Title: Every quantum helps: Operational advantage of quantum resources beyond convexity

Speakers: Kohdai Kuroiwa

Series: Perimeter Institute Quantum Discussions

Date: January 10, 2024 - 11:00 AM

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Abstract: As quantum technologies are expected to provide us with unprecedented benefits, identifying what quantum-mechanical properties are useful is a pivotal question. Quantum resource theories provide a unified framework to analyze such quantum properties, which has been successful in the understanding of fundamental properties such as entanglement and coherence. While these are examples of convex resources, for which quantum advantages can always be identified, many physical resources are described by a non-convex set of free states and their interpretation has so far remained elusive. In this work, we address the fundamental question of the usefulness of quantum resources without convexity assumption, by providing two operational interpretations of the generalized robustness resource measure in general resource theories. On the one hand, we characterize the generalized robustness in terms of a non-linear resource witness and reveal that any state is more advantageous than a free one in some multi-copy channel discrimination task. On the other hand, we consider a scenario where a theory is characterized by multiple constraints and show that the generalized robustness coincides with the worst-case advantage in a single-copy channel discrimination setting. We further extend these results to the weight resource measure and QRTs for quantum channels and quantum instruments. Based on these characterizations, we conclude that every quantum resource state shows a qualitative and quantitative advantage in discrimination problems in a general resource theory even without any assumption on the structure of the free sets. This talk is based on arXiv:2310.09154 and arXiv:2310.09321.

Zoom link https://pitp.zoom.us/j/97730859535?pwd=VExLK0hNN2FHNVFWUW12RUM3d05UUT09

EVERY QUANTUM HELPS: OPERATIONAL ADVANTAGE OF QUANTUM RESOURCES BEYOND CONVEXITY

Kohdai Kuroiwa, Ryuji Takagi, Gerardo Adesso, and Hayata Yamasaki

PI QI group meeting, 10th January 2024

arXiv:2310.09154

arXiv:2310.09321

QUANTUM RESOURCES

Quantum properties:

Resources to overcome restrictions on quantum operations

Example: entanglement, magic states, non-Gaussianity, etc

What quantum properties show "**advantage**" in information processing?

EXAMPLE: ENTANGLEMENT

- All bipartite entangled states can enhance a teleportation power of some other states. L. Masanes, Phys. Rev. Lett. 96, 150501 (2006)

- All entangled states are useful for some channel discrimination task. M. Piani and J. Watrous, Phys. Rev. Lett. 102, 250501 (2009)

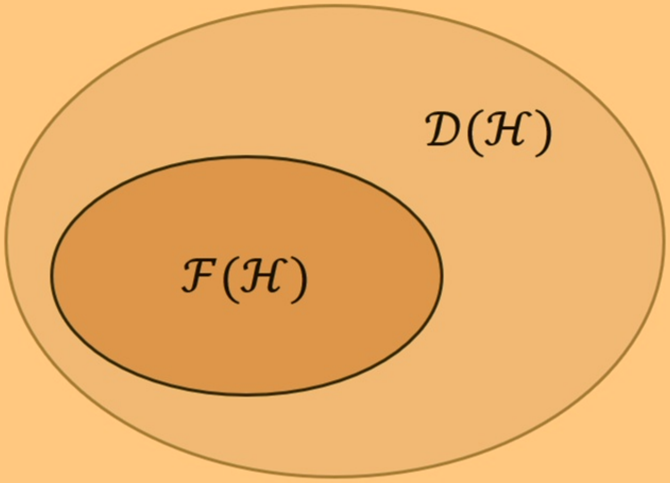
- All entangled states show non-zero "non-classicality" in teleportation. D. Cavalcanti, P. Skrzypczyk, and I. Šupić, Phys. Rev. Lett. 119, 110501 (2017)

QUANTUM RESOURCE THEORIES

E. Chitambar and G. Gour
Rev. Mod. Phys. 91, 025001 (2019)

- Quantum Resource Theories:
  Unified framework for analyzing these quantum properties.
  Manipulation and quantification of resources under a restricted class of operation

Main components



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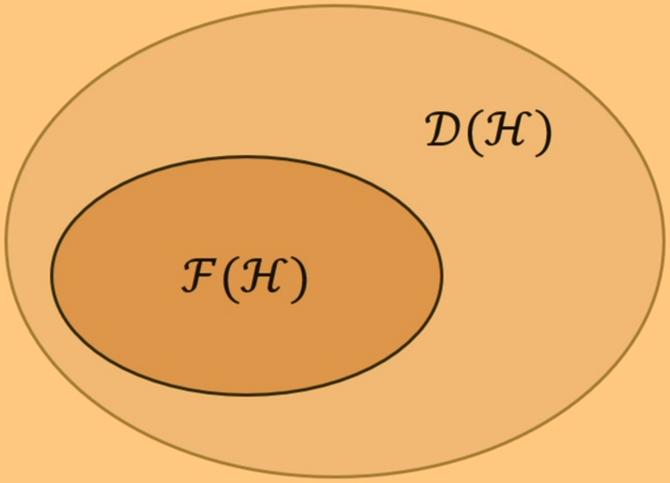
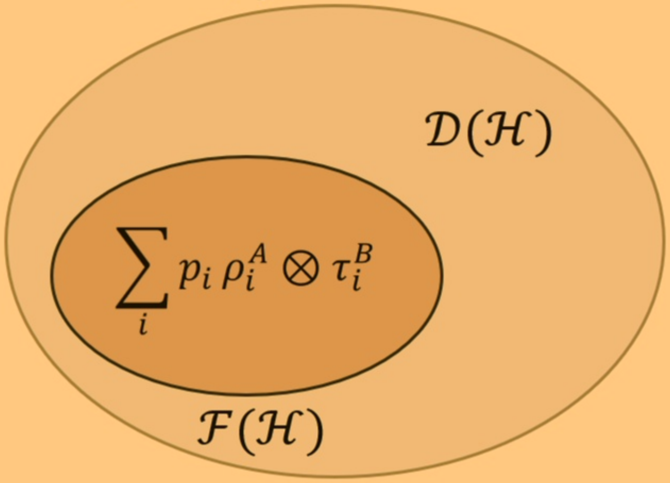
Main components

1. A class of quantum operations (free operations)
2. $\mathcal{F}(\mathcal{H}) \subseteq \mathcal{D}(\mathcal{H})$:
   States that can be prepared by free operations
   (free states)
3. $\mathcal{D}(\mathcal{H}) \backslash \mathcal{F}(\mathcal{H})$:
   Non-free states (resource states)



QUANTUM RESOURCE THEORY OF ENTANGLEMENT

- Free operations = LOCC (local operations + classical communication)
- $\mathcal{F}(\mathcal{H}^{AB}) = \left\{\sum_i p_i\, \rho_i^A \otimes \tau_i^B : \rho_i^A \in \mathcal{D}(\mathcal{H}^A), \tau_i^B \in \mathcal{D}(\mathcal{H}^B)\right\}$: Separable state
- $\mathcal{D}(\mathcal{H})\backslash\mathcal{F}(\mathcal{H})$: Entangled states



OTHER EXAMPLES OF RESOURCE THEORIES

- Resource theory of magic states:

  $\mathcal{F}(\mathcal{H}) = \mathrm{conv}\{U|0\rangle\langle 0|U^\dagger : U : \mathrm{Clifford}\}$: Stabilizer states

  V. Veitch *et al.*, New J. Phys. 16 013009 (2014)

- Resource theory of coherence:

