Title: Gravitational wave resonance in ultralight dark matter halos

Speakers: Paola Moreira Delgado

Series: Cosmology & Gravitation

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Abstract: In this talk I will show how the gravitational potentials generated by ultralight dark matter halos interact with gravitational waves, resonantly amplifying them. Significant amplifications can be achieved in dense dark matter environments, where the Floquet exponent is increased. The frequency of the amplified gravitational wave is equal to the axion mass when one requires resonance in the first band. For some masses considered, the gravitational wave frequencies fall within the Pulsar Timing Array range, representing an interesting possibility to test ultralight axions as dark matter candidates.

Zoom link

Gravitational wave resonance in ultralight dark matter halos

Based on *Phys.Rev.D* 108 (2023) 12, 123539

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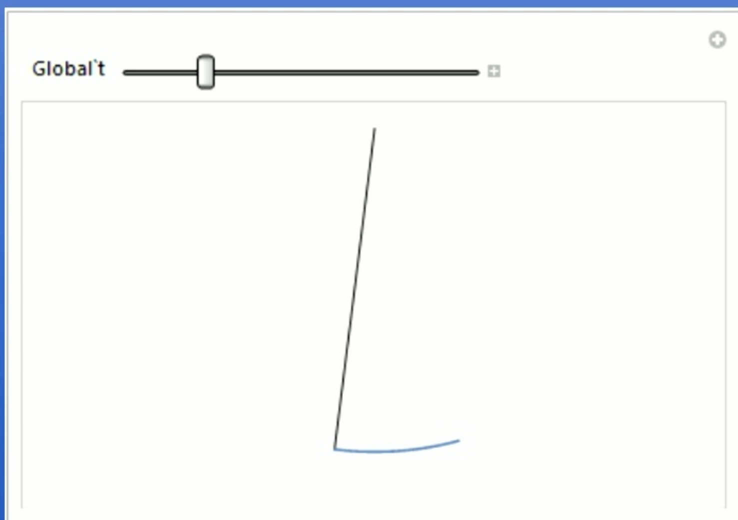
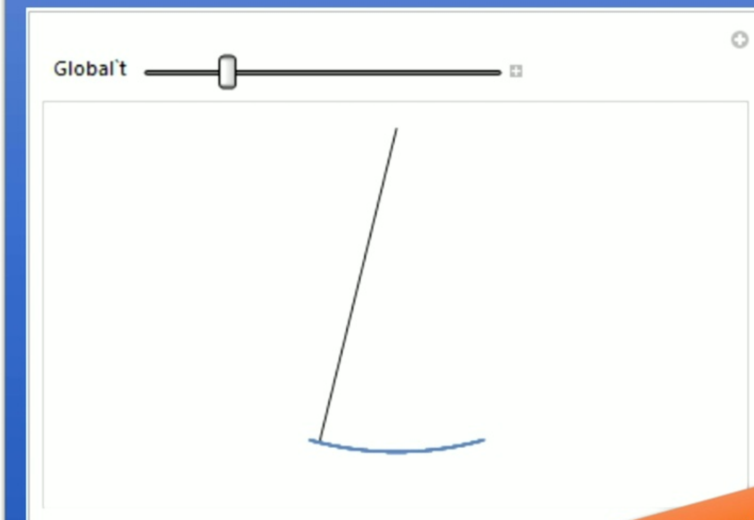
Image: geralt (pixabay.com)

Parametric resonance

$\ddot{x}(t) + Ax(t) = 0$



$\ddot{x}(t) + Ax(t) - 2q\cos(2t)\,x(t) = 0$



Mathieu equation

Why does this exponential instability take place?

$$x(\ddot{t}) + Ax(t) - 2q\cos(2t)\,x(t) = 0$$

$$\pi \equiv \dot{x} \qquad \mathrm{X} \equiv (x, \pi)^T \qquad \dot{X} = UX$$

$$U \equiv \begin{pmatrix} 0 & 1 \\ -A + 2q\cos(2t) & 0 \end{pmatrix}$$

Fundamental matrix of solutions: $O(t, t_0)$

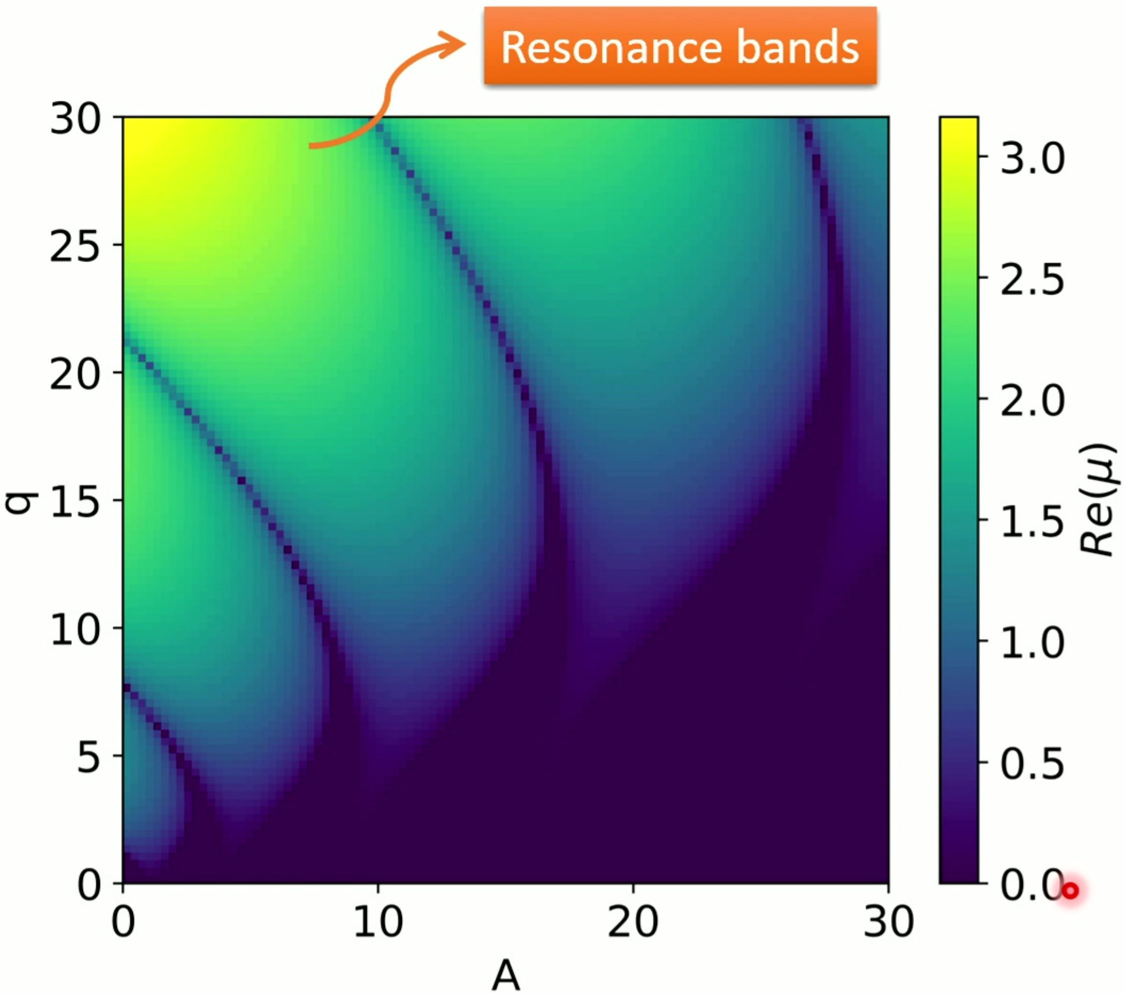
Solve $O(\dot{t}, t_0) = UO(t, t_0)$ from t_0 to $t_0 + T$
$O(t_0, t_0) = I$

