

Title: Asymptotically safe quantum gravity on foliated spacetimes - VIRTUAL

Speakers: Frank Saueressig

Series: Quantum Gravity

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Abstract: Incorporating time poses a challenge for all quantum gravity programs primarily build around Euclidean spacetimes. This talk surveys how the gravitational asymptotic safety program may address this challenge by encoding the gravitational degrees of freedom via the Arnowitt-Deser-Misner (ADM) decomposition of the spacetime metric. This formulation equips spacetime with a foliation structure singling out a preferred direction. This structure may then take the role of time in the Lorentzian framework. Within this setting, we will outline recent results related to the Wilsonian renormalization group flow of the graviton two-point function including the infrared attractors rendering the graviton massless. Furthermore, we will explain how these results generalize to Lorentzian signature spacetimes and briefly comment on potential applications in the context of cosmology.

Zoom link <https://pitp.zoom.us/j/98382991491?pwd=SmtpMTRTK2NUMEpUN3laa0pIWWs2UT09>



Asymptotically Safe Quantum Gravity on foliated spacetimes

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Foliated asymptotically safe gravity in the fluctuation approach
F. Saueressig, J. Wang, JHEP 09 (2023) 064, arXiv:2306.10408

Global Flows of Foliated Gravity-Matter Systems
G. Korver, F. Saueressig, J. Wang, arXiv:2401.xxxxx

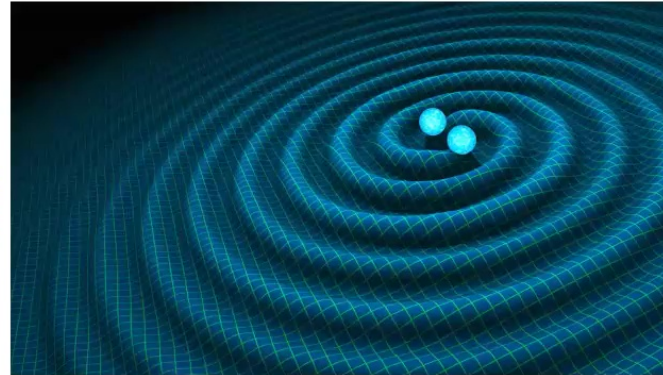
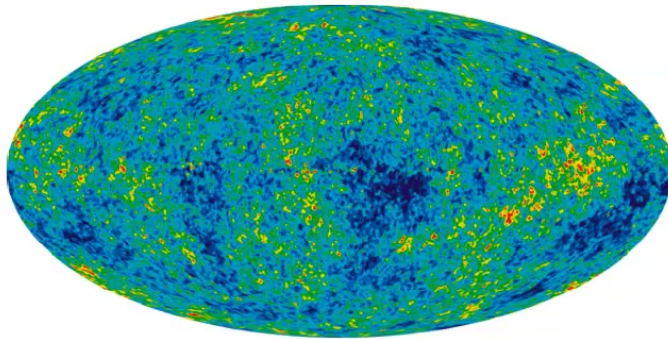
Quantum Gravity Seminar, Perimeter Institute, Jan. 11th, 2024

Outline

- ① introduction and motivation
 - gravitons in general relativity
 - embedding the graviton in quantum gravity
- ② basics of asymptotic safety
- ③ the renormalized graviton 2-point function
 - phase diagram of the graviton mass
 - IR fixed points
 - results for Lorentzian signature
- ④ summary and outlook

!!! Questions Welcome !!!

Motivation I: phenomenology from first principles



- propagation of gravitational waves in a flat background [today]
- tensor fluctuations in the CMB [future]

can we use these observations to constrain quantum gravity?

Gravitons in General Relativity

Gravitational waves (from: Introduction to GR)

describe gravitational waves propagating in flat space

- Einstein's equations in vacuum

$$R_{\mu\nu} = 0$$

- consider small perturbation around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- adopt harmonic gauge
- *massless* wave equation for the metric perturbations

$$-(-\partial_t^2 + \partial_x^2 + \partial_y^2 + \partial_z^2)h_{\mu\nu} = 0$$

+ additional constraints

Gravitational waves (from: Introduction to GR)

describe gravitational waves propagating in flat space

- example: gravitational wave propagating in the t - z -plane:

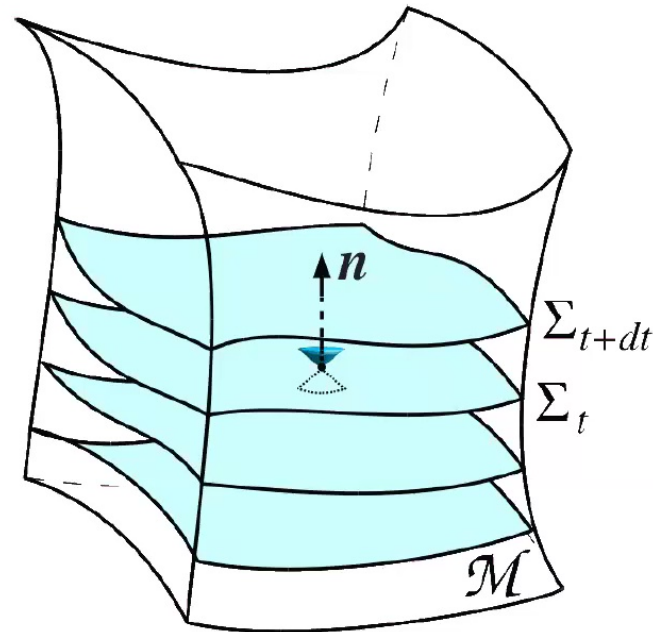
$$h_{\mu\nu}(t - z) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \phi_+(t - z) & \phi_\times(t - z) & 0 \\ 0 & \phi_\times(t - z) & -\phi_+(t - z) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- general properties:
 - two degrees of freedom (+ and \times -polarization)
 - travel with the speed of light ($c = 1$)
 - degrees of freedom: in spatial part (transverse-traceless sector)

Gravitons in (Euclidean) Quantum Gravity



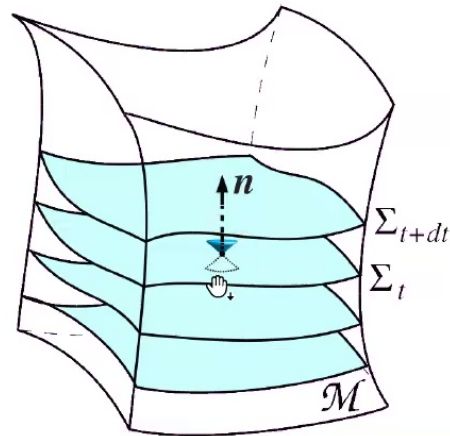
Equip spacetime with a foliation



Arnowitt-Deser-Misner (ADM) decomposition of the metric:

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= -N^2 dt^2 + \sigma_{ij} (N^i dt + dy^i) (N^j dt + dy^j) \end{aligned}$$


ADM-formalism



virtues:

- allows to transit between Lorentzian and Euclidean signature
 - provides a preferred “time”-direction
- closely related to observables (Bardeen variables)

Where is the graviton?