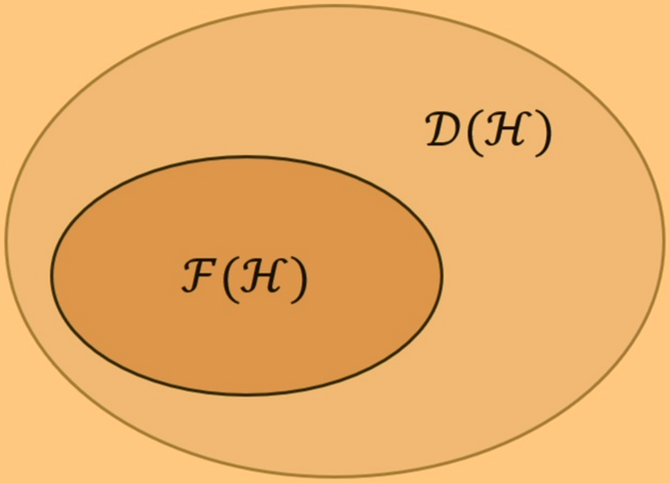
  $\mathcal{F}(\mathcal{H})$: Diagonal states in a fixed basis

  A. Streltsov *et al.*, Rev. Mod. Phys. 89, 041003 (2017)

- Resource theory of non-Gaussianity:

  $\mathcal{F}(\mathcal{H})$: Gaussian states (convex hull)

  R. Takagi and Q. Zhuang, Phys. Rev. A 97, 062337 (2018);
  F. Albarelli *et al.*, Phys. Rev. A 98, 052350 (2018).

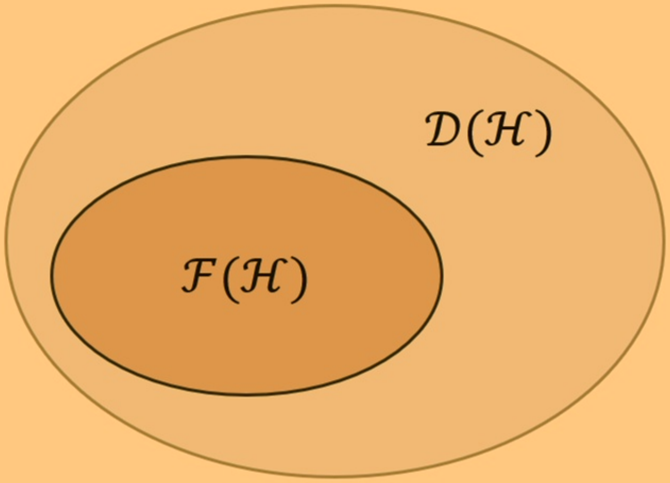


GENERAL QUANTUM RESOURCE THEORIES

Investigate universal properties shared among many quantum resources

$\mathcal{F}(\mathcal{H}) \subseteq \mathcal{D}(\mathcal{H})$: a subset of quantum states with some mathematical properties, e.g.,

1. Closedness
2. Convexity
3. Containing the maximally mixed state



OUR QUESTION

What quantum properties show "**advantage**" in information processing?

Are ALL resource states useful in some information processing task in quantum resource theories?

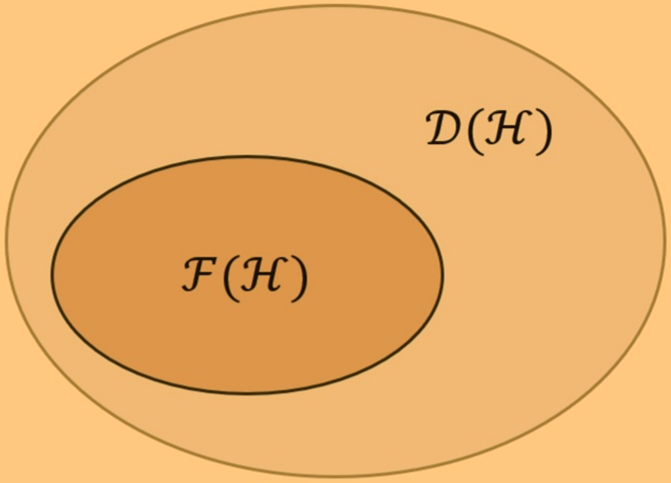
Previous approach: Usefulness of resources in convex resource theories

R. Takagi *et al.*, Phys. Rev. Lett. 122, 140402 (2019);
R. Takagi and B. Regula, Phys. Rev. X 9, 031053 (2019).

CONVEX RESOURCE THEORIES

Convex resource theories =
Quantum resource theories
with convex (and closed) $\mathcal{F}(\mathcal{H})$

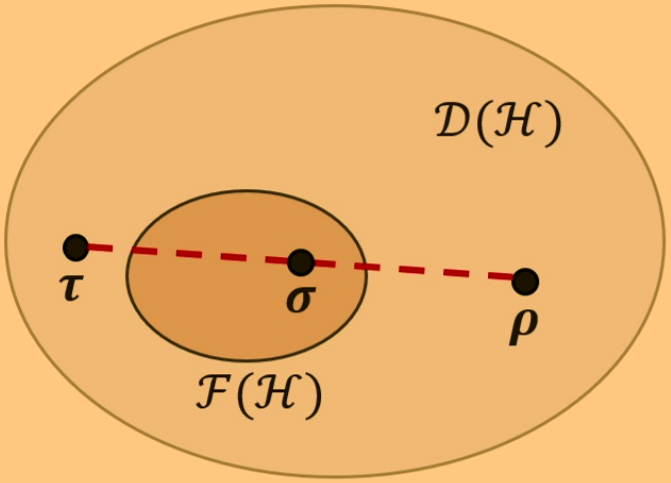
$$p\sigma_1 + (1-p)\sigma_2 \in \mathcal{F}(\mathcal{H}),$$
$$(\forall \sigma_1, \sigma_2 \in \mathcal{F}(\mathcal{H}), \forall p \in [0,1])$$



Examples: Entanglement, coherence, magic states

Z.W. Liu *et al.*, Phys. Rev. Lett. 123, 020401, (2019);
R. Takagi and B. Regula, Phys. Rev. X 9, 031053 (2019);
B. Regula *et al.*, Phys. Rev. A 101, 062315 (2020);
etc…

ROBUSTNESS OF RESOURCE



Generalized robustness

$$R_{\mathcal{F}(\mathcal{H})}(\rho) :=$$

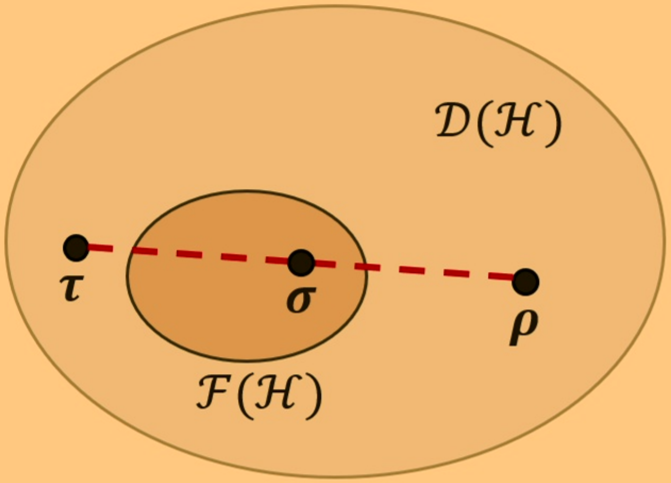
$$\min\left\{s \geq 0 \mid \frac{\rho+s\,\tau}{1+s} =: \sigma \in \mathcal{F}(\mathcal{H}), \tau \in \mathcal{D}(\mathcal{H})\right\}$$

Computability

SDP formulation for several convex resources:
coherence, asymmetry, magic states etc

