Title: Quantum Foundations Lecture

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Collection: Quantum Foundations

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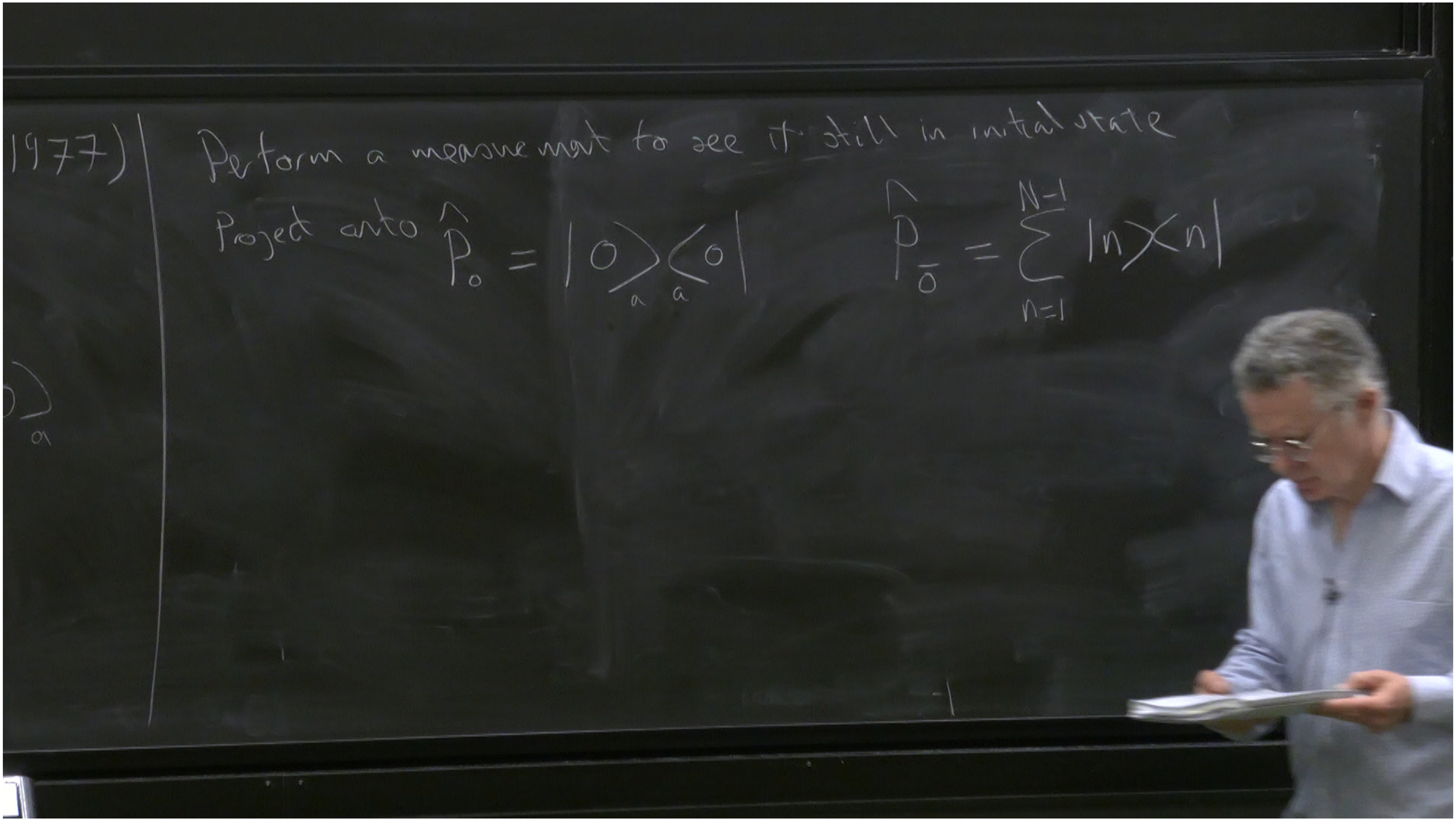
The quantum Zeno effect   Sudershan & Misra (1977)

Pe to

Start with system in some $|0\rangle_a$ at $t=0$.