- ADM-fields: $g_{\mu\nu} \mapsto \{N, N_i, \sigma_{ij}\}$ 

- introduce fluctuations in a flat background

$$\sigma_{ij} = \bar{\sigma}_{ij} + \hat{\sigma}_{ij}, \quad N_i = \bar{N}_i + \hat{N}_i, \quad N = \bar{N} + \hat{N}$$

with

$$\bar{\sigma}_{ij} = \delta_{ij}, \quad \bar{N} = 1, \quad \bar{N}_i = 0$$

- transverse-traceless decomposition of $\hat{\sigma}_{ij}$: ($\Delta = -\partial^2$)

$$\hat{\sigma}_{ij} = h_{ij} + \partial_i \frac{1}{\sqrt{\Delta}} v_j + \partial_j \frac{1}{\sqrt{\Delta}} v_i + \partial_i \partial_j \frac{1}{\Delta} E + \frac{1}{3} \delta_{ij} E + \frac{1}{3} \delta_{ij} \psi$$

- h_{ij} carries 2 degrees of freedom
- h_{ij} is gauge-invariant

Gravitons propagation in quantum gravity

object of interest: 2-point correlation function of h_{ij}

$$\Gamma = \frac{1}{2} \underbrace{\frac{1}{32\pi G}}_{\text{wave function renormalization}} \int dt d^3y h_{ij} \left[-\partial^2 + \underbrace{\mu^2}_{\text{graviton mass}} \right] h^{ij}$$

- gives the graviton propagator on a flat background
- variation with respect to $h_{ij} \implies$ massive wave equation

$$(-\partial^2 + \mu^2) h_{ij} = 0$$

phenomenology: gravitons are massless

$$\mu^2 = 0$$

can we derive this from asymptotic safety?

Motivation II: structural aspects of Asymptotic Safety

tension:

- most computations are Euclidean
- our world is Lorentzian




strategies for addressing this tension:

- ADM-decomposition + analytic continuation of lapse function
[E. Manrique, S. Rechenberger, F. Saueressig, arXiv:1102.5012]
- spectral functions on flat background
[J. Fehre, D. F. Litim, J. M. Pawłowski, M. Reichert, arXiv:2111.13232]
- algebraic quantum field theory + renormalization group
[E. D'Angelo, N. Drago, N. Pinamonti, K. Rejzner, arXiv:2202.07580]

Asymptotic Safety in the Handbook of Quantum Gravity

[editors: C. Bambi, L. Modesto and I.L. Shapiro, Springer Singapore]

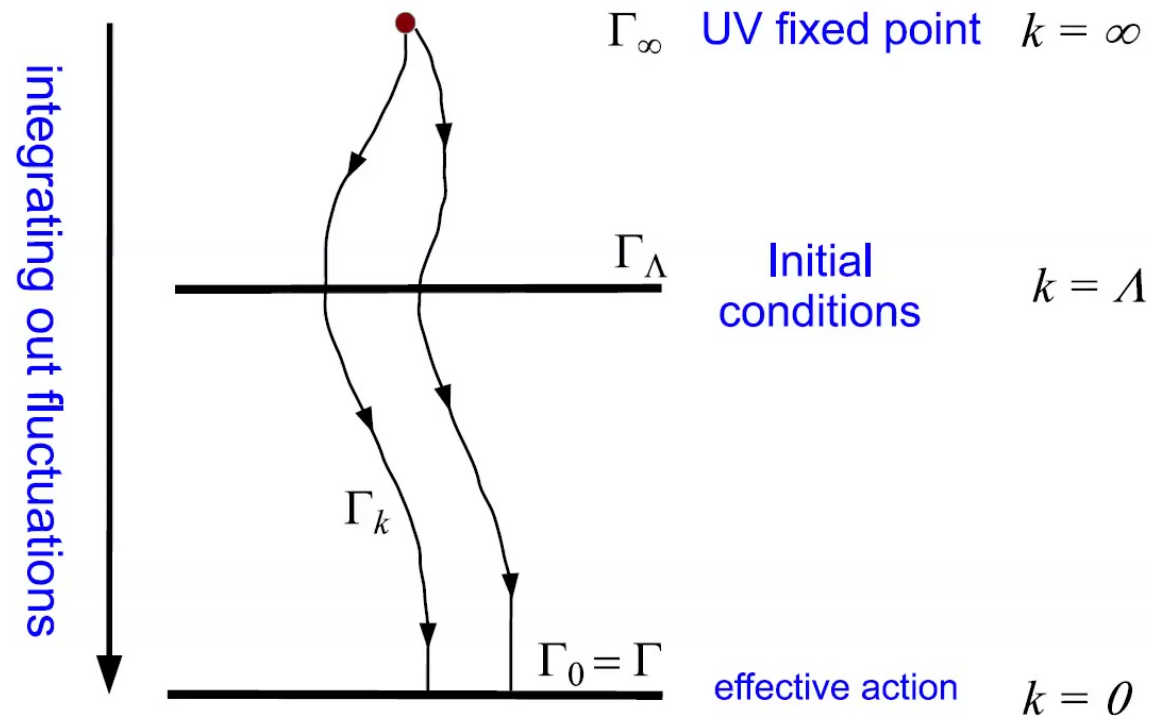
[R. Percacci - organizing the Asymptotic Safety Section]

