Title: Principle of Information Causality Rationalizes Quantum Composition

Speakers: Mir Alimuddin

Series: Quantum Foundations

Date: January 18, 2024 - 11:00 AM

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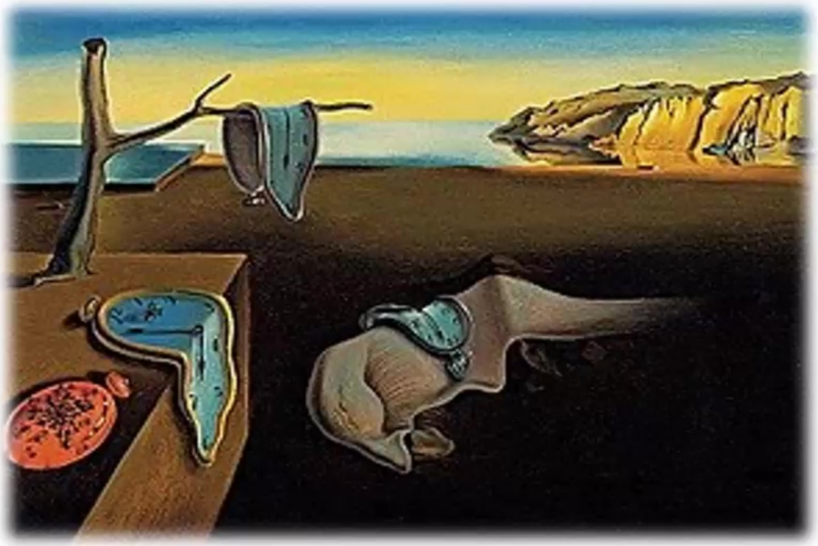
Abstract: The principle of information causality, proposed as a generalization of no signaling principle, has efficiently been applied to outcast beyond quantum correlations as unphysical. In this talk, we show that this principle, when utilized properly, can provide physical rationale toward structural derivation of multipartite quantum systems. In accordance with the no signaling condition, the state and effect spaces of a composite system can allow different possible mathematical descriptions, even when descriptions for the individual systems are assumed to be quantum. While in one extreme, namely, the maximal tensor product composition, the state space becomes quite exotic and permits composite states that are not allowed in quantum theory, the other extreme - minimal tensor product composition - contains only separable states, and the resulting theory allows only Bell local correlation. As we show, none of these compositions is commensurate with information causality, and hence cannot be the bona-fide description of nature. Information causality therefore promises an information-theoretical derivation of self duality of the state and effect cones for composite quantum systems.

Zoom link

Principle of Information Causality: Rationalizes Quantum Composition



The Persistence of Memory
(Salvador Dali)

Perimeter Institute

18th January, 2024

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Chanakya Post Doctoral Fellow (PDF)
S. N. Bose National Centre for Basic Sciences, Kolkata

Brief Introduction of the Candidate:

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July 2021 — RA - I — March 2022

PhD degree Awarded, July, 2022

April 2022 — PDF — March 2024

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Institute
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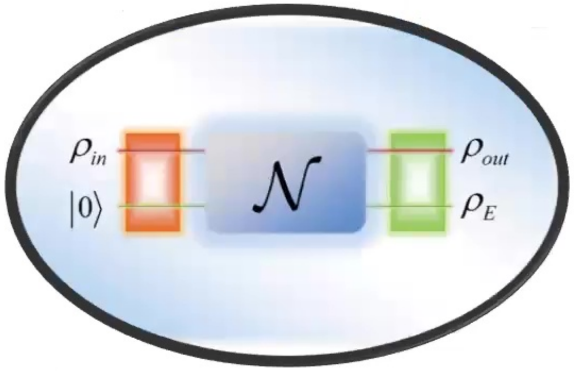
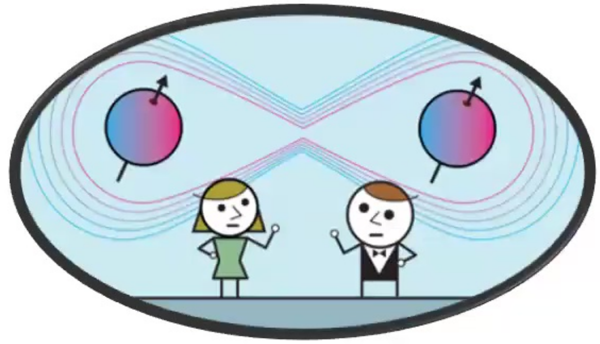
&
S. N. B. N. C. B.S

Composite Quanta: Recent Studies





Foundational Aspects

- ✓ PRL 128, 140401
- ✓ PRL 130, 110202
- ✓ PRA (Letter) 106, L040201
- ✓ PRA106, 062406

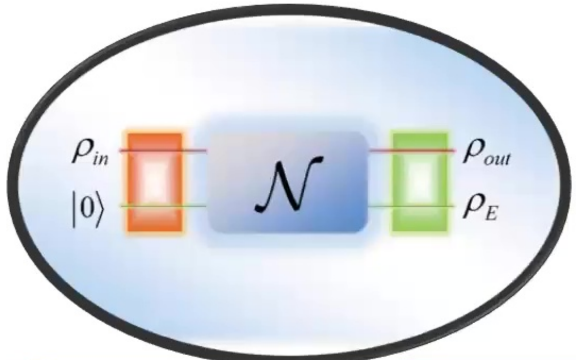
Quantum Information & Communication theory

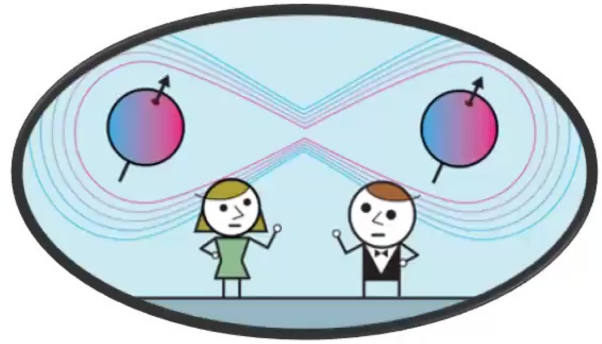
- ✓ PRL 126, 210505
- ✓ NJP 23, 033039
- ✓ Quantum 5, 569
- ✓ PRA 106, 012432
- ✓ PRA 105, 032407
- ✓ PRA 108, 052430
- ✓ arXiv:2309.17263
- ✓ arXiv:2311.17772
- ✓ arXiv:2202.06796