$$|\psi(\delta t)\rangle_a = e^{-i\hat{H}\delta t/\hbar}|0\rangle_a = \left(1 - \frac{i\hat{H}\delta t}{\hbar}\right)|0\rangle_a$$

$$= |0\rangle_a - \frac{i\hat{H}\delta t}{\hbar}|0\rangle_a + \ldots$$


1977)
Perform a measurement to see if still in initial state
Project onto $\hat{P}_0 = |0\rangle_a \langle 0|_a$
$\hat{P}_{\bar{0}} = \sum_{n=1}^{N-1} |n\rangle\langle n|$

1977)

Perform a measurement to see if still in initial state

Project onto $\hat{P}_0 = |0\rangle_a \langle 0|_a$ $\qquad \hat{P}_{\bar{0}} = \sum_{n=1}^{N-1} |n\rangle\langle n|$

$$\text{prob}(P_{\bar{0}}) = \langle \psi(\delta t) | \hat{P}_{\bar{0}} | \psi(\delta t) \rangle = k^2 (\delta t)^2$$

1977)

Perform a measurement to see if still in initial state

Project onto $\hat{P}_0 = |0\rangle_a \, {}_a\langle 0|$ $\qquad \hat{P}_{\bar{0}} = \sum_{n=1}^{N-1} |n\rangle\langle n|$

$$\text{prob}(P_{\bar{0}}) = \langle \psi(\delta t)|\hat{P}_{\bar{0}}|\psi(\delta t)\rangle = k^2(\delta t)^2$$

$$= \langle 0|\left(1 + \frac{i\hat{H}\delta t}{\hbar}\right)\sum_{n=1}^{N-1} |n\rangle\langle n|\left(1 - \frac{i\hat{H}\delta t}{\hbar}\right)|0\rangle$$

1977)

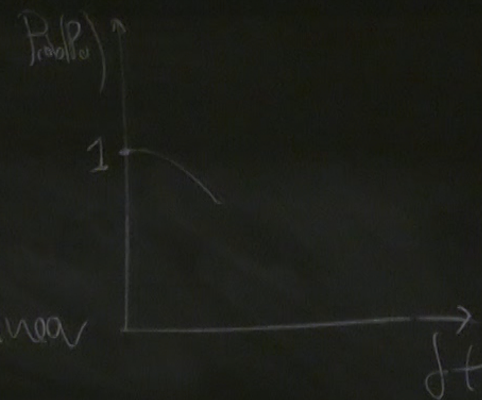
Perform a measurement to see it still in initial state

Project onto $\hat{P}_0 = |0\rangle_a\, {}_a\langle 0|$ $\quad \hat{P}_{\bar{0}} = \sum_{n=1}^{N-1} |n\rangle\langle n|$

$$\text{prob}(P_{\bar{0}}) = \langle \psi(\delta t)|\hat{P}_{\bar{0}}|\psi(\delta t)\rangle = k^2(\delta t)^2 = \sum_{n=1}^{N-1} |\langle 0|\hat{H}|n\rangle|^2 (\delta t)^2$$

$$= \langle 0|\left(\cancel{1} + \frac{i\hat{H}\delta t}{\hbar}\right)\sum_{n=1}^{N-1} |n\rangle\langle n|\left(\cancel{1} - \frac{i\hat{H}\delta t}{\hbar}\right)|0\rangle$$

$$\text{prob}(P_0) = 1 - k^2(\delta t)^2$$



For classical systems initial decay is linear

for example

$$\text{prob}(\delta t) = e^{-\delta t/\tau} = 1 - \frac{\delta t}{\tau} + \dots$$

leads to the Q. Zeno effect

we keep making measurements every δt

$$\text{prob}(0 \text{ after } T) \simeq \left(1 - k^2 (\delta t)^2\right)^{(T/\delta t)}$$

$$N = \frac{T}{\delta t} \qquad \simeq 1 - \left(\frac{T}{\delta t}\right) k^2 (\delta t)^2 = 1 - T k^2 \delta t$$

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We keep making measurements every δt

$$\text{prob}(0 \text{ after } T) \simeq \left(1 - k^2 (\delta t)^2\right)^{\left(T/\delta t\right)}$$

$$N = \frac{T}{\delta t}$$

$$\simeq 1 - \left(\frac{T}{\delta t}\right) k^2 (\delta t)^2 = 1 - T k^2 \delta t$$

$$\rightarrow 1 \text{ as } \delta t \rightarrow 0$$

Quantum optical interferometers

single mode, all same polarization.

