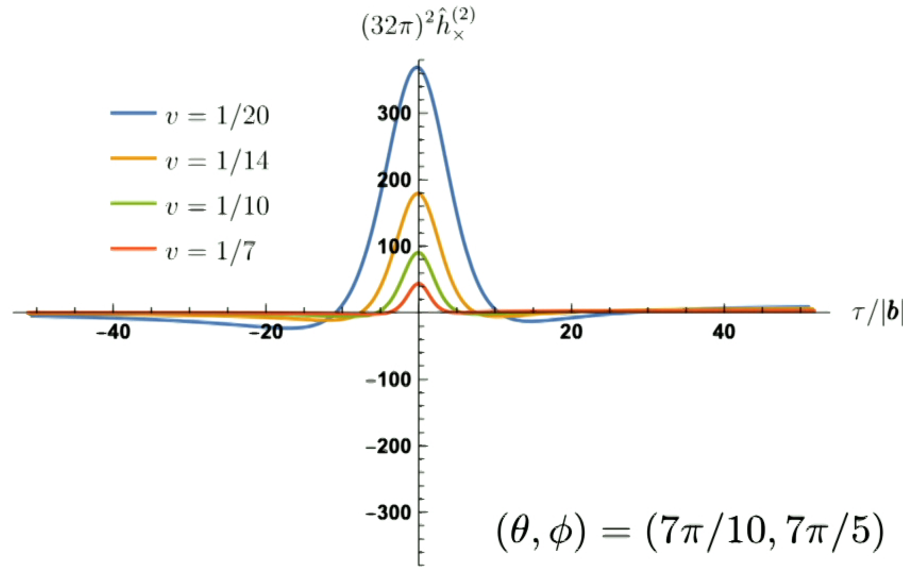

[Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini (JHEP 2023), Herderschee, Roiban, Teng (JHEP 2023), Elkhidir, O'Connell, Sergola, Vazquez-Holm (JHEP 2023), Caron-Huot, Giroux, Hannesdottir, SM (JHEP 2023a)]

Physical intuition for the result: Infrared divergence

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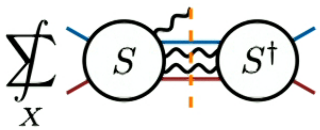
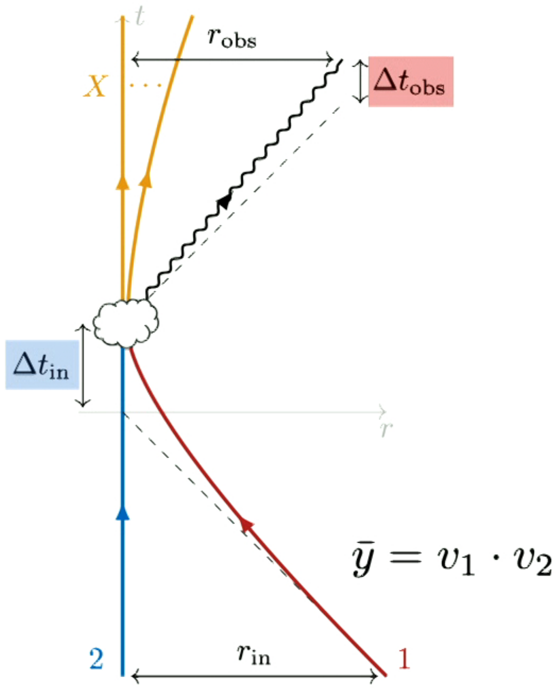
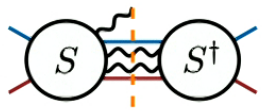


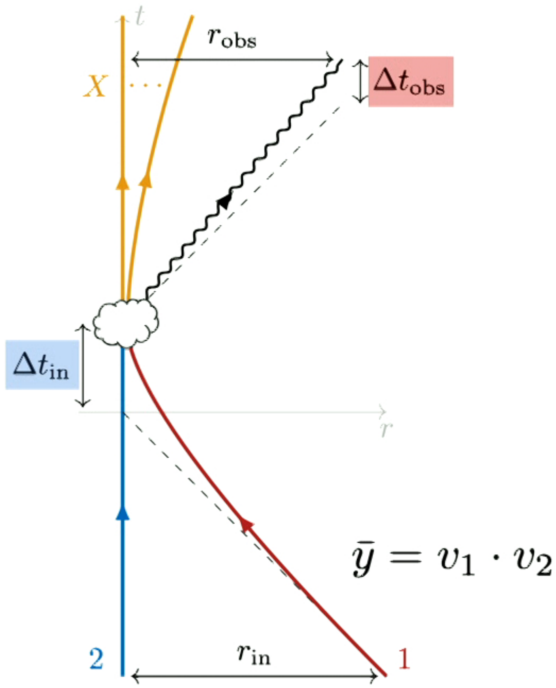
$$\sim \text{(tree-level)} \exp\left[-iG_N \hat{k}\cdot(\bar{p}_1+\bar{p}_2)\left(2+\frac{2-3\bar{y}^{-2}}{(1-\bar{y}^{-2})^{\frac{3}{2}}}\right)\log\frac{\Lambda}{\mu_{\mathrm{IR}}}\right]$$