Quantum Thermodynamics & Entanglement Theory

- ✓ PRL 129, 070601
- ✓ PRL 131, 030402
- ✓ PRE 100, 012147
- ✓ PRA 99, 052320
- ✓ PRA 101, 012115
- ✓ PRE 102, 022106
- ✓ PRA 102, 032215
- ✓ PRE 102, 012145
- ✓ arXiv:2305.15012

Composite Quanta: Recent Studies





Foundational Aspects

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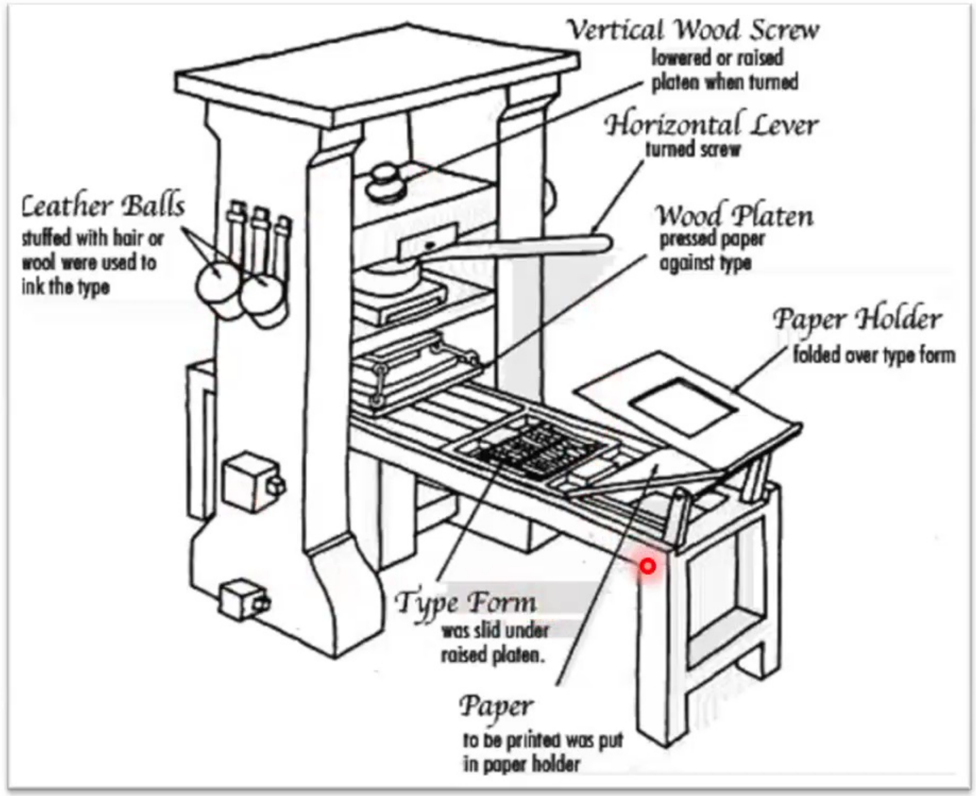
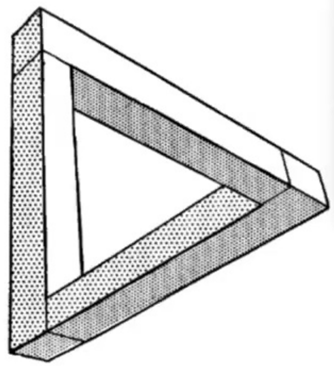
- ✓ PRE 102, 012145

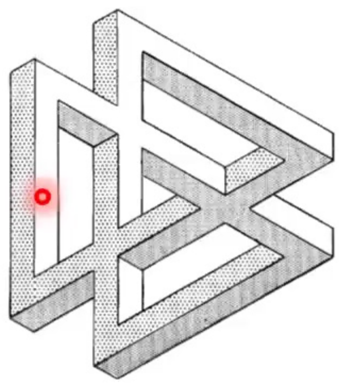
Motivation:

Gutenberg printing press
[1436-1440]



Impossible Composition

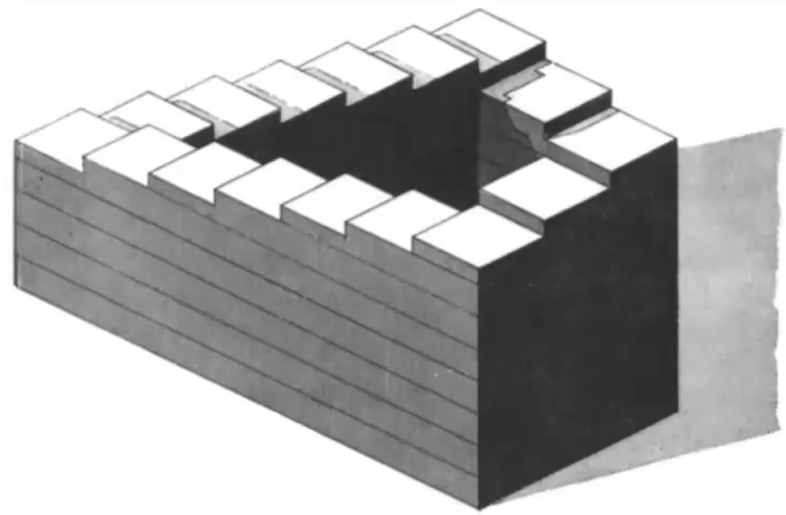




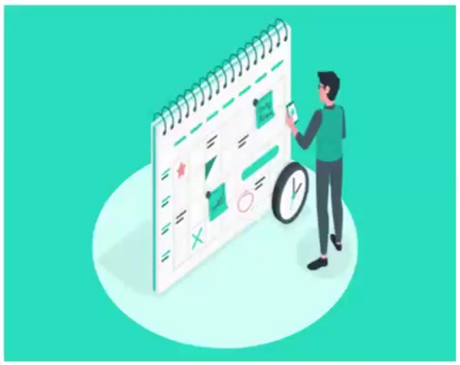
IMPOSSIBLE OBJECTS: A SPECIAL TYPE OF VISUAL ILLUSION

BY L. S. PENROSE AND R. PENROSE

(*University College, London, and Bedford College, London*)



