Title: What can we learn from elastic scattering of Cosmological Gravitational Wave Background? - VIRTUAL

Speakers: Morgane KÃ¶nig

Series: Cosmology & Gravitation

Date: January 16, 2024 - 11:00 AM

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Abstract: The current progress in gravitational wave detection opened a new exciting window in cosmology. It is natural to ask ourselves how we can best use this new tool to explore physics beyond the standard model. With this idea in mind, my collaborators and I asked what we could learn from Cosmological Gravitational Wave Backgrounds if they were to be detected to a certain accuracy. By drawing comparison to the cosmic microwave background, we investigate the impact of elastic scattering on any cosmological background. Specifically, we focus on quantifying spectral distortions in the energy density spectrum of CGWB attributed to interaction with beyond-the-standard-model particles. We will also explore the effect that elastic scattering of graviton of Primordial Black holes would have on CGWB in the regime where PBHs account for all the dark matter.

Zoom link https://pitp.zoom.us/j/98460268383?pwd=RytzWHd5dU1lenRhWG1NYXM3OVJpQT09

PERIMETER INSTITUTE

What can we learn from Cosmological Gravitational Wave Backgrounds?

arXiv:2309.15925v1 [gr-qc], arXiv:2308.00111v1 [gr-qc]
JCAP_12_041 JCAP_125P_0923

Morgane König

Jan 16th 2024

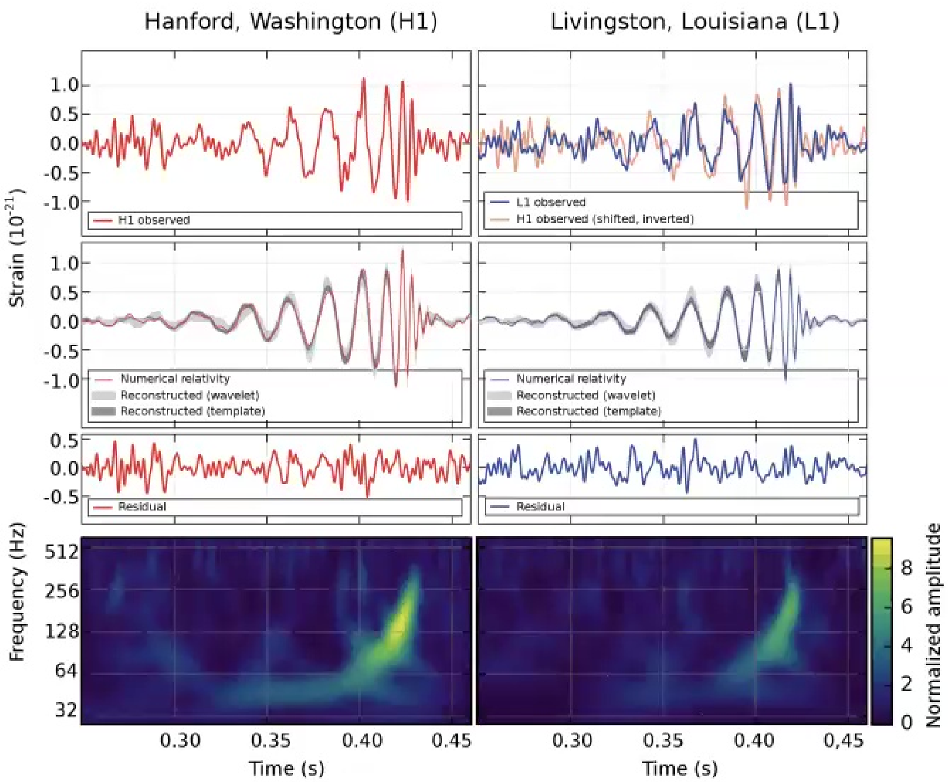
An SZ-Like Effect on Cosmological Gravitational Wave Backgrounds

Tatsuya Daniel,[a] Marcell Howard,[b,1] and Morgane König[c]

Elastic Scattering of Cosmological Gravitational Wave Backgrounds: Primordial Black Holes and Stellar Objects

Marcell Howard,[a,1] and Morgane König[b]