- *The Functional Renormalization Group in Quantum Gravity*,
F. Saueressig, arXiv:2302.14152
- *Quantum Gravity from dynamical metric fluctuations*,
J.M. Pawłowski, M. Reichert, arXiv:2309.10785 
- *Asymptotic safety of gravity with matter*,
A. Eichhorn, M. Schiffer, arXiv:2212.07456
- *Form Factors in Asymptotically Safe Quantum Gravity*,
B. Knorr, C. Ripken, F. Saueressig, arXiv:2210.16072
- *The Functional $f(R)$ Approximation*,
T.R. Morris, D. Stulga, arXiv:2210.11356
- *Perturbative approaches to non-perturbative quantum gravity*,
R. Martini, G.P. Vacca, O. Zanusso, arXiv:2210.13910
- *Quantum gravity and scale symmetry in cosmology*,
C. Wetterich, arXiv:2211.03596.
- *Black Holes in Asymptotically Safe Gravity*,
A. Platania, arXiv:2302.04272.

The Wilsonian Renormalization Group



integrate out fluctuations shell-by-shell in momentum space

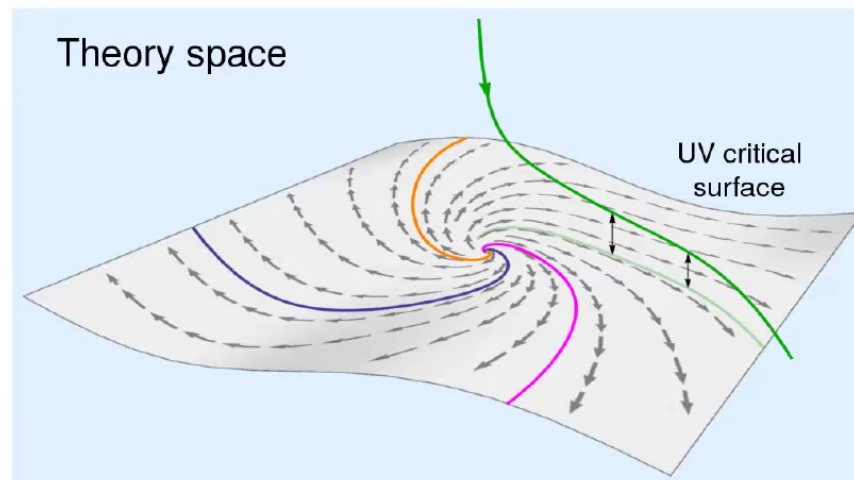


The arena

- **input**
 - a) field content (e.g., spacetime metric $g_{\mu\nu}$)
 - b) symmetries (e.g., coordinate transformations)
- **actions** = combinations of interaction monomials \mathcal{O}
 - build from the field content
 - compatible with symmetries (e.g., $\mathcal{O}[g] = \int d^4x \sqrt{g} R$)
- **theory space** = space containing all actions
 - coordinates: couplings $\{u_i\}$ (e.g., G, Λ)
- **Wilsonian renormalization group flow:**
 - couplings run when integrating out quantum fluctuations (e.g., $k\partial_k u_i = \beta_i(\{u_i\})$)

Fixed points of the Renormalization Group

high-energy behavior: controlled by renormalization group fixed point



- 2 classes of renormalization group trajectories:
 - relevant = end at the fixed point in the UV
 - irrelevant = go somewhere else

Renormalization group fixed points

suppose we have beta functions for dimensionless couplings $\{u_i\}$:

$$k\partial_k u_i = \beta_i(\{u_i\}), \quad i = 1, 2, \dots$$

definition: renormalization group fixed point $\{u_i^*\}$:

$$\beta_i(\{u_i^*\}) = 0, \quad \forall i$$

linearized RG flow in the vicinity of $\{u_i^*\}$:

$$k\partial_k u_i = \sum_j B_i^j (u_j - u_j^*), \quad B_i^j \equiv \left. \frac{\partial \beta_i}{\partial u_j} \right|_{u_i = u_i^*}$$

- B_i^j is the stability matrix of the fixed point

predictive power \iff spectrum of B_i^j

Renormalization group fixed points

linearized RG flow in the vicinity of $\{u_i^*\}$:

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solve the linearized equation:

- stability coefficients θ^l and right-eigenvectors V^l of B_i^j

$$B_i^j V_j^l = -\theta^l V_i^l$$

- solution:

$$u_i(k) = u_i^* + \sum_J C^J V_i^J \left(\frac{k_0}{k} \right)^{\theta^J}$$

- $\text{Re}(\theta^l) < 0$: UV-attractive direction (C^l : free parameter)
- $\text{Re}(\theta^l) > 0$: UV-repulsive direction (C^l : must be set to zero)

for each θ^l where $\text{Re}(\theta^l) < 0$ there is one free parameter

The Asymptotic Safety package

Reuter fixed point (NGFP)

- ensures absence of UV-divergences

NGFP has **finite number** of relevant parameters (predictivity)

effective action compatible with observations

- tests of general relativity
 - solar system tests, cosmology, gravitational waves, ...
- compatible with standard model of particle physics at 1 TeV

structural demands:

- resolution of spacetime singularities?
- unitarity?

Question:

? How do we get beta functions ?