Plan of the talk:



- ✓ **Operational theory framework (a.k.a. GPT)**
- ✓ **Space-like and time-like paradigms**
- ✓ **Pairwise distinguishability game**

  Results

  Phys. Rev. Lett. 128, 140401 (2022)
- ✓ **Information Causality**

  Results

  Phys. Rev. Lett. 130, 110202 (2023)
- ✓ **Conclusion**

Operational theory framework:

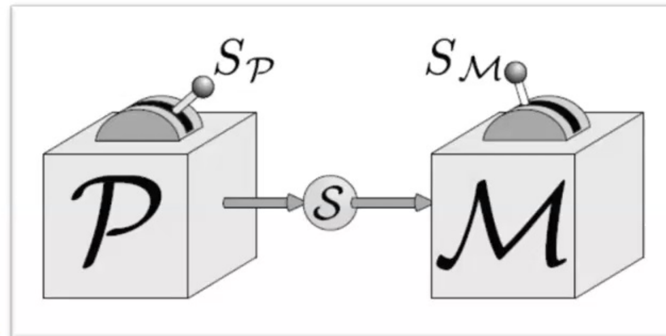
(a.k.a. generalized probability theory)

$$\mathcal{S} \equiv (\Omega, E, T)$$

Collection of Preparations (States)

Collection of Effects (Observables)

Collection of Transformations (Dynamics)



Prepare & Measure setup

- (Pic.) N. Harrigan and T. Rudolph; arXiv:0709.4266

Operational theory framework:

(a.k.a. generalized probability theory)

$\Omega \subset V_+$ **=>a convex set embedded in some real vector space**

(un-normalized states form a convex cone)

$E \subset V_+^*$ **=>dual cone**

$e \in E \; s.t. \; e: \Omega \to [0,1]$

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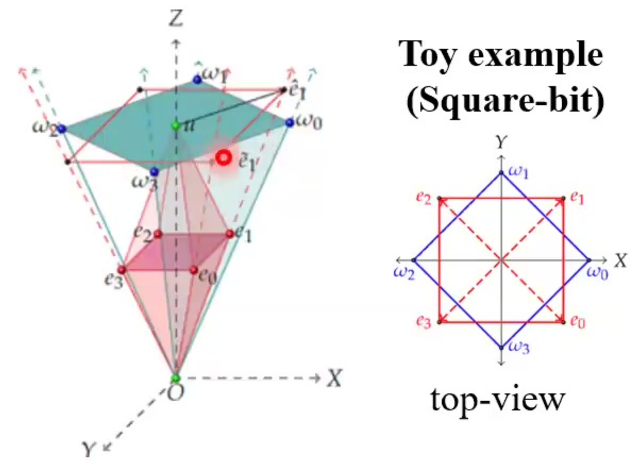


- Bhattacharya et al. Phys. Rev. Research **2**, 012068(R) (2020)
- Banik et al. Phys. Rev. A **92**, 030103(R) (2015)
- Martin Plávala Physics Report **1**, 1033 (2023)

Elementary quantum:

System

Hilbert space $\mathcal{H}$

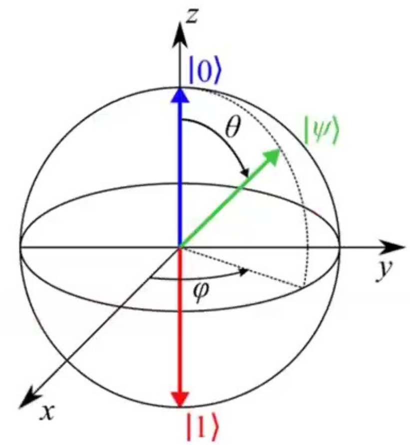
State Cone

$\mathcal{T}_+(\mathcal{H}) \subset \mathcal{T}(\mathcal{H})$

$\mathcal{T}(\mathcal{H})$: set of all hermitian operators

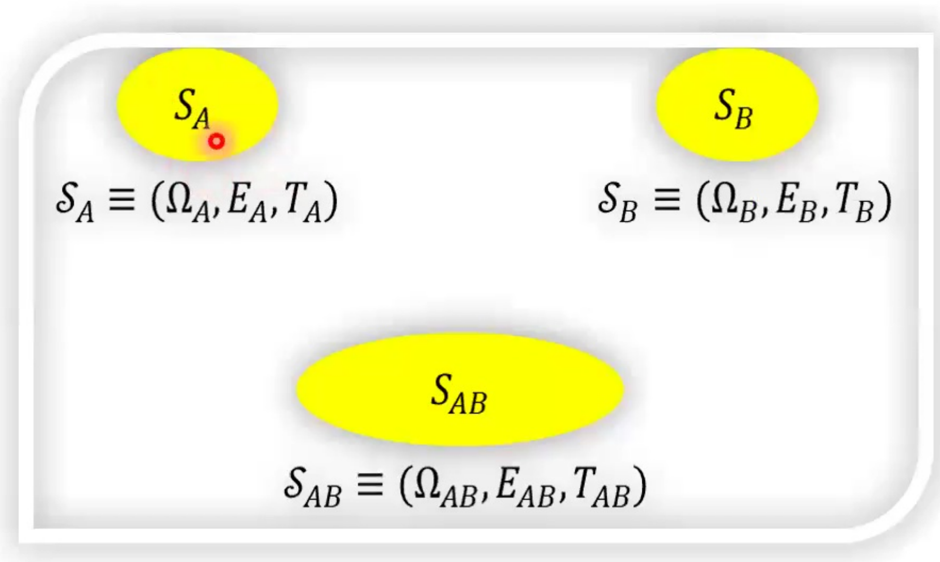
Measurement

$M \equiv \{\pi_i \mid \pi_i \in \mathcal{T}_+(\mathcal{H}),\ \sum_i \pi_i = \mathbf{I}_{\mathcal{H}}\}$