Eigenvalues $o^{\pm}$ $\rightarrow$

$$Re(\mu^{\pm}) = \frac{1}{T}\ln|o^{\pm}|$$

$$x(t) \propto \exp(\mu t)$$

Image: Freepik



Bands centered around:

A=1

A=4

A=9

...

$$\mu \propto \begin{cases} q \; if \; A \subset (1-q, 1+q) \\ q^2 if \; A \subset (4-q^2, 4+q^2) \end{cases}$$

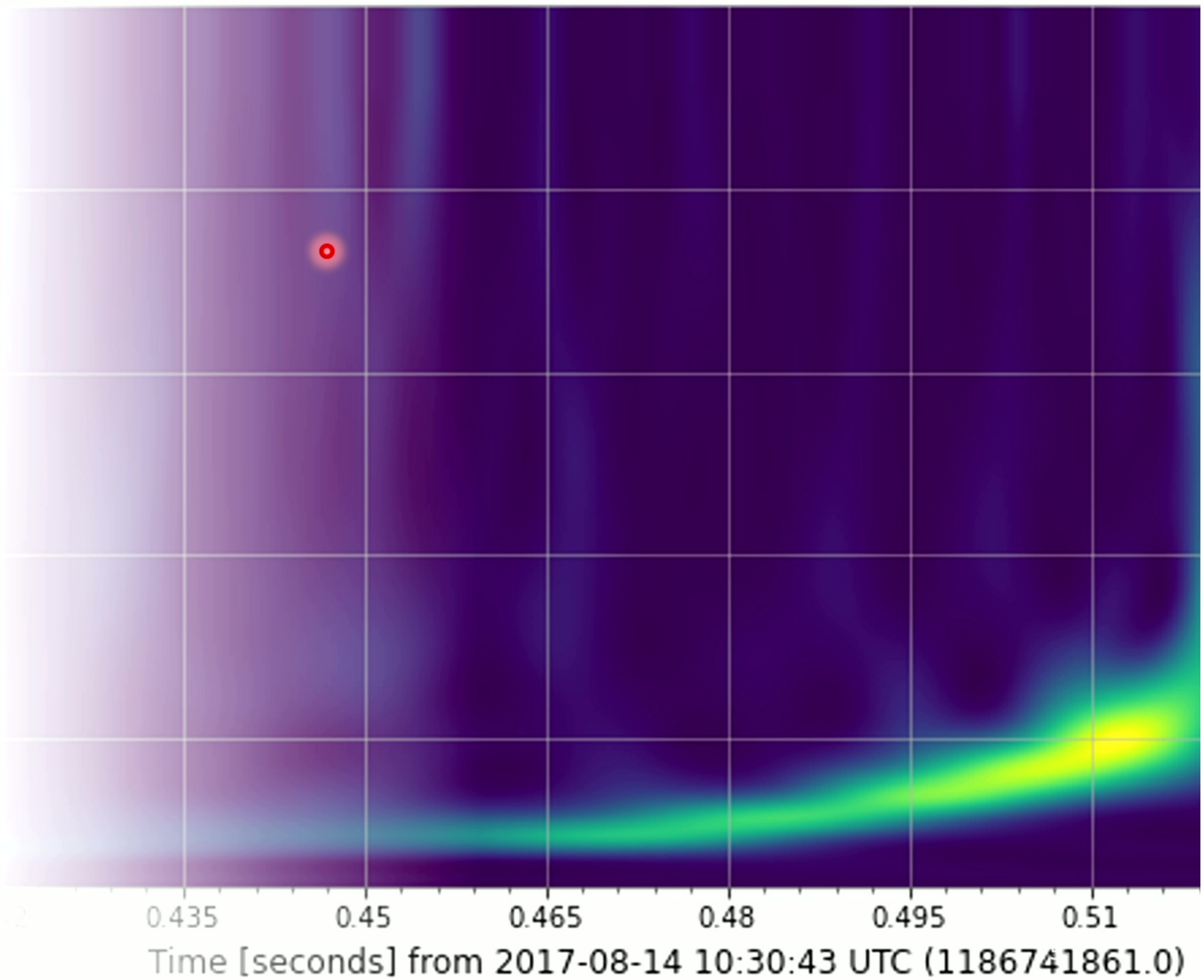
q ≪ 1: narrow band resonance

$x(t) \propto \exp(\mu t)$

In light of GW physics,

- Non-linear order: **interactions** between cosmological perturbations might lead to resonance.
- GWs are damped via resonance with photons: Phys.Dark Univ. 40 (2023) 101202, R. Brandenberger, PCMD, A. Ganz, C. Lin.

Are there scenarios where gravitational waves are amplified via parametric resonance?



ULDM halo

Why Ultra-Light Axions (ULAs) as dark matter?

- Incompatibilities between the CDM description and the observed data on sub-galactic scales.

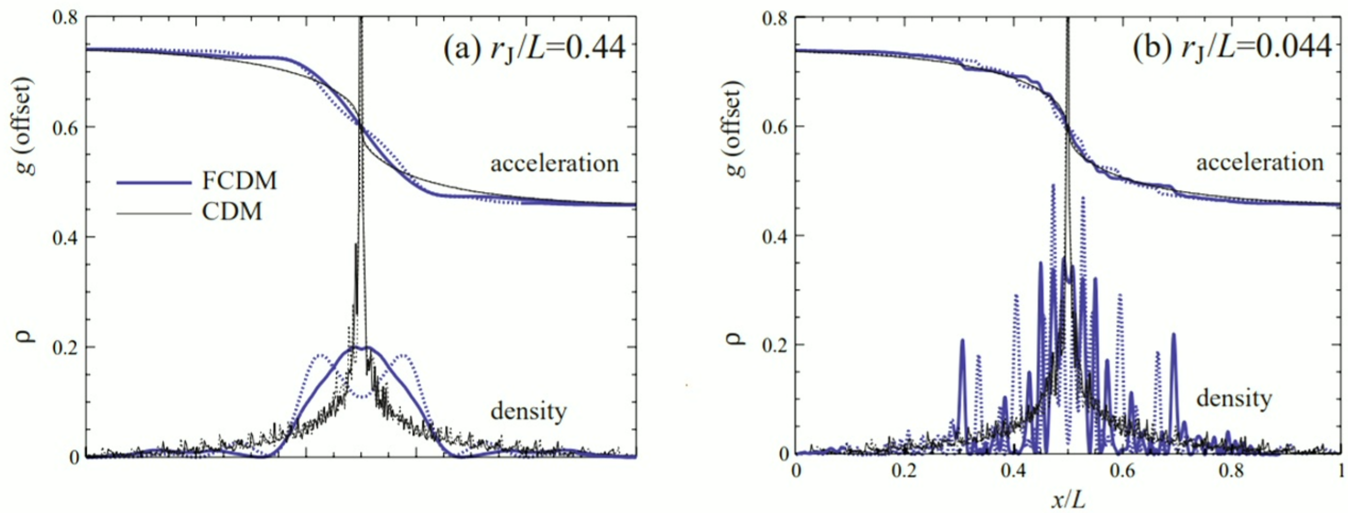


Figure from arXiv:astro-ph/0003365, Wayne Hu, Rennan Barkana, Andrei Gruzinov.