Physical intuition for the result: Infrared divergence

$\sum_X$ 

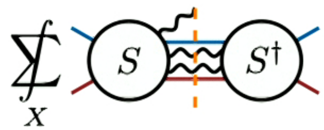
$$\sim \text{(tree-level)} \exp\left[-iG_N \hat{k}\cdot(\bar{p}_1 + \bar{p}_2)\left(2 + \frac{2-3\bar{y}^{-2}}{(1-\bar{y}^{-2})^{\frac{3}{2}}}\right)\log\frac{\Lambda}{\mu_{\rm IR}}\right]$$



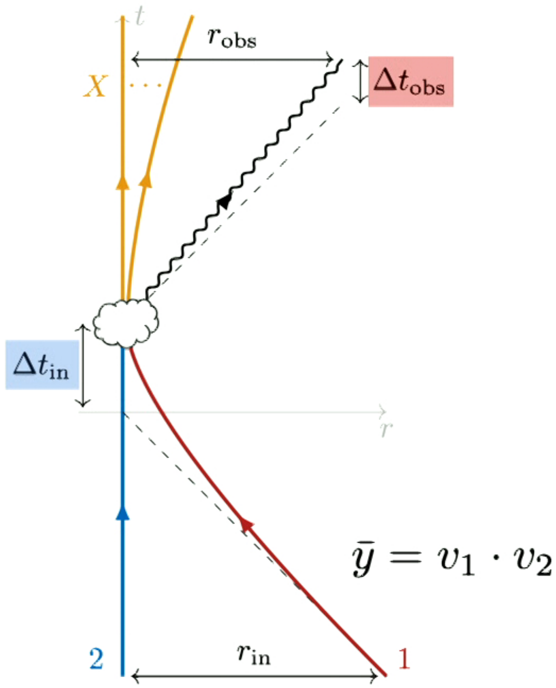
Two sources of time delays:

$$\Delta t_{\rm in} = -G_N\,\hat{k}\cdot(\bar{p}_1 + \bar{p}_2) \times \frac{2-3\bar{y}^{-2}}{(1-\bar{y}^{-2})^{\frac{3}{2}}}\log\frac{r_{\rm in}}{b}$$

Physical intuition for the result: Infrared divergence



$$\sim \text{(tree-level)} \exp\left[-iG_N \hat{k}\cdot(\bar{p}_1 + \bar{p}_2)\left(2 + \frac{2 - 3\bar{y}^{-2}}{(1-\bar{y}^{-2})^{\frac{3}{2}}}\right)\log\frac{\Lambda}{\mu_{\text{IR}}}\right]$$



Two sources of time delays:

$$\Delta t_{\text{in}} = -G_N\,\hat{k}\cdot(\bar{p}_1 + \bar{p}_2) \times \frac{2 - 3\bar{y}^{-2}}{(1-\bar{y}^{-2})^{\frac{3}{2}}}\log\frac{r_{\text{in}}}{b}$$

$$\Delta t_{\text{obs}} = -2G_N\,\hat{k}\cdot(\bar{p}_1 + \bar{p}_2)\log\frac{r_{\text{obs}}}{b}$$

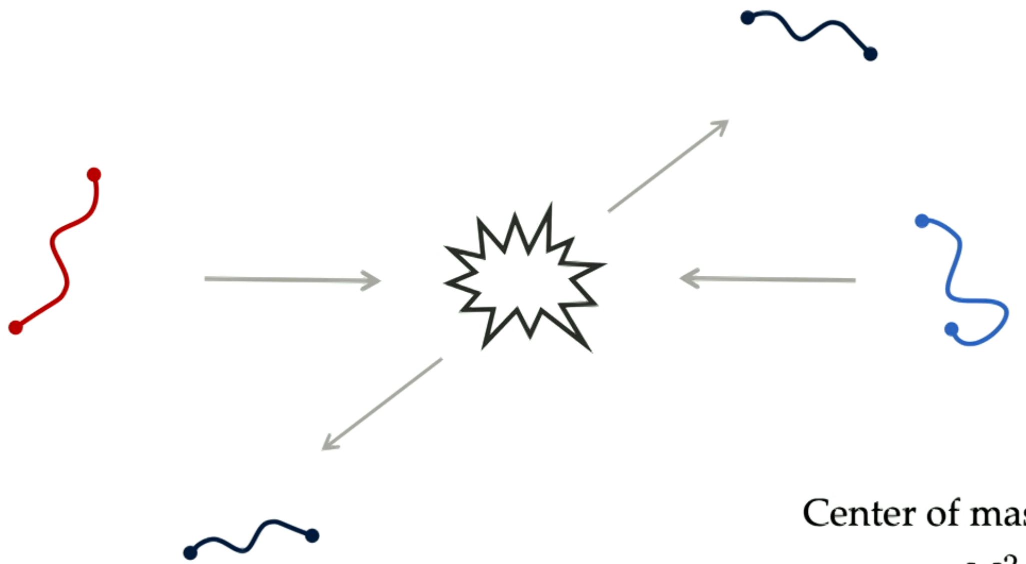
[Caron-Huot, Giroux, Hannesdottir, SM (JHEP 2023a)]
[cf. Sahoo, Sen (JHEP 2019)]