The Asymptotic Safety package

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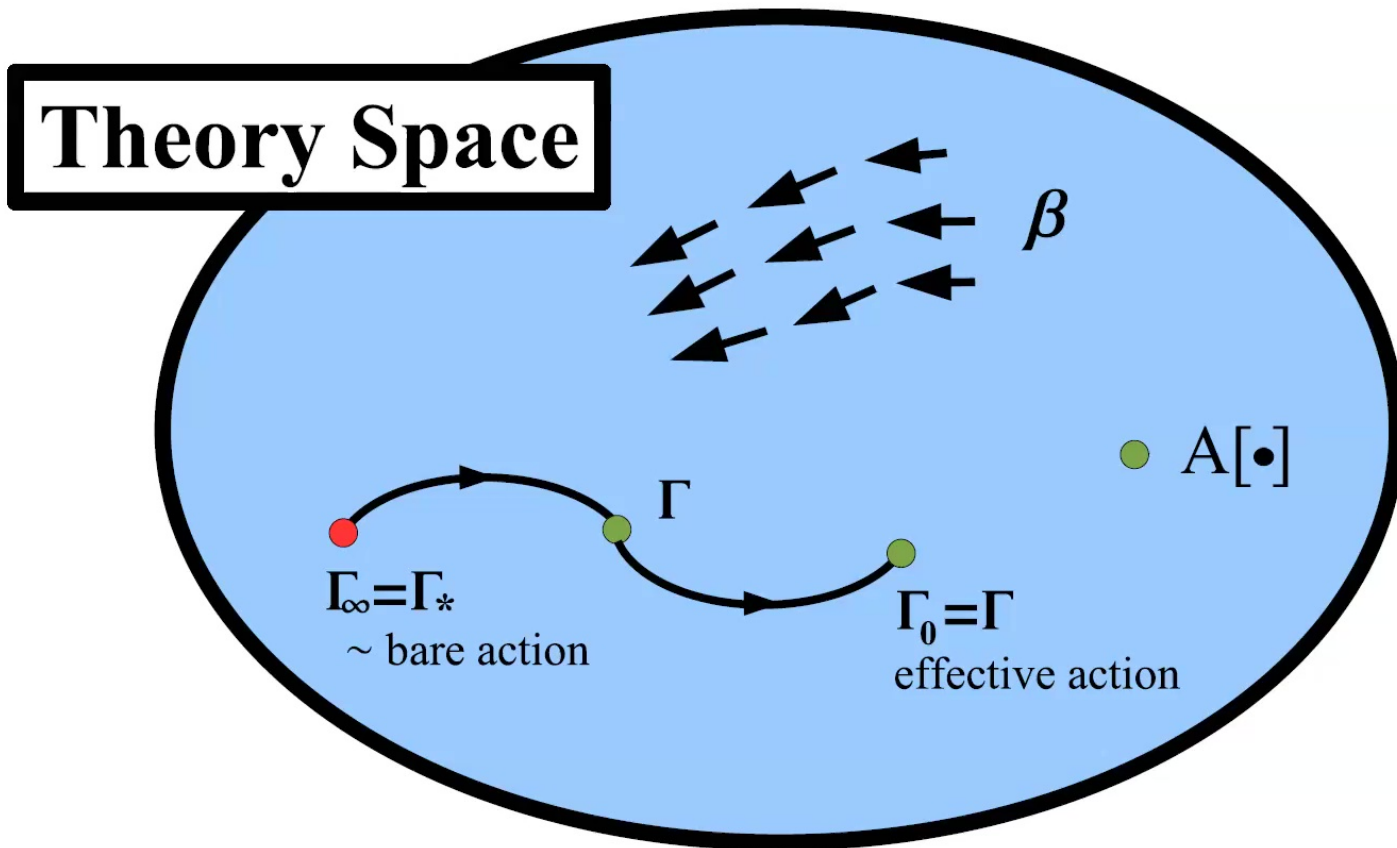
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Summary: structures of the program



Functional Renormalization Group Equations



Renormalization group flows beyond perturbation theory

Functional Renormalization Group Methods:

- Callan-Symanzik Equation
- Wegner-Houghton Equation
- Polchinski Equation
- Wetterich Equation

applicability:

[N. Depuis, et. al., Phys. Rept. 910 (2021) 1]

- scalar field theory, QCD, Quantum Gravity, condensed matter

flow equation for effective average action Γ_k

[C. Wetterich, Phys. Lett. **B301** (1993) 90; T. Morris, Int. J. Mod. Phys. A 9 (1994) 2411]

extension to gauge theory

[M. Reuter and C. Wetterich, Nucl. Phys. B 417 (1994) 181]

extension to gravity

[M. Reuter, Phys. Rev. D **57** (1998) 971, hep-th/9605030]

The Wetterich equation

Renormalization group equation for effective average action $\Gamma_k[\phi]$

governed by the Wetterich equation

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2}\text{Tr} \left[\left(\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi} + \mathcal{R}_k \right)^{-1} k\partial_k\mathcal{R}_k \right]$$

- encodes dependence of Γ_k on coarse-graining scale k
- formally exact equation
- flow: driven by quantum fluctuations with momentum $p^2 \approx k^2$
- no UV-regulator needed
- flexible in terms of field content

The Wetterich equation for gravity

[M. Reuter, Phys. Rev. D **57** (1998) 971, hep-th/9605030]

- **complications:**
 - diffeomorphism invariance
 - definition of a coarse graining scale
- **solution:** background field method: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
 - introduces background gauge fixing term + ghosts

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 - definition of a coarse graining scale
- **solution:** background field method: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
 - introduces background gauge fixing term + ghosts
- Wetterich equation: dependence of Γ_k on coarse graining scale

$$k\partial_k \Gamma_k[h, \bar{C}, C; \bar{g}] = \frac{1}{2} \text{STr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} k\partial_k \mathcal{R}_k \right]$$

✎

- $\Gamma_k^{(2)}$ = Hessian with respect to fluctuation fields
- “extra” \bar{g} -dependence necessary for formulating exact equation

Functional Renormalization Group Flows

approximations beyond perturbation theory

The Wetterich equation - approximate solutions

Wetterich equation:

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2}\text{Tr} \left[\left(\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi} + \mathcal{R}_k \right)^{-1} k\partial_k\mathcal{R}_k \right]$$

- formally exact **but can not be solved exactly**

common approximations:

- perturbation theory (expansion in a small coupling)
 - reproduces, e.g., one-loop results
- non-perturbative approximations (projections of the RG flow)
 - derivative expansion
 - curvature expansion
 - vertex expansion

Projecting the RG flow: general strategies

ansatz for Γ_k retaining a subset of all monomials

$$\Gamma_k[h_{\mu\nu}; \bar{g}_{\mu\nu}] = \sum_{i=1}^N \bar{u}_i(k) \mathcal{O}_i[h_{\mu\nu}; \bar{g}_{\mu\nu}]$$

⇒ substitute into Wetterich equation

⇒ project flow onto space spanned by \mathcal{O}_i gives beta functions

$$k\partial_k \bar{u}_i(k) = \beta_i(\bar{u}_i; k)$$

- projecting on background structures

$$k\partial_k \Gamma_k[h_{\mu\nu}; \bar{g}_{\mu\nu}]|_{h_{\mu\nu}=0} = \dots$$

- projecting on correlation functions of fluctuation fields

$$k\partial_k \Gamma_k^{(n,0)}[h_{\mu\nu}; \bar{g}_{\mu\nu}]|_{h_{\mu\nu}=0} = \dots$$

