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Speakers: Conner Dailey
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Abstract: Abstract TBA

Zoom link

Scattering Amplitudes in High Energy Limit of Projectable Hořava Gravity 2306.00102

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Grad Seminar Series 2024

Outline

1. Introduction and Motivation
2. Methods and Results
3. Projectable Hořava Gravity
4. Generalized Ward Identities
5. Calculating the Amplitudes
6. The Limit $\lambda \to \infty$
7. Conclusions and Outlook

Introduction and Motivation

- Hořava Gravity is a perturbatively renormalisable metric QFT of gravity* (2301.13580, 2307.13039)
- The Action:

$$S = \frac{1}{2G}\int dt\Big(T[\partial_t \text{metric}; \lambda] - V[\text{metric; other couplings}]\Big) \tag{1}$$

- The RG flow of the *projectable* version (pHG) at $\lambda = \infty$ has asymptotically free UV fixed points
- The pHG **does not** have a stable perturbative Minkowski vacuum (if the scalar gets strongly coupled it might give something similar to GR at low energies)
- The *non-projectable* version can reproduce the low energy phenomenology but it is a more complicated theory

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Methods and Results

Methods

- We perform the BRST quantization of the theory to build the Hilbert space of the scattering states
- We find the amplitudes using symbolic computer algebra

Results

- The BRST analysis of the modes is generalised to the case of non-relativistic theories and the gauge invariance of the amplitudes is confirmed
- The full set of tree-level 2 to 2 scattering amplitudes is computed
- The limit $\lambda \to \infty$ is shown to exist

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Formulating the Theory

- The renormalisabilty is achieved by separating spacetime into space *and* time: the theory is taken to be invariant under anisotropic scaling (like in crystals)

$$x \to b^{-1}\, x, \quad t \to b^{-d}\, t \tag{2}$$

- The theory can no longer be diffeomorphism invariant, however it can be symmetric under foliation preserving diffeomorphisms (FDiffs)

$$x \to \tilde{x} = x(\mathbf{x}, t), \quad t \to \tilde{t}(t), \quad \tilde{t}(t) - \text{monotonic function}$$

- The spacetime metric can therefore be written in terms of ADM-like variables

$$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \tag{3}$$

- The fields γ, N and N^i transform according to

$$N \to N\frac{dt}{d\tilde{t}}, \quad N^i \to \left(N^j \frac{\partial \tilde{x}^i}{\partial x^j} - \frac{\partial \tilde{x}}{\partial t}\right)\frac{dt}{d\tilde{t}}, \quad \gamma_{ij} \to \gamma_{kl}\frac{\partial x^k}{\partial \tilde{x}^i}\frac{\partial x^l}{\partial \tilde{x}^j}.$$

Formulating the Theory

- The projectability condition:

$$N = N(t) \quad \text{consistent with transformation law} \tag{4}$$

- The Action for projectable model

$$S = \frac{1}{2G}\int d^3x dt \sqrt{\gamma}\left(K_{ij}K_{ij} - \lambda K^2 - \mathcal{V}\right) \tag{5}$$

where

$$K_{ij} = \frac{\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i}{2}, \quad K \equiv \gamma^{ij}K_{ij}$$

is the extrinsic curvature (of the leafs) and

$$\begin{aligned} \mathcal{V}_{\text{3d}} &= 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij}R_{ij} \\ &+ \nu_1 R^3 + \nu_2 R R_{ij} R^{ij} + \nu_3 R_{ij}R_{jk}R_{ki} + \nu_4 \nabla_i R \nabla_i R + \nu_5 \nabla_i R_{jk} \nabla_i R_{jk} \end{aligned} \tag{6}$$

is the most general potential consisting only of relevant and marginal operators w. r. t. scaling (2)