The halo description (ground state):

$$ds^2 = -(1+2U)dt^2 + (1-2\bar{U})(dx^2+dy^2+dz^2) \qquad U, \bar{U} \ll 1$$

$$\phi(t) = \phi_0 \cos(mt)$$

$$\rho = \frac{1}{2}m^2\phi_0^2, \qquad p = -\rho\cos(2mt)$$

→

$$\begin{aligned} T &= T_0 + \delta T \\ U &= U_0 + \delta U \\ \bar{U} &= \bar{U}_0 + \delta\bar{U} \\ R &= R_0 + \delta R \end{aligned}$$

$$R = -6\ddot{\bar{U}} + 2\nabla^2(2\bar{U} - U)$$

Einstein equations →

$$\bar{U}_0 = U_0$$

$$2\nabla^2 U_0 = \rho \rightarrow U_0 \propto \frac{\rho}{k_a^2}$$

$$\delta T = 6\delta\ddot{\bar{U}} \rightarrow \delta\bar{U} = \frac{\rho}{8m^2}\cos(2mt), \quad \delta U = -\delta\bar{U}$$

Oscillating gravitational potentials 👀

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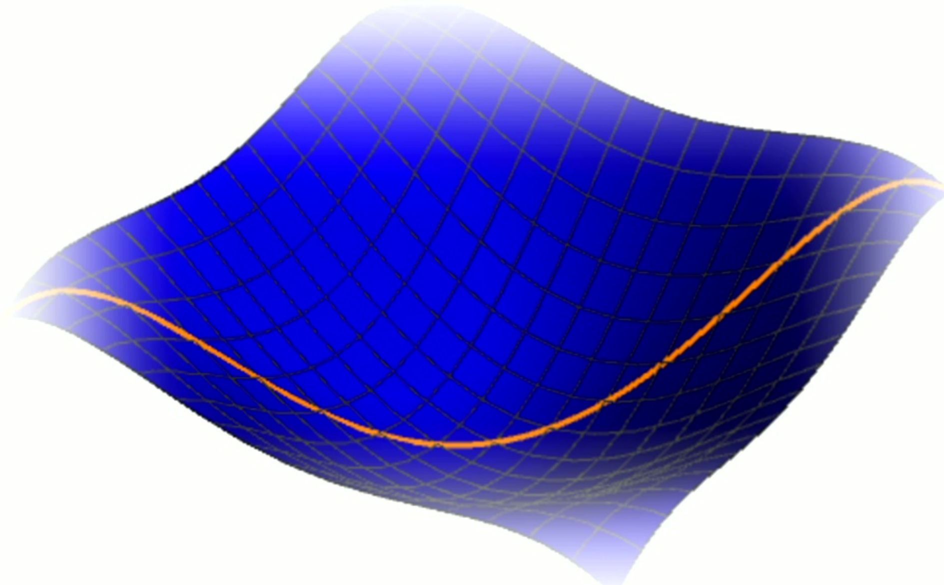
Gravitational Wave and ULDM halo interaction

Gravity bends gravity

From gravitational wave lensing:

$$h_{\mu\nu} = h\epsilon_{\mu\nu}$$

$$\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu h) = 0$$



In the context of ULDM:

$g^{\mu\nu}, g$ → includes oscillating gravitational potentials

h → gravitational wave as the oscillator

Interaction that might lead to a Mathieu equation!

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Expanding the equation of motion,

$$\ddot{h} - (1 + 2U + 2\bar{U})\nabla^2 h - \dot{U}\dot{h} - 3\dot{\bar{U}}\dot{h} +$$
$$+\partial_i h \partial_i \bar{U} - \partial_i h \partial_i U = 0$$

$$\tau \equiv mt$$
$$\bar{h} \equiv \exp(\delta U) h$$

To kill friction terms

$$\bar{h}_k'' + \frac{k^2}{m^2}\bar{h}_k - \frac{4}{m^2}\int d^3x \exp(-i\vec{k}\cdot\vec{x}) U_0 \nabla^2 \bar{h} +$$
$$- \frac{1}{2}\frac{\rho}{m^2}\cos(2\tau)\bar{h}_k = 0$$

$$A\bar{h}_k \equiv \frac{k^2}{m^2}\bar{h}_k - \frac{4}{m^2}\int d^3x \exp(-i\vec{k}\cdot\vec{x}) U_0 \nabla^2 \bar{h}$$
$$\simeq \frac{k^2}{m^2}\bar{h}_k$$

$$(\rho/k_a^2)(k^2/m^2)\bar{h}_k$$

$$\bar{h}_k'' + A\bar{h}_k - 2q\cos(2\tau)\bar{h}_k = 0$$

$q \equiv \rho/m^2/4$ ≪1

$$A\bar{h}_k \equiv \frac{k^2}{m^2}\bar{h}_k - \frac{4}{m^2}\int d^3x \exp(-i\vec{k}\cdot\vec{x})U_0\nabla^2\bar{h}$$
$$\simeq \frac{k^2}{m^2}\bar{h}_k$$

→ $k^2 = m^2$

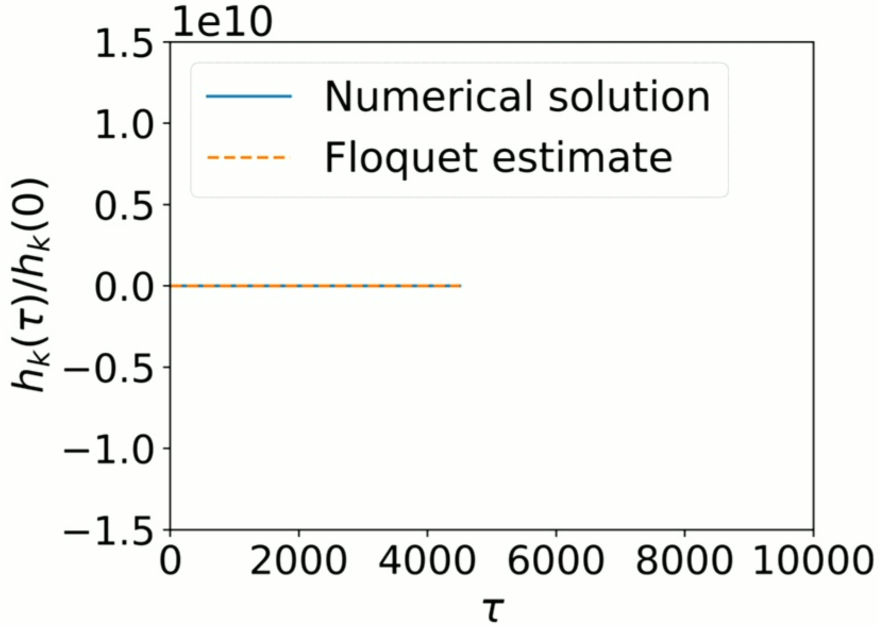
Floquet instability theory:

$$h_k \simeq \bar{h}_k \propto \exp(q\tau/2)$$

↓

$\exp(\delta U) \simeq 1$

Parametric resonance



$m = 10^{-22}\text{eV}$

$\rho = 10^{16} \times 0.4\text{GeV/cm}^3$

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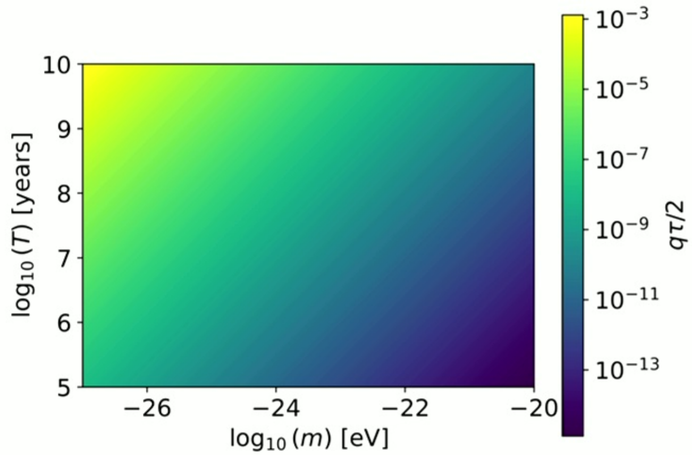
Amplification estimates