M. Steiner, Phys. Rev. A 67, 054305 (2003);
A. W. Harrow and M. A. Nielsen,
Phys. Rev. A 68, 012308 (2003).

ROBUSTNESS OF RESOURCE



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Generalized robustness

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Computability

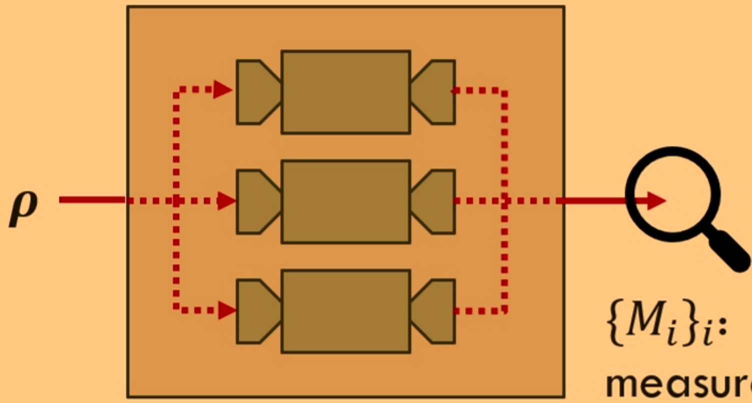
SDP formulation for several convex resources:
coherence, asymmetry, magic states etc

Faithfulness

$$R_{\mathcal{F}(\mathcal{H})}(\rho) = 0 \Leftrightarrow \rho \in \mathcal{F}(\mathcal{H})$$

CHANNEL DISCRIMINATION USING RESOURCE

$\{p_i, \Psi_i\}$: channel ensemble



$\{M_i\}_i$: measurement

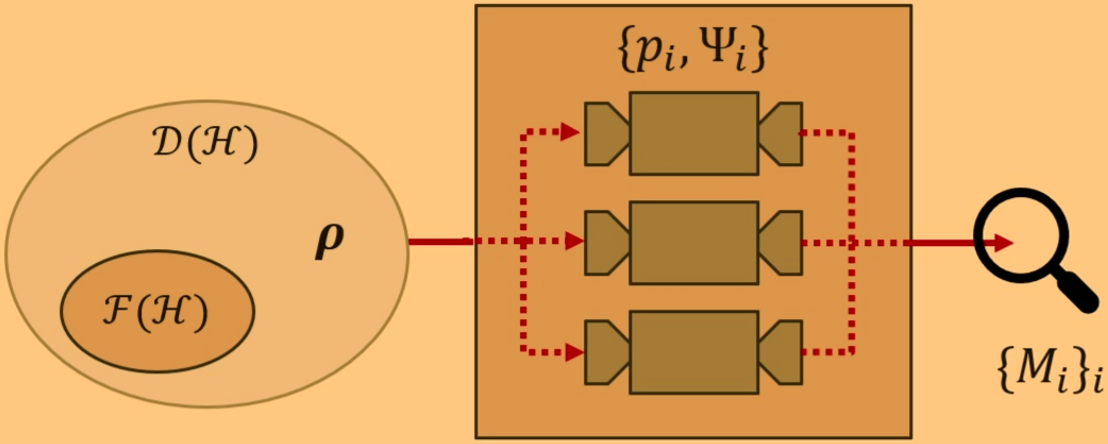
Goal:

to maximize the probability of successfully identifying the channel

$$p_{\text{succ}}(\rho, \{p_i, \Psi_i\}_i, \{M_i\}_i) = \sum_i p_i \mathrm{Tr}[M_i \Psi_i(\rho)]$$

USEFULNESS OF ALL CONVEX RESOURCES

R. Takagi *et al.*, Phys. Rev. Lett. 122, 140402 (2019);
R. Takagi and B. Regula, Phys. Rev. X 9, 031053 (2019).



$$p_{\mathrm{succ}}(\rho, \{p_i, \Psi_i\}_i, \{M_i\}_i) = \sum_i p_i \mathrm{Tr}[M_i \Psi_i(\rho)]$$

$$\max_{\{p_i, \Psi_i\}, \{M_i\}} \frac{p_{\mathrm{succ}}(\rho, \{p_i, \Psi_i\}, \{M_i\})}{\max_{\sigma \in \mathcal{F}(\mathcal{H})} p_{\mathrm{succ}}(\sigma, \{p_i, \Psi_i\}, \{M_i\})} = 1 + R_{\mathcal{F}}(\rho)$$

USEFULNESS OF ALL CONVEX RESOURCES

R. Takagi *et al.*, Phys. Rev. Lett. 122, 140402 (2019);
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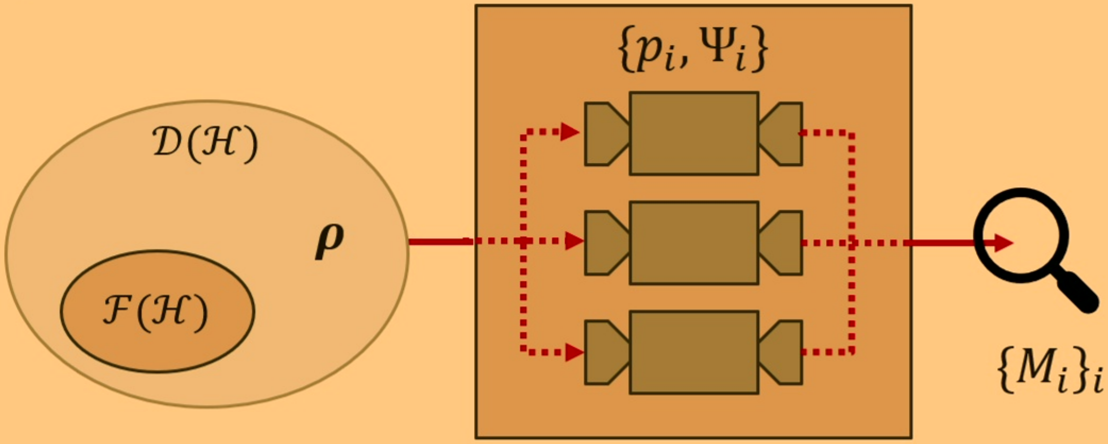


$$p_{\rm succ}(\rho, \{p_i, \Psi_i\}_i, \{M_i\}_i) = \sum_i p_i \mathrm{Tr}[M_i \Psi_i(\rho)]$$

Operational advantage of every resource state in convex resource theory

$$\boxed{\max_{\{p_i, \Psi_i\}, \{M_i\}} \frac{p_{\rm succ}(\rho, \{p_i, \Psi_i\}, \{M_i\})}{\max_{\sigma \in \mathcal{F}(\mathcal{H})} p_{\rm succ}(\sigma, \{p_i, \Psi_i\}, \{M_i\})} = 1 + R_{\mathcal{F}}(\rho)}$$

HOWEVER…

Not all physically-motivated quantum resources can be covered by this convex framework

G. Adesso, Metrological resources braced for the worst, Quantum Resources 2022.

Challenge in Establishing General QRTs

What are universal properties of quantum resources shared among QRTs?

General QRTs: Investigating universal properties shared among many resources

Previous works on general QRTs with **limited applicability**

Mathematical assumptions have been imposed to make the analysis tractable

e.g. uniqueness of a maximal resourceful state, convexity, finite-dimensionality

E.g., Z.W. Liu et al. (2019), R. Takagi et al. (2019), B. Regula et al. (2020),…

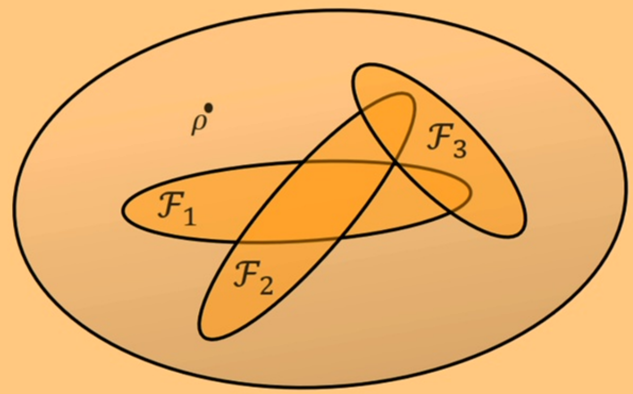
Challenge: **Physically well-motivated resources** may not satisfy assumptions to make analysis tractable

- **Non-uniqueness of max resources**: magic on qutrits, coherence with physically incoherent operations
- **Non-convexity**: non-Gaussianity, quantum discord, quantum Markov chain
- **Infinite-dimensionality**: non-Gaussianity

K. Kuroiwa, H. Yamasaki, General Quantum Resource Theories: Maximal Resources, Catalytic Replication, and Asymptotically Consistent Measures, Quantum Resources 2022.