BRST Quantization

- Ghosts and BRST transformation (Weinberg QFT Vol. II)

$$\mathrm{s}h_{ij} = \partial_i c_j + \partial_j c_i + \partial_i c^k h_{jk} + \partial_j c^k h_{ik} + c^k \partial_k h_{ij} \ , \quad \mathrm{s}N^i = \dot{c}^i - N^j \partial_j c^i + c^j \partial_j N^i \ , \tag{7a}$$

$$\mathrm{s}c^i = c^j \partial_j c^i \ , \quad \mathrm{s}\bar{c}_i = b_i \ , \quad \mathrm{s}b_i = 0 \ . \tag{7b}$$

- The quantum tree-level BRST-invariant action with gauge-fixing fermion Ψ,

$$S_q = S + \frac{1}{2G} \int d^3x dt \, \mathrm{s}\Psi \, , \quad \Psi = 2\bar{c}_i \, F^i - \bar{c}_i \, O^{ij} \, b_j \ , \quad \mathrm{s}S_q = 0 \tag{8}$$

where F^i are the gauge-fixing functions and O^{ij} is a non-degenerate operator:

$$F^i = \dot{N}^i + \frac{1}{2} O^{ij} \left(\partial_k h^k_j - \lambda \partial_j h \right) \ , \qquad O^{ij} = -\frac{1}{\sigma} \left(\delta^{ij} \Delta^2 + \xi \partial^i \Delta \partial^j \right) \ , \tag{9}$$

- Integrate out the non-dynamical Nakanishi–Lautrup field, and the action takes the form,

$$S_q = S + \int d^3x dt \left(\frac{1}{2G} F^i O^{-1}_{ij} F^j - \frac{1}{G} \bar{c}_i \, \mathrm{s}F^i \right) \ . \tag{10}$$

BRST Quantization

- Scatter on flat background

$$\gamma_{ij} = \delta_{ij} + h_{ij}, \quad N_i = 0 + N_i, \quad \Lambda = 0$$

- Spectrum

$$\omega_{tt}^2 = \nu_5 k^6 \,, \quad \omega^2 = \nu_s k^6, \tag{11}$$

$$\omega_1^2 = \frac{k^6}{2\sigma} \,, \quad \omega_0^2 = \frac{(1-\lambda)(1+\xi)}{\sigma} k^6 \tag{12}$$

- Quantum symmetry

$$[\hat{\Phi}_{\text{as}}, Q^{(2)}] = i(\mathbf{s}\hat{\Phi})_{\text{linearized}} \tag{13}$$

- Realization on the spectrum of in and out states

$$Q^{(2)} |\psi\rangle = 0 \,, \qquad |\psi_1\rangle \sim |\psi_2\rangle \quad \leftrightarrow \quad |\psi_1\rangle = |\psi_2\rangle + Q^{(2)} |\chi\rangle \ . \tag{14}$$

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General Considerations

- S-matrix

$$\langle q', out|\, q, in\rangle = \langle q', in|\, \mathcal{S}\, |q, in\rangle \ , \tag{15}$$

- Symmetry

$$[Q^{(2)}, \mathcal{S}] = 0 \implies \langle \psi'|\, \mathcal{S} Q^{(2)}\, |\chi\rangle = 0 \quad \text{for} \quad Q^{(2)}\, |\psi'\rangle = 0 \tag{16}$$

- In particular, for $|\chi\rangle$ we can take a physical state with added an anti-ghost,

$$|\chi\rangle = \bar{c}^{+}_{\mathbf{k}\alpha}\, |\psi\rangle \ . \tag{17}$$

- BRST transformation for the anti-ghosts:

$$i[Q^{(2)}, \bar{c}^{+}_{\mathbf{k}\alpha}]_{+} = i \sum_{a} \mathcal{C}_a\, \Phi^{a+}_{\mathbf{k}\alpha} \ , \tag{18}$$

- Substituting this into the symmetry condition we get

$$\sum_{a} \mathcal{C}_a\ \langle \psi'|\, \mathcal{S}\, \Phi^{a+}_{\mathbf{k}\alpha}\, |\psi\rangle = 0 \ . \tag{19}$$

