Title: Quantum Foundations Lecture

Speakers: Lucien Hardy

Collection: Quantum Foundations

Date: January 12, 2024 - 10:15 AM

URL: https://pirsa.org/24010061

# The no cloning theorem

~1982 Nick Herbert wrong FTL paper

Wootters Zurek, Ghirardi, Dieks.

# The no cloning theorem

~1982 Nick Herbert wrong FTL paper

Wootters Zurek, Ghirardi, Dieks.

$$|\psi\rangle_{a_1} |0\rangle_{a_2} |M\rangle_{C_4}$$

Imagine we have a machine, $M$, that can clone states in $\mathcal{H}_{a_1}$

so

$$\mathcal{H}_{a_1}\otimes\mathcal{H}_{b_3} = \mathcal{H}_{a_1}\otimes\underbrace{\mathcal{H}_{a_2}\otimes\mathcal{H}_{c_4}}_{\mathcal{H}_{b_3}}$$

$$|\psi\rangle_{a_1}|M\rangle_{b_3} \xrightarrow{U} |\psi\rangle_{a_1}|\psi\rangle_{a_2}|M_\psi\rangle_{c_4} \quad \forall\ |\psi\rangle \in \mathcal{H}_a$$

$$|\varphi\rangle_{a_1}|M\rangle_{b_3} \xrightarrow{U} |\varphi\rangle_{a_1}|\varphi\rangle_{a_2}|M_\varphi\rangle_{c_4}$$

$$|u\rangle \to U|u\rangle = |v\rangle$$

$$|w\rangle \to U|w\rangle = |s\rangle$$

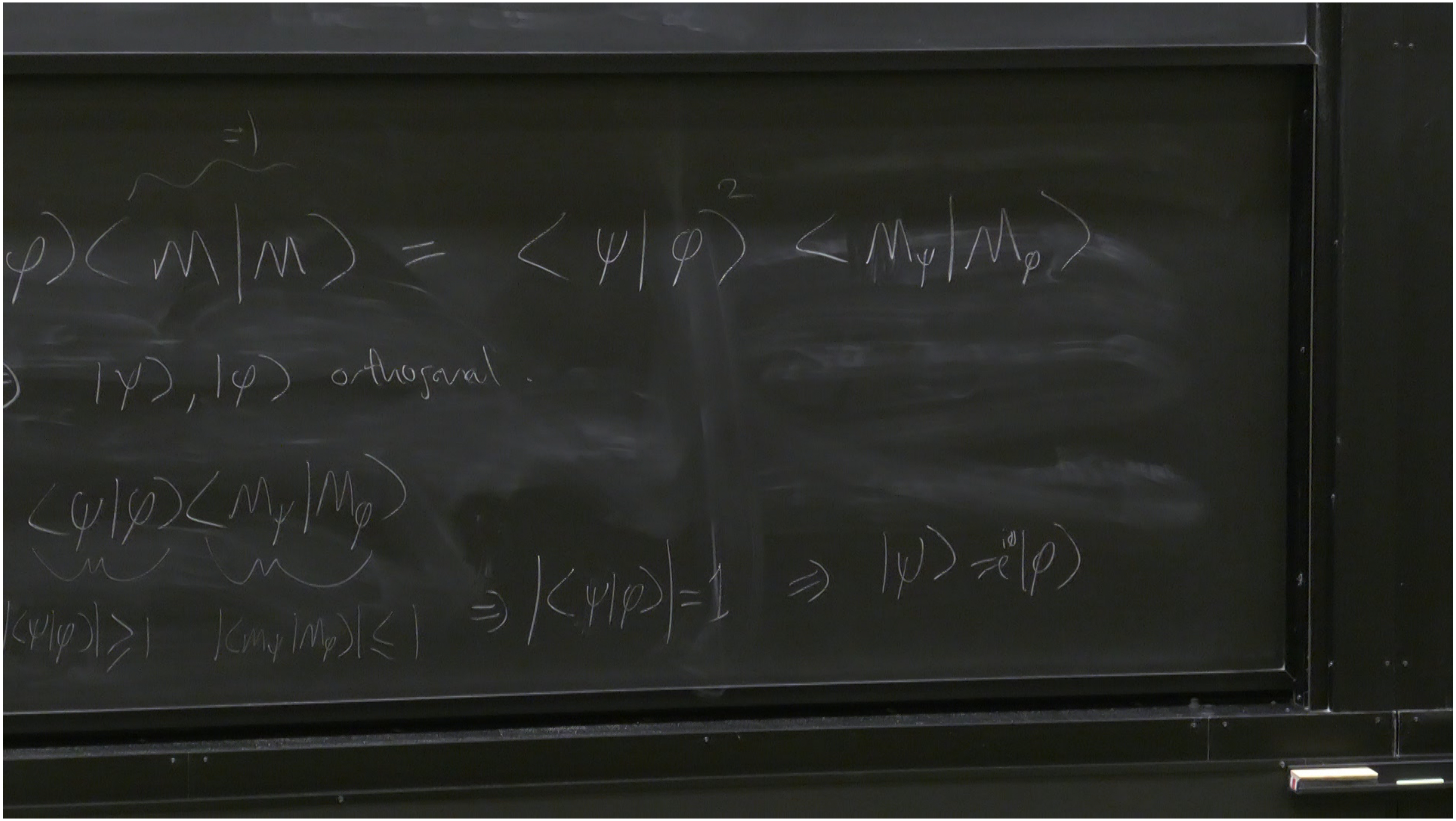
$$\langle v|s\rangle = \langle u|\underbrace{U^\dagger U}_{=\mathbb{1}}|w\rangle = \langle u|w\rangle$$