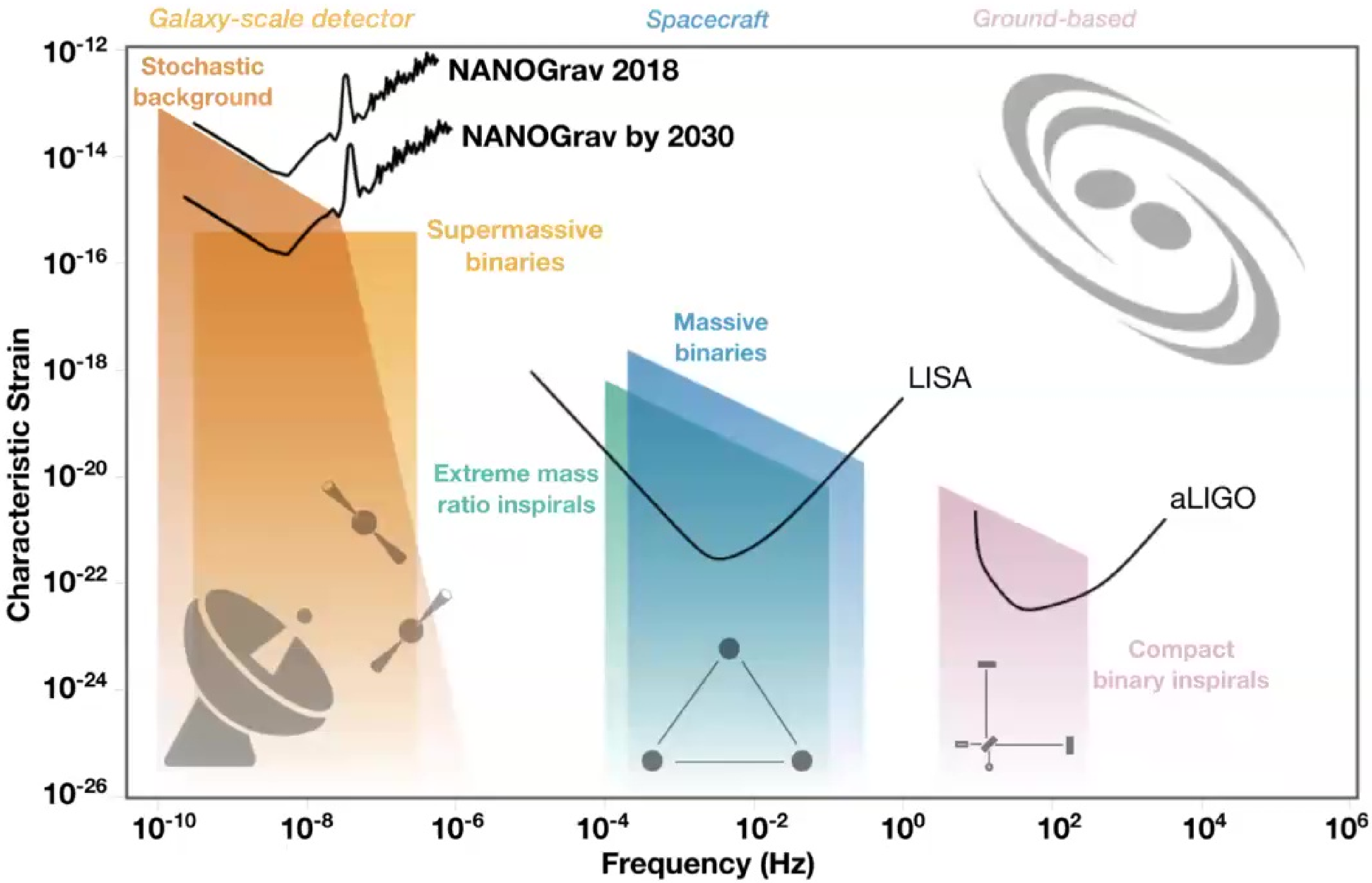
Gravitational Wave Detectors



Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) 2015

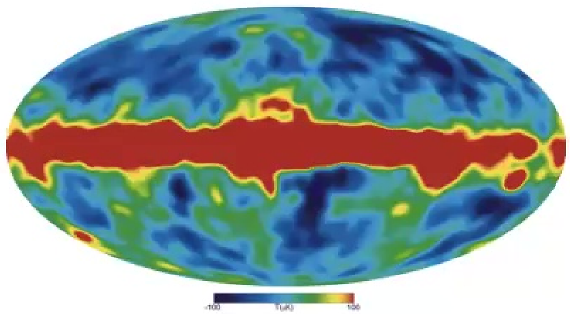
LIGO Hanford

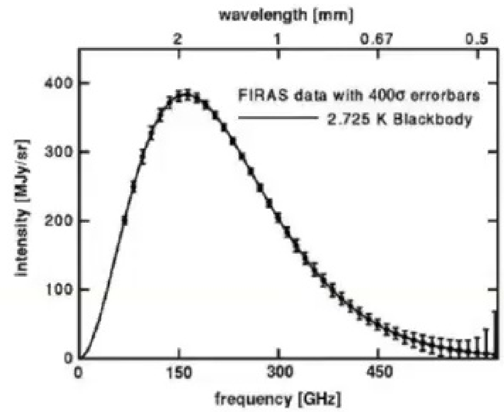
State of Gravitational Wave Detectors

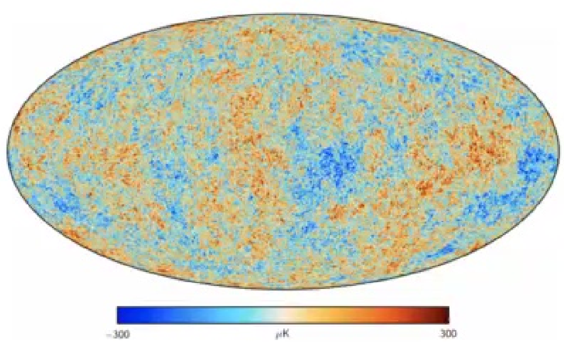


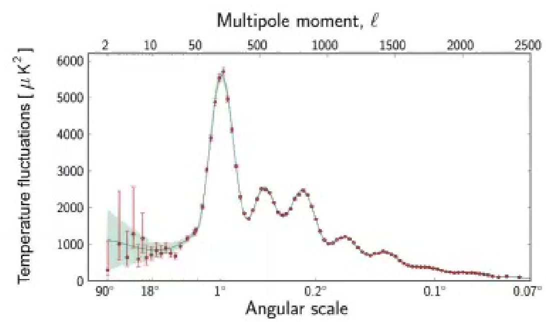
A Long Way to go

- 1965: Penzias and Wilson
- 1992: Cobe
- 2013 Planck









Stochastic Gravitational Wave Backgrounds

(Emerges from the incoherent superposition of a large number of astrophysical or cosmological sources)

- Stationarity : $\left\langle \tilde{h}_A^*(\nu)\tilde{h}_B(\nu')\right\rangle \propto \delta(\nu-\nu')$
- Gaussianity : All n-point correlators are or can be reduced to sums and products of the 2-point correlation function.
- Isotropy : $\left\langle \tilde{h}_A^*(\nu,\hat{\mathbf{n}})\tilde{h}_B(\nu',\hat{\mathbf{n}}')\right\rangle \propto \delta^2(\hat{\mathbf{n}},\hat{\mathbf{n}}')$
- Polarization : $\left\langle \tilde{h}_A^*(\nu,\hat{\mathbf{n}})\tilde{h}_B(\nu',\hat{\mathbf{n}}')\right\rangle \propto \delta_{AB}$

$$\left\langle \tilde{h}_A^*(\nu,\hat{\mathbf{n}})\tilde{h}_B(\nu',\hat{\mathbf{n}}')\right\rangle = \delta(\nu-\nu')\frac{\delta^2(\hat{\mathbf{n}},\hat{\mathbf{n}}')}{4\pi}\delta_{AB}\frac{S_h(\nu)}{2}$$

- Spectral density for the fractional density :

$$\Omega_{\rm GW}(\nu) = \frac{1}{\rho_c}\frac{{\rm d}\rho_{\rm GW}}{{\rm d}\log\nu} = \frac{(2\pi)^2}{3H_0^2}\nu^3 S_h(\nu)$$

Chiara Caprini and Daniel G Figueroa 2018 *Class. Quantum Grav.* **35** 163001

Stochastic Gravitational Wave Backgrounds

- Linearized perturbations around the conformal FLRW spacetime in the synchronous gauge

$$\mathrm{d}s^2 = a^2(\eta)[-\,\mathrm{d}\eta^2 + (\delta_{ij} + h_{ij}^{TT})\,\mathrm{d}r^i\,\mathrm{d}r^j]$$

- Equation of motion for the metric perturbation

$$h_{ij}^{\prime\prime TT}(\mathbf{r},\eta) + 2\mathcal{H}h_{ij}^{\prime TT}(\mathbf{r},\eta) - \nabla^2 h_{ij}^{TT}(\mathbf{r},\eta) = 0$$

- In Fourier space

$$\tilde{h}_A'' + 2\mathcal{H}\tilde{h}_A' + k^2\tilde{h}_A = 0$$

- Primordial spectrum

$$\mathcal{P}(k) = A_T(k_*)\left(\frac{k}{k_*}\right)^{n_T}$$

Stochastic Gravitational Wave Backgrounds

Connection with the primordial background

- Observables of interest

$$\Omega_{\rm GW}(\nu) = \frac{1}{\rho_c}\frac{{\rm d}\rho_{\rm GW}}{{\rm d}\log\nu} = \frac{(2\pi)^2}{3H_0^2}\nu^3 S_h(\nu)$$

- Primordial vs present day

$$\Omega_{\rm GW}(\nu) = |\mathcal{T}_{\rm GW}(\nu)|^2\Omega^P_{\rm GW}(\nu, z)$$

$$\Omega_{\rm GW}(\nu) = \frac{\pi^2}{3H_0^2}\nu^2|\mathcal{T}_{\rm GW}(\nu)|^2\mathcal{P}(\nu)$$

$$\Omega_{\rm GW}(\nu) = \frac{8\pi G}{3}\frac{(4\pi)^2}{H_0^2}\nu^4 n(\nu)$$

Production Mechanisms

Cosmological sources

- Inflation
- Phase transition
- Topological defects
- Cosmic strings

Caldwell, R., Cui, Y., Guo, HK. *et al. Gen Relativ Gravit* **54**, 156 (2022)

Kosowsky, Turner, and Watkins Phys. Rev. Lett. **69**, 2026 – 1992

Jones-Smith, Krauss, and Mathur Phys. Rev. Lett. **100**, 131302 – 2008

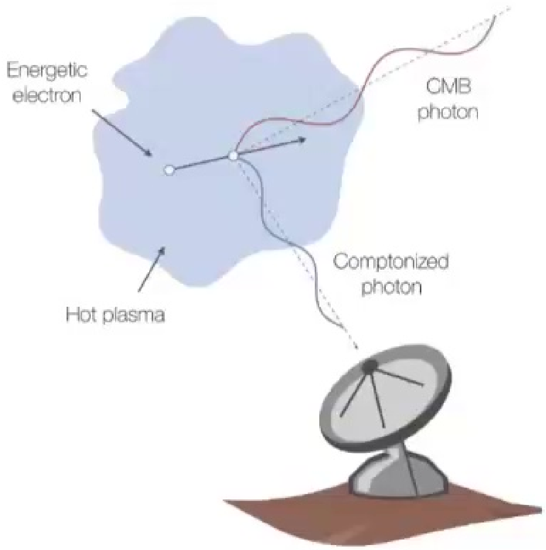
Astrophysical Contamination

- Binary Black holes
- Binary Neutron stars
- Super massive Black holes in galactic nuclei
- Binary stars in the galaxy