Normalized states of a qubit

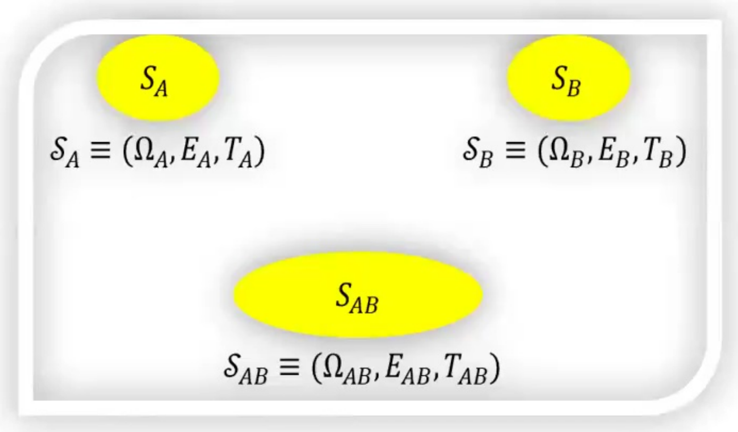
Composite systems in GPT:



Natural Conditions:

- $\forall\, \omega_{AB} \in \Omega_{AB}$ & $e_{AB} \in E_{AB}$ $e_{AB}(\omega_{AB}) \geq 0$
  Consistency condition
- *Joint Probabilities* $p(ab|xy)$ *must obey*
  No signaling condition
- *Marginal Statistics specifies the state uniquely*
  (Local Tomography)

Composite systems in GPT:



Maximal Tensor Product $\boldsymbol{\Omega_A \otimes_{\{max\}} \Omega_B}$

$$e_A \otimes e_B(\omega_{AB}) \geq 0 \; where \; e_A \in E_A, e_B \in E_B$$

Minimal Tensor Product $\boldsymbol{\Omega_A \otimes_{\{min\}} \Omega_B}$

$$e_{AB}(\omega_A \otimes \omega_B) \geq 0 \; where \; \omega_A \in \Omega_A, \omega_B \in \Omega_B$$

- Namioka and R. R. Phelps, Pac. J. Math. 31, 469 (1969)
- G. P. Barker, Linear Multilinear Algebra 4, 191 (1976)
- G. P. Barker, Linear Algebra Appl. 39, 263, (1981)
- Aubrun et al. Geom. Funct. Anal. 31, 181 (2021)

Compositions of two quanta:

SEP: minimal composition of state cone

$\overline{\text{SEP}}$: maximal composition of state cone

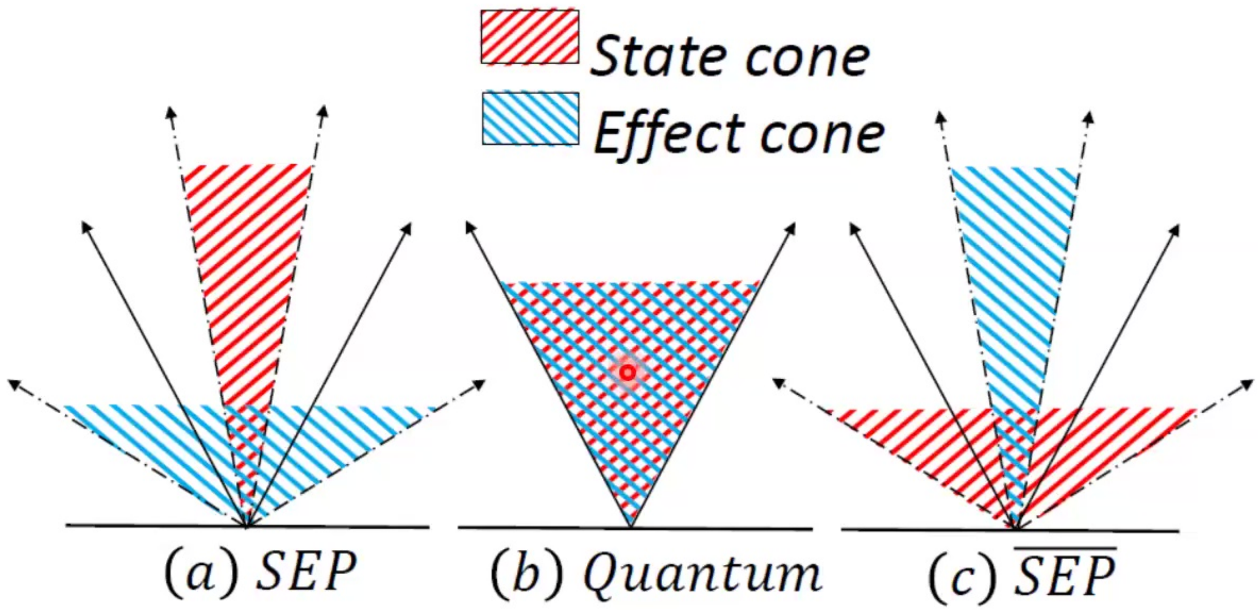
State Cone

$$\left(V_{AB}^{SEP}\right)_+ := \left\{ \sum_i \pi_i^A \otimes \pi_i^B \mid \forall\, i,\ \pi_i^A \in \mathcal{T}_+(\mathcal{H}_A)\ \&\ \pi_i^B \in \mathcal{T}_+(\mathcal{H}_B) \right\}$$