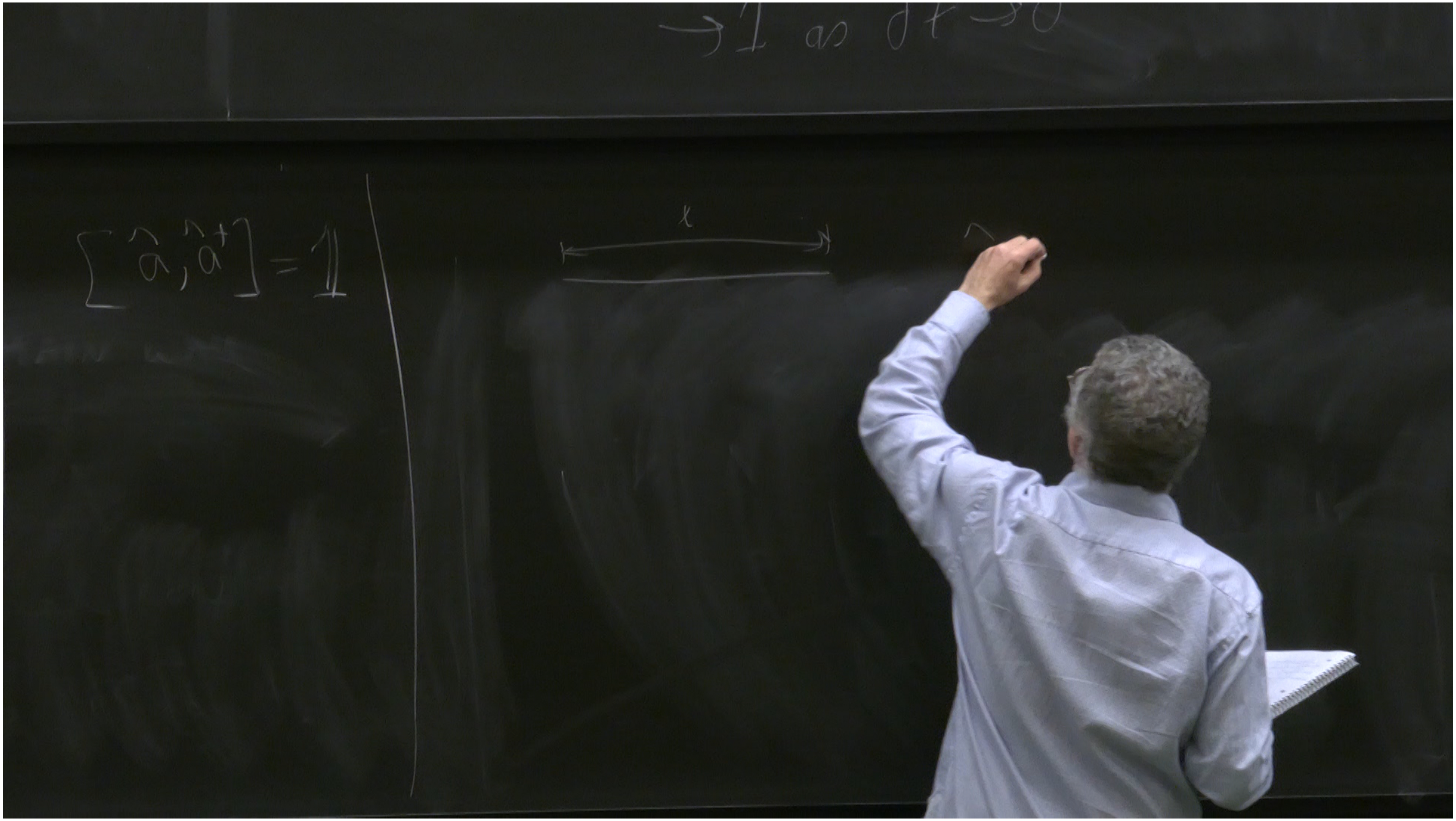
Use creation, $\hat{a}^{+}$, and annihilation operators $\hat{a}$