$\rho = f\rho_{DM}$ $\quad f = 1$ → ULAs as the totality of dark matter

density in the solar region ← $\rho = 0.4\text{GeV}/\text{cm}^3$, $m \simeq 10^{-22}\text{eV}$ → 3.9×10^{17} years

time estimated via Floquet theory to achieve O(1) amplification



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Amplification estimates

$\rho = f\rho_{DM}$ $\quad f = 1$ → ULAs as the totality of dark matter

density in the solar region ← $\rho = 0.4 \mathrm{GeV/cm^3}$

$m \simeq 10^{-22} \mathrm{eV}$

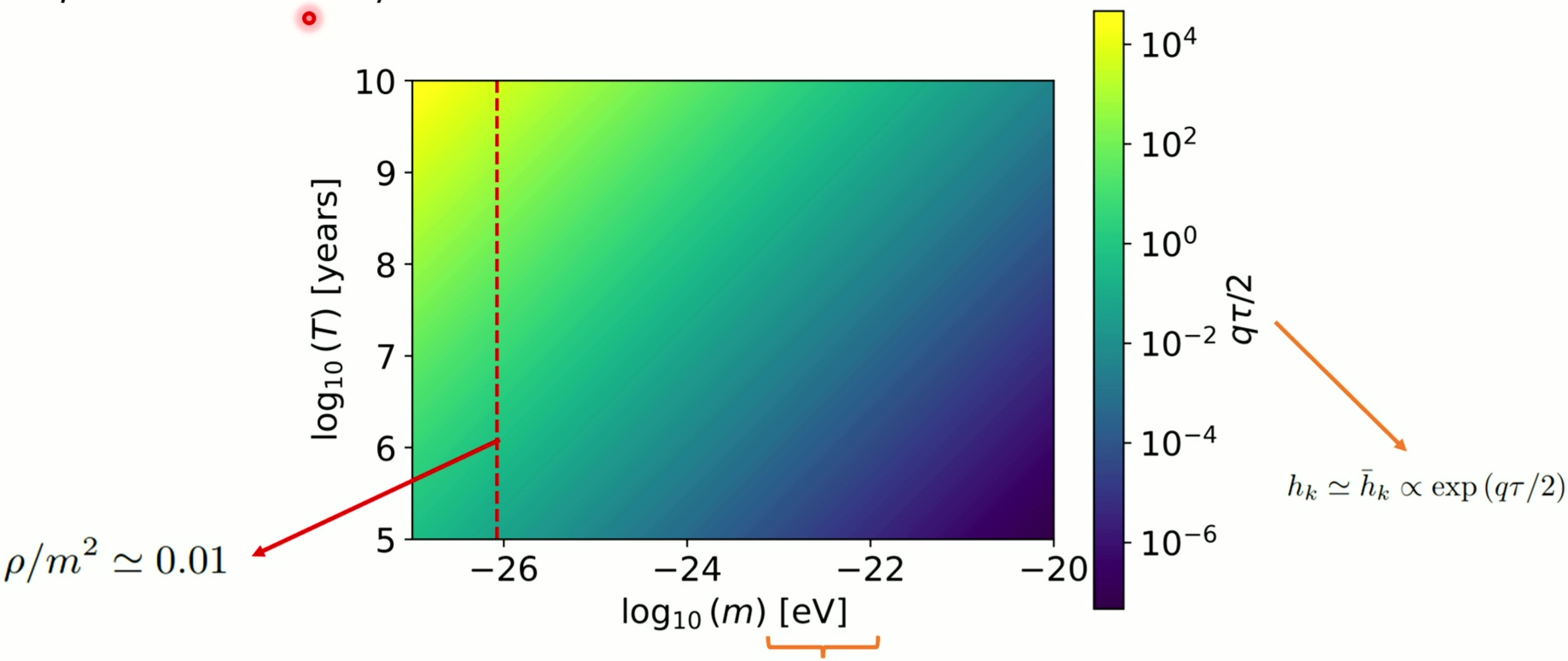
3.9×10^{17} years → time estimated via Floquet theory to achieve O(1) amplification

→ **Higher densities** are required to reduce the waiting time.

- arXiv:2212.05664 [astro-ph.HE], Man Ho Chan, Chak Man Lee.
- arXiv:2103.12439 [astro-ph.HE], Sourabh Nampalliwar, Saurabh K., Kimet Jusufi, Qiang Wu, Mubasher Jamil, Paolo Salucci.

In **extremely dense regions** the amplifications might reach significant values:

$\rho \simeq 1.4 \times 10^7 GeV/cm^3$



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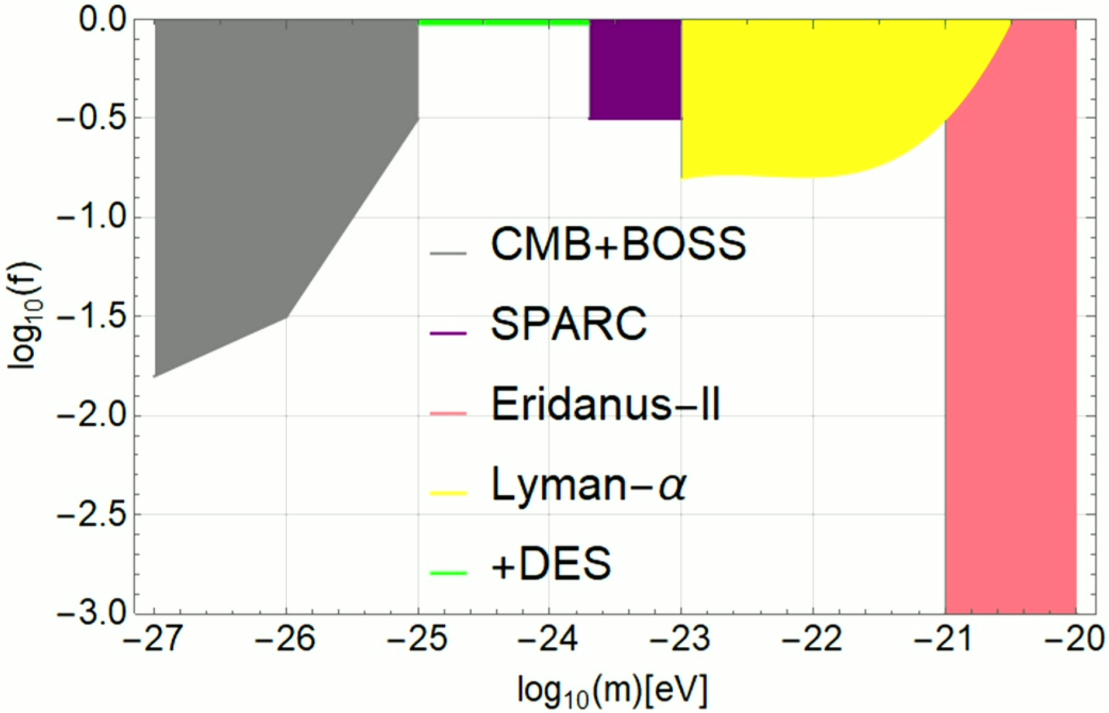
What about the constraints already imposed to the ULA fraction as dark matter?