Effect Cone

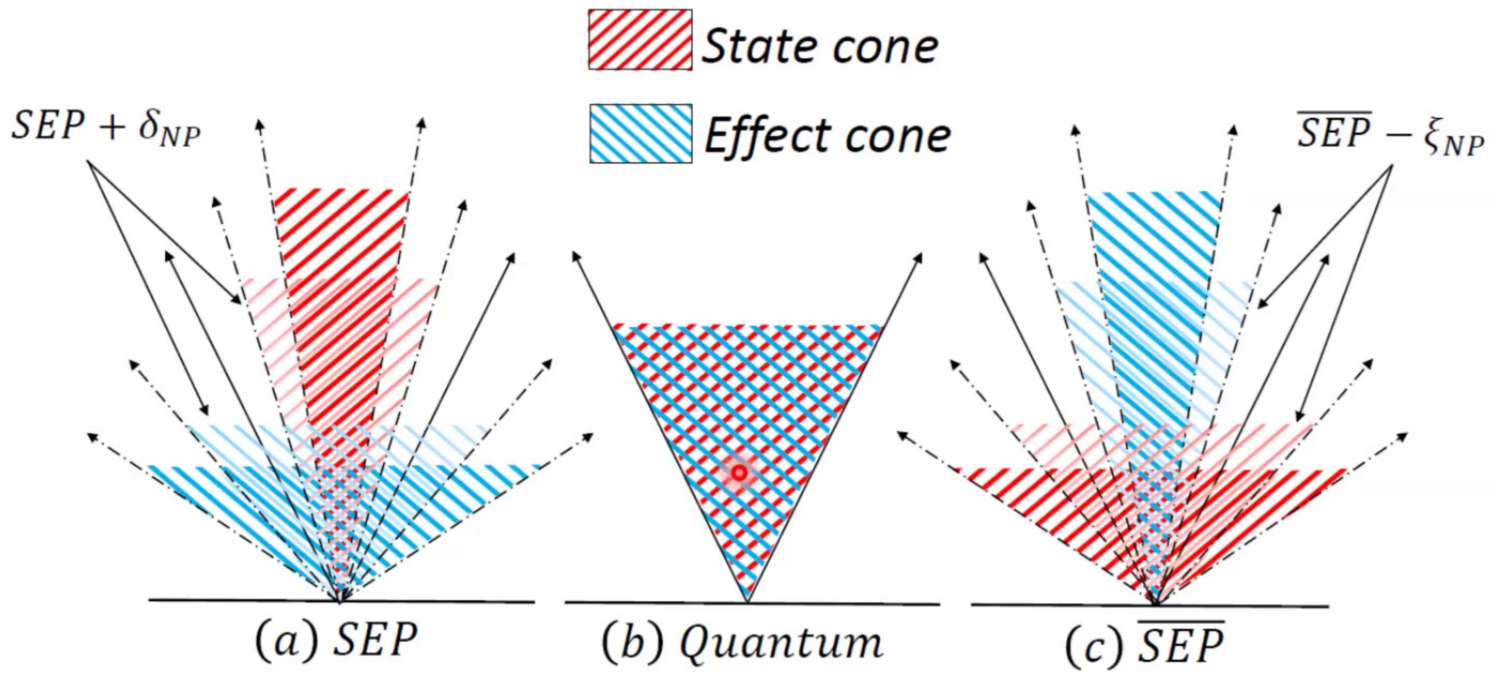
$$\left(V_{AB}^{SEP}\right)_+^\star := \left\{ Y \in \mathcal{T}(\mathcal{H}_A \otimes \mathcal{H}_B) \mid \mathrm{Tr}(XY) \geq 0\ \forall\, X \in \left(V_{AB}^{SEP}\right)_+ \right\}$$

Compositions of two quanta:

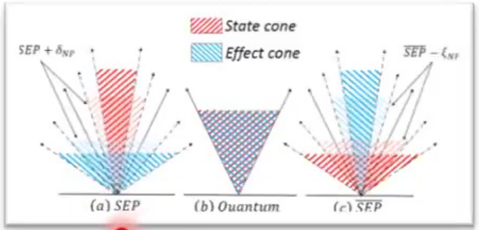


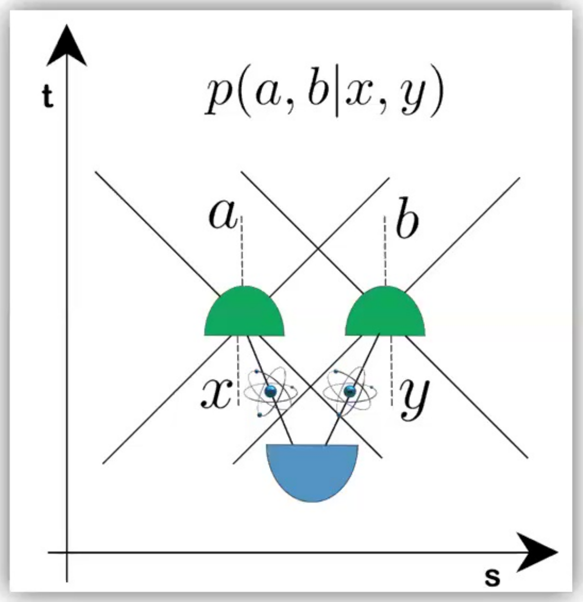
Compositions of two quanta:

Operational Distinction??



Spacelike Correlations:





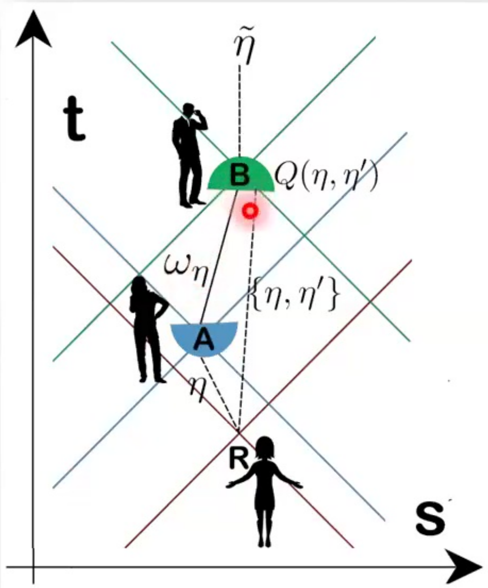
Space-like scenario

- Any bipartite space-like correlation obtained in any of these compositions is also achievable in Q.
- Two quantum systems satisfying the no signaling principle cannot have a beyond quantum space-like separated correlation within its description.
- Experiments involving such space-like correlations are no good to make distinction(s) among different models.

Barnum et al. Phys. Rev. Lett. **104**, 140401 (2010)

Pairwise distinguishability game:

$\mathcal{P}_D^{[n]}$



- In each run, the Referee provides a randomly chosen message $\eta \in \mathcal{X}$ to Alice and accordingly asks Bob the question $\mathrm{Q}(\eta, \eta')$.

- Alice and Bob do not share any correlation, but Alice is allowed to communicate Bob as she is in the past light cone of Bob.

- Alice applies an encoding map $\mathcal{E} : \mathcal{X} \mapsto \{\omega_\eta\}_{\eta \in \mathcal{X}} \subset \Omega$, and sends the encoded state to Bob.

Few definitions:

Definition 1 (Distinguishability). *A set of states* $\{\omega_i\} \subset \Omega$ *are called perfectly distinguishable if there exists a measurement* $M \equiv \{e_i \mid \sum_i e_i = u\}$ *such that* $e_i(\omega_j) = \delta_{ij}$.