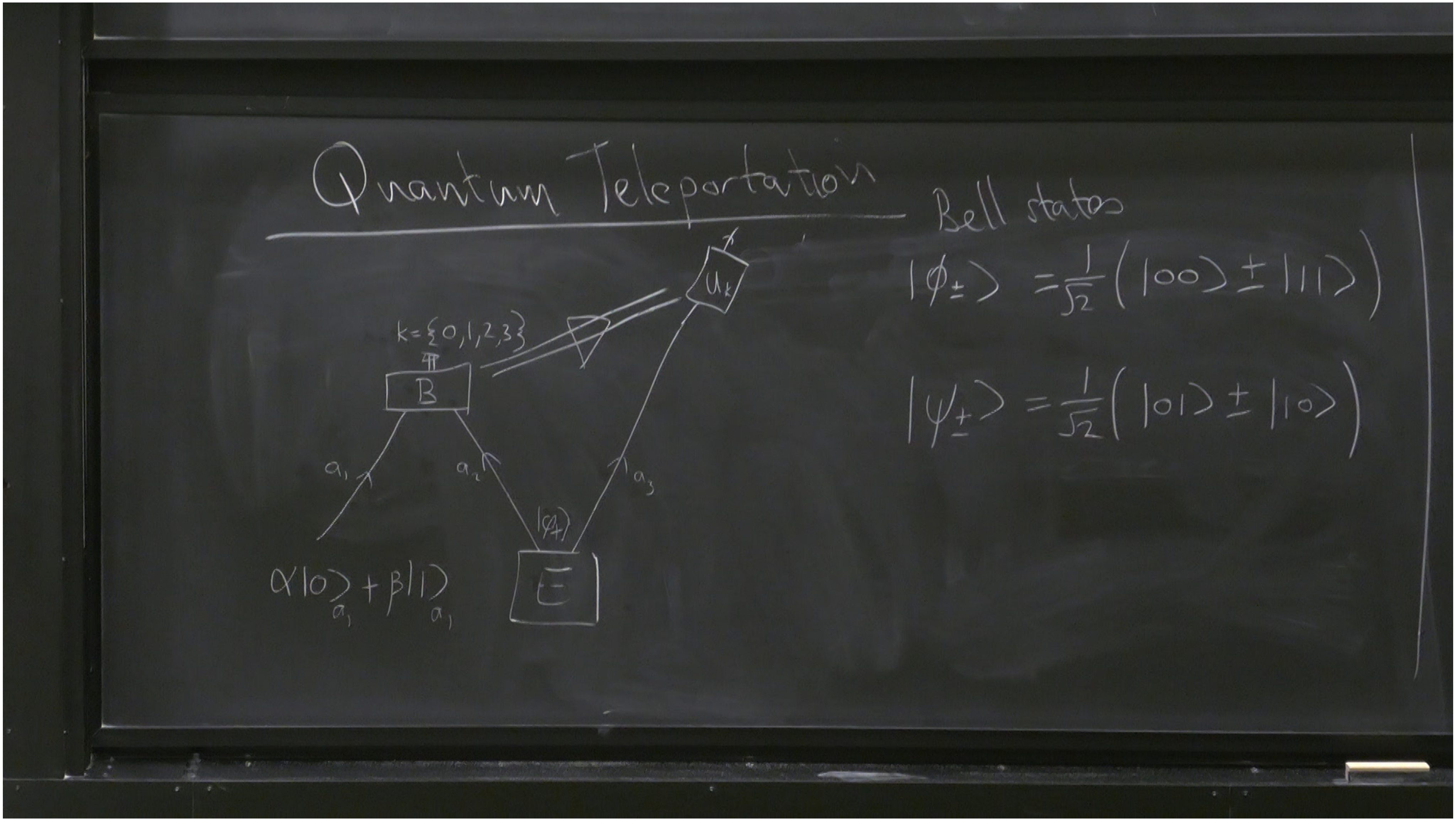
$$1 = \langle\psi|\varphi\rangle \overbrace{\langle M|M\rangle}^{=1} = \langle\psi|\varphi\rangle^2 \langle M_\psi|M_\varphi\rangle$$