Example: projecting the RG flow

truncate Γ_k retaining a (finite) subset of all monomials

$$\Gamma_k[\phi] \approx \sum_{i=1}^N \bar{u}_i(k) \mathcal{O}_i[\phi]$$


Examples:



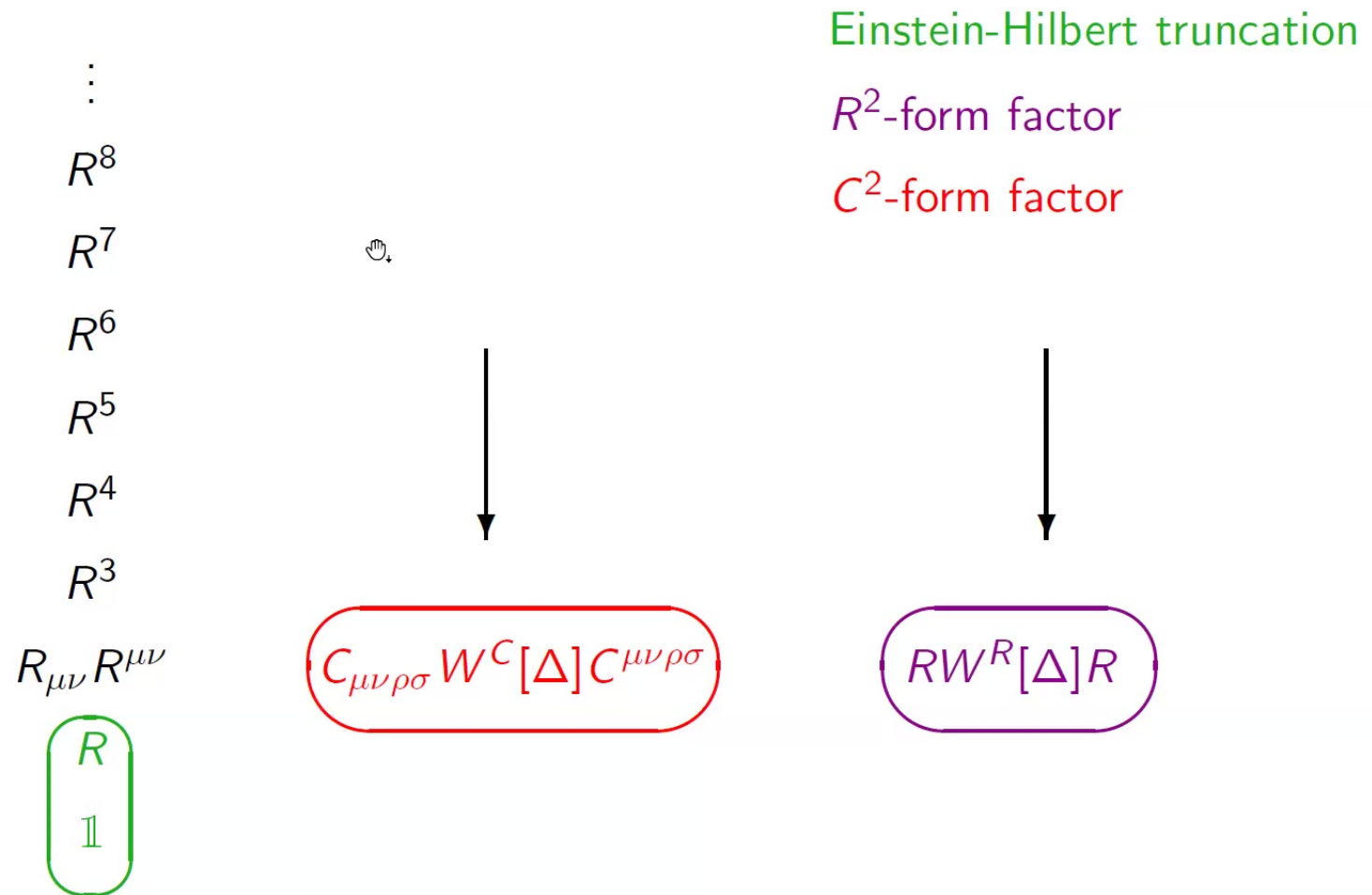
$\mathcal{O}_i[\phi]$	$\bar{u}_i(k)$	
$\frac{1}{2} \int d^d x (\partial_\mu \phi)(\partial^\mu \phi)$	Z_k	wave-function renormalization
$\frac{1}{2} \int d^d x \phi^2$	m_k^2	mass term
$\frac{1}{4!} \int d^d x \phi^4$	λ_k	ϕ^4 -selfinteraction

Ordering principle I: derivative expansion of $\Gamma_k^{\text{grav}}[g]$

Einstein-Hilbert truncation

\vdots	\vdots		
R^8	$C_{\mu\nu\rho\sigma}\Delta^6 C^{\mu\nu\rho\sigma}$	$R\Delta^6 R$	
R^7	$C_{\mu\nu\rho\sigma}\Delta^5 C^{\mu\nu\rho\sigma}$	$R\Delta^5 R$	
R^6	$C_{\mu\nu\rho\sigma}\Delta^4 C^{\mu\nu\rho\sigma}$	$R\Delta^4 R$	+ ...
R^5	$C_{\mu\nu\rho\sigma}\Delta^3 C^{\mu\nu\rho\sigma}$	$R\Delta^3 R$	+ ...
R^4	$C_{\mu\nu\rho\sigma}\Delta^2 C^{\mu\nu\rho\sigma}$	$R\Delta^2 R$	+ many more
R^3	$C_{\mu\nu\rho\sigma}\Delta C^{\mu\nu\rho\sigma}$	$R\Delta R$	+ 5 more
$R_{\mu\nu}R^{\mu\nu}$	$C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$	R^2	
			