NON CONVEX RESOURCES

Free states: $\mathcal{F} = \bigcup_j \mathcal{F}_j$



NON-CONVEX RESOURCE THEORIES

Quantum discord:

$$\mathcal{F} = \bigcup_{\{|a_k\rangle\}} \left\{ \sum_k p_k |a_k\rangle\langle a_k| \otimes \rho_k \right\}$$

A. Bera *et al.*, Rep. Prog. Phys. 81, 024001 (2017).

Quantum non-Markovianity:

$$\mathcal{F} = \{\rho^{ABC} : I(A:C|B) = 0\}$$

E. Wakakuwa, arXiv:1709.07248 [quant-ph] (2017).

Thermodynamics with multiple baths:

$$\mathcal{F} = \bigcup_k \left\{ \frac{e^{-\beta_k H}}{\mathrm{Tr}[e^{-\beta_k H}]} \right\}$$

L. A. Correa *et al.*, Sci. Rep. 4, 3949 (2014);
H. Wojewódka- Ściążko *et al.*, arXiv:2304.09559 [quant-ph] (2023).

CONVEXITY ITSELF MAY BE RESOURCE

PHYSICAL REVIEW LETTERS **121,** 190504 (2018)

Quantifying Resources in General Resource Theory with Catalysts

Anurag Anshu,[1,*] Min-Hsiu Hsieh,[2,†] and Rahul Jain[3,‡]
[1]*Center for Quantum Technologies, National University of Singapore, 21 Lower Kent Ridge Rd, 119077 Singapore*
[2]*Centre for Quantum Software and Information, Faculty of Engineering and Information Technology, University of Technology Sydney, Sydney, NSW 2007, Australia*
[3]*Centre for Quantum Technologies and Department of Computer Science, National University of Singapore and MajuLab CNRS-UNS-NUS-NTU International Joint Research Unit, UMI 3654, Singapore*

Investigated "randomness cost" needed for transformation with catalysts

PHYSICAL REVIEW A **72**, 032317 (2005)

Quantum, classical, and total amount of correlations in a quantum state

Berry Groisman,[1,*] Sandu Popescu,[1,2,†] and Andreas Winter[3,‡]
[1]*H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, United Kingdom*
[2]*Hewlett-Packard Laboratories, Stoke Gifford, Bristol BS12 6QZ, United Kingdom*
[3]*Department of Mathematics, University of Bristol, Bristol BS8 1TW, United Kingdom*
(Received 1 February 2005; published 13 September 2005)

Investigating erasure of classical correlation (convex combination)

OUR FOCUS

Are **all** quantum resources
without "convex assumption" useful at all?

➡ Yes!! (shown in this talk)

WHY CONVEX WORKS WELL

Main technique used in convex resource theories:
Convex optimization

$$R_{\mathcal{F}(\mathcal{H})} = \min\left\{ s \geq 0 \mid \frac{\rho + s\,\tau}{1+s} =: \sigma \in \mathcal{F}(\mathcal{H}), \tau \in \mathcal{D}(\mathcal{H}) \right\}$$

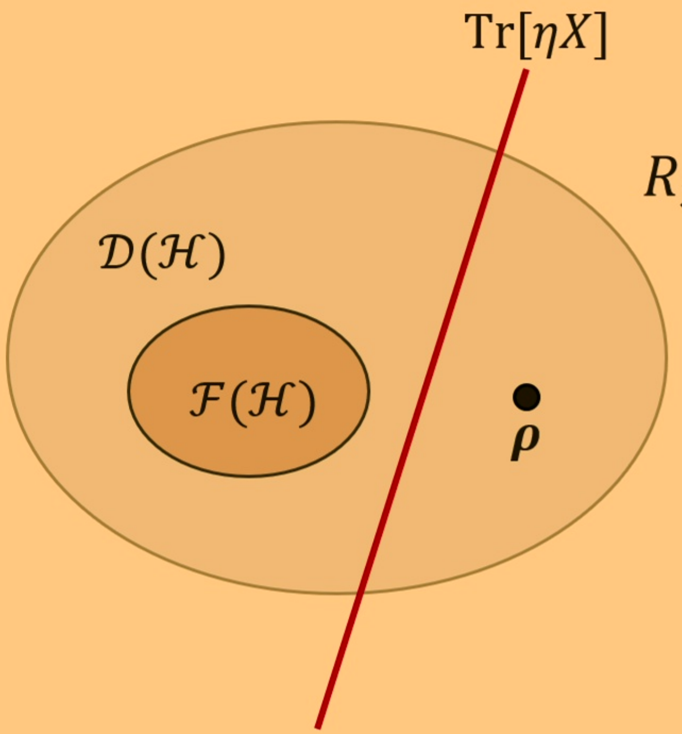
$\Updownarrow$ Convex dual

Maximize: $\mathrm{Tr}[\rho X] - 1$
subject to:

$X \geq 0$

$\mathrm{Tr}[\sigma X] \leq 1 \;\; \forall \sigma \in \mathcal{F}$

WHY CONVEX WORKS WELL



Main technique used in convex resource theories: Convex optimization

$$R_{\mathcal{F}(\mathcal{H})} = \min\left\{ s \geq 0 \mid \frac{\rho + s\,\tau}{1+s} =: \sigma \in \mathcal{F}(\mathcal{H}), \tau \in \mathcal{D}(\mathcal{H}) \right\}$$

Strong duality $\Updownarrow$ Convex dual

$R_{\mathcal{F}(\mathcal{H})} =$ Maximize: $\mathrm{Tr}[\rho X] - 1$

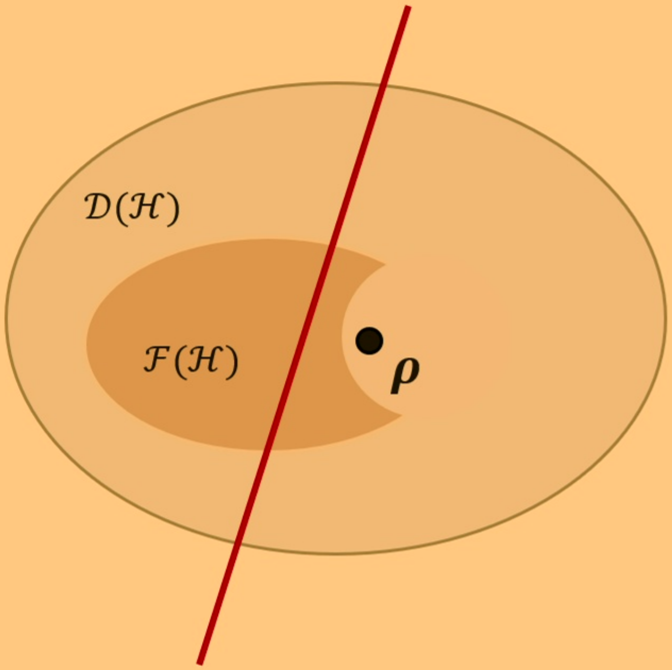
subject to:

$X \geq 0$

$\mathrm{Tr}[\sigma X] \leq 1 \;\; \forall \sigma \in \mathcal{F}$

Linear resource witness!

WHY IT FAILS FOR NON-CONVEX



When $\mathcal{F}(\mathcal{H})$ is non-convex

Linear resource witness does NOT exist in general…

How can we resolve this issue?