Summary of the 2nd part

- Crossing connects different **observables**

- Used to obtain new results in **gravitational-wave physics** and beyond

String theory as a practical window for quantum gravity



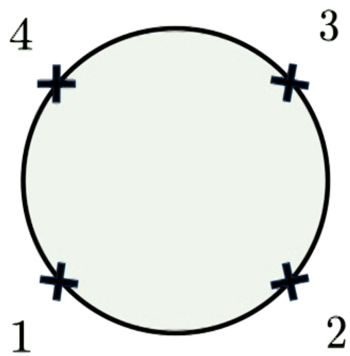
Center of mass energy

$$s \sim M^2_{\text{Planck}}$$

[Gross, Mende (Nucl. Phys. B 1988)]

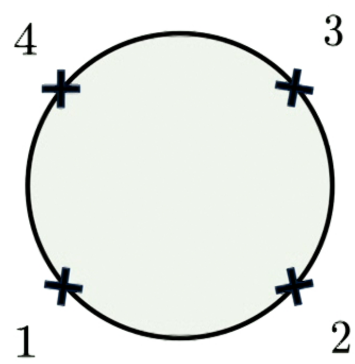
Euclidean vs. Lorentzian worldsheets

Euclidean vs. Lorentzian worldsheets



Euclidean worldsheet
(Riemann surface)

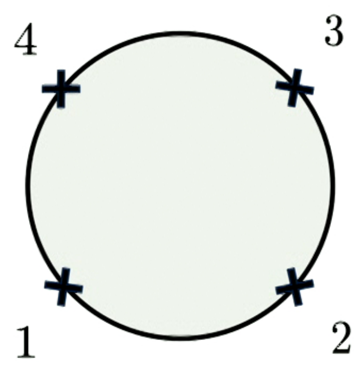
Euclidean vs. Lorentzian worldsheets



Euclidean worldsheet
(Riemann surface)

- Textbook formulation
- Easy to use CFT technology

Euclidean vs. Lorentzian worldsheets



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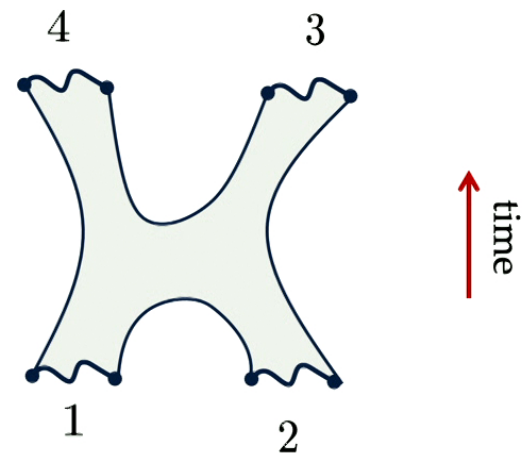

Lorentzian worldsheet

- Textbook formulation
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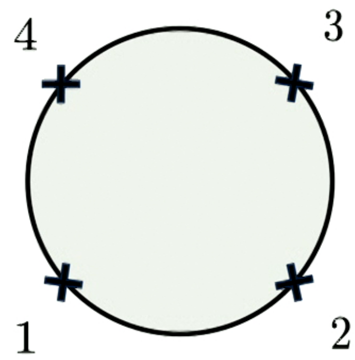
Euclidean vs. Lorentzian worldsheets



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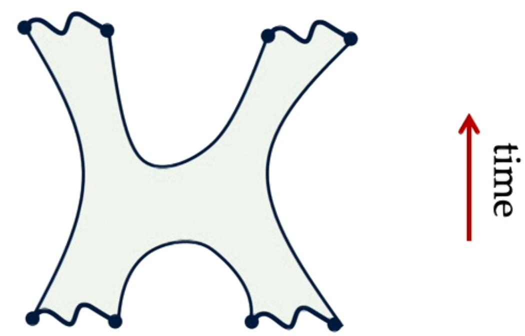
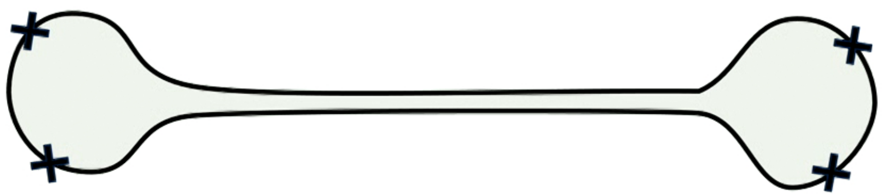