Ordering principle II: curvature expansion



Ordering principle I: derivative expansion of $\Gamma_k^{\text{grav}}[g]$

Einstein-Hilbert truncation

\vdots	\vdots		
R^8	$C_{\mu\nu\rho\sigma}\Delta^6 C^{\mu\nu\rho\sigma}$	$R\Delta^6 R$	
R^7	$C_{\mu\nu\rho\sigma}\Delta^5 C^{\mu\nu\rho\sigma}$	$R\Delta^5 R$	
R^6	$C_{\mu\nu\rho\sigma}\Delta^4 C^{\mu\nu\rho\sigma}$	$R\Delta^4 R$	+ ...
R^5	$C_{\mu\nu\rho\sigma}\Delta^3 C^{\mu\nu\rho\sigma}$	$R\Delta^3 R$	+ ...
R^4	$C_{\mu\nu\rho\sigma}\Delta^2 C^{\mu\nu\rho\sigma}$	$R\Delta^2 R$	+ many more
R^3	$C_{\mu\nu\rho\sigma}\Delta C^{\mu\nu\rho\sigma}$	$R\Delta R$	+ 5 more
$R_{\mu\nu}R^{\mu\nu}$	$C_{\mu\nu\rho\sigma}\overset{\updownarrow}{C}^{\mu\nu\rho\sigma}$	R^2	
$\begin{matrix} R \\ \mathbb{1} \end{matrix}$			

Ordering principle III: vertex expansion

expand Γ_k in powers of the fluctuation field

$$\Gamma_k[h; \bar{g}] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \Gamma_k^{(n)}[0; \bar{g}](p_1, \dots, p_n) \cdot (h)^n$$

projection: make a selection on



- vertices $\Gamma_k^{(n)}[0; \bar{g}](p_1, \dots, p_n)$ retained in Γ_k
- momentum-dependence of the vertices
- closure of the RG flow

Which approximation should I use?

back-of-the-envelope computations

- qualitative results (existence of fixed points, ...)

derivative expansion

- precision computations of critical exponents
- convergence of results

☞ approximations tracking momentum dependence

- degrees of freedom present at UV-fixed points
- stability properties of propagators
- asymptotically safe scattering amplitudes

Physics from the effective average action

physics should be analyzed based on $\Gamma = \lim_{k \rightarrow 0} \Gamma_k$

quantum field theory:

- all quantum fluctuations should be integrated out

conceptual consequences for the energy dependence of couplings:

- k -dependence is not observable
- energy-dependence of couplings \implies form factors in Γ
 - Newton's coupling: no energy dependence
 - cosmological constant: no energy dependence
- resolves puzzle about gravity-mediated scattering processes

[J. Donoghue, Front. in Phys. 8 (2020) 56]

Computing the renormalized correlator

use the Wilsonian renormalization group

Wetterich equation adapted to ADM variables ($t = \ln k$)

[E. Manrique, S. Rechenberger, F. Saueressig, arXiv:1102.5012]

$$\partial_t \Gamma_k = \frac{1}{2} \text{STr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \partial_t \mathcal{R}_k \right]$$

- limits:
 - $k \rightarrow \infty$: bare theory [predictive power is here!]
 - $k \rightarrow 0$: renormalized couplings [physics is here!]
- solving the flow: impose initial condition Γ_Λ
 - no need to specify a bare theory [results from computation!]
 - also works in an effective field theory setting

Graviton propagation in quantum gravity

object of interest: 2-point correlation function of h_{ij}

$$\Gamma_k = \frac{1}{2} \underbrace{\frac{1}{32\pi G_k}}_{\text{wavefunction renormalization}} \int dt d^3y h_{ij} \left[-\partial^2 + \underbrace{\mu_k^2}_{\text{graviton mass}} \right] h^{ij}$$

- promote couplings to be k -dependent
- use correlator as the left-hand side of the flow equation

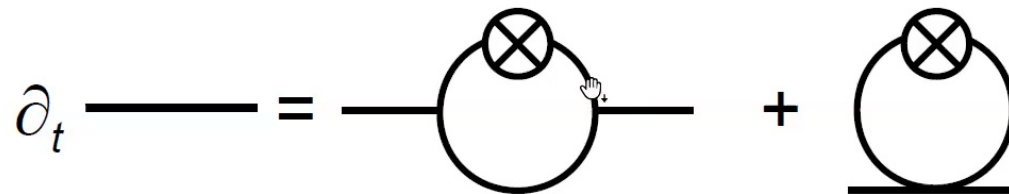
strategy for extracting physics:

- renormalized couplings = end-point of a RG trajectory ($k = 0$)

Flow of a two-point function

take two functional derivatives with respect to the graviton h_{ij} :

$$\partial_t \Gamma_k^{(2)} = \text{STr} \left[\mathcal{G} \Gamma_k^{(3)} \mathcal{G} \Gamma_k^{(3)} \mathcal{G} \partial_t \mathcal{R}_k \right] - \frac{1}{2} \text{STr} \left[\mathcal{G} \Gamma_k^{(4)} \mathcal{G} \partial_t \mathcal{R}_k \right].$$



here $\mathcal{G} \equiv \left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1}$ is the regulated propagator

- flow of $\Gamma_k^{(2)}$ determined by 3-point and 4-point vertex

Closure of the flow equation

ansatz for Γ_k (gravity + scalar matter + $U(1)$ -gauge fields):

$$\Gamma_k \simeq \Gamma_k^{\text{EH}} + \Gamma_k^{\text{gf}} + \mathcal{S}^{\text{ghost}} + \mathcal{S}^{\text{scalar}} + \mathcal{S}^{\text{vector}} .$$

where

$$\Gamma_k^{\text{EH}} = \frac{1}{16\pi G_k} \int dt d^3y N \sqrt{\sigma} \left(K^{ij} K_{ij} - K^2 - {}^{(3)}R + 2\Lambda_k \right)$$

$$\mathcal{S}^{\text{scalar}} = \frac{1}{2} \sum_{n=1}^{N_s} \int dt d^3y N \sqrt{\sigma} g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) \quad \text{↵}$$

$$\mathcal{S}^{\text{vector}} = \frac{1}{4} \sum_{n=1}^{N_v} \int dt d^3y N \sqrt{\sigma} F_{\mu\nu} F^{\alpha\beta} + \mathcal{S}^{\text{gauge-fixing}} + \mathcal{S}^{\text{ghosts}}$$