1. Non-linear resource witness
2. Decomposition of $\mathcal{F}(\mathcal{H})$ into several convex sets

Result1:
Advantage Based on Multicopy Witness

MULTICOPY WITNESS

Main idea: Construct "non-linear" witness based on generalized robustness instead of conventional linear witness

$$R_{\mathcal{F}(\mathcal{H})}(\rho) := \min\left\{s \geq 0 \mid \frac{\rho + s\,\tau}{1+s} =: \sigma \in \mathcal{F}(\mathcal{H}), \tau \in \mathcal{D}(\mathcal{H})\right\}$$

$\overset{?}{\Longrightarrow}$

For some Hermitian operator W and positive integer m,

- $\mathrm{Tr}\left[\rho^{\otimes m} W\right] < 0$
- $\mathrm{Tr}\left[\sigma^{\otimes m} W\right] \geq 0, \forall \sigma \in \mathcal{F}(\mathcal{H})$

MULTICOPY WITNESS: 1-QUBIT CASE

$$R_{\mathcal{F}(\mathcal{H})}(\rho) \coloneqq \min\left\{s \geq 0 \mid \frac{\rho + s\,\tau}{1+s} =: \sigma \in \mathcal{F}(\mathcal{H}), \tau \in \mathcal{D}(\mathcal{H})\right\}$$

For a fixed s,

consider the set of quantum states $\eta = \frac{\rho + s\phi}{1+s}$ for some pure state ϕ

MULTICOPY WITNESS: 1-QUBIT CASE

$$R_{\mathcal{F}(\mathcal{H})}(\rho) := \min\left\{s \geq 0 \mid \frac{\rho + s\,\tau}{1+s} =: \sigma \in \mathcal{F}(\mathcal{H}), \tau \in \mathcal{D}(\mathcal{H})\right\}$$

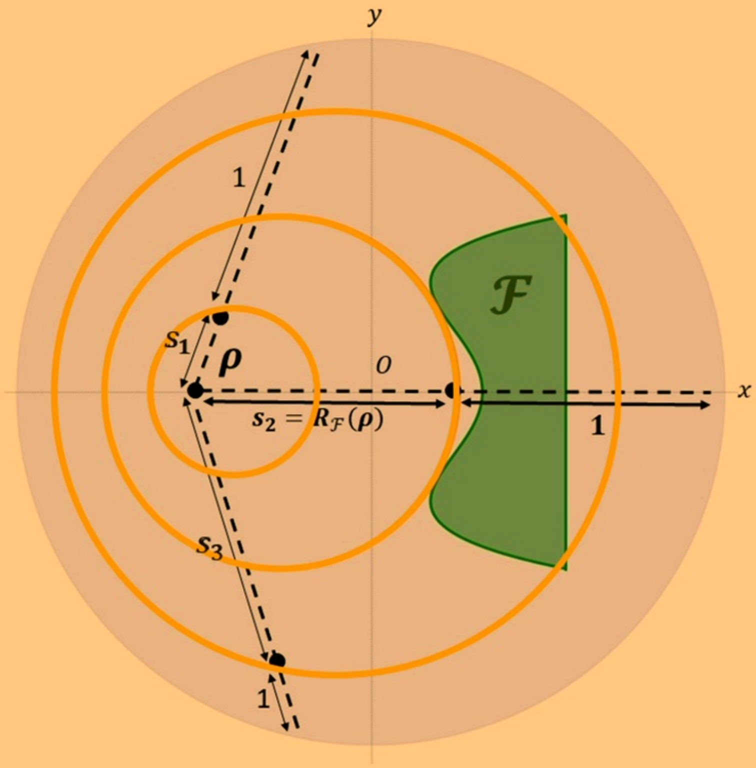
For a fixed s,

consider the set of quantum states $\eta = \frac{\rho+s\phi}{1+s}$ for some pure state ϕ

Use this set as a curve separating ρ from $\mathcal{F}(\mathcal{H})$

MULTICOPY WITNESS: 1-QUBIT CASE

The set of quantum states $\eta = \frac{\rho+s\phi}{1+s}$ for some pure state ϕ



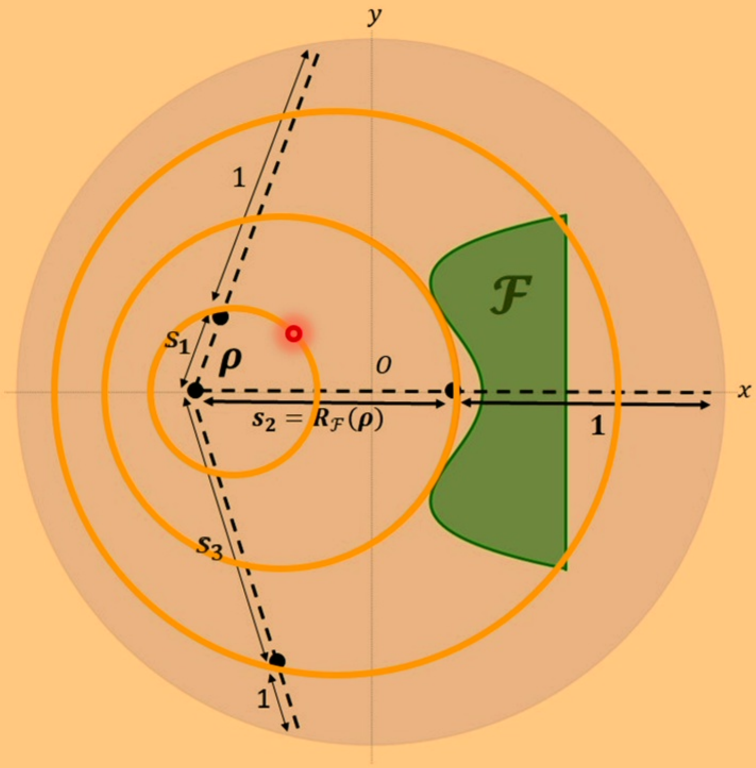
MULTICOPY WITNESS: 1-QUBIT CASE

The set of quantum states $\eta = \frac{\rho + s\phi}{1+s}$ for some pure state ϕ

The curve of (x_1, x_2, x_3):

$$\frac{1}{4} - \sum_{i=1}^{3} \frac{\left((1+s)x_i - r_i\right)^2}{4s^2} = 0$$

(x_1, x_2, x_3): Bloch vector of η
(r_1, r_2, r_3): Bloch vector of ρ



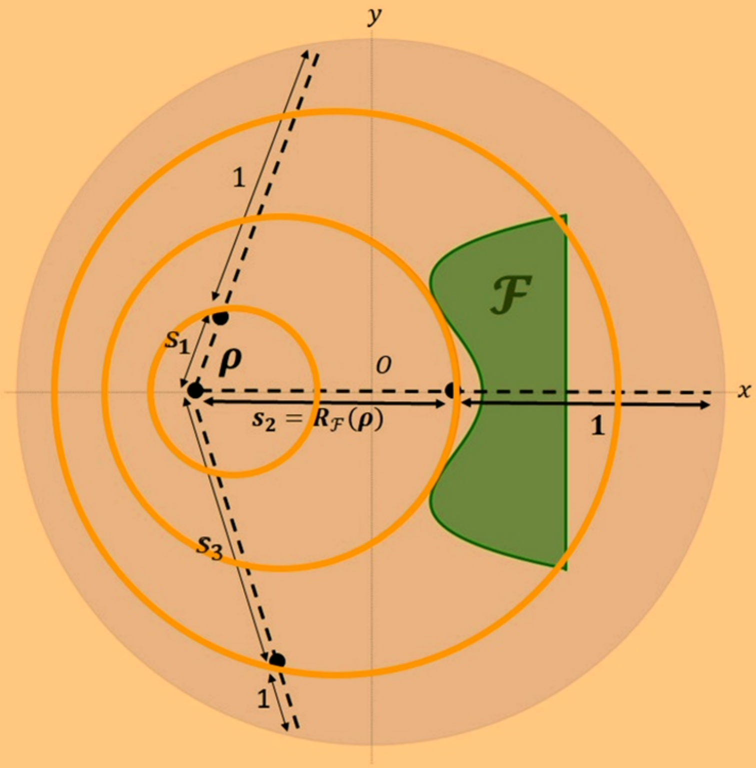
MULTICOPY WITNESS: 1-QUBIT CASE

The set of (x_1, x_2, x_3) satisfying

$$\frac{1}{4} - \sum_{i=1}^{3} \frac{\left((1+s)x_i - r_i\right)^2}{4s^2} = 0$$

We can construct Hermitian operator W_s with

$$\mathrm{Tr}\left[\eta^{\otimes 2} W_s\right] = \frac{1}{4} - \sum_{i=1}^{3} \frac{\left((1+s)x_i - r_i\right)^2}{4s^2}$$