B. P. Abbott *et al. (LIGO and Virgo)* Phys. Rev. Lett. **116**, 061102 –2016

B. P. Abbott *et al.* (LIGO and Virgo) Phys. Rev. Lett.**119**, 161101 – 2017

Sunyaev Zeldovich Effect

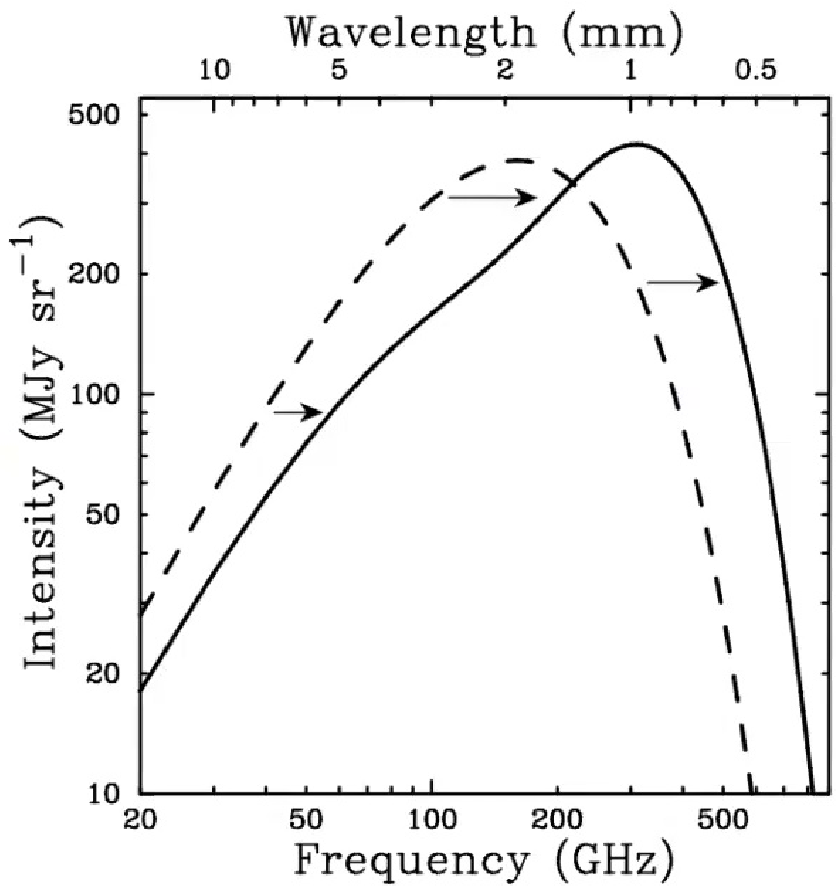


Mroczkowski et al. 2019

- Distortion on the CMB by solving the Kompaneets equations

$$\frac{\Delta n}{n_0} = \frac{\Delta J}{J_0} = xy\,\frac{e^x}{e^x - 1}\left\{\frac{x}{\tanh(x/2)} - 4\right\}$$

R.A. Sunyaev and Y.B. Zeldovich,
Distortions of the Background Radiation Spectrum,
Nature, 223 (1969) 721



Calstrom, Holder, Reese Annu. Rev. Astronomy & Astrophysics 2002 40

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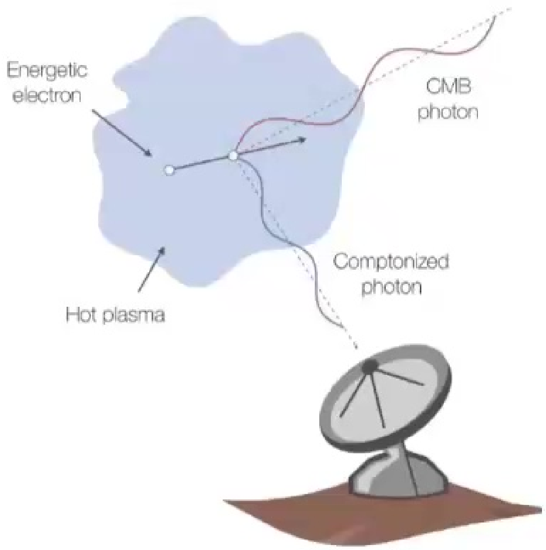
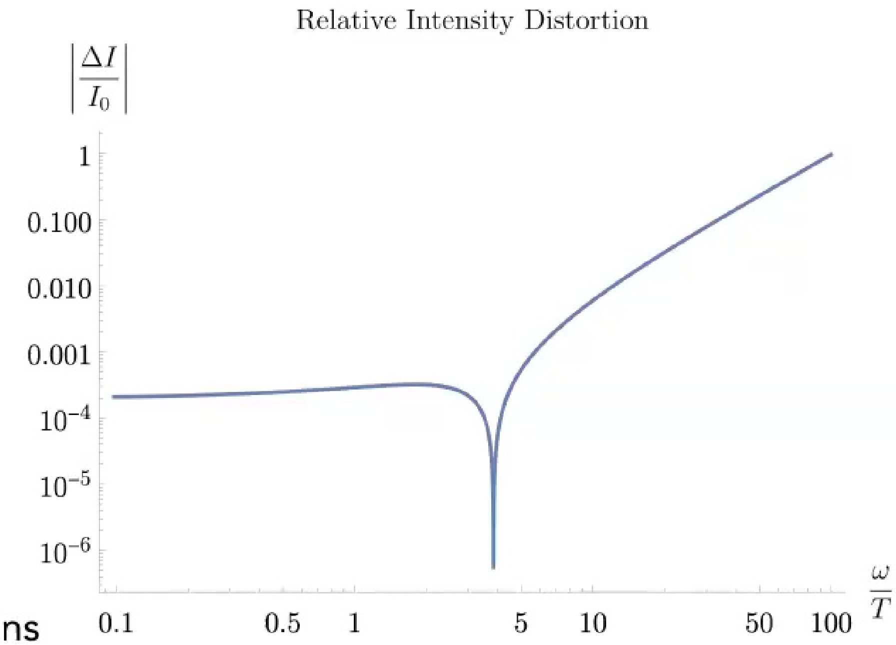


Mroczkowski et al. 2019



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R.A. Sunyaev and Y.B. Zeldovich,
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Nature, 223 (1969) 721

Kompaneets Equations

- The relaxation of a photon bath to thermal equilibrium via Compton scattering with electrons

$$\frac{\partial n}{\partial t} = \frac{k_B T_e}{m_e c^2} \frac{n_e \sigma_e}{c} (x_e)^4 \left(\frac{\partial n}{\partial x_e} + n + n^2 \right)$$

- Stimulated emission

  Affects the photons distribution $n(\mathbf{k}, t)$

- Compton scattering

  Affects the photons+electrons distribution $n(\mathbf{p}, \mathbf{k}; t)$

SOVIET PHYSICS JETP VOLUME 4, NUMBER 5 JUNE, 1957

The Establishment of Thermal Equilibrium between Quanta and Electrons*

A. S. KOMPANEETS

Institute of Chemical Physics, Academy of Sciences, USSR

(Submitted to JETP editor October 15, 1955)

J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 876-885 (November, 1956)

The role of the Compton effect in the establishment of equilibrium between quanta and electrons is considered in the nonrelativistic approximation.