$\hat{a}|0\rangle_a = 0 \qquad \hat{a}|n\rangle_a = \sqrt{n}\,|n-1\rangle_a,$
$(n>0)$

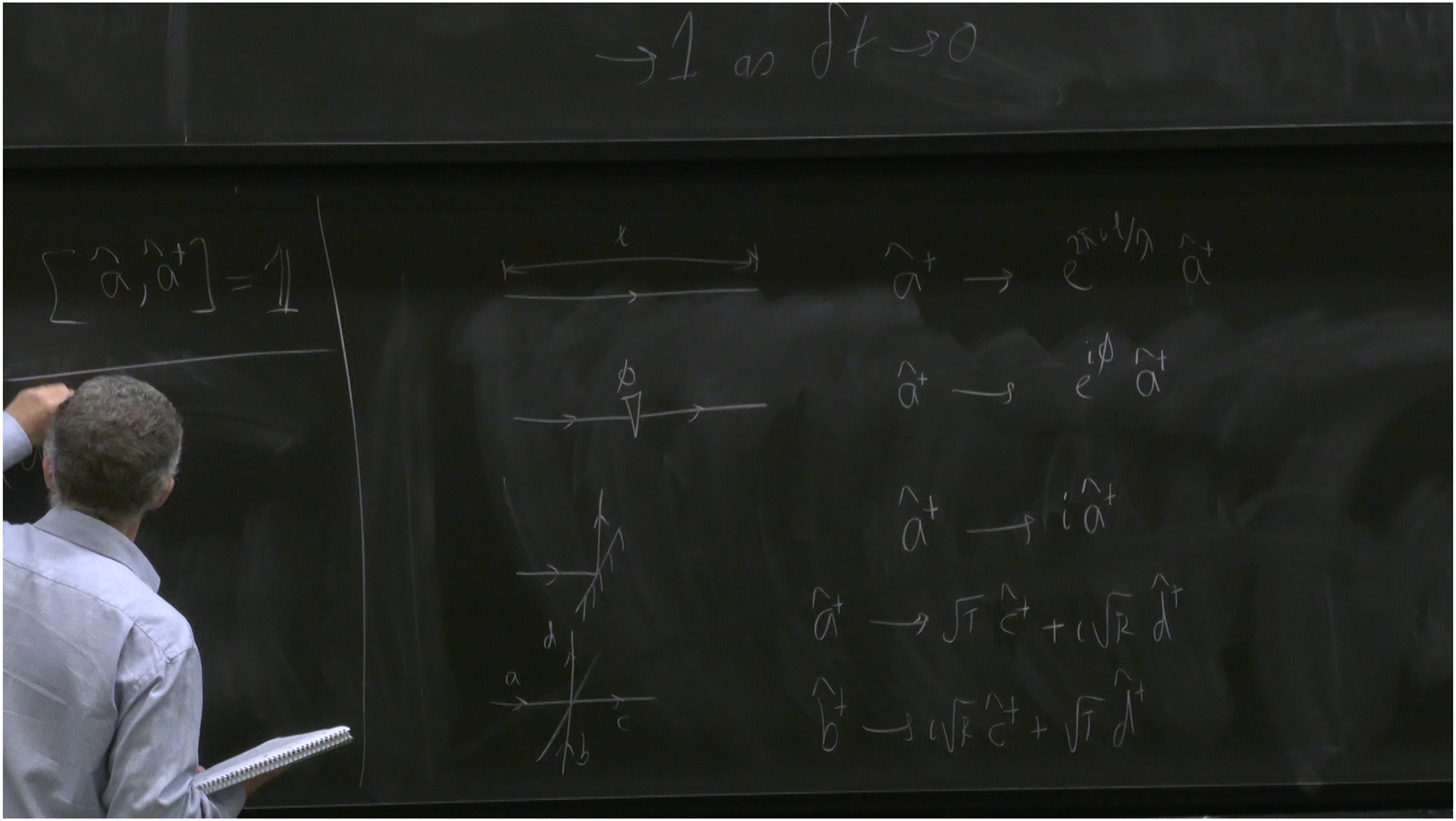
$\hat{a}^{+}|n\rangle_a = \sqrt{n+1}\,|n+1\rangle_a$