$\langle\psi|\varphi\rangle = 0 \Rightarrow |\psi\rangle, |\varphi\rangle$ orthogonal.

Now assume $\langle\psi|\varphi\rangle \neq 0$

$$1 = \underbrace{\langle\psi|\varphi\rangle}_{|\langle\psi|\varphi\rangle| \geq 1} \underbrace{\langle M_\psi|M_\varphi\rangle}_{|\langle M_\psi|M_\varphi\rangle| \leq 1} \Rightarrow |\langle\psi|\varphi\rangle| = 1$$


$\varphi\rangle\langle M|M\rangle = \langle\psi|\varphi\rangle^2 \langle M_\psi|M_\varphi\rangle$
$\Rightarrow |\psi\rangle, |\varphi\rangle$ orthogonal.
$\langle\psi|\varphi\rangle\langle M_\psi|M_\varphi\rangle$
$|\langle\psi|\varphi\rangle| \geq 1$ $|\langle M_\psi|M_\varphi\rangle| \leq 1$
$\Rightarrow |\langle\psi|\varphi\rangle| = 1 \Rightarrow |\psi\rangle = e^{i\theta}|\varphi\rangle$


Quantum Teleportation
Bell states
$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)$
$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)$
$k = \{0,1,2,3\}$
B
$U_k$
$a_1$
$a_2$
$a_3$
$|\phi_+\rangle$
E
$\alpha|0\rangle_{a_1} + \beta|1\rangle_{a_1}$

leportation


Bell states

$$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)$$

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)$$

$$U_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad U_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$U_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad U_3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\left(\alpha|0\rangle_{a_1} + \beta|1\rangle_{a_1}\right)|\phi_+\rangle_{a_2a_3}$$

$$= \frac{1}{2}\underbrace{|\phi_+\rangle_{a_1a_2}}_{k=0}\left(\alpha|0\rangle_{a_3} + \beta|1\rangle_{a_3}\right) +$$

$$+ \frac{1}{2}|\psi_+\rangle_{a_1a_2}\left(\beta|0\rangle_{a_3} + \alpha|1\rangle_{a_3}\right) +$$

$$\left(\alpha|0\rangle_{a_1} + \beta|1\rangle_{a_1}\right)|\phi_+\rangle_{a_2 a_3}$$

$$= \frac{1}{2}|\phi_+\rangle_{a_1 a_2}\left(\alpha|0\rangle_{a_3} + \beta|1\rangle_{a_3}\right) + \frac{1}{2}|\phi_-\rangle_{a_1 a_2}\left(\alpha|0\rangle_{a_3} - \beta|1\rangle_{a_3}\right)$$

$k=0$

$$+ \frac{1}{2}|\psi_+\rangle_{a_1 a_2}\left(\beta|0\rangle_{a_3} + \alpha|1\rangle_{a_3}\right) + \frac{1}{2}|\psi_-\rangle_{a_1 a_2}\left(-\beta|0\rangle_{a_3} + \alpha|1\rangle_{a_3}\right)$$