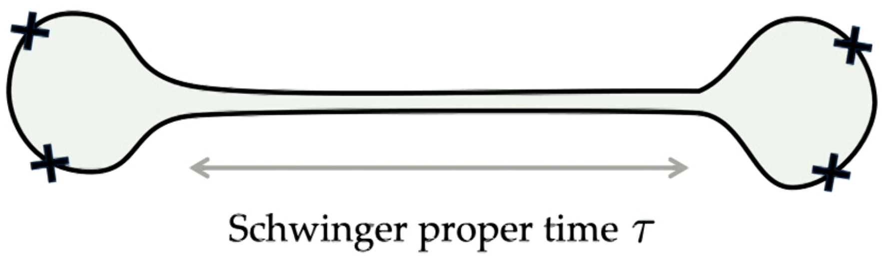
Lorentzian worldsheet

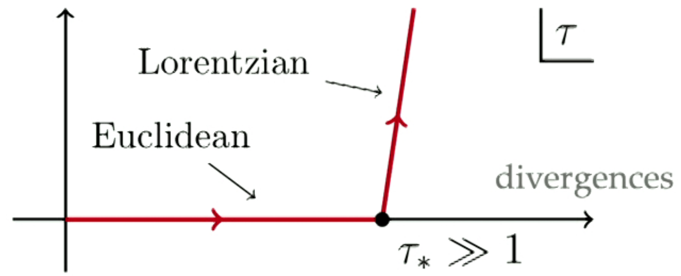
- Not well understood
- Essential for practical computations!

Wick rotation of the worldsheet



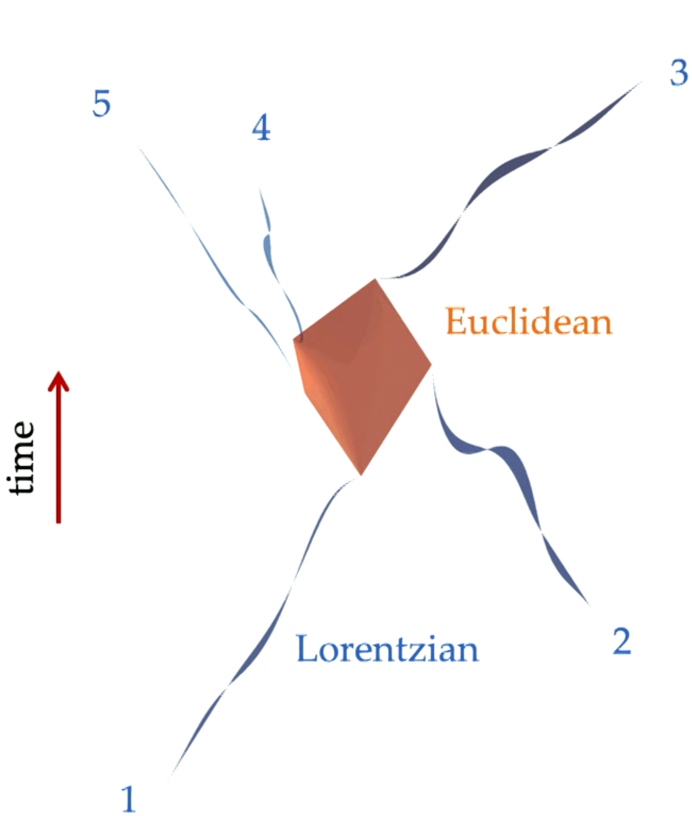
Wick rotation of the worldsheet



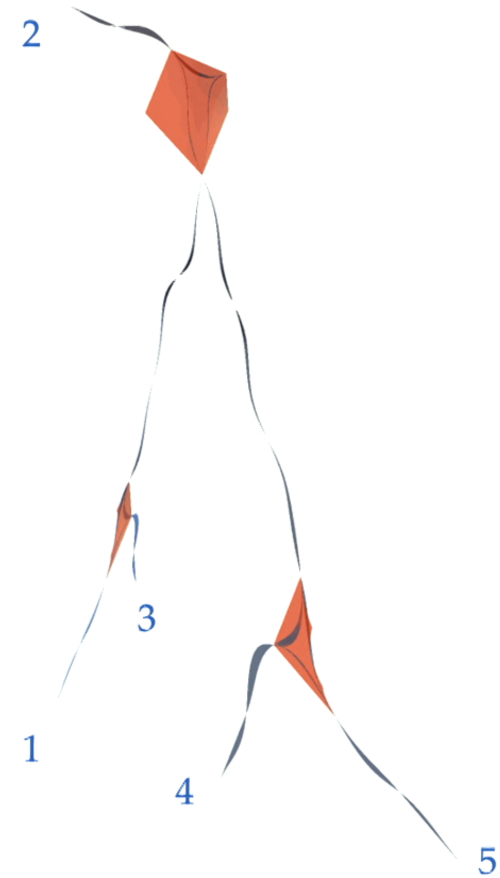


[Witten (JHEP 2015)]