Freire Oliveira, Maes and Meerts, Astroparticle Physics 133 (2021) 102644

Kompaneets Equations

- Expressed as a function of a dimensionless parameter y

In the limit of small x_e

$$\frac{\partial n}{\partial y} = \frac{1}{x_e^2}\frac{\partial}{\partial x_e}x_e^4\left(\frac{\partial n}{\partial x_e} + \cancel{n + n^2}\right) \quad \text{with} \quad y = \frac{k_B T_e}{m_e c^2}\frac{ct}{\lambda_e} = \int n_e \sigma_T \, \mathrm{d}\ell \, \frac{k_B T_e}{m_e c^2}$$

Optical depth

- At low optical depth and low temperature i.e. when y is small

$$\Delta n = xy\frac{e^x}{(e^x-1)^2}(x\coth(x/2) - 4)$$

SZ Effect

Birkinshaw Phys.Rept.310:97-195,1999

Kompaneets Equations

- Master equation for photons and electrons :

$$\frac{\partial n}{\partial t}(\mathbf{p},\mathbf{k};t) = \int \mathrm{d}^3\mathbf{p}'\,\mathrm{d}^3\mathbf{k}'\,\{n(\mathbf{p}',\mathbf{k}';t)w(p',k' \to p,k)(1+n(\mathbf{k},t)) - n(\mathbf{p},\mathbf{k};t)w(p,k \to p',k')(1+n(\mathbf{k}',t))]$$

Non covariant transition rate

- Transition rate :

$$w(p,k \to p',k')\,\mathrm{d}^3\mathbf{p}'\,\mathrm{d}^3\mathbf{k}' = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\mathbf{p},\mathbf{k})\,\mathrm{d}\Omega\,c\left(1 - \frac{\mathbf{v}}{c}\cdot\hat{\mathbf{n}}\right)$$

Differential Compton cross section

Kompaneets Equations

- Electrons are in thermal equilibrium
- Photons are soft i.e $\hbar\omega \ll m_e c^2$ but $\hbar\omega \sim k_B T_e$
- Electrons are non relativistic ie $|\mathbf{p}| \ll m_e c$

Assumptions

- Master equation

$$\frac{\partial n}{\partial t} = [\partial_x n + n(1+n)]I_1 + \left[\frac{1}{2}\partial_{xx} n + (1+n)\left(\frac{n}{2} + \partial_x n\right)\right] I_2$$

with

$$I_\ell(x) = c\int \mathrm{d}^3\mathbf{p}\,\mathrm{d}\Omega\left(1 - \frac{\mathbf{v}}{c}\cdot\hat{\mathbf{n}}\right)\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\mathcal{N}_{eq}(|\mathbf{p}|)\Delta^\ell$$

where $\Delta = (x' - x) = \frac{\hbar(\omega' - \omega)}{k_B T}$

Momentum distribution of electrons

Gravitational SZ Effect

- Cosmic Microwave Background
- Photon Electron Compton scattering
- Solve the Kompaneets equations

- Cosmic Gravitational Wave Backgrounds
- Graviton BSM particle Compton scattering
- Solve the Gravitational Kompaneets equations

Gravitational SZ Effect

- Cosmic Microwave Background
- Photon Electron Compton scattering
- Solve the Kompaneets equations

- Cosmic Gravitational Wave Backgrounds
- Graviton BSM particle Compton scattering
- Solve the Gravitational Kompaneets equations

Much bigger effect for PBHs !!!!!

Gravitational Kompaneets Equations

- BSM particles are in thermal equilibrium
- Gravitons are soft i.e $\hbar\omega \ll m\,c^2$ but $\hbar\omega \sim k_B T$
- BSM particle are non relativistic ie $|\mathbf{p}| \ll m\,c$

S. Boughn and T. Rothman Class.Quant.Grav. 23 (2006) 5839-5852

Assumptions

- Compton scattering
- Stimulated emission

- Master equation

$$\frac{\partial n}{\partial t} = \frac{J_2(x,\lambda;s)}{2}\frac{\partial}{\partial x}\left[\frac{\partial n}{\partial x} + n(1+n)\right] + \left(J_1(x,\lambda;s) + \frac{J_2(x,\lambda;s)}{2}\right)\left[\frac{\partial n}{\partial x} + n(1+n)\right]$$

with

$$J_\ell(x,\lambda;s) = 2\pi\int_{\theta_{\min}(\lambda)}^{\theta_{\max}} \sin\theta\,\mathrm{d}\theta \int \mathrm{d}^3\mathbf{p}\left(1 - \frac{\mathbf{p}}{m}\cdot\hat{\mathbf{n}}\right)\frac{\mathrm{d}\sigma_s(\mathbf{p},x,\theta)}{\mathrm{d}\theta}\mathcal{N}_{\mathrm{eq}}(\mathbf{p})\Delta^\ell(x,\theta)$$

$$\frac{n_\chi}{(2\pi m T)^{3/2}}\exp\left(-\frac{\mathbf{p}^2}{2mT}\right)\mathrm{d}^3\mathbf{p}$$

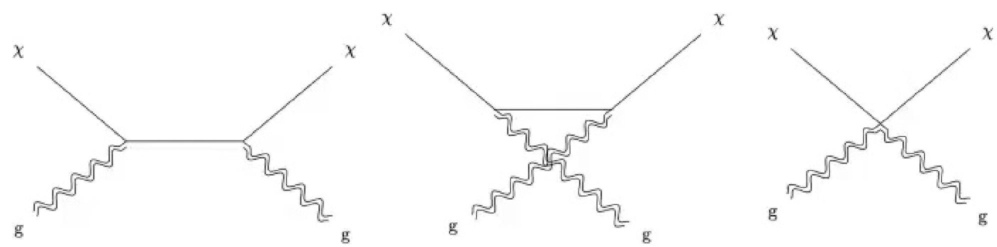
Momentum distribution of BSM particles

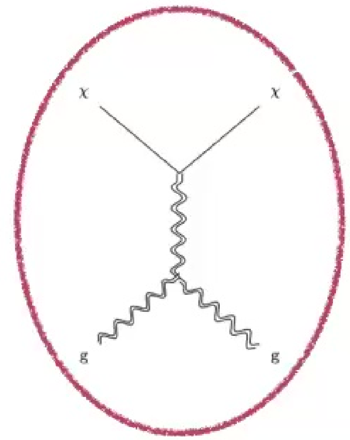
GR as an EFT

- Unique theory of a massless spin 2 particle
- EFT is a valid description for low Energy phenomenon below the Planck scale

GR as an EFT

- Graviton scattering





Forward Scattering limit

$$t \equiv -(k-k')^2 \sim 0$$

Regularization

$$\bar{\Box} h_{\mu\nu} + 2\bar{R}_{\mu\alpha\nu\beta} h^{\alpha\beta} = 0$$

$$h_{\mu\nu}(x) = \left(A_{\mu\nu}(x) + \epsilon B_{\mu\nu}(x) + \ldots\right) e^{i\theta(x)/\epsilon}$$

Geometric optics is valid in the following regime:

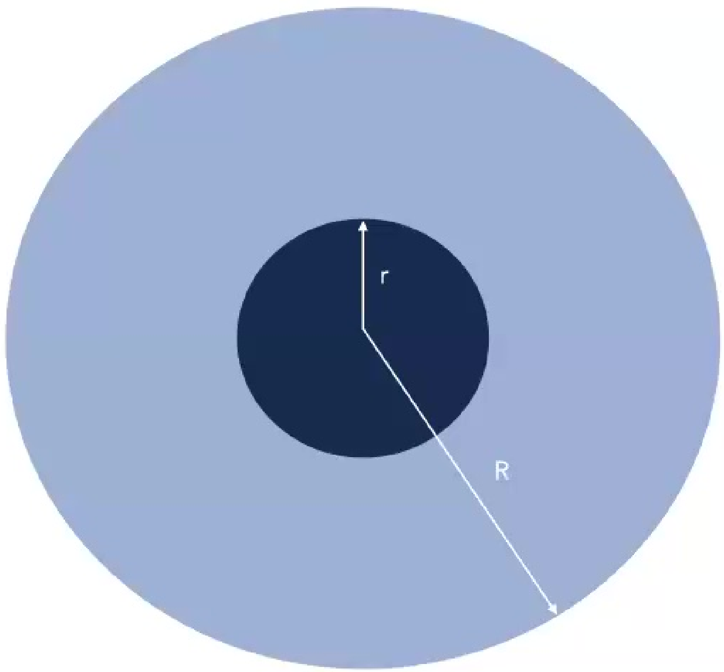
$$\frac{1}{\lambda^2} \gg \sqrt{\bar{R}_{\mu\alpha\nu\beta}\bar{R}^{\mu\alpha\nu\beta}} \equiv \sqrt{K}$$

The vicinity of an object can be described by a Schwarzschild metric

$$\mathrm{d}s^2 = -\left(\frac{1-\frac{Gm}{2r}}{1+\frac{Gm}{2r}}\right)^2 \mathrm{d}t^2 + \left(1+\frac{Gm}{2r}\right)^4 \left[\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2(\theta)\,\mathrm{d}\varphi^2\right]$$

$$K(r) \simeq \frac{12 r_s^2}{r^6}$$

Upper bound for wavelike effects $R_\lambda = \left(2\sqrt{3} r_s \lambda_{\mathrm{GO}}^2\right)^{\frac{1}{3}}$