Definition 2 (Operational dimension). *Given a system* $\mathcal{S} \equiv (\Omega, E)$, *the maximal cardinality of the set of states that can be perfectly distinguished (by a single measurement) is known as the operational dimension of the system.*

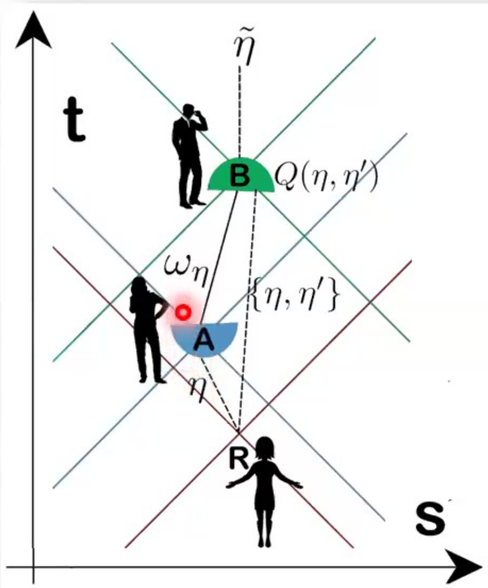
Definition 3 (Information dimension). *Given a system* $\mathcal{S} \equiv (\Omega, E)$,, *the maximal cardinality of the set of states that can be perfectly distinguished pairwise in known as the information dimension of the system.*

Results:

Proposition 1. *Perfect winning of the game* $\mathcal{P}_{\mathrm{D}}^{[n]}$ *requires Alice to encode her message* $\eta \in \mathcal{N}$ *on a set of states* $\{\omega_\eta\}_{\eta\in\mathcal{N}} \subset \Omega$ *of some system* $\mathcal{S} \equiv (\Omega, E)$ *such that the states within the set* $\{\omega_\eta\}_{\eta\in\mathcal{N}}$ *are pairwise distinguishable.*

Pairwise distinguishability game:

$\mathcal{P}_D^{[n]}$



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Theorem 1. *Four qubits communication from Alice to Bob is required for winning the game $\mathcal{P}_D^{[12]}$ when quantum composition is considered among the elementary systems, whereas two SEP-bits (i.e. two qubits in SEP composition) suffice for winning this game.*

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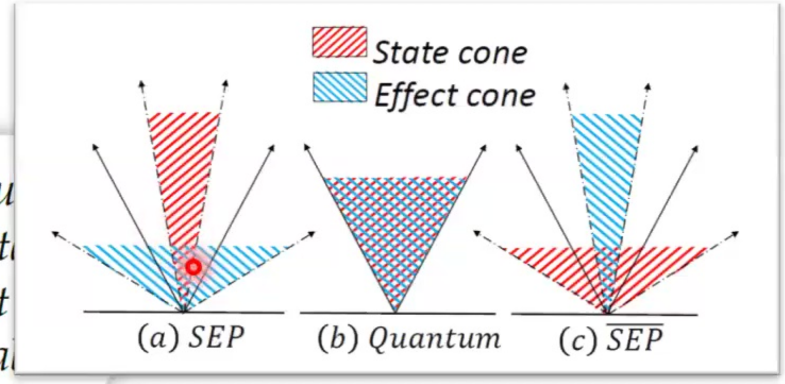

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SEP composition allows stronger than quantum correlation in time-like scenario

Results:

Lemma 1. *The game $\mathcal{P}_D^{[n]}$ cannot be won pe*

PHYSICAL REVIEW LETTERS **128,** 140401 (2022)

Composition of Multipartite Quantum Systems: Perspective from Timelike Paradigm

Sahil Gopalkrishna Naik,[1] Edwin Peter Lobo,[1] Samrat Sen,[1] Ram Krishna Patra,[1] Mir Alimuddin,[1] Tamal Guha,[2] Some Sankar Bhattacharya,[3] and Manik Banik[1]

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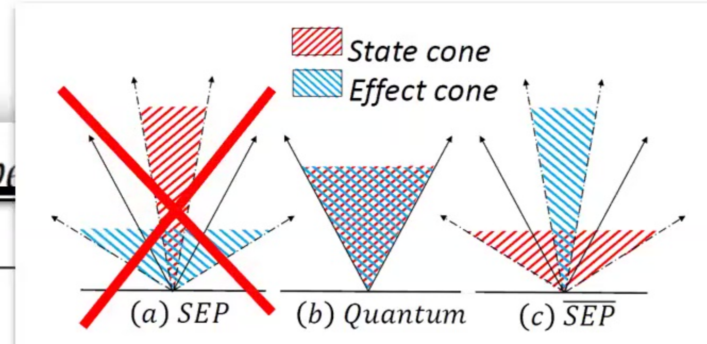
(Received 21 July 2021; revised 18 January 2022; accepted 11 March 2022; published 8 April 2022)

Figuring out the physical rationale behind natural selection of quantum theory is one of the most acclaimed quests in quantum foundational research. This pursuit has inspired several axiomatic initiatives to derive a mathematical formulation of the theory by identifying the general structure of state and effect



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pon

with k qubits communication from Alice to Bob if the composition rule is quantum.

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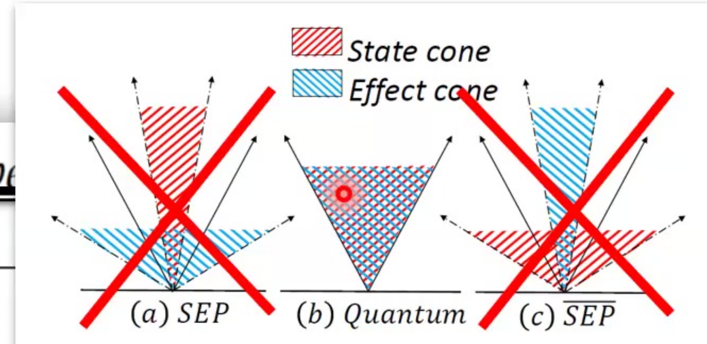
(Received 21 July 2021; revised

Figuring out the physical rational
acclaimed quests in quantum foundat
to derive a mathematical formulation



PHYSICAL REVIEW A **106**, 062406 (2022)

Timelike correlations and quantum tensor product structure

Samrat Sen,[1] Edwin Peter Lobo,[2] Ram Krishna Patra,[1] Sahil Gopalkrishna Naik,[1] Anandamay Das Bhowmik,[3] Mir Alimuddin,[1] and Manik Banik[1]

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(Received 12 August 2022; accepted 16 November 2022; published 7 December 2022)

Results:

Lemma 1. *The game* $\mathcal{P}_D^{[n]}$ *cannot be won perfectly by communicating the encoded states chosen from the composite system* $\mathbb{C}^2 \otimes_{\min} \mathbb{C}^2$ *whenever* $n > 12$.