Dancing worldsheets

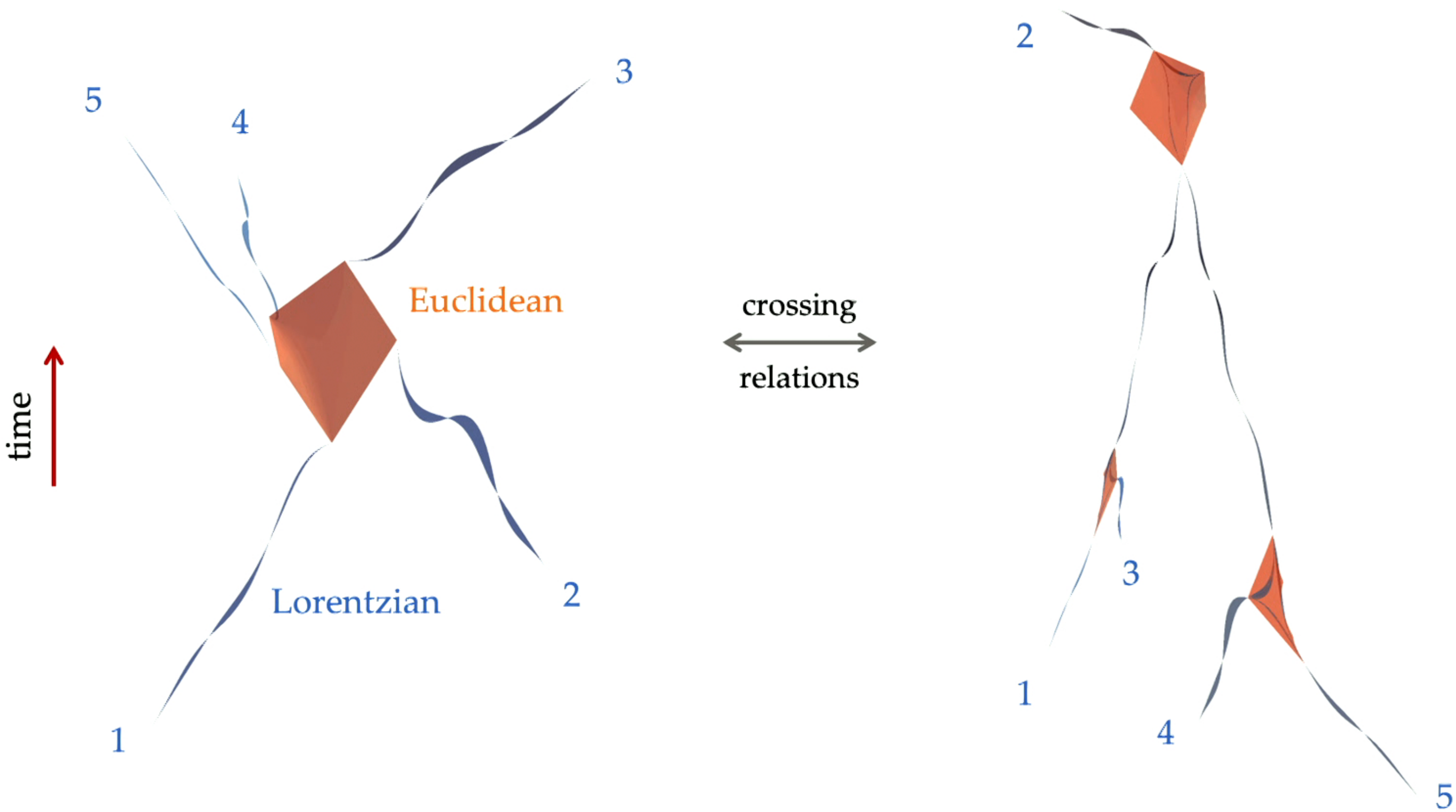


crossing
↔
relations



Quantum corrections to the string amplitude

Dancing worldsheets



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Quantum corrections to the string amplitude

```
NIntegrate[(θ1[z2 - z1, τ] θ1[z4 - z3, τ] / (θ1[z3 - z1, τ] θ1[z4 - z2, τ]))^-s (θ1[z3 - z2, τ] θ1[z4 - z1, τ] / (θ1[z3 - z1, τ] θ1[z4 - z2, τ]))^-t
  /. {s → 3 / 2, t → -1 / 2, z4 → 1, τ → I Imτ},
  {Imτ, 0, ∞}, {z1, 0, 1}, {z2, z1, 1}, {z3, z2, 1}]
```

Textbook formula for type-I four-point amplitude [Green, Schwarz (Nucl. Phys. B 1982)]

Quantum corrections to the string amplitude

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NIntegrate[(θ1[z2 - z1, τ] θ1[z4 - z3, τ] / (θ1[z3 - z1, τ] θ1[z4 - z2, τ]))^-s (θ1[z3 - z2, τ] θ1[z4 - z1, τ] / (θ1[z3 - z1, τ] θ1[z4 - z2, τ]))^-t
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```

Textbook formula for type-I four-point amplitude [Green, Schwarz (Nucl. Phys. B 1982)]

··· NIntegrate: Catastrophic loss of precision in the global error estimate due to insufficient WorkingPrecision or divergent integral.