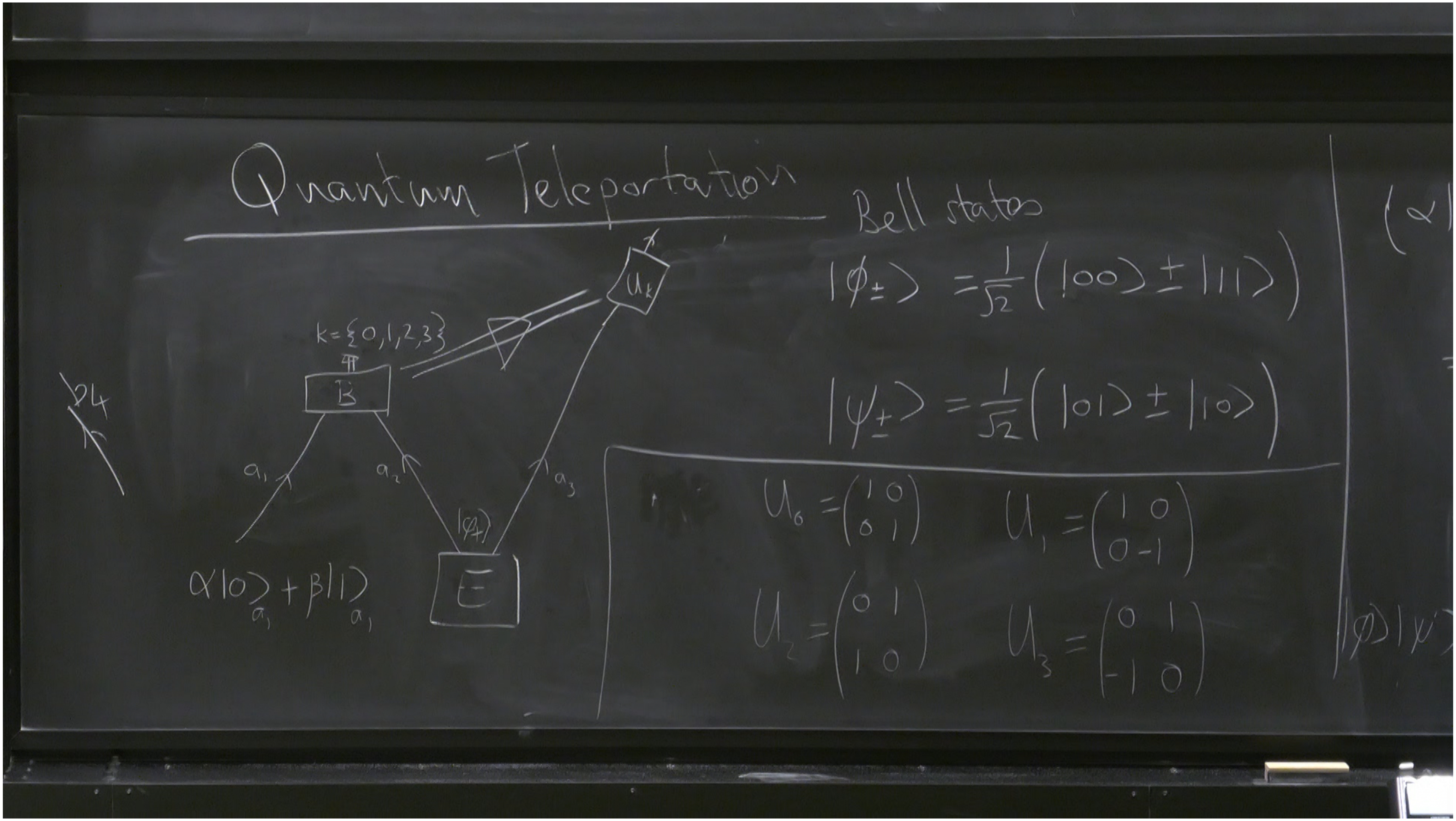
$$|\phi\rangle|\psi\rangle \to |\phi\rangle|\psi'\rangle$$