G. Cusin, R. Durrer, P. G. Ferreira 2018

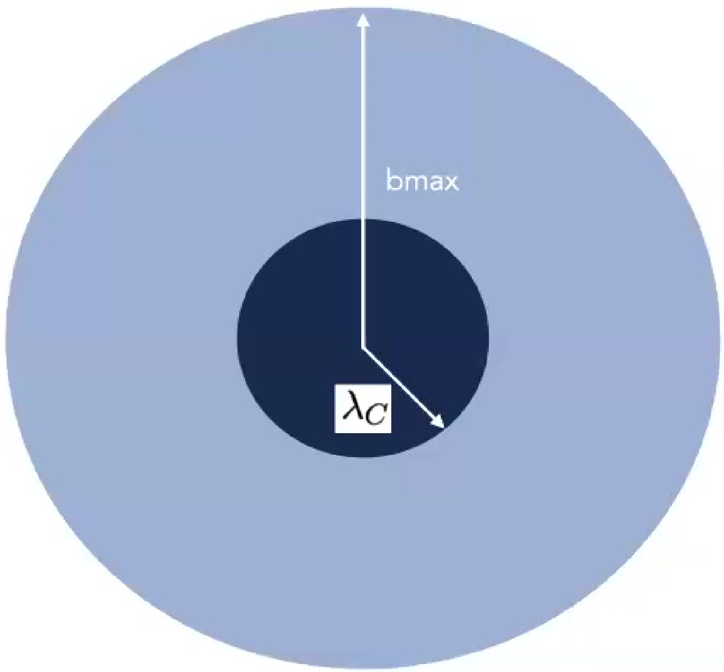
Regularization

A more pertinent length scale is the Compton wavelength :

$$\frac{r_s}{\lambda_C} \sim \frac{m^2}{M_{\rm Pl}^2}$$

Thus, we set the minimum angle at:

$$\theta_{\rm min} = \frac{\lambda_C}{b_{\rm max}} = \left(\frac{2Gm}{\lambda_{\rm GO}}\right)^{2/3} \frac{1}{Gm^2}$$



Gravitational Kompaneets Equations

- Problems :

  IR divergences (similar as the Coulomb divergences)

$$J_\ell(x,\lambda;s) = 2\pi \int_{\theta_{\min}(\lambda)}^{\theta_{\max}} \sin\theta\, \mathrm{d}\theta \int \mathrm{d}^3\mathbf{p} \left(1 - \frac{\mathbf{p}}{m}\cdot\hat{\mathbf{n}}\right) \frac{\mathrm{d}\sigma_s(\mathbf{p},x,\theta)}{\mathrm{d}\theta} \mathcal{N}_{\mathrm{eq}}(\mathbf{p}) \Delta^\ell(x,\theta)$$

- Geometric Optics approximation:

(Defines a region for which wave like effects are relevant)

Garoffolo, arXiv e-prints (2022) arXiv:2210.05718

$$K(r) \simeq \frac{12 r_s^2}{r^6}$$

Kretschman Scalar

$$b_{\max}(\lambda) = R_\lambda = \left(2\sqrt{3} r_s \lambda_{\mathrm{GO}}^2\right)^{\frac{1}{3}}$$

Impact parameter

- Solution :

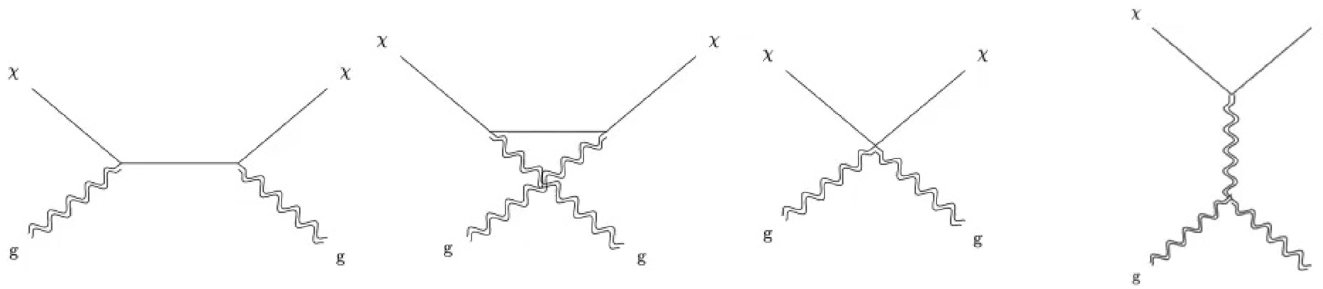
  Setting the lower cutoff angle at the geometric limit

$$\theta_{\min}(\lambda) = \frac{\lambda_C}{b_{\max}(\lambda)} = \left(\frac{2Gm}{\lambda_{\mathrm{GO}}}\right)^{2/3} \frac{1}{Gm^2}$$

Cusin, Durrer, and Ferreira. Phys. Rev. D **99**, 023534

GR as an EFT

- Graviton scattering



- The scalar case : $S_{s=0} = -\frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\left(g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi + m^2\chi^2\right)$

$$\frac{\mathrm{d}\sigma_s(\mathbf{p},\hat{\mathbf{n}},\hat{\mathbf{n}}')}{\mathrm{d}\Omega} = \frac{\sigma_s(\lambda)}{(4\pi)^2}\frac{1}{(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2}\left(\frac{\omega'(\mathbf{p},\hat{\mathbf{n}},\hat{\mathbf{n}}')}{\omega}\right)^2\overline{|\mathcal{M}_s|^2} \quad \text{with} \quad \frac{\omega'(\mathbf{p},\hat{\mathbf{n}},\hat{\mathbf{n}}')}{\omega} = \frac{E-\mathbf{p}\cdot\hat{\mathbf{n}}}{E-\mathbf{p}\cdot\hat{\mathbf{n}}'+\omega(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')}$$

$$|\mathcal{M}_{s=0}|^2 = \frac{\kappa^4}{8}\left[\frac{m^8(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')^2}{(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2(E-\mathbf{p}\cdot\hat{\mathbf{n}}')^2} + \frac{[2(E-\mathbf{p}\cdot\hat{\mathbf{n}})(E-\mathbf{p}\cdot\hat{\mathbf{n}}')-m^2(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')]^4}{(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')^2(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2(E-\mathbf{p}\cdot\hat{\mathbf{n}}')^2}\right]$$

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  (Defines a region for which wave like effects are relevant)

$$K(r) \simeq \frac{12 r_s^2}{r^6} \quad \Rightarrow \quad b_{\max}(\lambda) = R_\lambda = \left(2\sqrt{3} r_s \lambda_{\mathrm{GO}}^2\right)^{\frac{1}{3}}$$

Kretschman Scalar — Impact parameter

Garoffolo, arXiv e-prints (2022) arXiv:2210.05718

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Cusin, Durrer, and Ferreira. Phys. Rev. D **99**, 023534

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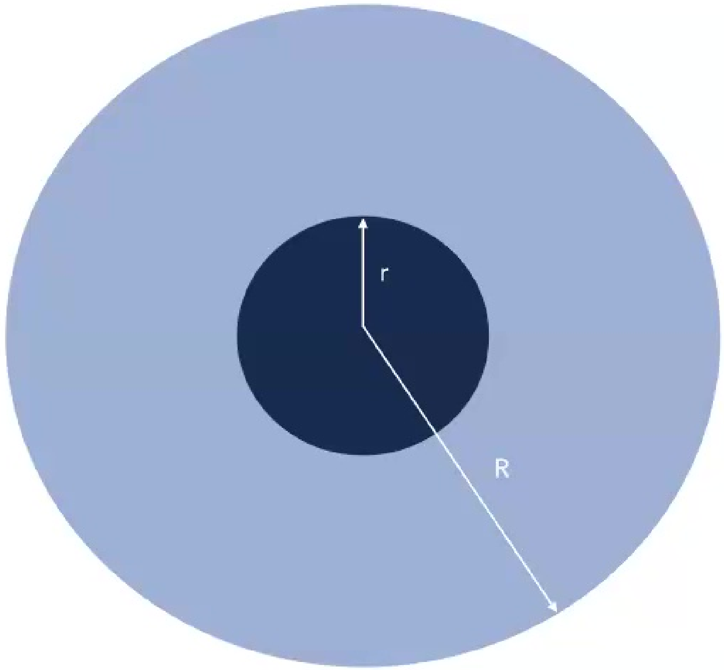

