Title: Gravitational Physics Lecture

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Collection: Gravitational Physics

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Equatorial geodesics remain equatorial

$$\eta = \frac{\left((r^2+a^2)E - ah\right)^2}{r^2\Delta} - \frac{1}{r^2}(aE-h)^2 - \frac{r^2\dot{r}^2}{\Delta}$$

co- & counter rotating

$$V_{eff}(r) = \eta - E^2 - \frac{2GM\eta}{r} + \frac{a^2(\eta - E^2)+h^2}{r^2} - \frac{2GM}{r^3}(h-aE)^2$$

Null

$$1 + V = \frac{h^2 - a^2}{r^2} - \frac{2GM}{r^3}(h-a)^2 = 1 - \dot{r}^2$$

Consider the limit $a \to GM$

& look at $\dot{r}^2 = 0$, at $r_0 \geq a$

$$1 = \frac{h^2 - a^2}{r_0^2} - \frac{2a}{r_0^3}(h-a)^2$$

$$= \frac{(h-a)}{r_0^3}\left((h+a)r_0 - 2a(h-a)\right)$$

$E)^2$

Solved by $r_0 = h - a$

$\Rightarrow \quad h \geq 2a$.

$$\frac{h^2-a^2}{r^2} - \frac{2GM}{r^3}(h-a)^2 = 1-\dot{r}^2$$

the limit $a \to GM$

at $\dot{r}^2 = 0$, at $r_0 \geq a$

$$\frac{h^2-a^2}{r_0^2} - \frac{2a}{r_0^3}(h-a)^2$$

$$= \frac{(h-a)}{r_0^3}\left((h+a)r_0 - 2a(h-a)\right)$$

Solved by $r_0 = h-a$

$\Rightarrow h \geq 2a$

$$V'(r_0) = -\frac{2(h^2-a^2)}{r_0^3} + \frac{6a(h-a)^2}{r_0^4}$$

$$= -\frac{2(h+a)}{r_0^2} + \frac{6a}{r_0^2}$$

$$= \frac{4a-2h}{r_0^2} \leq 0$$

$$\frac{h^2-a^2}{r^2} - \frac{2GM}{r^3}(h-a)^2 = 1-\dot{r}^2$$

the limit $a \to GM$

at $\dot{r}^2 = 0$, at $r_0 \geq a$

$$\frac{h^2-a^2}{r_0^2} - \frac{2a}{r_0^3}(h-a)^2$$

$$= \frac{(h-a)}{r_0^3}\left((h+a)r_0 - 2a(h-a)\right)$$

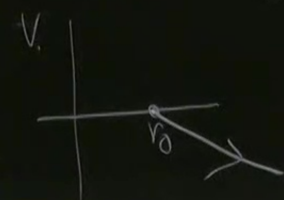
Solved by $r_0 = h-a$

$\Rightarrow \quad h \geq 2a$

$$V'(r_0) = -2\frac{(h^2-a^2)}{r_0^3} + \frac{6a(h-a)^2}{r_0^4}$$

$$= -2\frac{(h+a)}{r_0^2} + \frac{6a}{r_0^2}$$

$$= \frac{4a-2h}{r_0^2} \leq 0$$



GEODESIC ESCAPES to ∞

Geodesics reach ∞

outside $r_0 = a$

— EVENT HORIZON

Is a null surface

SCH $ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - r^2 d\Omega_{II}^2$

$$- dr^2 \Big/ \left(1 - \frac{2GM}{r}\right)$$

Define Tortoise coord.

$$r^* = \int \frac{dr}{\underbrace{1-\frac{2GM}{r}}_{f(r)}} = r + \underbrace{2GM}_{r_+} \log \frac{(r-2GM)}{2GM}$$

$$ds^2 \to f\,(dt^2 - dr^{*2}) \cdots$$

Define Kruskal null coords

$$U = -r_+ e^{-\frac{(t-r^*)}{2r_+}} \; ; \; V = r_+ e^{\frac{t+r^*}{2r_+}}$$

$$\ldots g\,\frac{(r-2GM)}{2GM}$$

...rds

$$e^{\frac{t+r^*}{2r_+}}$$

$r \to r_+ \quad r^* \to -\infty$

at fixed t, $U, V \to 0$

But if $t = t_0 - r^*$

$t \to \infty$ as well, $U \to 0$

at finite V.