Quantum corrections to the string amplitude

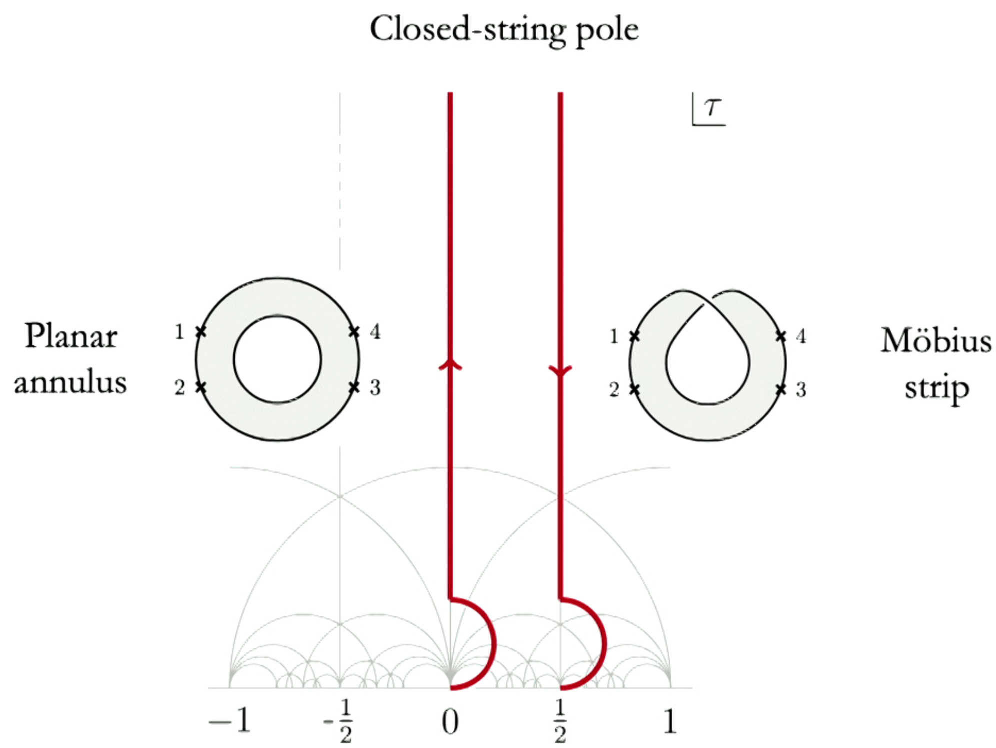
```
NIntegrate[(θ1[z2 - z1, τ] θ1[z4 - z3, τ] / (θ1[z3 - z1, τ] θ1[z4 - z2, τ]))^-s (θ1[z3 - z2, τ] θ1[z4 - z1, τ] / (θ1[z3 - z1, τ] θ1[z4 - z2, τ]))^-t
  /. {s → 3 / 2, t → -1 / 2, z4 → 1, τ → I Imτ},
  {Imτ, 0, ∞}, {z1, 0, 1}, {z2, z1, 1}, {z3, z2, 1}]
```

Textbook formula for type-I four-point amplitude [Green, Schwarz (Nucl. Phys. B 1982)]

··· NIntegrate : Catastrophic loss of precision in the global error estimate due to insufficient WorkingPrecision or divergent integral.

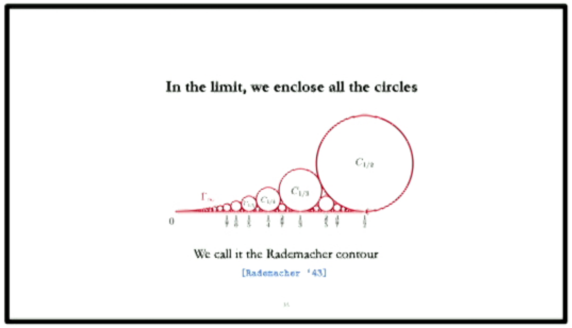
Textbook formulas
can't be used in practice!

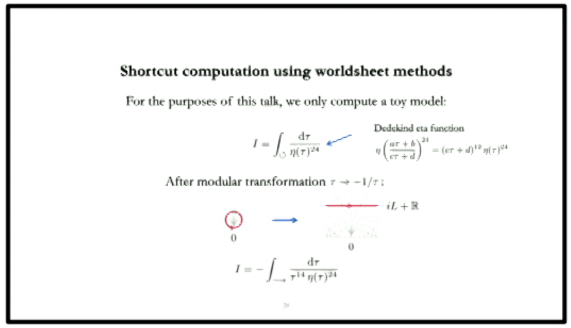
Implementing the Lorentzian time evolution