MULTICOPY WITNESS: 1-QUBIT CASE

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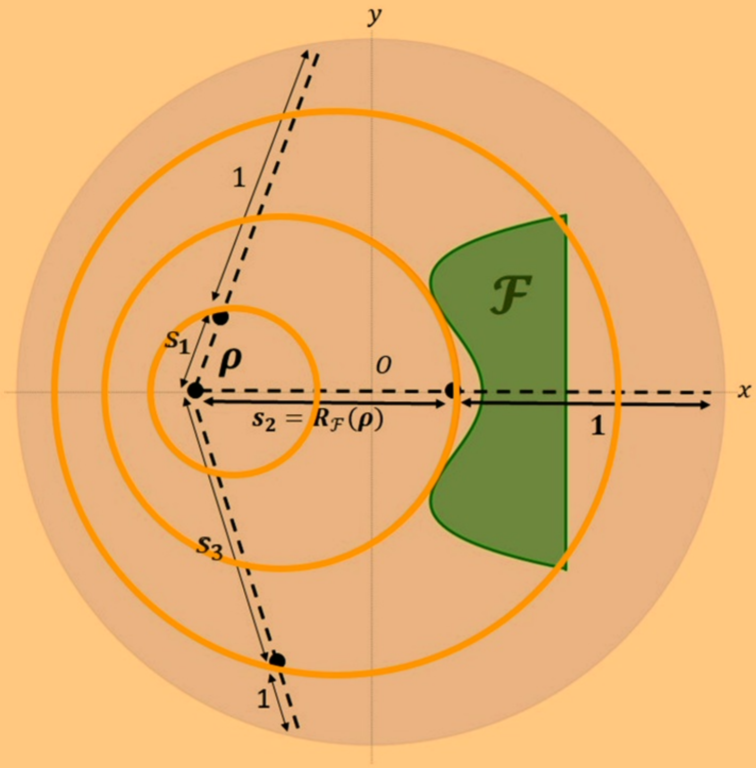
$$\mathrm{Tr}\left[\eta^{\otimes 2} W_s\right] = \frac{1}{4} - \sum_{i=1}^{3} \frac{\left((1+s)x_i - r_i\right)^2}{4s^2}$$

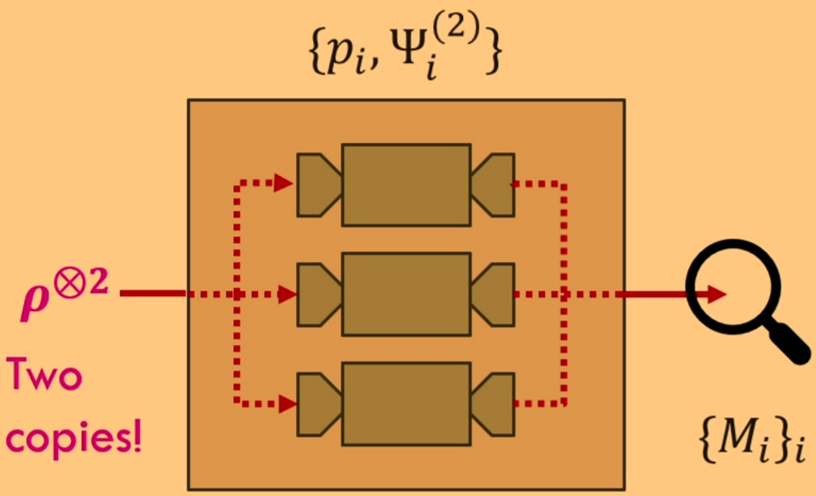
W_s satisfies

- $\mathrm{Tr}\left[\rho^{\otimes 2} W_s\right] < 0$
- $\mathrm{Tr}\left[\sigma^{\otimes 2} W_s\right] \geq 0, \forall \sigma \in \mathcal{F}(\mathcal{H})$

for $s < R_{\mathcal{F}}(\rho)$



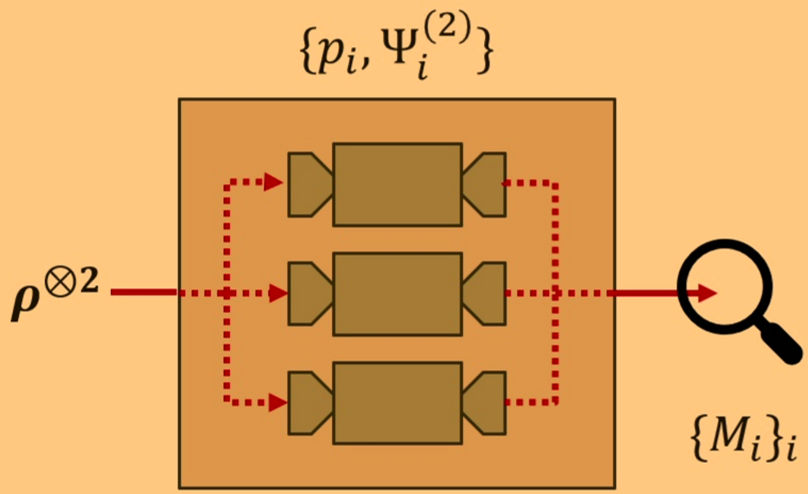
TWO-COPY CHANNEL DISCRIMINATION



$$p_{\mathrm{succ}}\left(\rho^{\otimes 2}, \left\{p_i, \Psi_i^{(2)}\right\}_i, \{M_i\}_i\right)$$
$$= \sum_i p_i \mathrm{Tr}\left[M_i \Psi_i^{(2)}(\rho^{\otimes 2})\right]$$

ADVANTAGE IN TWO-COPY CHANNEL DISCRIMINATION

Based on the quadratic witness W_s ($s < R_{\mathcal{F}(\mathcal{H})}(\rho)$),
we design a two-copy channel discrimination task



cf. M. Piani and J. Watrous, Phys. Rev. Lett. 102, 250501 (2009)

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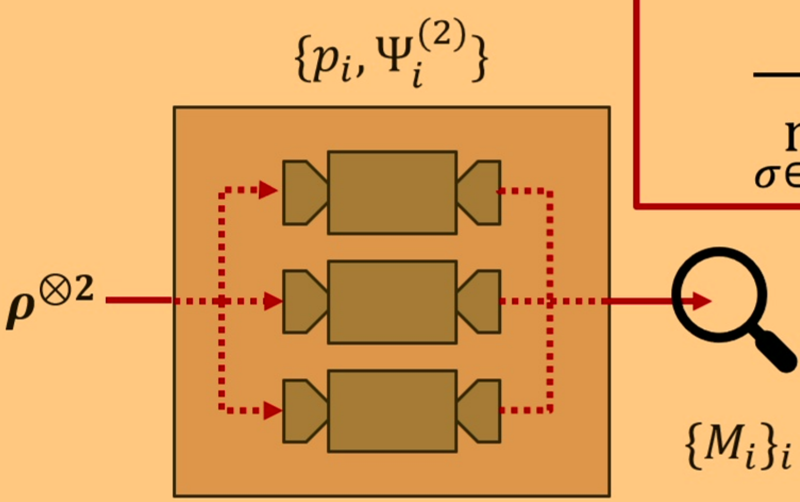

$$\boxed{\frac{\max_{\{M_i\}} p_{\text{succ}}\left(\rho^{\otimes 2}, \left\{p_i, \Psi_i^{(2)}\right\}, \{M_i\}\right)}{\max_{\sigma \in \mathcal{F}(\mathcal{H})} \max_{\{M_i\}} p_{\text{succ}}\left(\sigma^{\otimes 2}, \left\{p_i, \Psi_i^{(2)}\right\}, \{M_i\}\right)} > 1}$$

Advantage of every resource state based on the quadratic witness!