Examples

- Relativistic Yang-Mills in 3+1 dimensions in Feynman gauge

$$\mathcal{L}_q^{\mathrm{YM}} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \frac{1}{2} (\partial_\mu A^a_\mu)^2 + \bar{c}^a \partial_\mu D_\mu c^a \,, \tag{20}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \,, \qquad D_\mu c^a = \partial_\mu c^a + g f^{abc} A^b_\mu c^c \,, \tag{21}$$

g is the coupling constant, and f^{abc} are the structure constants

- The theory is straightforwardly quantized

$$A^a_\mu(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_{\mathbf{k}}} \, A^a_{\mu\,\mathbf{k}} \, \mathrm{e}^{-i\omega_{\mathbf{k}} t + i\mathbf{k}\mathbf{x}} + \text{h.c} \,, \tag{22a}$$

$$[A^a_{\mu\,\mathbf{k}}, A^{b+}_{\nu\,\mathbf{k}'}] = 2\omega_{\mathbf{k}} \eta_{\mu\nu} \delta^{ab} (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \,, \tag{22b}$$

where $\omega_{\mathbf{k}} = k$.

Examples

- The BRST transformation of the anti-ghost coincides, up to a sign, with the gauge-fixing function,

$$i[Q^{(2)}, \bar{c}^a]_+ = \mathrm{s}\bar{c}^a = -\partial_\mu A^a_\mu \ , \tag{23}$$

whence we read off

$$i[Q^{(2)}, \bar{c}^{a+}_{\mathbf{k}}]_+ = -ik_\mu A^{a+}_{\mu\,\mathbf{k}} \ . \tag{24}$$

Substituting this expression into Eqs. (19) we find

$$k_\mu \, \langle \psi' | \, \mathcal{S} \, A^{a+}_{\mu\,\mathbf{k}} \, |\psi\rangle = k_\mu \, \langle \psi' | \, A^{a}_{\mu\,\mathbf{k}} \, \mathcal{S} \, |\psi\rangle = 0 \ . \tag{25}$$

On the other hand, the scattering amplitudes involving a physical gluon with helicity ± 1 in the initial or final state are given by

$$e^{(\pm 1)}_\mu \, \langle \psi' | \, \mathcal{S} \, A^{a+}_{\mu\,\mathbf{k}} \, |\psi\rangle \ , \qquad e^{(\pm 1)*}_\mu \, \langle \psi' | \, A^{a}_{\mu\,\mathbf{k}} \, \mathcal{S} \, |\psi\rangle \tag{26}$$

Examples

- Non-relativistic Yang–Mills theory (in $4+1$) with the Lagrangian

$$\mathcal{L}^{\mathrm{YM}} = \frac{1}{2} F^a_{i0} F^a_{i0} - \frac{\kappa_1}{4} D_i F^a_{jk}\, D_i F^a_{jk} - \frac{\kappa_2}{2} D_i F^a_{ik}\, D_j F^a_{jk} - g\frac{\kappa_3}{3}\, f^{abc} F^a_{ij} F^b_{jk} F^c_{ki} \tag{27}$$

- Gauge fixing:

$$S_{g.f.} \sim \int F^a O^{-1}_{ab} F^b\,, \quad F^a = \dot{\mathcal{A}}^a + \xi\, \Delta \partial_i A^a_i\;, \qquad O^{-1}_{ab} = \frac{\delta_{ab}}{\xi \Delta} \tag{28}$$

- The dispersion relations are different for the transverse and longitudinal modes, as expected in theories without Lorentz invariance:

$$\omega^2_{\mathbf{k}1} = (\kappa_1 + \kappa_2) k^4\;, \qquad \omega^2_{\mathbf{k}0} = \xi k^4 \tag{29}$$