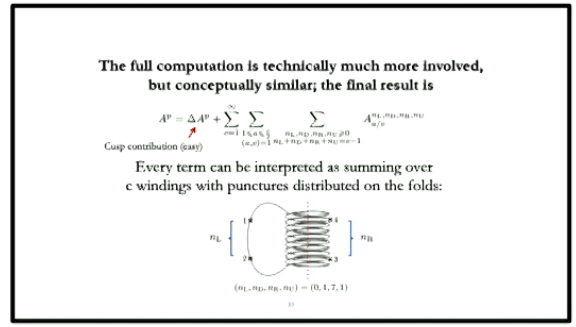


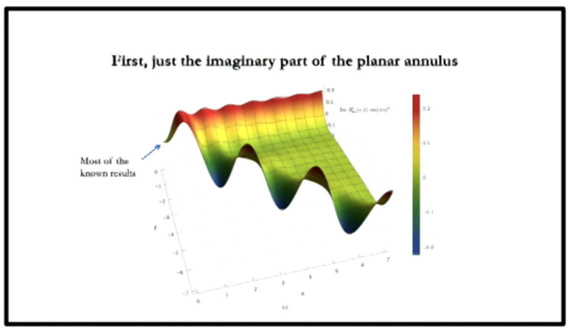
[Eberhardt, SM (SciPost 2023b)]

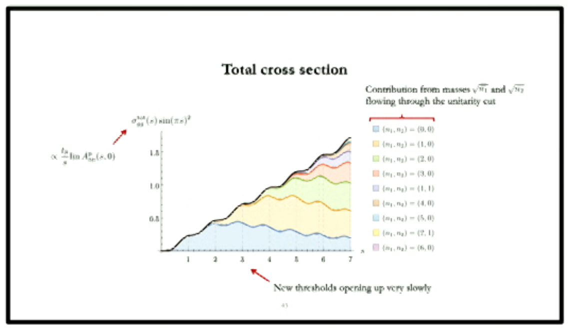
Start of an adventure into the physics of string scattering