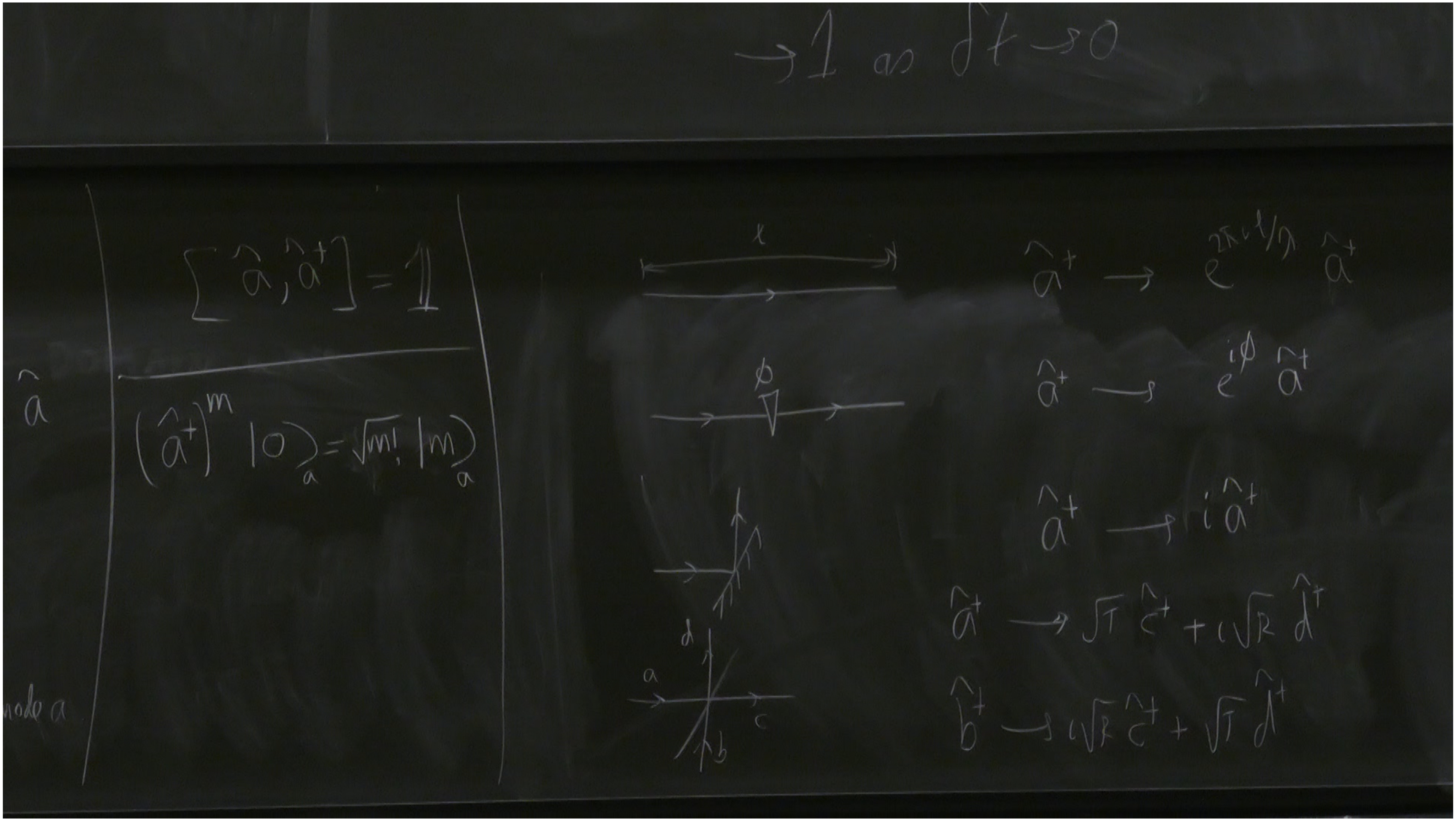
$|n\rangle_a$ means
n photons in mode a

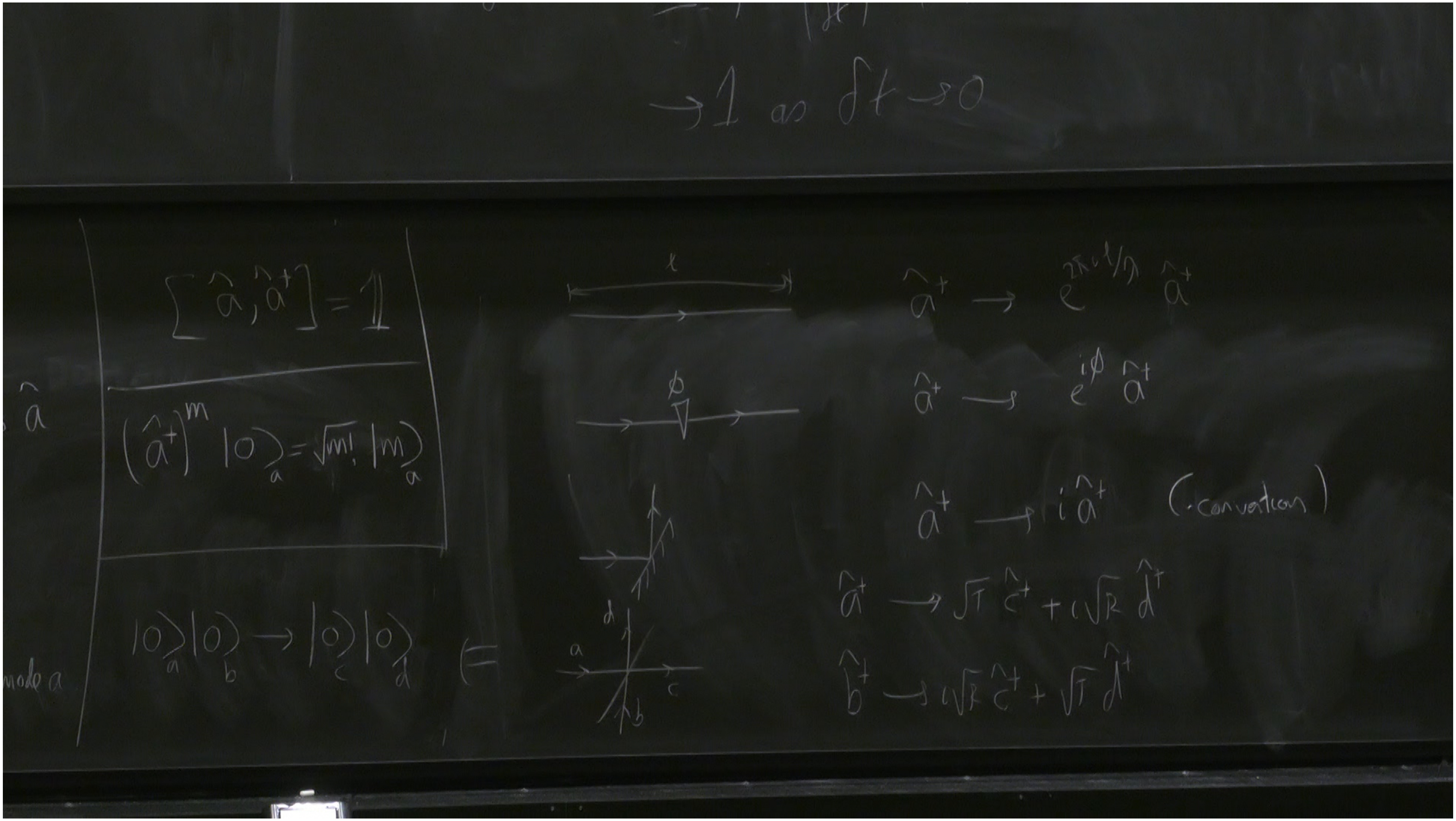
$[\hat{a}, \hat{a}^{+}] = 1$



→ 1 as δt → 0
[â, â†] = 1
ℓ


→ 1 as δt → 0
[â, â†] = 1
l
φ
â† → e^{2πil/λ} â†
â† → e^{iφ} â†
â† → i â†
â† → √T ĉ† + i√R d̂†
b̂† → i√R ĉ† + √T d̂†
d
a
c
b