G. Cusin, R. Durrer, P. G. Ferreira 2018

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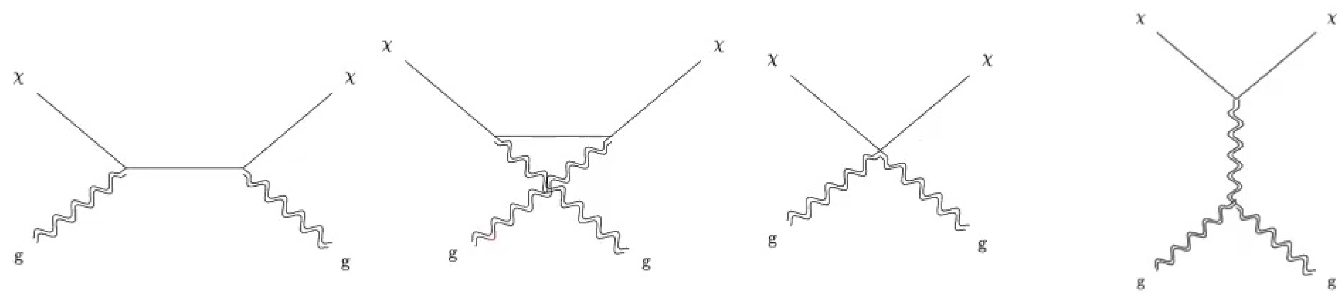

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Solving the Gravitational SZ effect

- In the case of single field inflation : $n(x) = A_T x^\alpha$ $\alpha = -2$

- Master equation :

Average energy transferred

Average energy^2 transferred

$$\frac{\partial n}{\partial t} = A_T(k_*)\left[J_1(x,\lambda;s) + \frac{J_2(x,\lambda;s)}{2} + \frac{\alpha(J_1(x,\lambda;s) + J_2(x,\lambda;s))}{x} + \frac{\alpha(\alpha-1)J_2(x,\lambda;s)}{2x^2}\right]x^\alpha$$
$$+ A_T^2(k_*)\left(J_1(x,\lambda;s) + \frac{J_2(x,\lambda;s)}{2} + \frac{\alpha J_2(x,\lambda;s)}{x}\right)x^{2\alpha},$$

with

$$J_\ell(x,\lambda;s) = 2\pi\int_{\theta_{\min}(\lambda)}^{\theta_{\max}} \sin\theta\, \mathrm{d}\theta \int \mathrm{d}^3\mathbf{p}\left(1 - \frac{\mathbf{p}}{m}\cdot\hat{\mathbf{n}}\right)\frac{\mathrm{d}\sigma_s(\mathbf{p},x,\theta)}{\mathrm{d}\theta}\mathcal{N}_{\mathrm{eq}}(\mathbf{p})\Delta^\ell(x,\theta)$$

Maxwell Boltzman momentum distribution

$$\mathcal{N}(\mathbf{p})\,\mathrm{d}^3\mathbf{p} = \mathcal{N}_{\mathrm{eq}}(|\mathbf{p}|)\,\mathrm{d}^3\mathbf{p} = \frac{n_\chi}{(2\pi mT)^{3/2}}\exp\left(-\frac{\mathbf{p}^2}{2mT}\right)\mathrm{d}^3\mathbf{p}$$

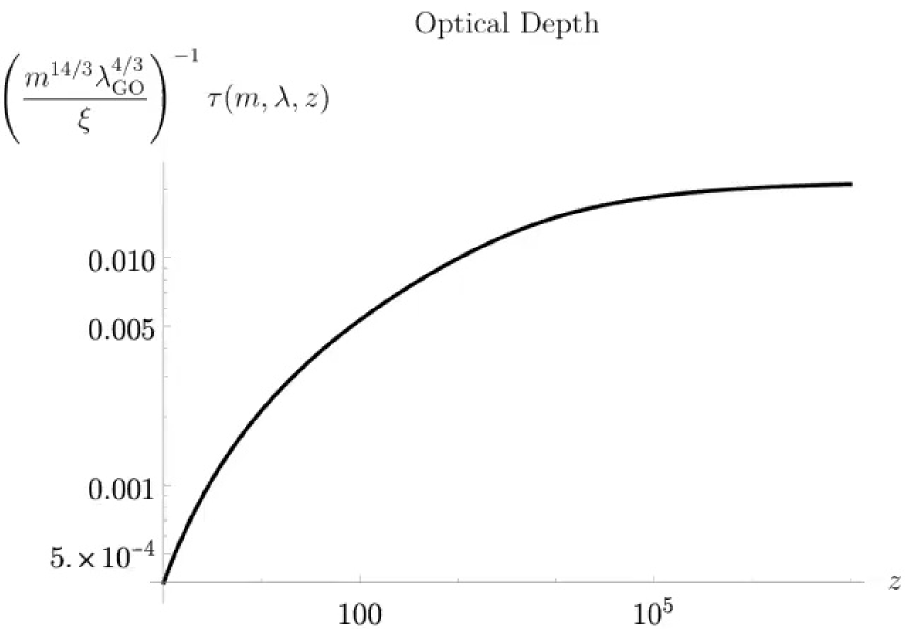
Solving the Gravitational SZ effect

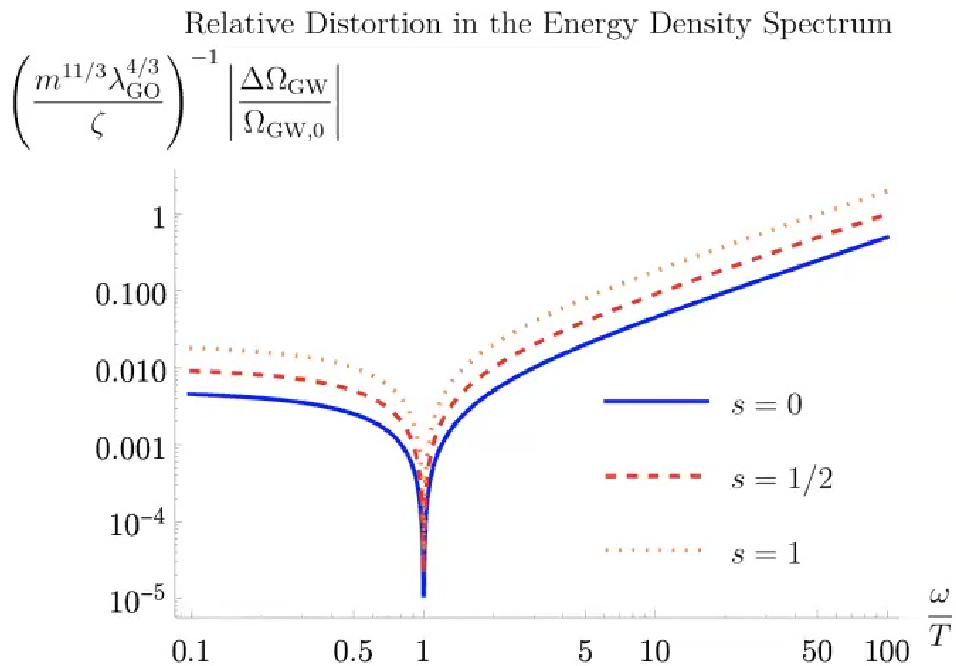
- Master equation :

$$\Delta n(x,z;s) = y\frac{16}{\pi}\left(\frac{H}{M_{\rm Pl}}\right)^2\left[\tilde{J}_1(x,\lambda;s)+\frac{1}{2}\tilde{J}_2(x,\lambda;s)-\frac{2(\tilde{J}_1(x,\lambda;s)+\tilde{J}_2(x,\lambda;s))}{x}+\frac{3\tilde{J}_2(x,\lambda;s)}{x^2}\right]x^{-2}$$