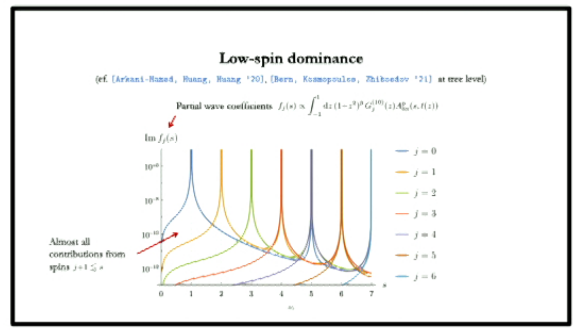




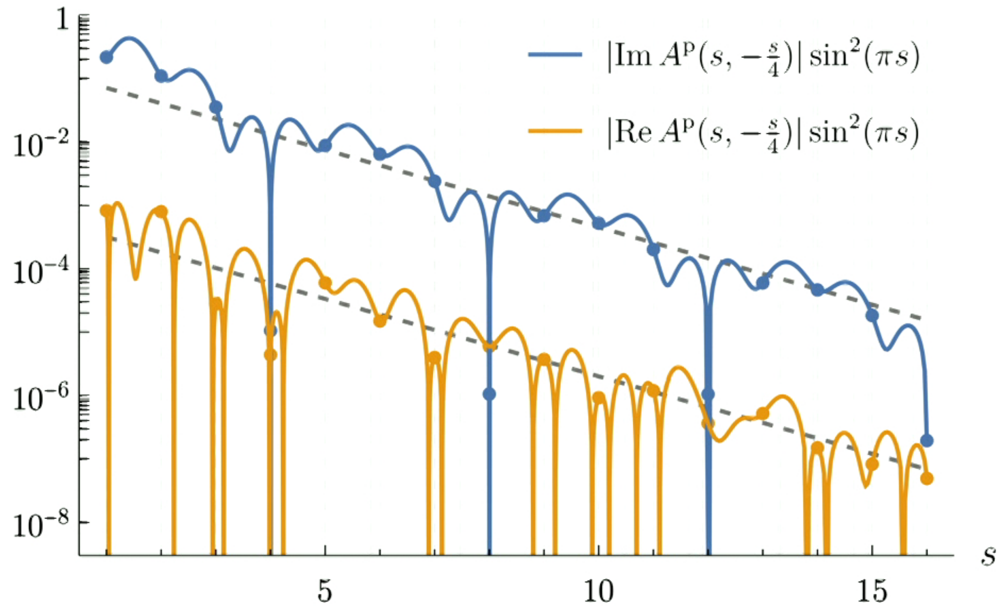








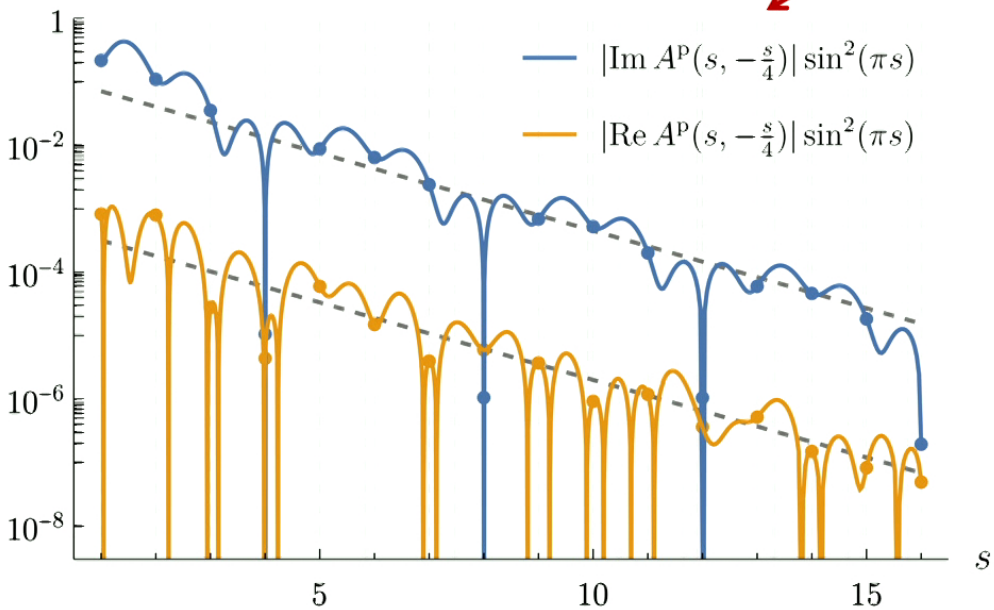
Highlight: First evaluation of a quantum string amplitude at finite energy



[Eberhardt, SM (SciPost 2023b)]

Highlight: First evaluation of a quantum string amplitude at finite energy

Fixed scattering angle $\theta = \frac{\pi}{3}$

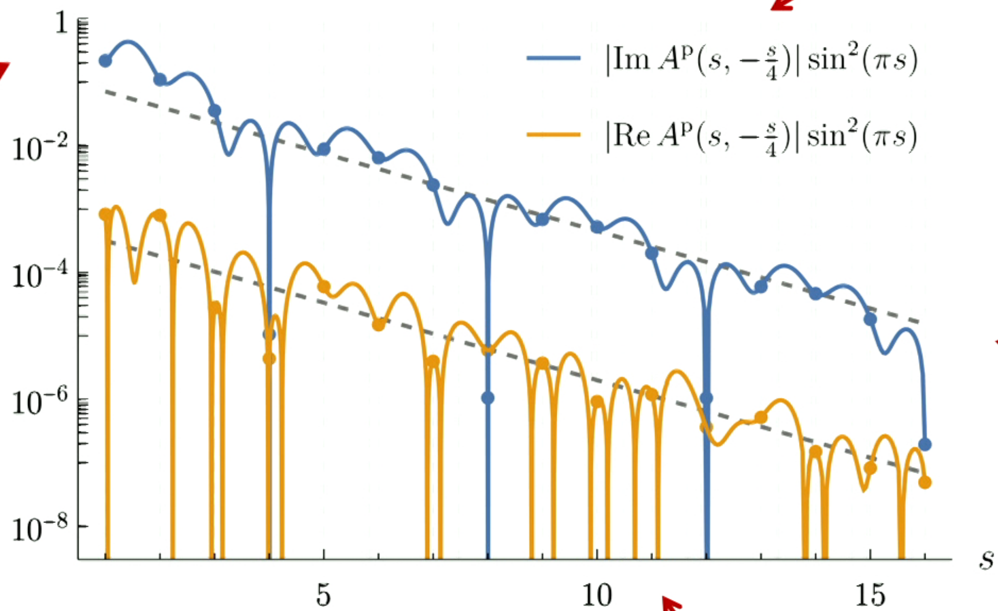


[Eberhardt, SM (SciPost 2023b)]

Highlight: First evaluation of a quantum string amplitude at finite energy

Fixed scattering angle $\theta = \frac{\pi}{3}$

Logarithmic scale



Probing Planck-scale chaos?

[Eberhardt, SM (SciPost 2023b)]

Zeros

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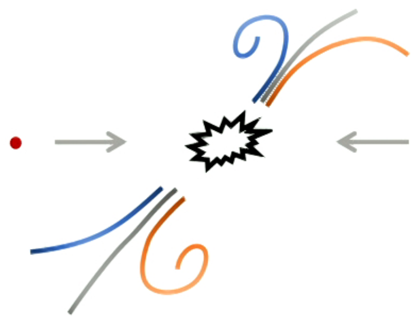
Summary of the 3rd part

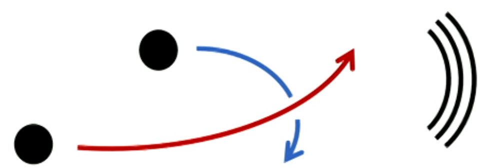
- Essential to understand **Lorentzian worldsheets**
- Used to extract **physics** of string scattering

Summary of the talk

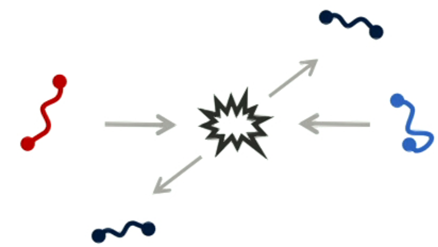
- Particle physics



- Gravitational waves



- String theory



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Thank you