- generates graviton 2-point function
- vertices: Feynman rules constructed from Γ_k

Beta functions and interacting fixed points

introduce dimensionless couplings

$$g_k = k^2 G_k, \quad \lambda_k = k^{-2} \Lambda_k, \quad \mu_k^2 = -2k^2 \lambda_k$$

beta functions from evaluating the projected flow equation

$$\partial_t \lambda_k = \beta_\lambda(g_k, \lambda_k; N_S, N_V), \quad \partial_t g_k = \beta_g(g_k, \lambda_k; N_S, N_V)$$

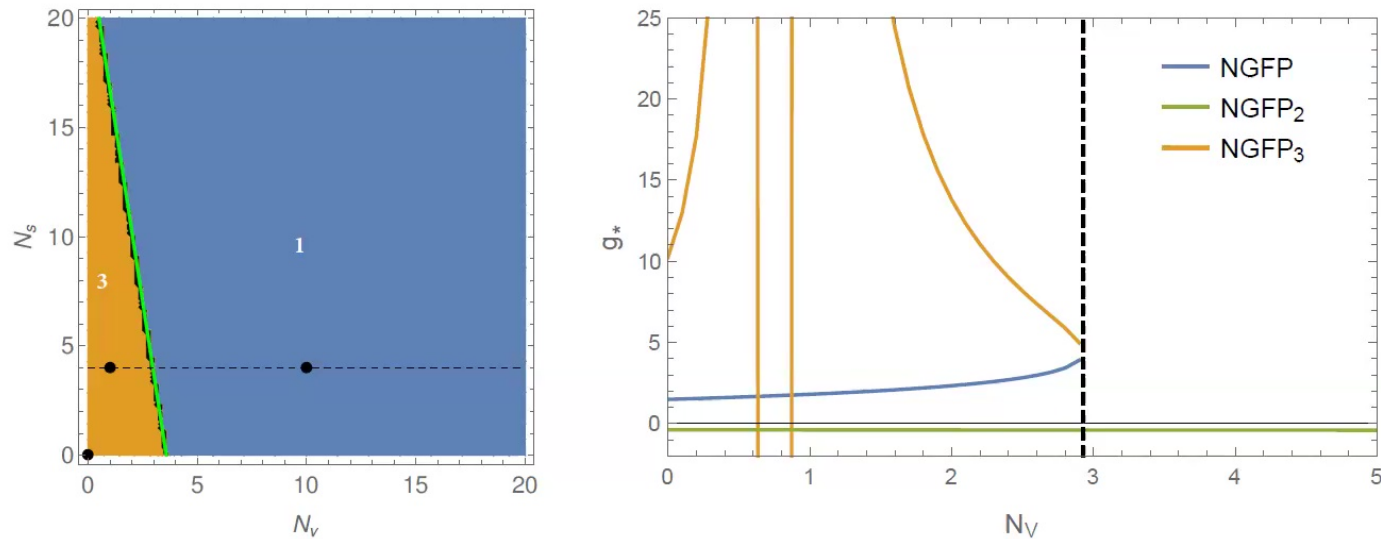
- depend on the number of matter fields N_S, N_V
- very complicated expressions

special interest: non-Gaussian fixed points (NGFPs)

$$\beta_g(g_*, \lambda_*) = 0, \quad \beta_\lambda(g_*, \lambda_*) = 0$$

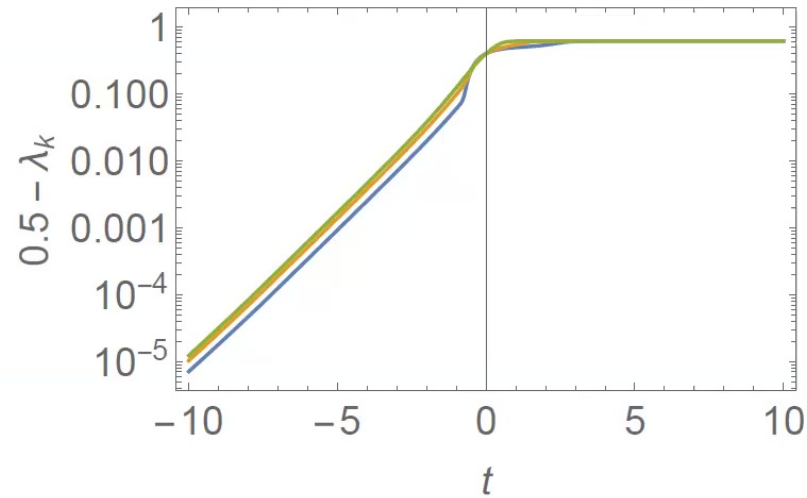
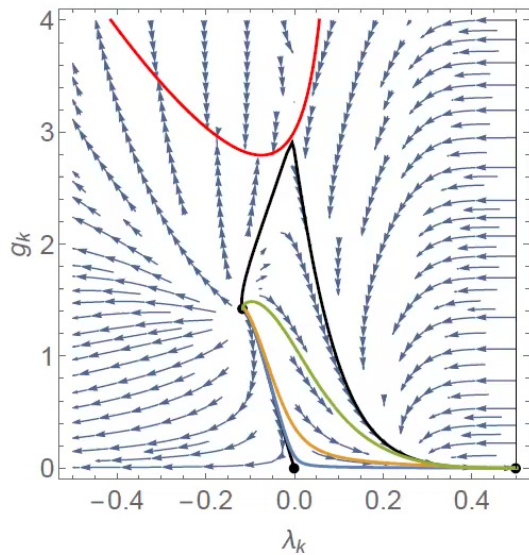
- allow to take $k \rightarrow \infty$ in a controlled way