Examples

- The BRST transformation of the anti-ghost is

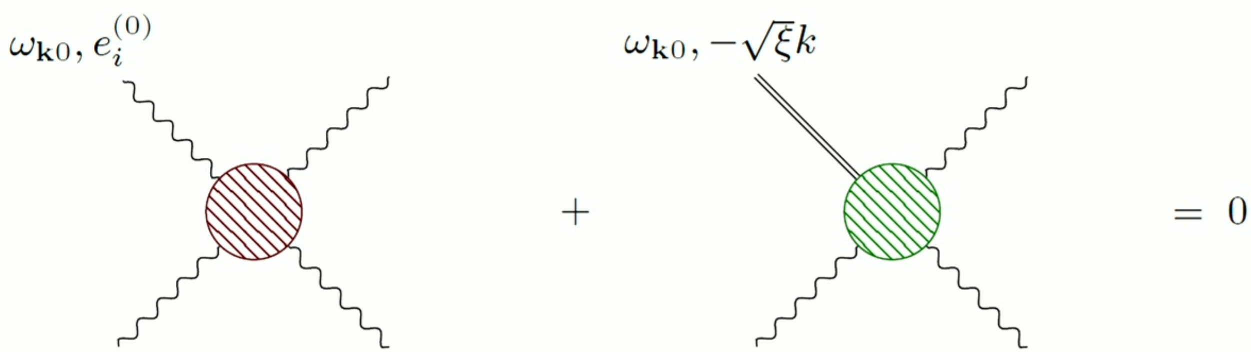
$$i[Q^{(2)}, \bar{c}^a]_+ = \mathbf{s}\bar{c}^a = \frac{1}{\xi\,\Delta}\left(\dot{\mathcal{A}}^a + \xi\Delta\partial_i A_i^a\right) \ . \tag{30}$$

For creation-annihilation operators

$$i[Q^{(2)}, \bar{c}_{\mathbf{k}}^{a+}]_+ = ik\left(\mathcal{A}_{\mathbf{k}}^{a+} + A_{\mathbf{k}0}^{a+}\right) \ . \tag{31}$$

Incidentally, this has the same form as in the relativistic case, cf. Eq. (24)! Hence, the constraint (19) becomes

$$\langle\psi'|\,\mathcal{S}\,A_{\mathbf{k}0}^{a+}\,|\psi\rangle + \langle\psi'|\,\mathcal{S}\,\mathcal{A}_{\mathbf{k}}^{a+}\,|\psi\rangle = 0\ , \tag{32}$$

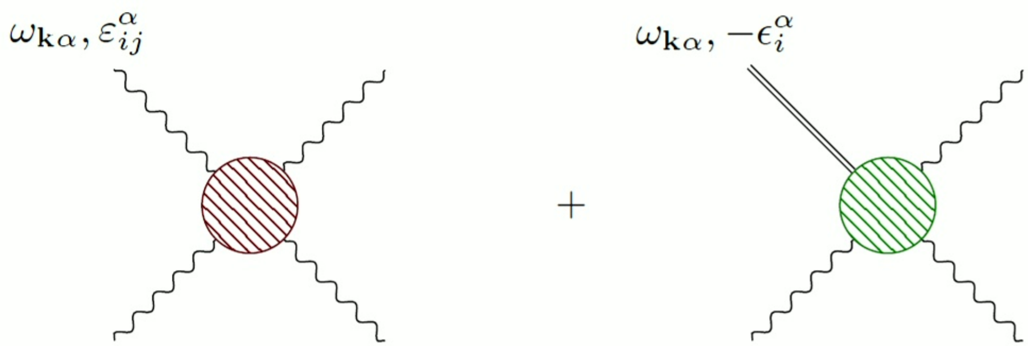


Hořava Gravity

- BRST transformation (for the third time)

$$i[Q^{(2)}, \bar{c}_i]_+ = -\frac{\sigma}{\Delta^2}\dot{N}_i + \frac{\sigma\xi}{(1+\xi)\Delta^3}\partial_i\partial_j\dot{N}_j + \frac{1}{2}(\partial_j h_{ij} - \lambda\partial_i h) \tag{33}$$