→ 1 as δt → 0
[â, â†] = 1
(â†)^m |0⟩_a = √m! |m⟩_a
â
mode a
ℓ
ϕ
a
b
c
d
â† → e^{2πiℓ/λ} â†
â† → e^{iϕ} â†
â† → i â†
â† → √T ĉ† + i√R d̂†
b̂† → i√R ĉ† + √T d̂†



$\to 1$ as $\delta t \to 0$
$[\hat{a}, \hat{a}^\dagger] = \mathbb{1}$
$(\hat{a}^\dagger)^m |0\rangle_a = \sqrt{m!}\, |m\rangle_a$
$|0\rangle_a |0\rangle_b \to |0\rangle_c |0\rangle_d$
$\hat{a}^\dagger \to e^{2\pi i \ell/\lambda}\, \hat{a}^\dagger$
$\hat{a}^\dagger \to e^{i\phi}\, \hat{a}^\dagger$
$\hat{a}^\dagger \to i\hat{a}^\dagger$ (convention)
$\hat{a}^\dagger \to \sqrt{T}\, \hat{c}^\dagger + i\sqrt{R}\, \hat{d}^\dagger$
$\hat{b}^\dagger \to i\sqrt{R}\, \hat{c}^\dagger + \sqrt{T}\, \hat{d}^\dagger$

Lasers

$\boxed{\text{Laser}} \rightarrow a$

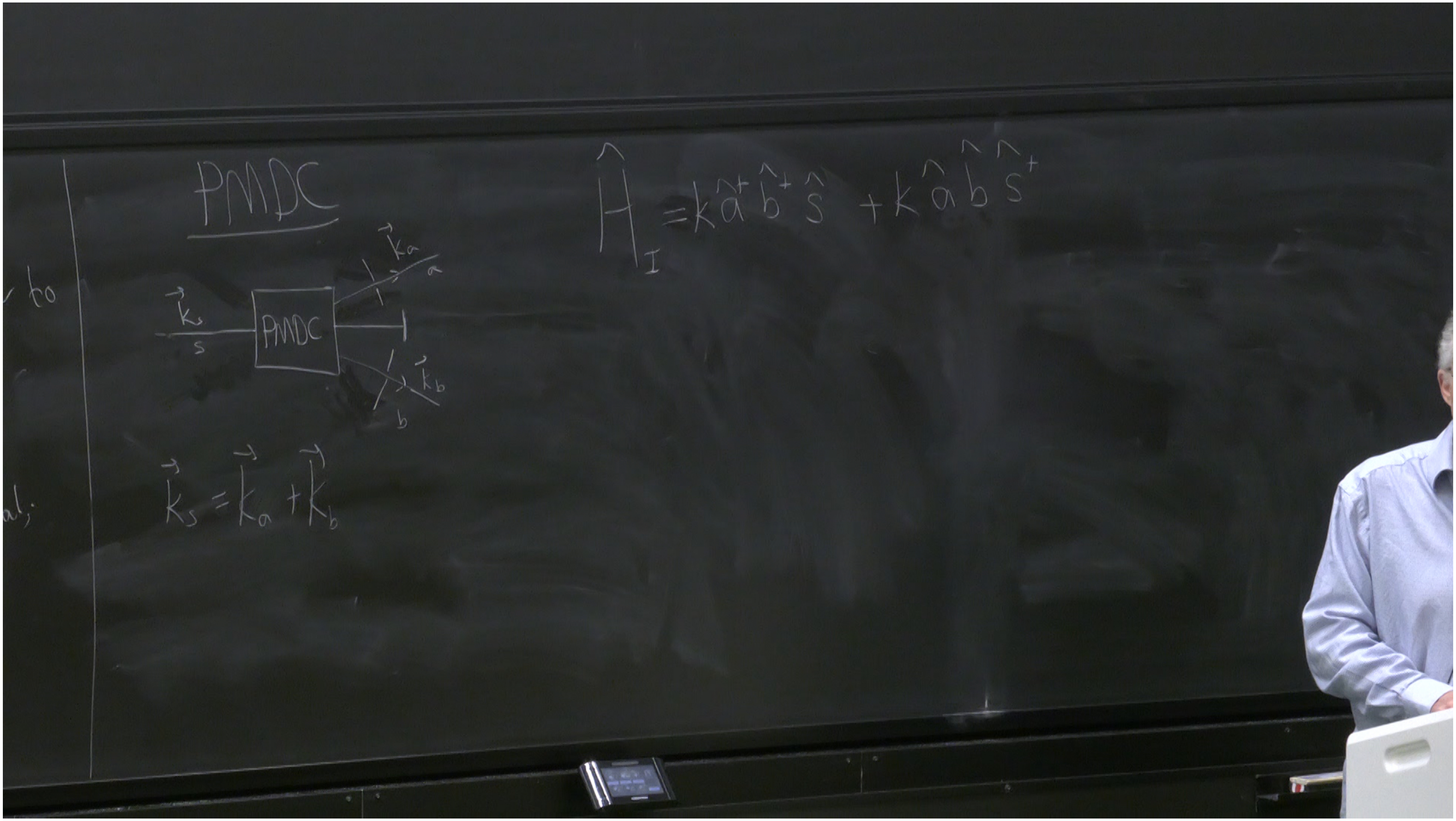
we take the state produced by Laser to be a coherent state