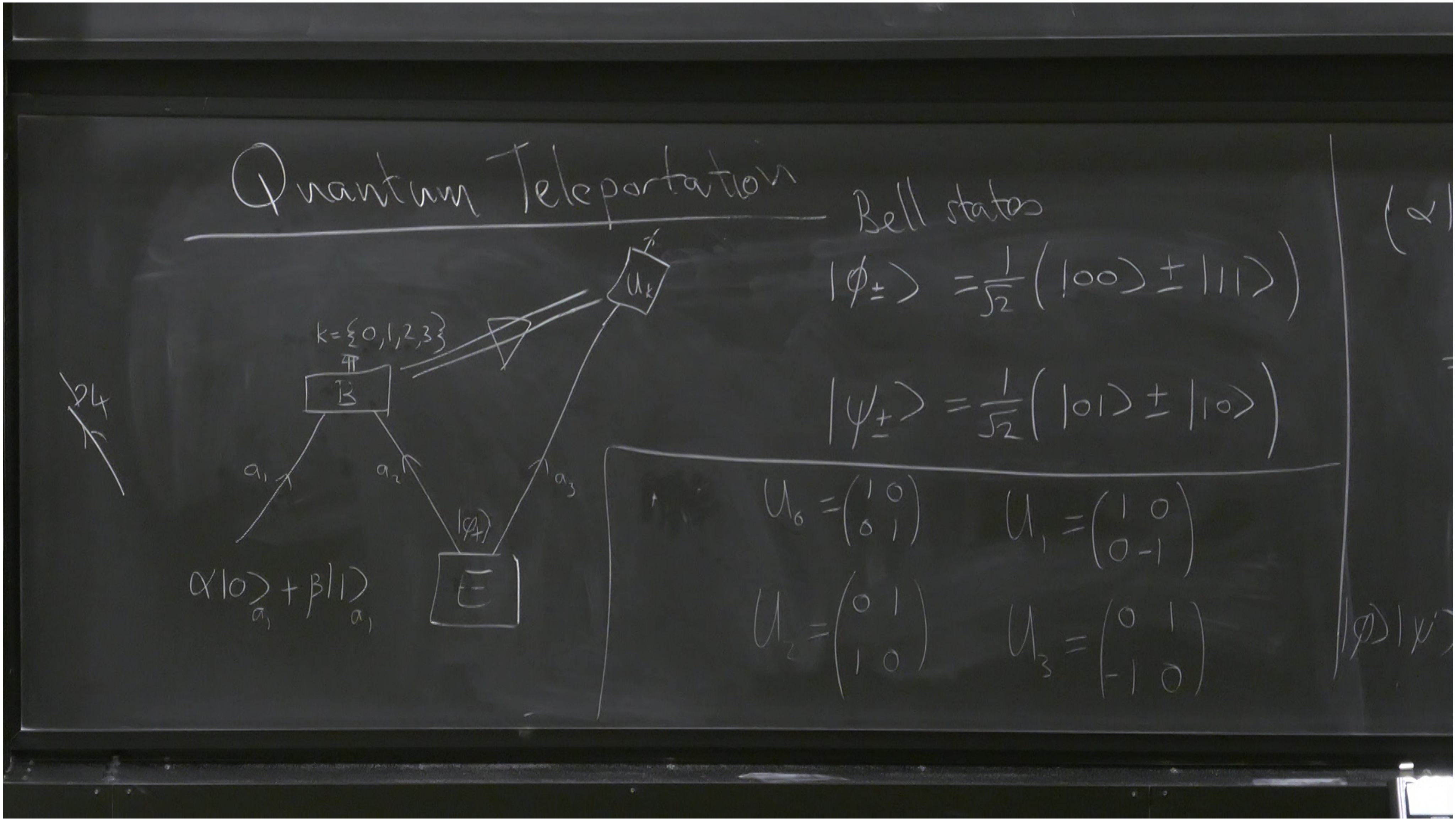
$\rangle \pm |11\rangle)$

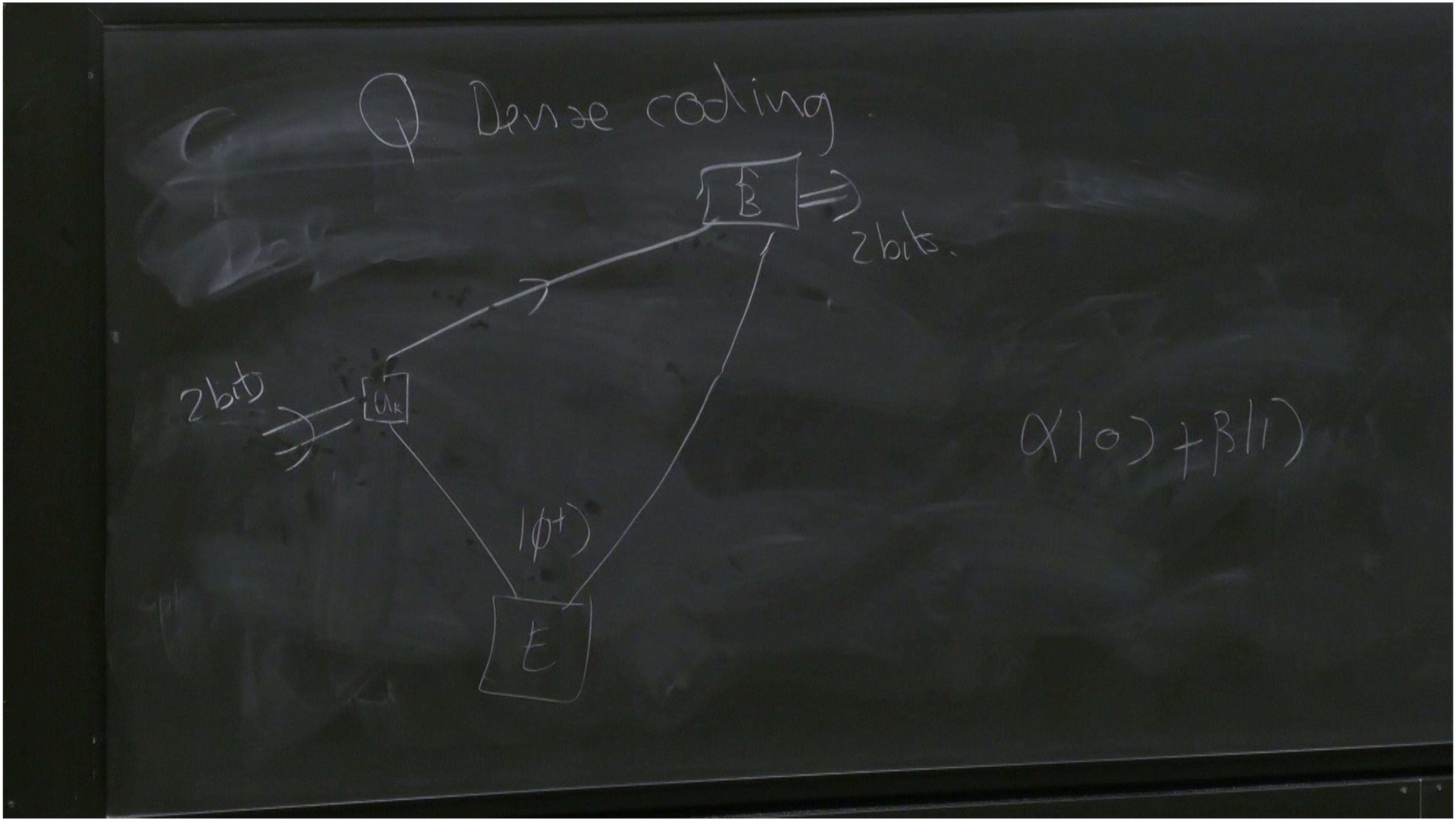
$\rangle \pm |10\rangle)$

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$


Quantum Teleportation
Bell states
$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$
$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$
$k = \{0,1,2,3\}$
B
E
$U_k$
$a_1$
$a_2$
$a_3$
$|\phi_+\rangle$
$\alpha|0\rangle_{a_1} + \beta|1\rangle_{a_1}$
$U_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$U_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$U_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$U_3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$


Quantum Teleportation
Bell states
|φ±⟩ = 1/√2 (|00⟩ ± |11⟩)
|ψ±⟩ = 1/√2 (|01⟩ ± |10⟩)
k = {0,1,2,3}
B
U_k
a_1
a_2
a_3
|φ+⟩
E
α|0⟩_{a_1} + β|1⟩_{a_1}
U_0 = (1 0; 0 1)
U_1 = (1 0; 0 -1)
U_2 = (0 1; 1 0)
U_3 = (0 1; -1 0)


Q Dense coding
B
2 bits.
2 bits
$U_k$
$|\phi^+\rangle$
E
$\alpha|0\rangle+\beta|1\rangle$