Fixed points in the presence of matter fields



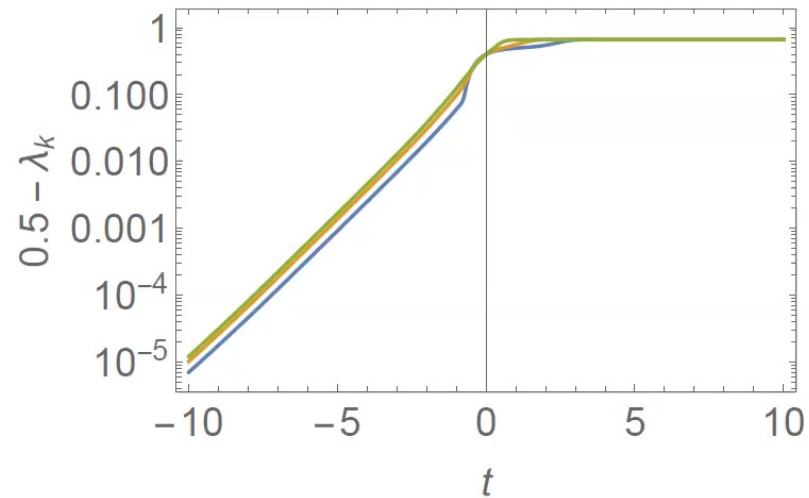
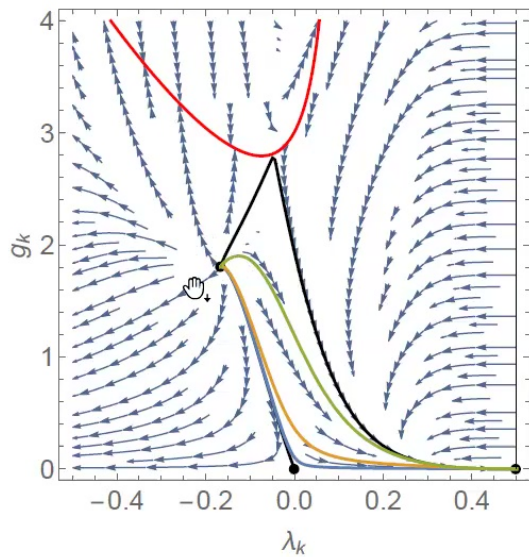
- NGFP (Asymptotic Safety): exists in the orange region
- green line: NGFP annihilates into the complex plain

Phase diagram I: pure gravity



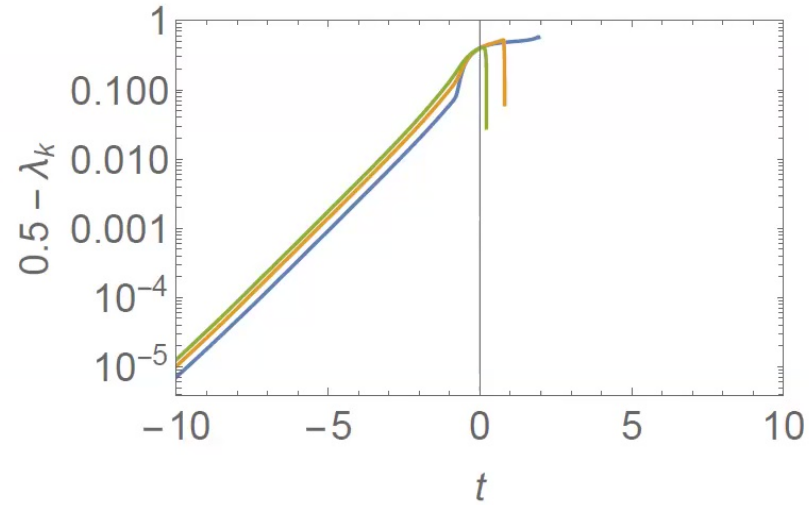
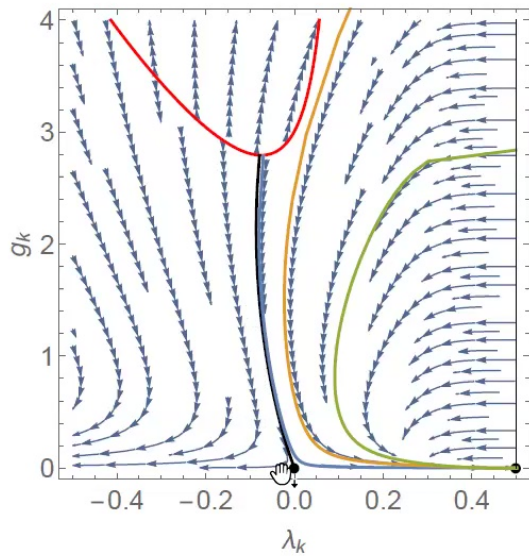
- RG flow: dominated by NGFP, GFP, IR-FP
- IR-FP is an attractor for $k \rightarrow 0$
 \implies zero renormalized graviton mass!

Phase diagram II: $N_s = 4, N_v = 1$



- qualitatively identical to the pure gravity case!

Phase diagram II: $N_s = 4, N_v = 10$



- NGFP has annihilated
- IR-FP persists also in the absence of a UV-completion
 \implies zero renormalized graviton mass!

RG flows at Lorentzian signature

properties of the ADM-decomposition

- the foliation allows to decompose $p^\mu \mapsto \{p^0, \vec{p}\}$
- we can use regulators $\mathcal{R}_k(\vec{p}^2)$
 - pro: does not modify analytic structure of propagators
 - con: regularization is not Lorentz covariant

analytic continuation of the Lapse function (no complex geometry)

$$N(t, \vec{y}) \mapsto iN(t, \vec{y})$$

!! beta functions for both signatures are identical !!

- NGFP carries over from Euclidean signature
- uniqueness of Lorentzian construction from Euclidean setting

Summary . . .

graviton 2-point function:

$$\Gamma_k = \frac{1}{64\pi G_k} \int d^4x h_{ij} [-\partial^2 + \mu_k^2] h^{ij} .$$

RG flow exhibits two remarkable fixed points:

- **non-Gaussian fixed point:** (high-energy completion)
 - exists for pure gravity
 - generalizes to gravity-matter systems
 - ⇒ bounds on matter degrees of freedom
- **IR fixed point:** (renormalized couplings)
 - yields $\mu^2 = 0$ dynamically
 - independent of UV-completion (EFT property?)

Outlook

phenomenology of cosmological observables:

- flat background \implies cosmological background
- generalize momentum dependence of 2-point function
- 2-point correlators \implies 3-point correlators (non-Gaussianities)

transitioning to Lorentzian signature:

- implement analytic continuation of Lapse function
- study vacuum dependence of the Lorentzian flow equation



Questions?