Note

$$UV = -r_+^2 \, e^{r^*/r_+}$$

$$\longrightarrow 0 \text{ at } r_+$$

$$dU = -\frac{(dt - dr^*)}{2r_+}\, U$$

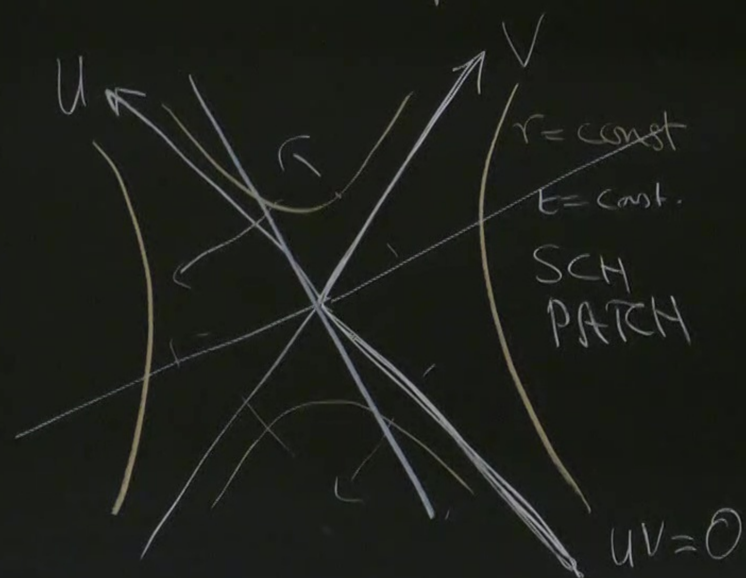
$$dV = \frac{dt + dr^*}{2r_+}\, V$$

$$\Rightarrow dU\,dV = -\frac{UV}{4r_+^2}\left(dt^2 - dr^{*2}\right)$$

$$= \frac{e^{r^*/r_+}}{4}\left(dt^2 - dr^{*2}\right)$$

$$-dr^2/\left(1-\frac{2GM}{r}\right) \qquad U = -r_+ e^{\ldots/2r_+} \; ;$$

$$ds^2_{SCH} = \frac{8GM}{r} e^{-r/r_+} dU\,dV - r^2 d\Omega^2_{II}$$



$r = \text{const} \leftrightarrow UV = \text{const}$

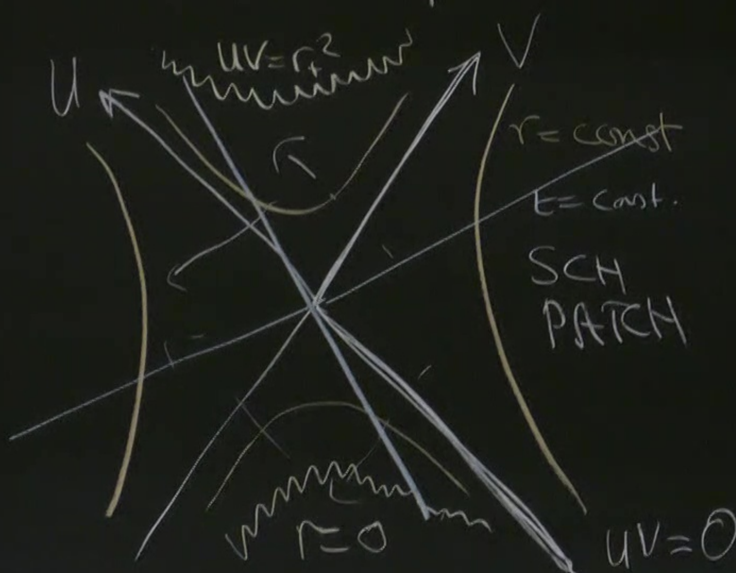
$t = \text{const} \leftrightarrow U/V = \text{const}$