- **CMB+BOSS**: Planck and LSS bounds from galaxy clustering.
- **SPARC**: bounds from galaxy rotation curves.
- **Eridanus-II**: bounds from Ultrafaint Dwarf Galaxy Eridanus II.
- **Lyman-α**: bounds from Lyman-α forest.
- **+DES**: bounds from galaxy weak lensing and Planck.

Implicitly considered:

- bounds from the UV luminosity function and optical depth to reionization.
- bounds from the M87 black hole spin.

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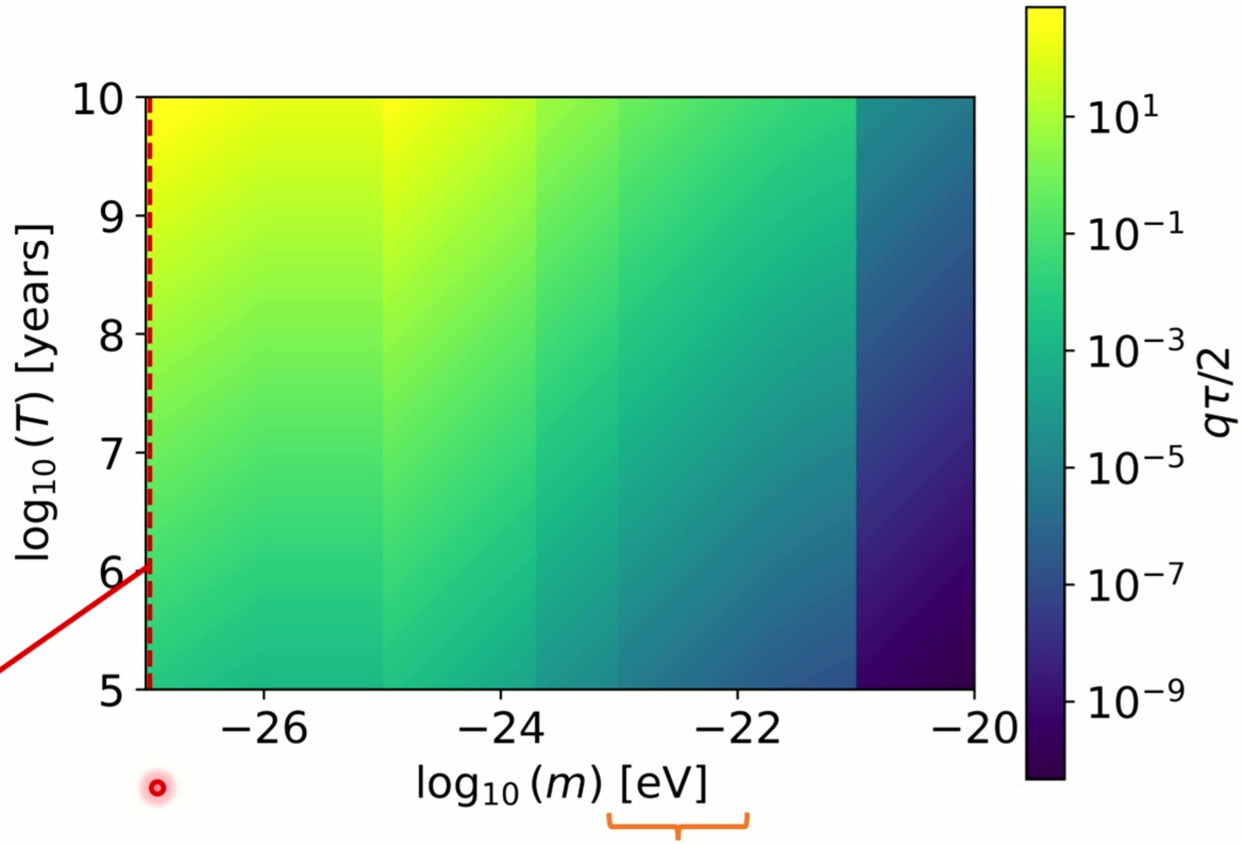
Considering **constraints:** $\rho = f\rho_{DM}$ $\rho \simeq 1.4 \times 10^7 GeV/cm^3$



$\rho/m^2 \simeq 0.01$

$h_k \simeq \bar{h}_k \propto \exp(q\tau/2)$

PTA frequency range

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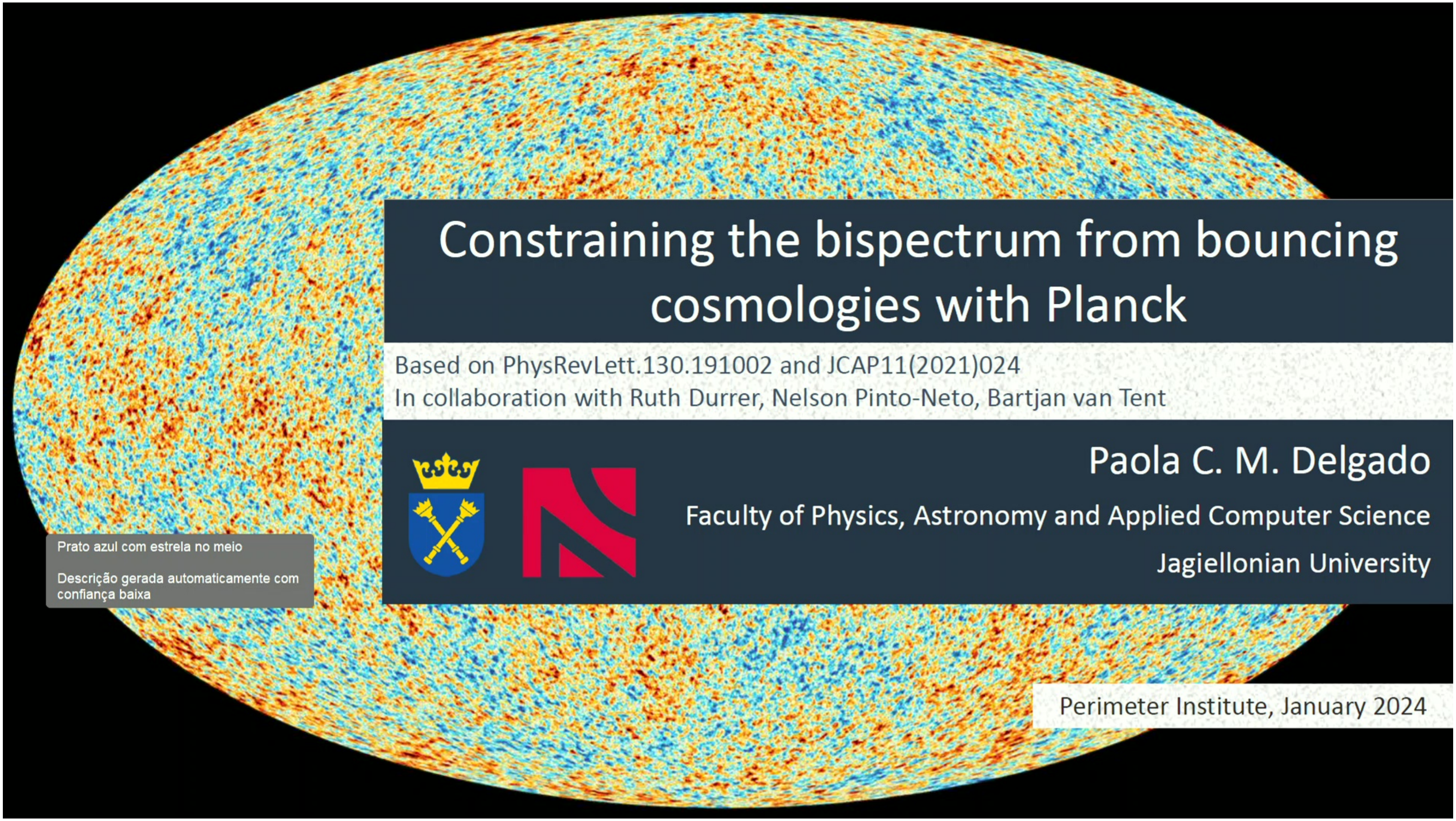
Summary and open questions