- Pictorially (α denotes various polarizations)



$= 0$ for $\alpha = 0, \ \pm 1$

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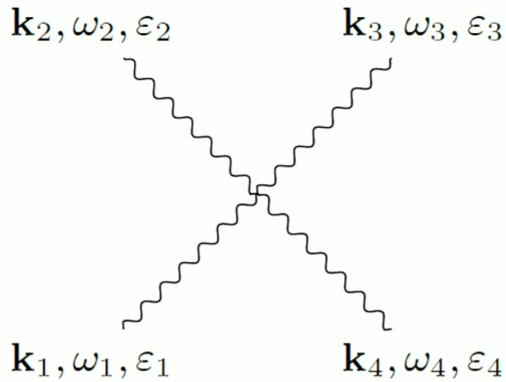
Feynman Diagrams

$i\mathcal{M}(\mathbf{k}_I, \omega_I, \alpha_I) \;=$



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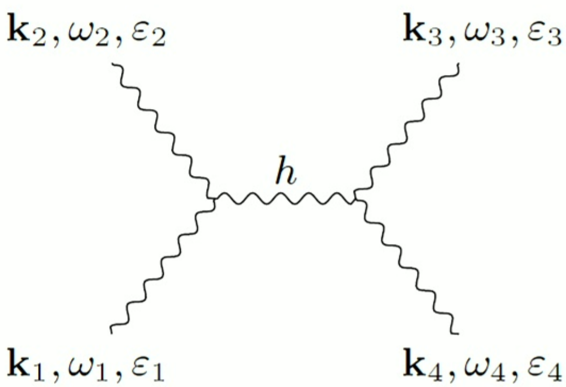

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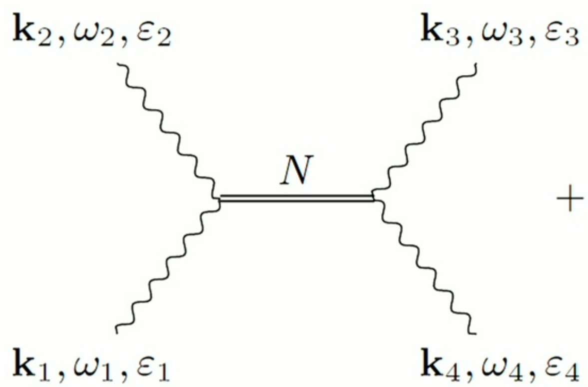

$+ \quad (t, u)$

Overview

- Use standard Feynman rules (and Mathematica) to compute the amplitudes
- Similar to GR

$$\mathcal{M} = \sum \text{contractions of } \varepsilon_i \text{ and } \mathbf{k}_i \tag{34}$$

- In HG the structure is richer due to the presence of higher powers of momenta (higher spatial derivatives) in the vertices. Extra terms:

$$(\mathbf{k}_3\varepsilon_1\varepsilon_2\mathbf{k}_4)(\mathbf{k}_1\varepsilon_3\mathbf{k}_1)(\mathbf{k}_2\varepsilon_4\mathbf{k}_2)\ , \qquad (\mathbf{k}_2\varepsilon_1\mathbf{k}_2)(\mathbf{k}_1\varepsilon_2\mathbf{k}_1)(\mathbf{k}_4\varepsilon_3\mathbf{k}_4)(\mathbf{k}_3\varepsilon_4\mathbf{k}_3) \tag{35}$$

- Independent of gauge parameters if on-shell

Head-On Amplitudes

- The helicity amplitudes have the form

$$\mathcal{M}_{\alpha_1\alpha_2,\alpha_3\alpha_4} = GE^2 f_{\alpha_1\alpha_2,\alpha_3\alpha_4}(\cos\theta; u_s, v_a, \lambda) \quad (36)$$