$r = r_+ \leftrightarrow UV = 0$

$r = 0 \longleftrightarrow UV = r_+^2$

$$- dr^2 \Big/ \left(1 - \frac{2GM}{r}\right) \qquad U = -r_+ e^{\dots\, 2r_+} \; ; \; V =$$

$$ds^2_{Sch} = \frac{8GM}{r} e^{-r/r_+} \, dU \, dV - r^2 d\Omega^2_{II}$$



$r = \text{const} \iff UV = \text{const}$

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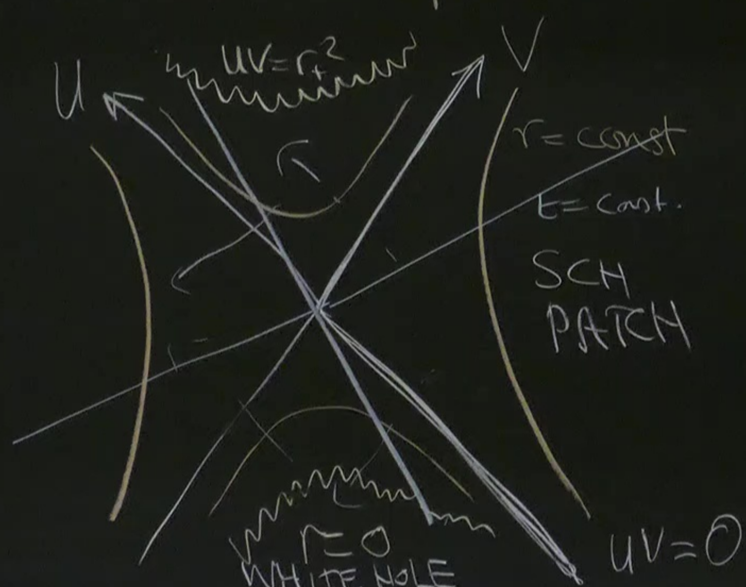
$r = r_+ \iff UV = 0$

$r = 0 \iff UV = r_+^2$

$- dr^2 / \left(1 - \frac{2GM}{r}\right)$ | $U = -r_+ e^{\ldots 2r_+}$;

$$ds^2_{SCH} = \frac{8GM}{r} e^{-r/r_+} \, dU dV - r^2 d\Omega_{II}^2$$



$r = \text{const} \iff UV = \text{const}$

$t = \text{const} \iff U/V = \text{const.}$

$r = r_+ \iff UV = 0$

$r = 0 \iff UV = r_+^2$

Thi…
ex…
t
t

$V = r_+ e^{\frac{\dots}{2r_+}}$ | $UV = -r_+^2 e^{\dots}$ $\to 0$ at r_+ | $\frac{\dots}{4}$

his is the maximal analytic extension of SCH.

$t = t_0 - r^*$ $U \to 0$, finite V

FUTURE EVENT HORIZON

$t = t_0 + r^*$ $V \to 0$, finite U

PAST EVENT HORIZON

Penrose Carter diagrams

Define new chart. $\begin{cases} p = \arctan V/r_+ \\ q = \arctan U/r_+ \end{cases}$

$V \in (-\infty, \infty) \longrightarrow p \in [-\pi/2, \pi/2]$

$V = 0 \Leftrightarrow p = 0$

$U = r_+ \Leftrightarrow q = \pi/4$

$V = r_+ e^{\frac{\cdots}{2r_+}}$

$UV = -r_+ e \cdots$

$\rightarrow 0$ at r_+

$\frac{\cdots}{4}$

[T]his is the maximal analytic [e]xtension of SCH.

$t = t_0 - r^*$ $\quad U \to 0$, finite V

FUTURE EVENT HORIZON

$t = t_0 + r^*$ $\quad V \to 0$, finite U.

PAST EVENT HORIZON.