Theorem 2. $2k$ *number of SEP-bits are sufficient for winning the game* $\mathcal{P}_D^{[12^k]}$ *perfectly, whereas it requires* $2k + \lceil k \log_2 3 \rceil$ *number of qubits, with* $k \in \mathbb{Z}_+$.

Proposition 2. *For* $n > 2^k$, *the game* $\mathcal{P}_D^{[n]}$ *cannot be won with* k *qubits communication from Alice to Bob if the composition rule is quantum.*

Information Causality:

Vol 461 | 22 October 2009 | doi:10.1038/nature08400 nature

LETTERS

Information causality as a physical principle

Marcin Pawłowski[1], Tomasz Paterek[2], Dagomir Kaszlikowski[2], Valerio Scarani[2], Andreas Winter[2,3]
& Marek Żukowski[1]

- IC can be envisaged as a generalization of the no-signaling condition.
- It limits the information gain that a receiver (say Bob) can reach about a previously unknown to him data set of a sender (say Alice), by using two types of resources:
  - (i) all his local resources that might be correlated with the sender (**Type-I resource**)
  - (ii) some physical system carrying bounded amount of information from Alice to Bob (**Type-II resource**)

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- Both these resources can further be of different kinds – **classical**, **quantum**, and **beyond quantum**.

Information Causality:

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  - (i) all his local resources that might be correlated with the sender (**Type-I resource**)
  - (ii) some physical system carrying bounded amount of information from Alice to Bob (**Type-II resource**)
- Both these resources can further be of different kinds – **classical**, **quantum**, and **beyond quantum**.
- IC principle provides a way to test their physicality.

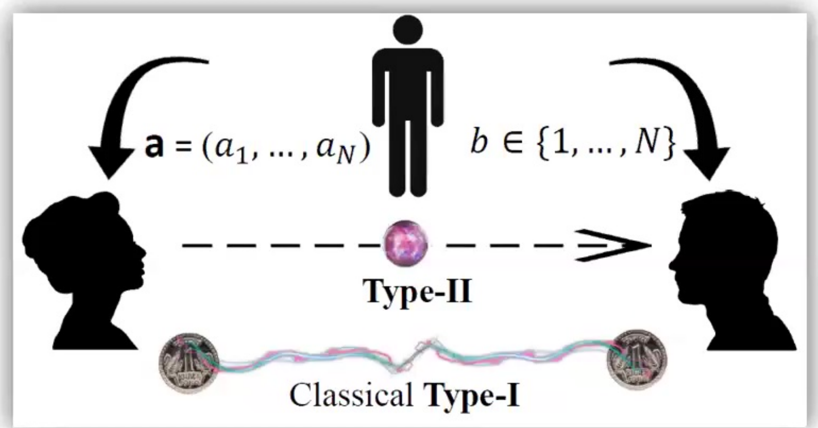
Information Causality:

- Although the correlated resources (**Type-I resource**) by themselves have no communication utility, as shown in the seminal superdense coding paper a quantum correlation viz. entanglement can double up the communication capacity of a quantum channel
- The power of entanglement, however, is limited in a way as it **cannot** enhance communication capacity of a classical channel
- Principle of IC generalizes this **no-go** by limiting Bob's information gain to be at most m bits when m classical bits are communicated by Alice to him, and he is allowed to use any of his local resources that might be correlated with Alice.
- Several NS correlations violate this principle and thus considered as unphysical.

In essence, restricting the **Type-II resources** to be classical, the IC principle discards some of the **Type-I resources** as **unphysical**.

We study the reverse scenario, i.e., **Type-II resource** is varied among several possibilities while restricting the **Type-I resource** to be classical. More particularly, we ask the question whether **unphysicality** of some **Type-II resources** can be established through information principles.

Information Causality:



IC-N game

- Alice receives a string of N independent bits, **a = (a1, · · · , aN)** randomly sampled from **{0, 1}^N**
- Bob receives a random value of **b ∈ {1, · · · , N }.**
- Bob's aim is to correctly guess the value of the **b^th** bit of Alice
- Alice and Bob may possess some classical shared randomness . The efficiency of the collaborative strategy is quantified through

$$I_N = \sum_{k=1}^{N} I\left(a_k : \beta \mid b = k\right)$$

Information Causality:

$$I_N \leq \Theta$$

Holevo capacity of a system captures the optimal rate of transfer of classical information when asymptotically many copies of the system are transferred.

Results:

Proposition 1. *The information capacity of the minimal composition of k local quantum systems, each of dimension d, is* d^k, *i.e.*, $\mathcal{I}\left((\mathbb{C}^d)^{\otimes_{\min} k}\right) = d^k$.

Proposition 2. *The information capacity of the maximal composition of k local quantum systems, each of dimension d, is* d^k, *i.e.*, $\mathcal{I}\left((\mathbb{C}^d)^{\otimes_{\max} k}\right) = d^k$.