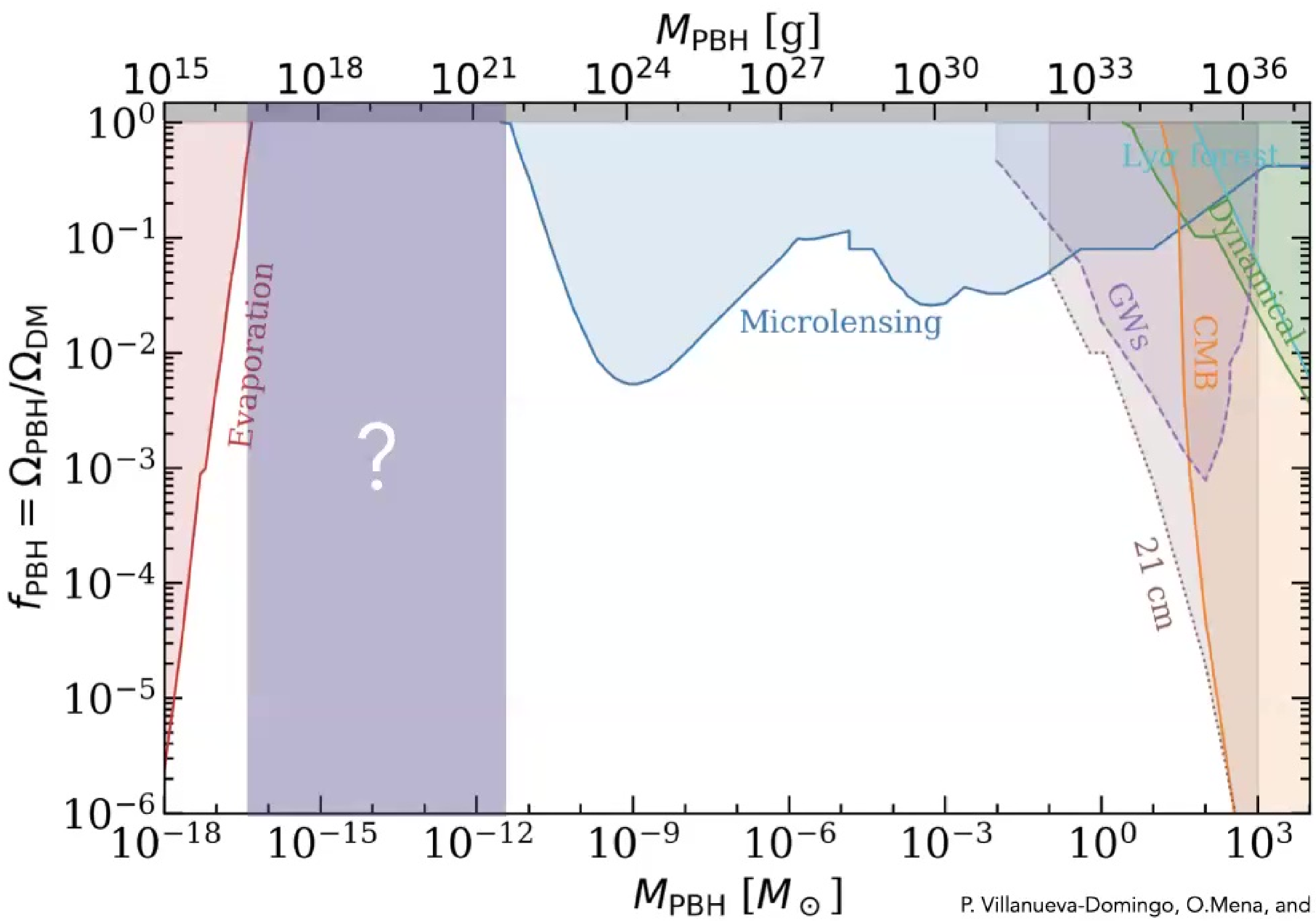
$$\frac{\Delta\Omega_{\rm GW}(x,z;s)}{\Omega_{\rm GW,0}(x,z)} = \frac{12\pi(G^5 m^{11}\lambda_{\rm GO}^4)^{1/3}H_0 T\Omega_\chi}{\Omega_M^{1/2}}g(x,z;s) = \frac{m^{11/3}\lambda_{\rm GO}^{4/3}}{\zeta}g(x,z;s)$$

Results : BSM particles $m = 10\,\mathrm{TeV}$





PBHs as a Dark Matter Candidate



P. Villanueva-Domingo, O.Mena, and S.Palomares-Ruiz 2021

PBHs

Non thermal → What temperature should we consider? → What mass distribution should we consider?

↓

Monochromatic mass distribution
M Kleban and C Norton 2023

←

$$\mathcal{N}(M,\mathbf{v}) = \sigma(M_H)\exp\left(-\frac{\delta_c^2}{2\sigma^2(M_H)}\right)\exp\left(-\frac{\mathbf{v}^2}{2\sigma_v^2}\right)$$