- Gravitational waves are **amplified** via parametric resonance with the oscillating gravitational potentials of ULDM halos.
- Significant amplifications nowadays can only be achieved in **very dense regions**.
- Possible **GW sources**: primordial perturbations and supermassive black hole binaries (10^{-8}Hz to 10^{-13}Hz).
- Upper bound on the amplifications (**$h \ll 1$**).

- Can **modified gravity** or the **GW background** boost the resonance?
- Can we **detect** the resonant amplification (e.g. PTA)?


Constraining the bispectrum from bouncing cosmologies with Planck
Based on PhysRevLett.130.191002 and JCAP11(2021)024
In collaboration with Ruth Durrer, Nelson Pinto-Neto, Bartjan van Tent
Paola C. M. Delgado
Faculty of Physics, Astronomy and Applied Computer Science
Jagiellonian University
Perimeter Institute, January 2024
Prato azul com estrela no meio
Descrição gerada automaticamente com confiança baixa

CMB anomalies on large scales

Power suppression:

$$S_{1/2} \equiv \int_{-1}^{1/2} [C(\theta)]^2 d(\cos\theta),$$

$$S_{1/2} \approx 1500\mu K^4 \qquad \Lambda CDM:\ 45000\mu K^4$$

Dipolar anomaly:

Method	A	Direction (l, b) [°]
Commander . .	0.067 ± 0.023	$(230, -18) \pm 31$
NILC	0.069 ± 0.022	$(228, -17) \pm 30$
SEVEM	0.067 ± 0.023	$(230, -17) \pm 31$
SMICA	0.069 ± 0.022	$(228, -18) \pm 30$
SEVEM-100 . .	0.070 ± 0.023	$(231, -19) \pm 30$
SEVEM-143 . .	0.068 ± 0.023	$(230, -17) \pm 31$
SEVEM-217 . .	0.069 ± 0.023	$(229, -20) \pm 31$

Table from Planck 2015 (arXiv:1506.07135)

Parity asymmetry:

$$R^{TT}(\ell_{\max}) = \frac{D_+(\ell_{\max})}{D_-(\ell_{\max})}$$

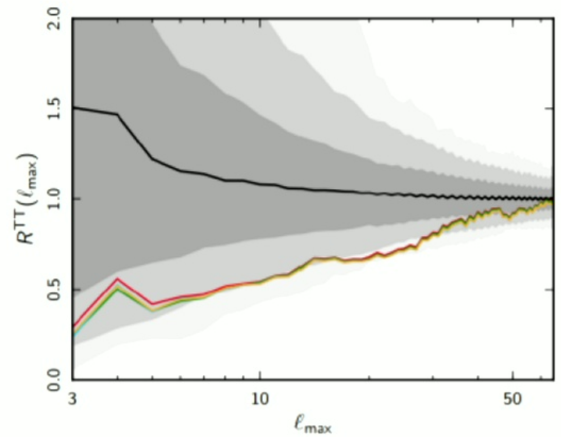


Figure from Planck 2015 (arXiv:1506.07135)

Bounce proposal to mitigate CMB anomalies

Bounce preceding inflation, I. Agullo, D. Kranas, V. Sreenath (arXiv:2005.01796).
Scale factor around the bounce:

$$a(t) = a_b(1 + bt^2)^n$$
$$R_b = 12nb$$

Adiabatic vacuum in the far past ⟶ Non-Gaussianities

$$C_\ell^{mod} = C_\ell\,(1 + \Delta_\ell) \qquad \sigma_0^2(\ell) = \frac{1}{C_\ell^2}\frac{1}{8\pi^2}\int dq\; q^2 P_\phi(q) |C_{\ell\ell}^0(q)|^2$$

$$C_{\ell\ell}^0(q) \propto f_{\rm nl}$$

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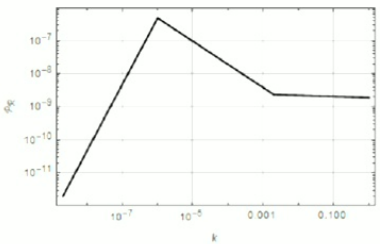
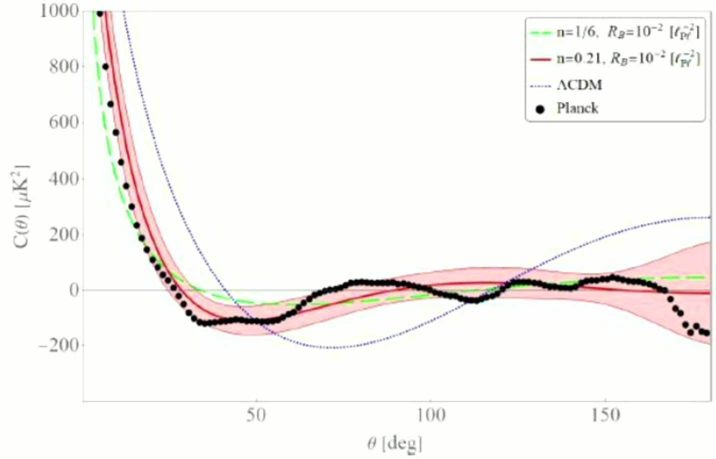
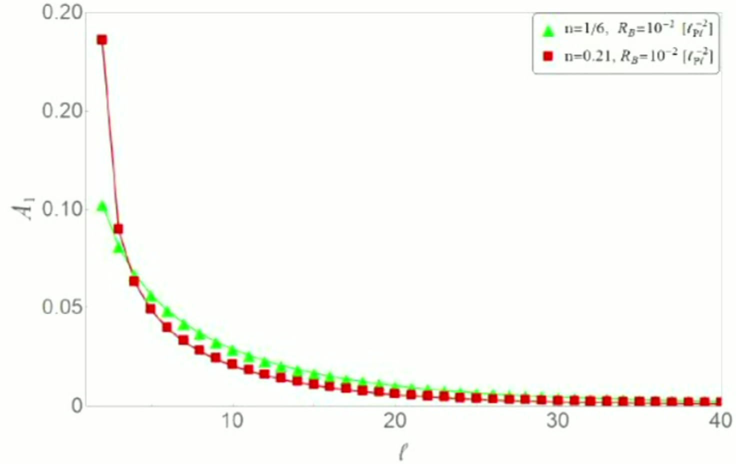
n	γ	q	$f_{\rm nl}$ for $R_B = 1\ l_{Pl}^{-2}$	$f_{\rm nl}$ for $R_B = 10^{-3}\ l_{Pl}^{-2}$
$1/6$	0.6468	-0.7	3326	8518
0.21	0.751	-1.24	959	4372