Results:

Theorem 1. *Minimal tensor product composition of two elementary quantum violates IC principle.*

Input string	Alice's encoding	Bob's decoding $1^{st} \to \mathcal{M}_1$	$2^{nd} \to \mathcal{M}_2$	$3^{rd} \to \mathcal{M}_3$
000	$\lvert+\rangle\lvert+\rangle$	0	0	random
001	$\lvert-\rangle\lvert-\rangle$	0	0	random
010	$\lvert0\rangle\lvert0\rangle$	0	1	0
011	$\lvert1\rangle\lvert1\rangle$	0	1	1
100	$\lvert1\rangle\lvert0\rangle$	1	0	0
101	$\lvert0\rangle\lvert1\rangle$	1	0	1
110	$\lvert+\rangle\lvert-\rangle$	1	1	random
111	$\lvert-\rangle\lvert+\rangle$	1	1	random
$I(a_k : \beta \mid b = k)$		1	1	≈ 0.19
I_3		$:= \sum_{k=1}^{3} I(a_k : \beta \mid b = k) \approx 2.19 > 2$		

$$E_1 := \begin{pmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 1 \end{pmatrix}, \quad E_2 := \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix},$$

$$E_3 := \mathbf{1}_2 \otimes \frac{1}{2}(\mathbf{1}_2 + \sigma_z),$$

Results:

Theorem 2. *Maximal tensor product composition of two elementary quantum violates IC principle.*

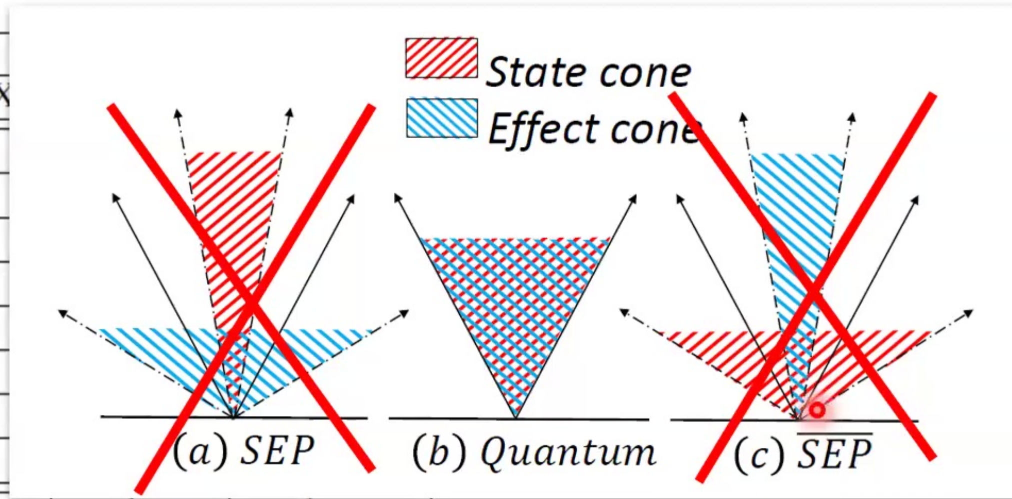
Input string	Alice's encoding	Bob's decoding		
		$1^{st} \to XX$	$2^{nd} \to YY$	$3^{rd} \to ZZ$
000	$\Gamma(\phi^+)$	0	0	0
001	ψ^+	0	0	1
010	ϕ^+	0	1	0
011	$\Gamma(\psi^+)$	0	1	1
100	ϕ^-	1	0	0
101	$\Gamma(\psi^-)$	1	0	1
110	$\Gamma(\phi^-)$	1	1	0
111	ψ^-	1	1	1
$I(a_k : \beta \mid b = k)$		1	1	1
I_3		$:= \sum_{k=1}^{3} I(a_k : \beta \mid b = k) = 3 > 2$		

$$\Gamma(\phi^+) = (\mathbf{1} \otimes \mathbf{1} + \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z)/4,$$
$$\psi^+ = (\mathbf{1} \otimes \mathbf{1} + \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y - \sigma_z \otimes \sigma_z)/4,$$
$$\phi^+ = (\mathbf{1} \otimes \mathbf{1} + \sigma_x \otimes \sigma_x - \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z)/4,$$
$$\Gamma(\psi^+) = (\mathbf{1} \otimes \mathbf{1} + \sigma_x \otimes \sigma_x - \sigma_y \otimes \sigma_y - \sigma_z \otimes \sigma_z)/4,$$
$$\phi^- = (\mathbf{1} \otimes \mathbf{1} - \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z)/4,$$
$$\Gamma(\psi^-) = (\mathbf{1} \otimes \mathbf{1} - \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y - \sigma_z \otimes \sigma_z)/4,$$
$$\Gamma(\phi^-) = (\mathbf{1} \otimes \mathbf{1} - \sigma_x \otimes \sigma_x - \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z)/4,$$
$$\psi^- = (\mathbf{1} \otimes \mathbf{1} - \sigma_x \otimes \sigma_x - \sigma_y \otimes \sigma_y - \sigma_z \otimes \sigma_z)/4,$$

Results:

Theorem 2. *Maximal tensor product composition of two elementary quantum violates IC principle.*

Input string	Alice's encoding	$1^{st} \to X$		
000	$\Gamma(\phi^+)$	0		
001	ψ^+	0		
010	ϕ^+	0		
011	$\Gamma(\psi^+)$	0		
100	ϕ^-	1		
101	$\Gamma(\psi^-)$	1		
110	$\Gamma(\phi^-)$	1		
111	ψ^-	1		
$I(a_k : \beta \mid b = k)$		1	1	1
I_3		$:= \sum_{k=1}^{3} I(a_k : \beta \mid b = k) = 3 > 2$		



$+ \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z)/4,$

$+ \sigma_y \otimes \sigma_y - \sigma_z \otimes \sigma_z)/4,$

$- \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z)/4,$

$- \sigma_y \otimes \sigma_y - \sigma_z \otimes \sigma_z)/4,$

$+ \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z)/4,$

$+ \sigma_y \otimes \sigma_y - \sigma_z \otimes \sigma_z)/4,$

$- \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z)/4,$

$\psi^- = (\mathbf{1} \otimes \mathbf{1} - \sigma_x \otimes \sigma_x - \sigma_y \otimes \sigma_y - \sigma_z \otimes \sigma_z)/4,$

Conclusions:

- The idea of composition becomes important while constructing theories in Physics. Using time-like correlations we show that some compositions for multipartite quantum systems are unphysical.

- We have also shown that principle of Information Causality plays crucial role in isolating the quantum composition. In the process Information Causality can discard a theory that admits only Bell local correlations. This might make Information Causality champion over the other principles.

- Our works thus bring some physical justifying towards self-dual structure for multipartite quantum systems from information theoretic principles.

Santhal Family
Ramkinkar Baij (1938)

" A creative artist works on his (her) next composition because he (she) was not satisfied with his (her) previous one"

--Dmitri Shostakovich
(a Soviet-era Russian composer and pianist)

Thank You