Penrose Carter diagrams

Define new chart. $\begin{cases} p = \arctan V/r_+ \\ q = \arctan U/r_+ \end{cases}$

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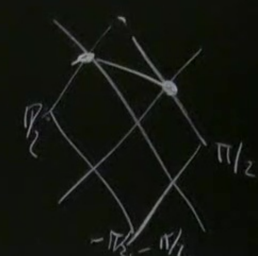
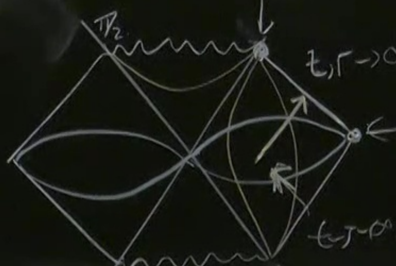


$$\tan(p+q) = \frac{\tan p + \tan q}{1 - \tan p \tan q} = \frac{U+V}{r_+^2 - UV}.$$

$$UV \to r_+^2, \quad p+q \to \pm \pi/2$$



$t \to \infty$, finite r $\quad i^+$ future t-like ∞

$t, r \to \infty$. $\quad \mathscr{I}^+$ future null ∞.

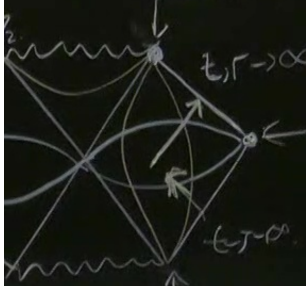
spatial ∞ $\quad i^0$

$t, -r \to \infty$. $\quad \mathscr{I}^-$ past null ∞.

$t \to -\infty$, r finite $\quad i^-$ past t-like ∞

$$+q) = \frac{\tan p + \tan q}{1 - \tan p \tan q} = \frac{U+V}{r_+^2 - UV}.$$

$\to r_+^2 \,, \quad p+q \to \pm \pi/2$



$t \to \infty$, finite r — i^+ future t-like ∞

$t, r \to \infty$ — $\mathscr{I}^+$ future null ∞

spatial ∞ — i^0

$\mathscr{I}^-$ past null ∞

$t, r \to -\infty$

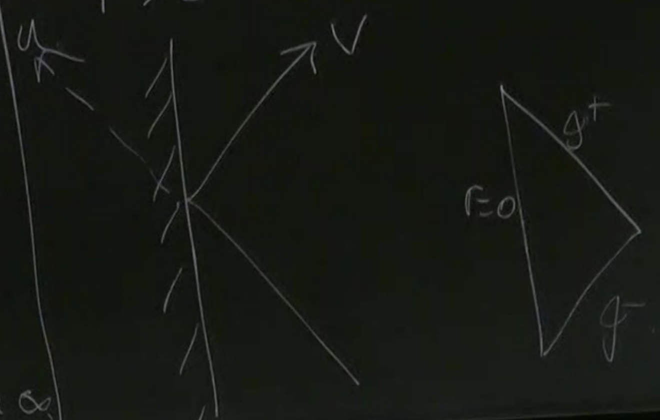
$t \to -\infty$, r finite — i^- past t-like ∞