GENERAL CASE

See arXiv:2310.09321 for more details!

Our analysis can be extended to general d-dimensional cases.

We can construct a family $\left(W_s^{(m)}\right)_{m=2}^{d}$ of Hermitian operators with

- $\max_m \mathrm{Tr}\left[\rho^{\otimes m} W_s^{(m)}\right] < 0$
- $\max_m \mathrm{Tr}\left[\sigma^{\otimes m} W_s^{(m)}\right] \geq 0, \forall \sigma \in \mathcal{F}(\mathcal{H})$

$\Leftrightarrow$ $s < R_{\mathcal{F}(\mathcal{H})}$

We can construct a family of channel ensembles with

$$\boxed{\max_m \min_{\sigma \in \mathcal{F}(\mathcal{H})} \frac{\max_{\{M_i\}} p_{\mathrm{succ}}\left(\rho^{\otimes m}, \left\{p_i, \Psi_i^{(m)}\right\}, \{M_i\}\right)}{\max_{\{M_i\}} p_{\mathrm{succ}}\left(\sigma^{\otimes m}, \left\{p_i, \Psi_i^{(m)}\right\}, \{M_i\}\right)} > 1}$$

ADVANTAGE IN TWO-COPY CHANNEL DISCRIMINATION

Based on the quadratic witness W_s ($s < R_{\mathcal{F}(\mathcal{H})}(\rho)$),
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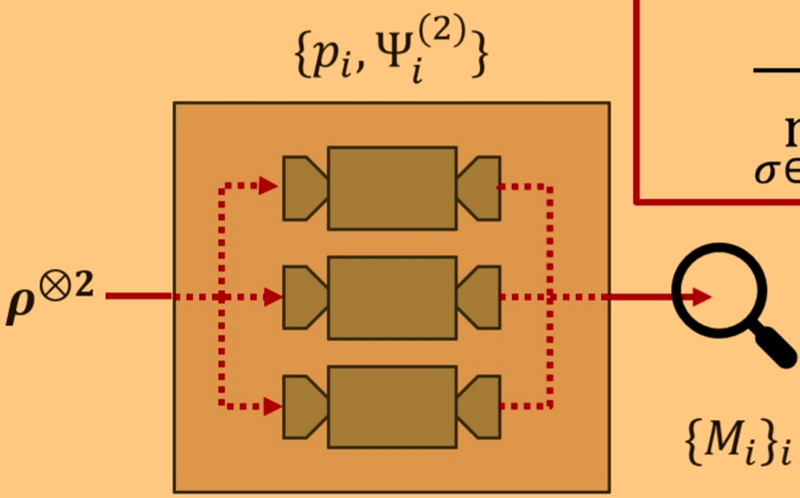
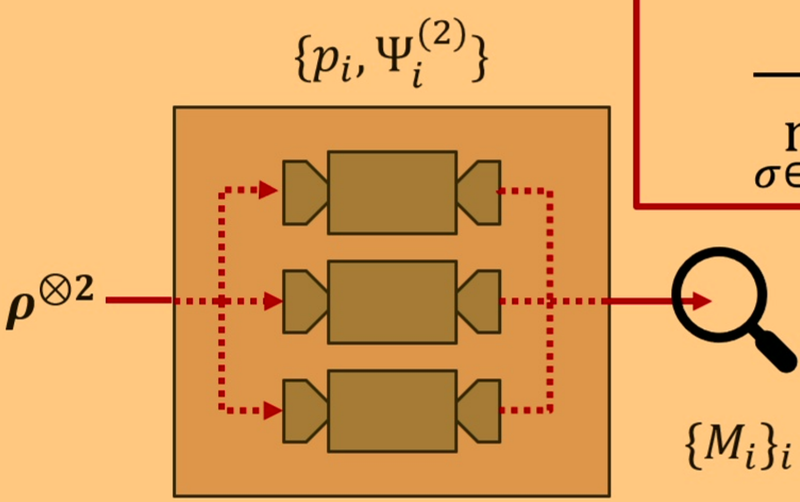


$$\frac{\max_{\{M_i\}} p_{\text{succ}}\left(\rho^{\otimes 2}, \left\{p_i, \Psi_i^{(2)}\right\}, \{M_i\}\right)}{\max_{\sigma \in \mathcal{F}(\mathcal{H})} \max_{\{M_i\}} p_{\text{succ}}\left(\sigma^{\otimes 2}, \left\{p_i, \Psi_i^{(2)}\right\}, \{M_i\}\right)} > 1$$

Advantage of every resource state
based on the quadratic witness!

ADVANTAGE IN TWO-COPY CHANNEL DISCRIMINATION

Based on the quadratic witness W_s $(s < R_{\mathcal{F}(\mathcal{H})}(\rho))$,
we design a two-copy channel discrimination task

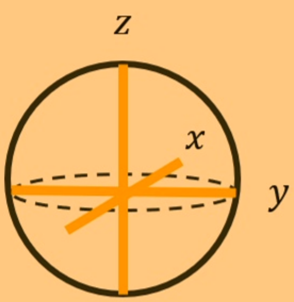


$$\frac{\max_{\{M_i\}} p_{\mathrm{succ}}\left(\rho^{\otimes 2}, \left\{p_i, \Psi_i^{(2)}\right\}, \{M_i\}\right)}{\max_{\sigma \in \mathcal{F}(\mathcal{H})} \max_{\{M_i\}} p_{\mathrm{succ}}\left(\sigma^{\otimes 2}, \left\{p_i, \Psi_i^{(2)}\right\}, \{M_i\}\right)} > 1$$

Advantage of every resource state based on the quadratic witness!

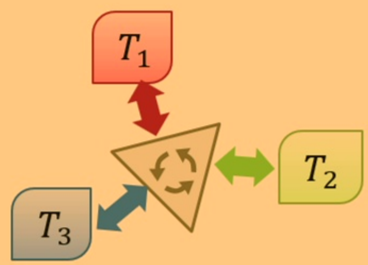
MULTIPLE SETS OF FREE STATES

Coherence in multiple bases



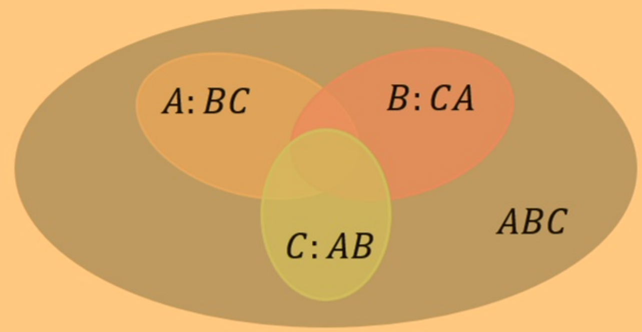
$$\mathcal{F} = \bigcup_{\alpha=x,y,z} \{\rho: \text{diagonal in } \alpha \text{ basis}\}$$