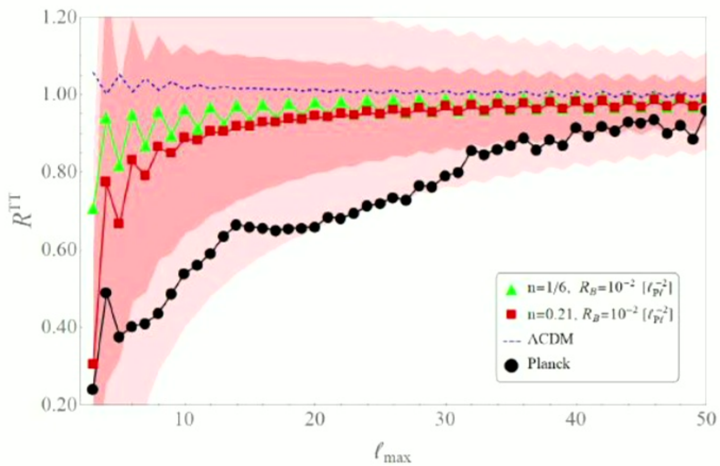


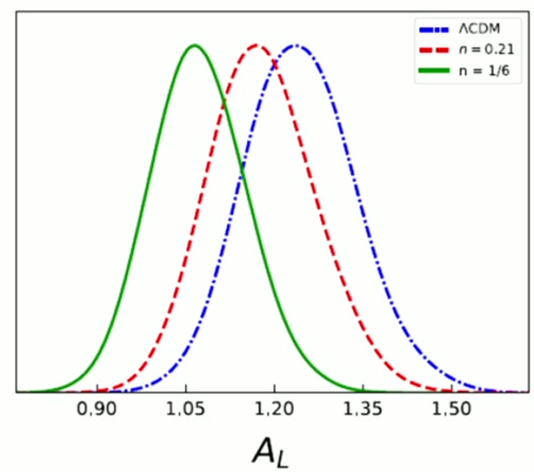
$$\mathscr{P}_{\mathscr{R}}(k) = A_s \begin{cases} (k/k_i)^2 (k_i/k_b)^q & \text{if } k \leq k_i \\ (k/k_b)^q & \text{if } k_i < k \leq k_b \\ (k/k_b)^{n_s - 1} & \text{if } k > k_b \end{cases}$$

$$B(k_1, k_2, k_3) = \frac{3}{5}(2\pi^2)^2 f_{\rm nl} \left[\frac{\mathscr{P}_{\mathscr{R}}(k_1)}{k_1^3} \frac{\mathscr{P}_{\mathscr{R}}(k_2)}{k_2^3} + \frac{\mathscr{P}_{\mathscr{R}}(k_1)}{k_1^3} \frac{\mathscr{P}_{\mathscr{R}}(k_3)}{k_3^3} + \frac{\mathscr{P}_{\mathscr{R}}(k_3)}{k_3^3} \frac{\mathscr{P}_{\mathscr{R}}(k_2)}{k_2^3} \right] \times \exp\left(-\gamma \frac{k_1 + k_2 + k_3}{k_b} \right)$$









Figures from I. Agullo, D. Kranas, V. Sreenath. (arXiv: 2005.01796)

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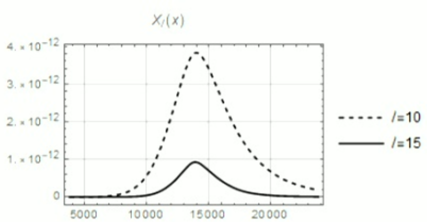
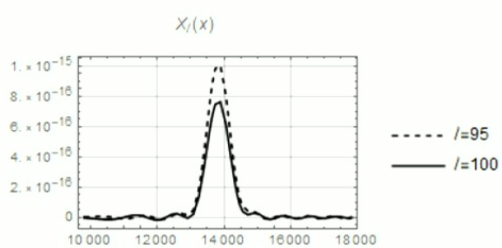
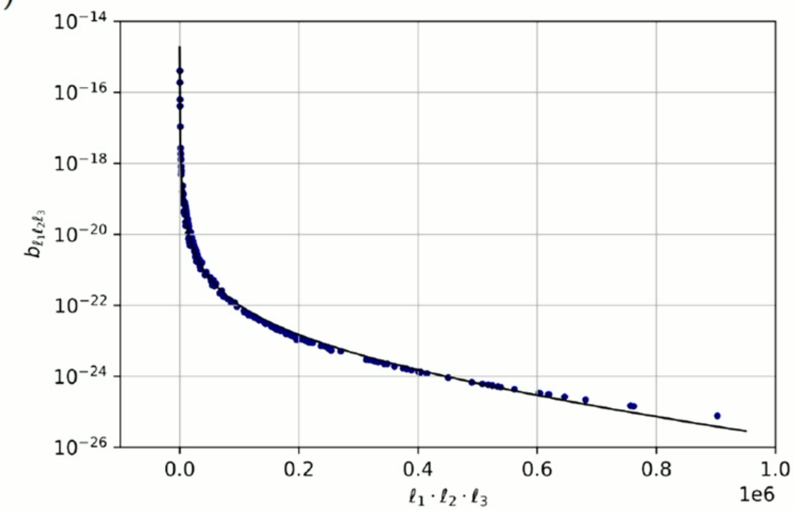
The CMB reduced bispectrum

$$b_{\ell_1\ell_2\ell_3} = \left(\frac{2}{\pi}\right)^3 \int_0^\infty dx\, x^2 \int_0^\infty dk_1 \int_0^\infty dk_2 \int_0^\infty dk_3 \times \left[\prod_{j=1}^{3} \mathscr{T}(k_j,\ell_j) j_{\ell_j}(k_j x)\right] (k_1 k_2 k_3)^2 B(k_1,k_2,k_3)$$