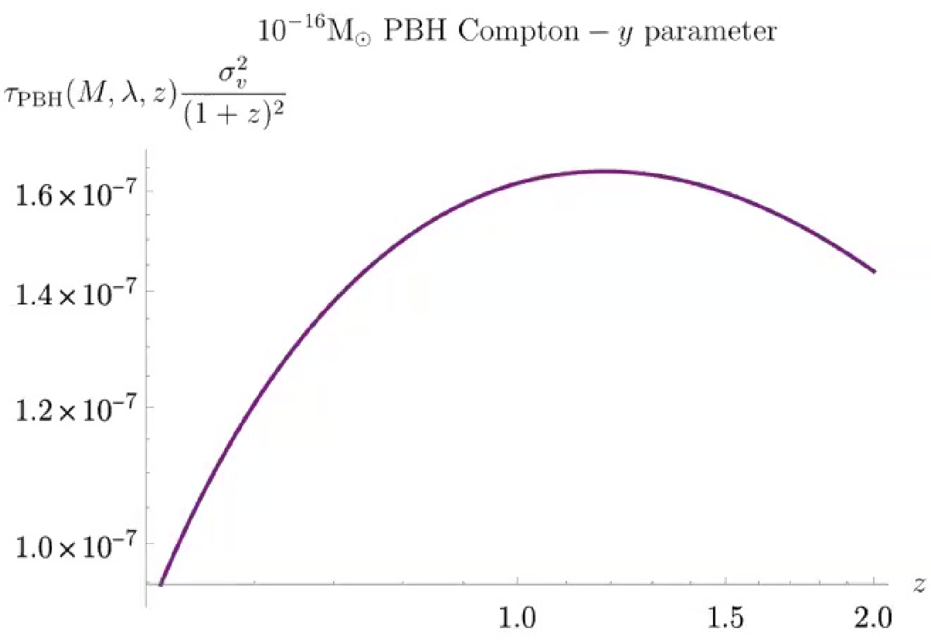
Press-Schechter mass distribution

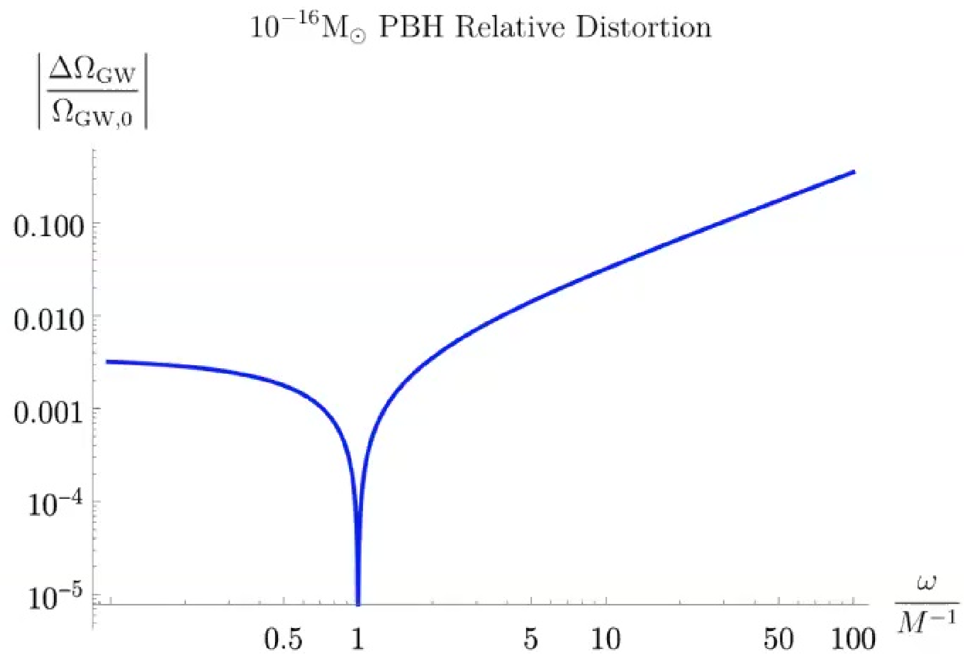
Hawking Temperature: $T_H \equiv (8\pi G M)^{-1}$

For PBHs with a mass $M = 10^{-16} M_\odot$

It corresponds to a temperature $T_H \sim \mathcal{O}(\mathrm{keV})$

Results : PBHs





Enhancement

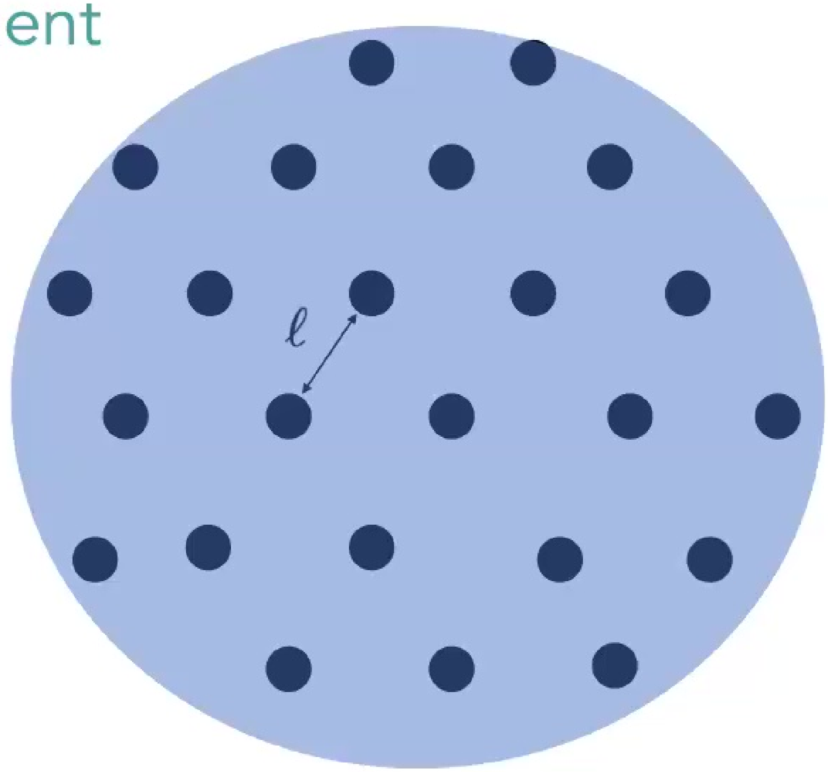
From PBHs as single "particles" to PBHs clusters

N : number of PBHs that the GW "sees"

The mass is increased by: $M \to NM$

And the cross section by: $\sigma_{\rm GC} \to N^{2/3}\sigma_{\rm GC}$

The average spacing is $\ell = \left(\frac{M}{\rho_{\rm CDM}}\right)^{1/3} \simeq 10^{-3}\,{\rm pc}$



Coherent enhancement factor $N = \frac{\lambda_{\rm GO}}{\ell} \simeq 70$

(With $\lambda_{\rm GO} = 0.1\,{\rm pc}$)

The spectral distortion goes from

$0.3\% \longrightarrow 5\%$

Gravitational Kompaneets Equations

- BSM particles are in thermal equilibrium
- Gravitons are soft i.e $\hbar\omega \ll m\ c^2$ but $\hbar\omega \sim k_B T$
- BSM particle are non relativistic ie $|\mathbf{p}| \ll m\ c$

S. Boughn and T. Rothman Class.Quant.Grav. 23 (2006) 5839-5852

Assumptions

- Compton scattering
- Stimulated emission

- Master equation

$$\frac{\partial n}{\partial t} = \frac{J_2(x,\lambda;s)}{2}\frac{\partial}{\partial x}\left[\frac{\partial n}{\partial x} + n(1+n)\right] + \left(J_1(x,\lambda;s) + \frac{J_2(x,\lambda;s)}{2}\right)\left[\frac{\partial n}{\partial x} + n(1+n)\right]$$

$$\frac{n_\chi}{(2\pi m T)^{3/2}} \exp\left(-\frac{\mathbf{p}^2}{2mT}\right) \mathrm{d}^3\mathbf{p}$$

with $$J_\ell(x,\lambda;s) = 2\pi \int_{\theta_{\min}(\lambda)}^{\theta_{\max}} \sin\theta\, \mathrm{d}\theta \int \mathrm{d}^3\mathbf{p}\left(1 - \frac{\mathbf{p}}{m}\cdot\hat{\mathbf{n}}\right)\frac{\mathrm{d}\sigma_s(\mathbf{p},x,\theta)}{\mathrm{d}\theta}\mathcal{N}_{\rm eq}(\mathbf{p})\Delta^\ell(x,\theta)$$

Momentum distribution of BSM particles

Enhancement

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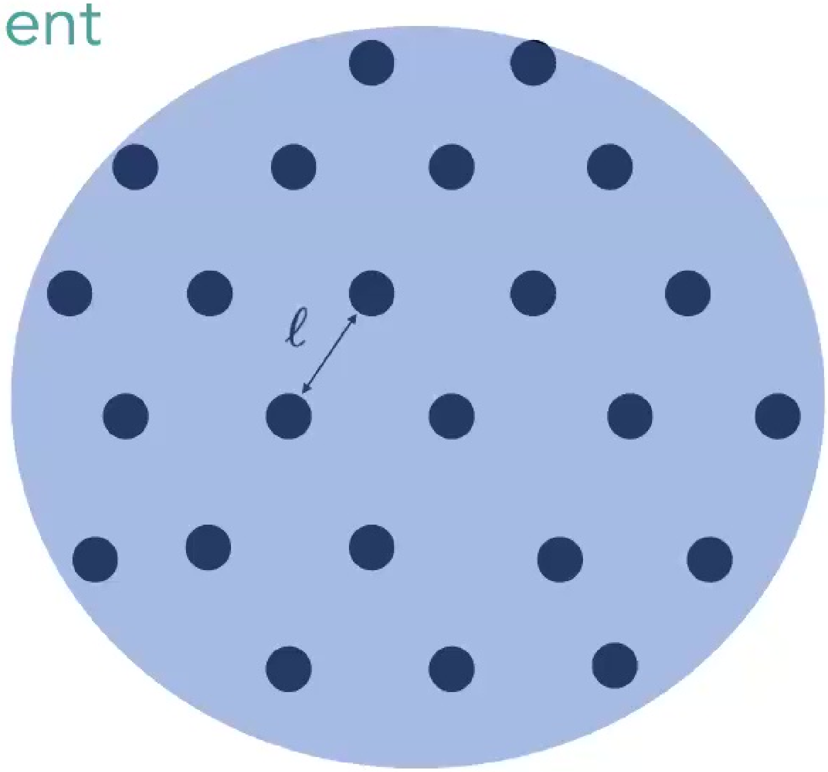

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Future Directions

- Explore different CGWB sources
- Address the cutoff issue
- Look at astrophysical events
- DM composites
- Expands the formalism to other interaction processes

Thank you