$$|\alpha\rangle$$

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

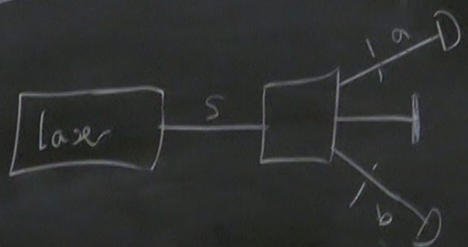
α is, in general, complex.

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle_a$$


PMDC
$\vec{k}_a$
a
$\vec{k}_s$
s
PMDC
$\vec{k}_b$
b
$\vec{k}_s = \vec{k}_a + \vec{k}_b$
$\hat{H}_I = k\hat{a}^{+}\hat{b}^{+}\hat{s} + k\hat{a}\hat{b}\hat{s}^{+}$

Experiments

Basic



$$|\alpha\rangle_s |0\rangle_a |0\rangle_b \longrightarrow e^{-i\hat{H}_I \delta t/\hbar} |\alpha\rangle_s |0\rangle_a |0\rangle_b$$

$$\simeq \left(1 - \frac{i\hat{H}_I \delta t}{\hbar}\right) |\alpha\rangle_s |0\rangle_a |0\rangle_b$$

$$= \left(1 - ik\hat{a}^\dagger \hat{b}^\dagger s \frac{\delta t}{\hbar} - ik\hat{a}\hat{b} s^\dagger \frac{\delta t}{\hbar}\right.$$

$|n\rangle_a$

$$|\alpha\rangle_s|0\rangle_a|0\rangle_b \longrightarrow e^{i\hat{H}_I\delta t/\hbar}|\alpha\rangle_s|0\rangle_a|0\rangle_b$$

$$\simeq \left(1 - \frac{i\hat{H}_I\delta t}{\hbar}\right)|\alpha\rangle_s|0\rangle_a|0\rangle_b$$

$$= \left(1 - ik\hat{a}^\dagger\hat{b}^\dagger\hat{s}\frac{\delta t}{\hbar} - \cancel{ik\hat{a}\hat{b}\hat{s}^\dagger\frac{\delta t}{\hbar}}\right)|\alpha\rangle_s|0\rangle_a|0\rangle_b$$

$$= |\alpha\rangle_s|0\rangle_a|0\rangle_b - i\delta t\,\alpha k|\alpha\rangle_s|1\rangle_a|1\rangle_b$$