Thermodynamics with several thermal baths



$$\mathcal{F} = \bigcup_{i=1,2,3} \left\{\frac{\mathrm{e}^{-\beta_i H}}{\mathrm{Tr}[\mathrm{e}^{-\beta_i H}]}\right\}$$

Bi-separable entanglement



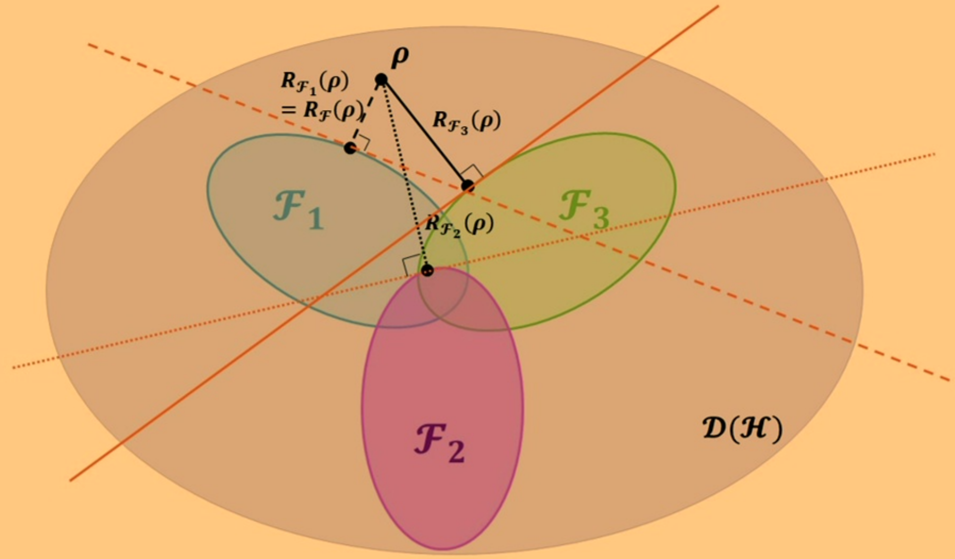
$$\mathcal{F} = \bigcup_{\gamma=A:BC,B:CA,C:AB} \{\rho: \gamma - \text{bi} - \text{separab}$$

WORST-CASE ROBUSTNESS

Convex decomposition

$$\mathcal{F}(\mathcal{H}) = \bigcup_j \mathcal{F}_j(\mathcal{H})$$

$\mathcal{F}_j(\mathcal{H})$: Convex

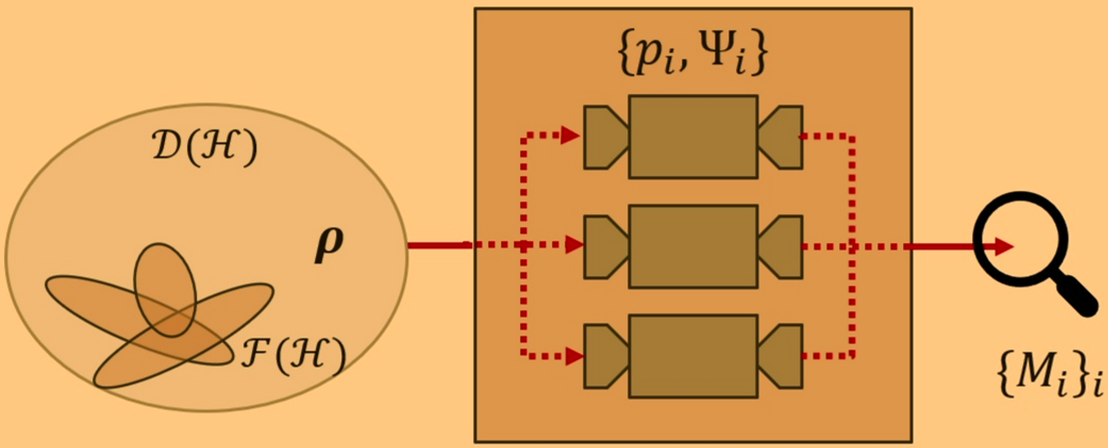


Characterization of robustness by worst-case scenario

$$\boxed{R_{\mathcal{F}(\mathcal{H})}(\rho) = \inf_j R_{\mathcal{F}_j(\mathcal{H})}(\rho)}$$

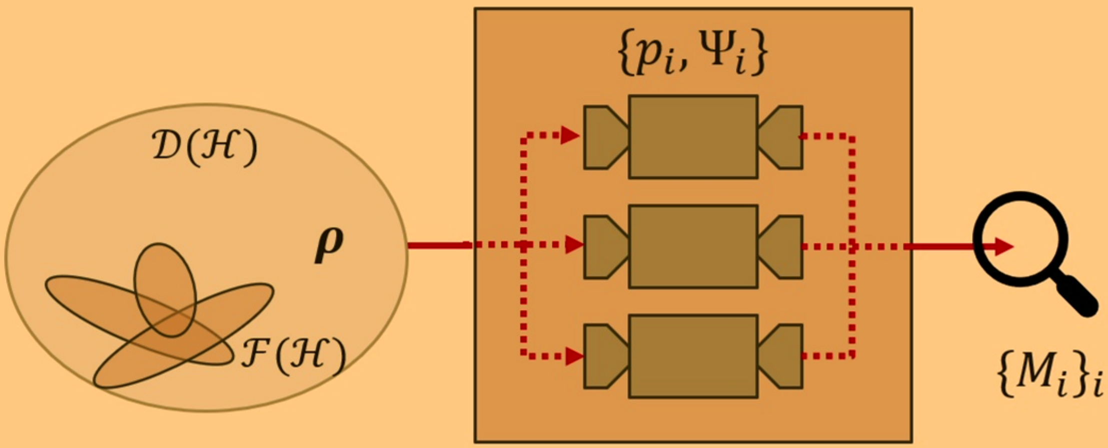
WORST-CASE ADVANTAGE

$$\max_{\{p_i,\Psi_i\},\{M_i\}} \frac{p_{\text{succ}}(\rho,\{p_i,\Psi_i\},\{M_i\})}{\max_{\sigma\in\mathcal{F}_j(\mathcal{H})} p_{\text{succ}}(\sigma,\{\mathrm{p}_i,\Psi_i\},\{M_i\})} = 1 + \quad R_{\mathcal{F}_j(\mathcal{H})}(\rho)$$



WORST-CASE ADVANTAGE

$$\inf_j \max_{\{p_i,\Psi_i\},\{M_i\}} \frac{p_{\text{succ}}(\rho,\{p_i,\Psi_i\},\{M_i\})}{\max_{\sigma\in\mathcal{F}_j(\mathcal{H})} p_{\text{succ}}(\sigma,\{\mathrm{p}_i,\Psi_i\},\{M_i\})} = 1 + \inf_j R_{\mathcal{F}_j(\mathcal{H})}(\rho) = 1 + R_{\mathcal{F}}(\rho)$$



SUMMARY AND DISCUSSION

1. We showed usefulness of every quantum resource without convexity assumption based on (i) Multi-copy witness and (ii) Worst-case robustness.

2. Our analysis can be extended to weight-based resource measure, and dynamical resources represented by quantum channels and instruments.

A. F. Ducuara and P. Skrzypczyk, Phys. Rev. Lett. 125, 110401 (2020);
R. Uola *et al*. Phys. Rev. Lett. 125, 110402 (2020)

FUTURE DIRECTION

1. Our multi-copy witness depends on the dimension. Can we design m-copy witness for constant m?
   Any extension to an infinite-dimensional case?

B. Regula *et al.*, Phys. Rev. Lett. 126, 110403 (2021);
L. Lami *et al.*, Phys. Rev. A 103, 032424 (2021)

2. Can we design another task for which the generalized robustness has an operational meaning without considering decomposition into convex sets?

Thank you!