- Scaling $\sim E^2$ is compatible with unitarity
- Cross section

$$\frac{d\sigma_{\alpha_1\alpha_2,\alpha_3\alpha_4}}{\sin\theta\, d\theta} = \frac{G^2}{72\pi\,\nu_5\,k^2}\left|f_{\alpha_1\alpha_2,\alpha_3\alpha_4}\right|^2 \propto \lambda^2_{\text{de Broglie}}. \quad (37)$$

  diverges at small angles signaling the necessity of an infrared regulator

Head-On Amplitudes

- Higher degree of collinear divergence
- Partially compensated by the orbital wave functions from the angular momentum conservation
- Poles at non-zero angles due to the decay
- Decay makes final states unstable - problem with $\mathcal{S}$-matrix definition

Head-On Amplitudes

- General Relativity ($x = \cos\theta$)

$$f_{++,++} = f_{--,--} = \frac{1}{1-x^2} \tag{38}$$

- Hořava Gravity:

$$\begin{aligned} f_{++,++} =& f_{--,--} = \\ =& \frac{1}{512\hat{u}_s^2(1-x^2)^3}\Bigg[x^8\Big(-161 - 320v_2^2 + v_2(464 - 720v_3) + 39\hat{u}_s^2 - 9v_3^2(45 - 11\hat{u}_s^2) \\ &+ 6v_3(87 - 85\hat{u}_s^2)\Big) + 4x^6\Big(231 + 443\hat{u}_s^2 - 72v_3^2\hat{u}_s^2 - 16v_2(21 - 8\hat{u}_s^2) \\ &+ 6v_3(63 - 53\hat{u}_s^2)\Big) + 2x^4\Big(-287 + 448v_2^2 - 4783\hat{u}_s^2 - 16v_2(49 - 63v_3 + 48\hat{u}_s^2) \\ &+ 63v_3^2(9 + \hat{u}_s^2) - 6v_3(147 + 295\hat{u}_s^2)\Big) - 4x^2\Big(581 + 128v_2^2 - 6343\hat{u}_s^2 \\ &- 16v_2(35 - 18v_3 + 24\hat{u}_s^2) + 54v_3^2(3 - \hat{u}_s^2) - 6v_3(105 + 269\hat{u}_s^2)\Big) - 169 - 64v_2^2 \\ &- 19921\hat{u}_s^2 - 9v_3^2(9 + 17\hat{u}_s^2) + 16v_2(13 - 9v_3 + 32\hat{u}_s^2) + 6v_3(39 - 613\hat{u}_s^2)\Bigg] \end{aligned} \tag{39}$$

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The Limit $\lambda \to \infty$

- Explicit cancellation of potentially divergent terms
- Manifest regularization of the theory by integrating in an auxiliary non-dynamical scalar field χ:

$$-\frac{\lambda}{2G}\sqrt{\gamma}\,K^2 \quad \longrightarrow \quad \frac{\sqrt{\gamma}}{G}\left[-\chi K + \frac{\chi^2}{2\lambda}\right] \tag{40}$$

Take the limit (with other couplings fixed) and get for the action of HG,

$$S \xrightarrow[\lambda\to\infty]{} S' = \frac{1}{2G}\int d^3x dt \sqrt{\gamma}\left(K_{ij}K^{ij} - 2\chi K - \mathcal{V}\right) \tag{41}$$

- The field χ takes the role of a Lagrange multiplier constraining the extrinsic curvature to be traceless, $K = 0$
- Propagators in the modified theory are regular: theory is well-defined (albeit inconvenient to use)

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Conclusions and Outlook

- Obtained generalised Ward identities for non-relativistic gauge theories
- Computed the full set of tree level 2 to 2 scattering amplitudes
- We checked explicitly the finiteness of the amplitudes in the limit $\lambda \to \infty$ and reformulated theory to make it manifestly regular in this limit. This establishes the $\lambda \to \infty$ as a viable location for asymptotically free UV fixed points
- What about the ground state of pHG?
- Proof of the renormalizability of the non-projectable version