$$\mathscr{T}(k,\ell) \simeq \frac{1}{5} j_\ell(k(t_0 - t_{\rm dec}))$$



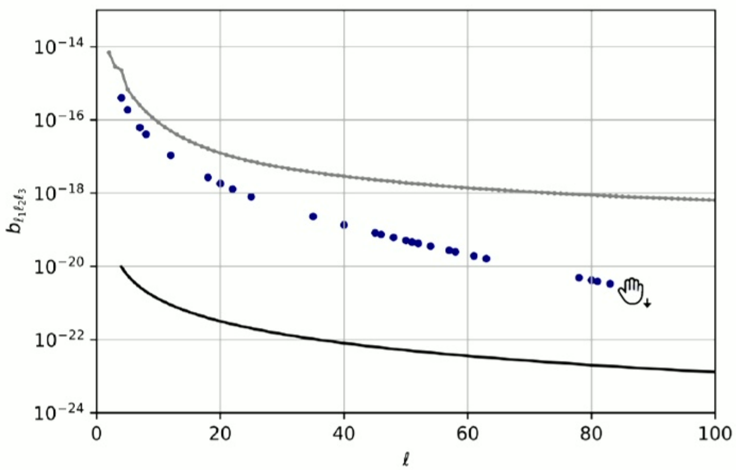


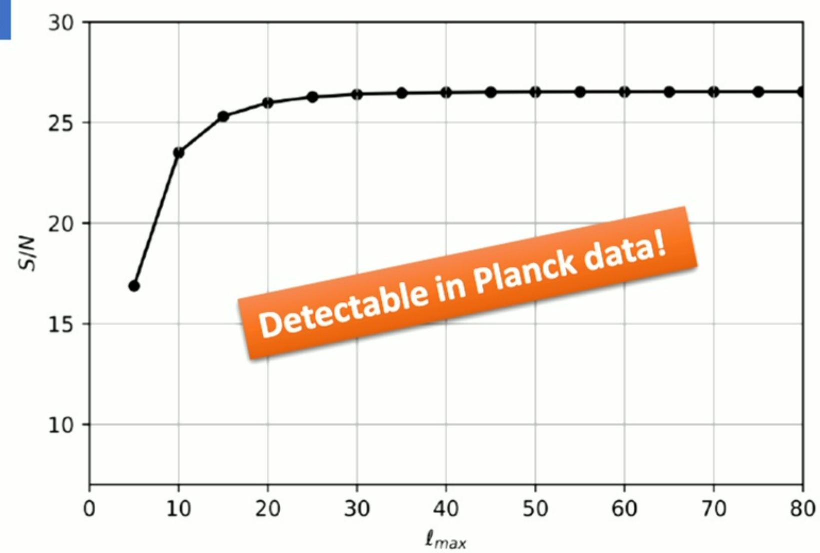


The **signal to noise** ratio (70% of sky coverage):

$$\mathrm{var}\left(b_{\ell_1\ell_2\ell_3}\right) \simeq g_{\ell_1\ell_2\ell_3}^{-2} C_{\ell_1} C_{\ell_2} C_{\ell_3} \left(1 + \delta_{\ell_1\ell_2} + \delta_{\ell_1\ell_3} + \delta_{\ell_3\ell_2} + 2\delta_{\ell_1\ell_2}\delta_{\ell_2\ell_3}\right)$$

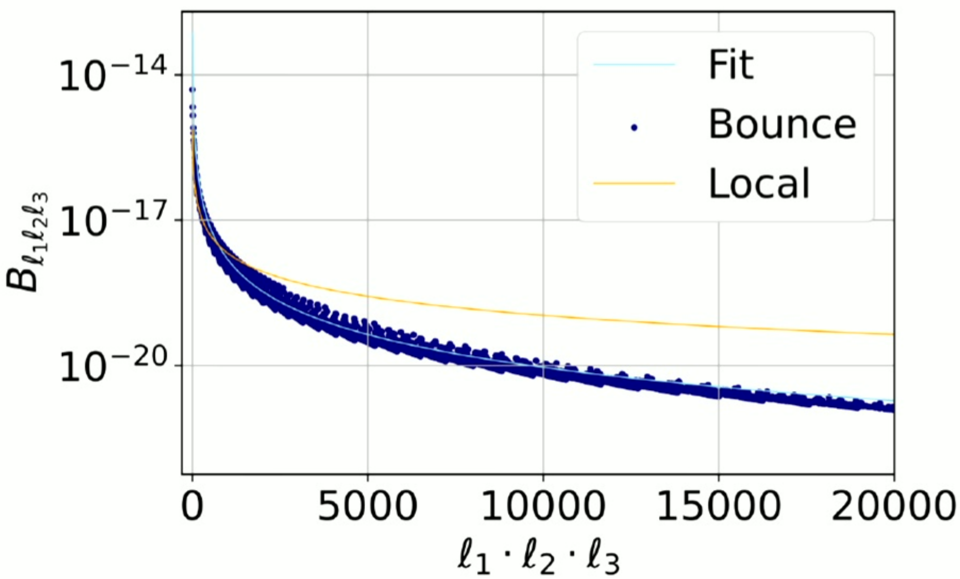
$$\left(\frac{S}{N}\right)^2 (\ell_{\max}) = \sum_{\ell_1\ell_2\ell_3=2}^{\ell_{\max}} \frac{b^2_{\ell_1\ell_2\ell_3}}{\mathrm{var}\left(b_{\ell_1\ell_2\ell_3}\right)}$$

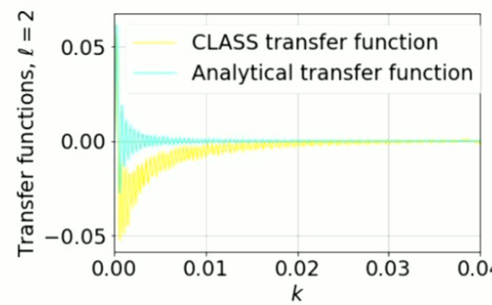


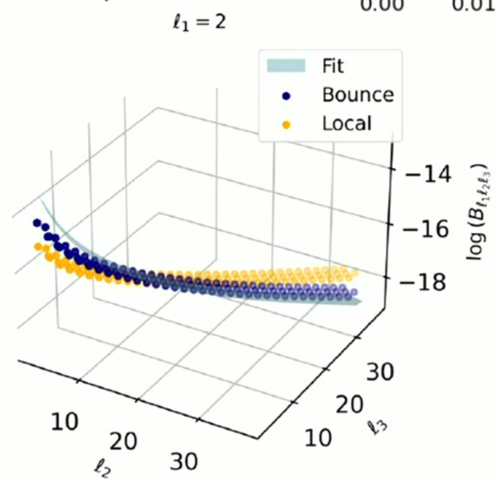


Comparison with Planck data

Bispectrum with the **full transfer function**:







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$$f_{\rm nl} = \frac{\langle B^{\rm th}, B^{\rm obs} \rangle}{\langle B^{\rm th}, B^{\rm th} \rangle}$$

$$\langle B^A, B^B \rangle = \sum_{\ell_1 \le \ell_2 \le \ell_3} \frac{B^A_{\ell_1\ell_2\ell_3} B^B_{\ell_1\ell_2\ell_3}}{V_{\ell_1\ell_2\ell_3}}$$

$$C_{IJ} = \frac{F_{IJ}}{\sqrt{F_{II} F_{JJ}}}$$

$$F_{IJ} = \langle B^I, B^J \rangle$$

	bouncing ($q = -0.7$)	bouncing ($q = -1.24$)
local	0.013	0.006
equilateral	0.006	-0.002
orthogonal	-0.039	-0.028
point sources	-10^{-10}	-10^{-11}
CIB	-10^{-7}	-10^{-8}
lensing	-0.002	-0.001
bouncing ($q = -0.7$)		0.91

template	$f_{\rm nl}$
bouncing ($q = -0.7$)	160 ± 260
bouncing ($q = -1.24$)	19 ± 34

Excluded by 6.4σ and 14σ respectively!

Summary

- Despite the fact that the bispectrum of these models decays exponentially below the pivot scale, they are **excluded by the Planck data** with high significances.
- Similar results for **other values of n (or q)** considered.
- Similar results if we consider a **p-value of 10%** to solve the power suppression anomaly. Less significantly excluded if we consider a **p-value of 5%**.
- This shows the **sensitivity of the Planck data** to scales beyond the pivot scale.
- Exploring quantum cosmology through existing data isn't hopeless; we might uncover some valuable insights using what we already have.