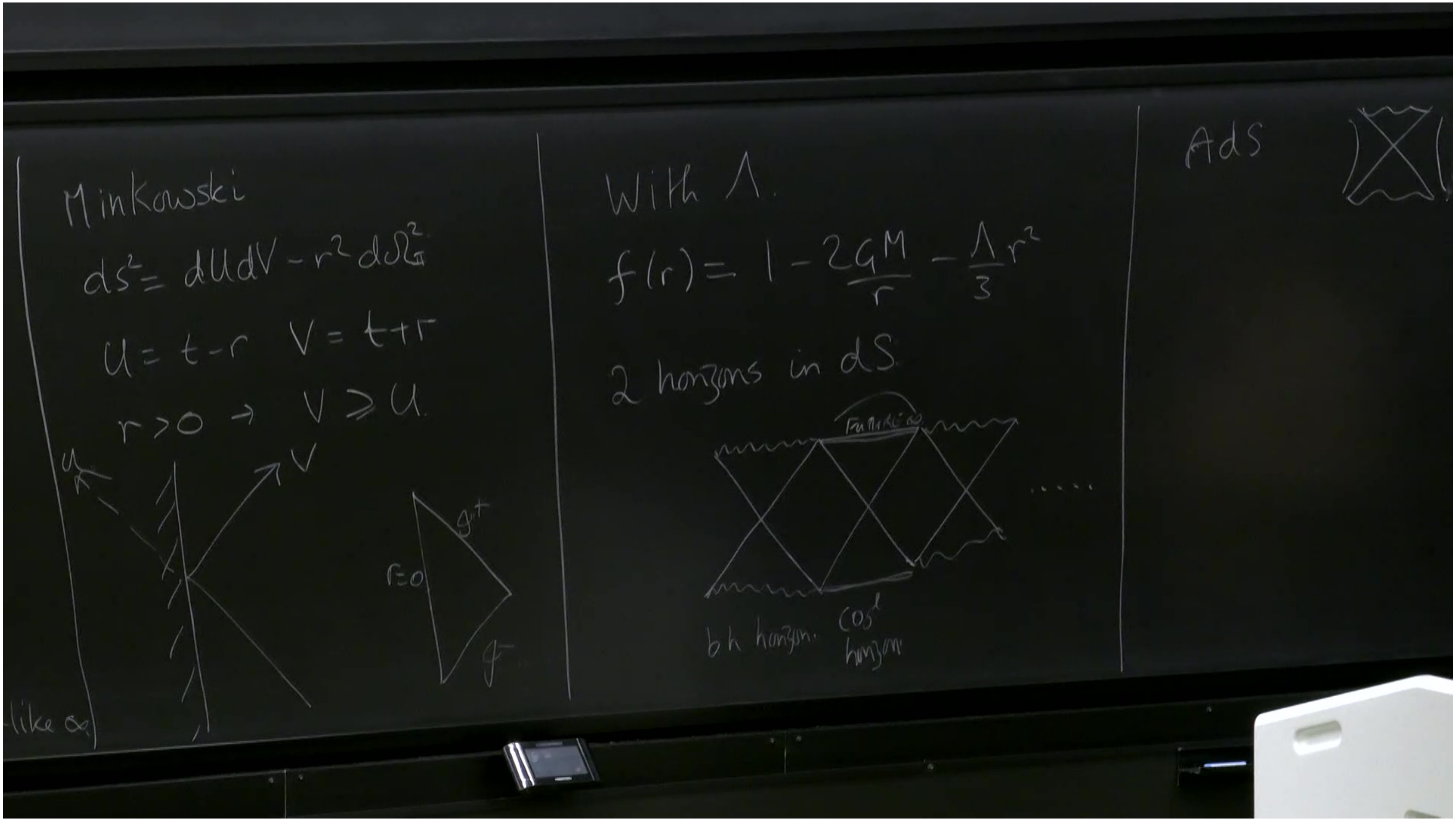
Minkowski

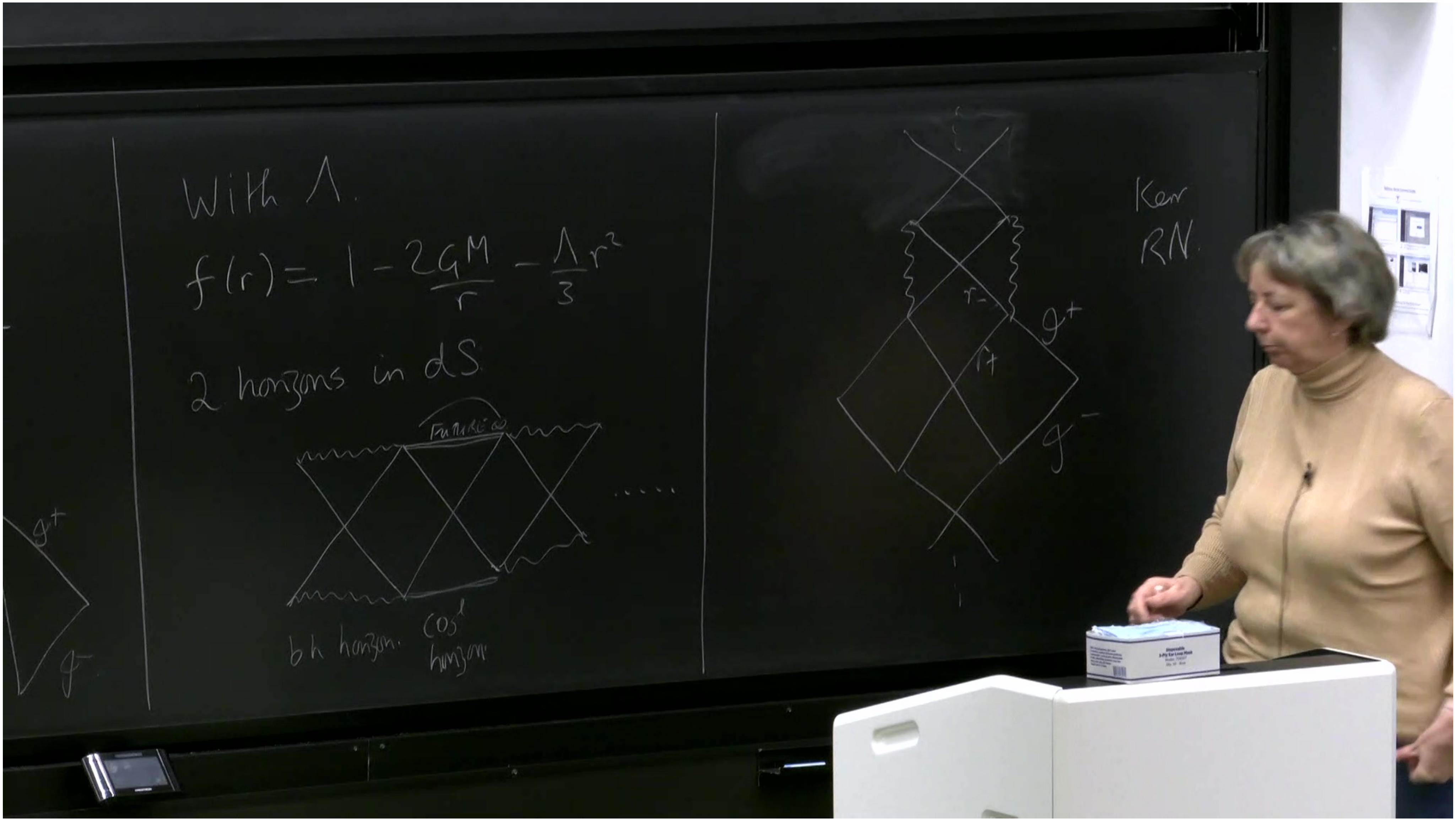
$$ds^2 = dU dV - r^2 d\Omega_2^2$$

$U = t - r \quad V = t + r$

$r > 0 \to V \geq U$




Minkowski
ds² = dUdV − r² dΩ²
U = t − r   V = t + r
r > 0 → V ≥ U
r=0
With Λ
f(r) = 1 − 2GM/r − Λ/3 r²
2 horizons in dS
bh horizon
cos horizon
AdS
like ∞


With Λ.
$f(r) = 1 - \frac{2GM}{r} - \frac{\Lambda}{3} r^2$
2 horizons in dS
FUTURE ∞
b h horizon.
cos^l horizon
Kerr
RN.
r_-
r_+
$\mathcal{I}^+$
$\mathcal{I}^-$

Return to Kerr. Area of event horizon is

$$A = 4\pi(r_+^2 + a^2) < A_{SCH}$$

Consider throwing in an object

$$\delta A = 8\pi(r_+ \delta r_+ + a\,\delta a)$$

Kerr. Area of

n is

$r_+^2 + a^2) < A_{SCH}$

in an object

$\pi(r_+ \delta r_+ + a\delta a)$

$\pi\left(r_+\left[G\delta M + \frac{G^2 M\delta M - a\delta a}{r_+ - GM}\right] + a\delta a\right)$

$$8\pi\left(\frac{r_+^2 G\delta M}{r_+ - GM} - \frac{GMa\delta a}{r_+ - GM}\right).$$

Note $a = J/M$.

Kerr. Area of

n is

$(r_+^2 + a^2) < A_{SCH}$

in an object

$\pi\left(r_+ \delta r_+ + a\,\delta a\right)$

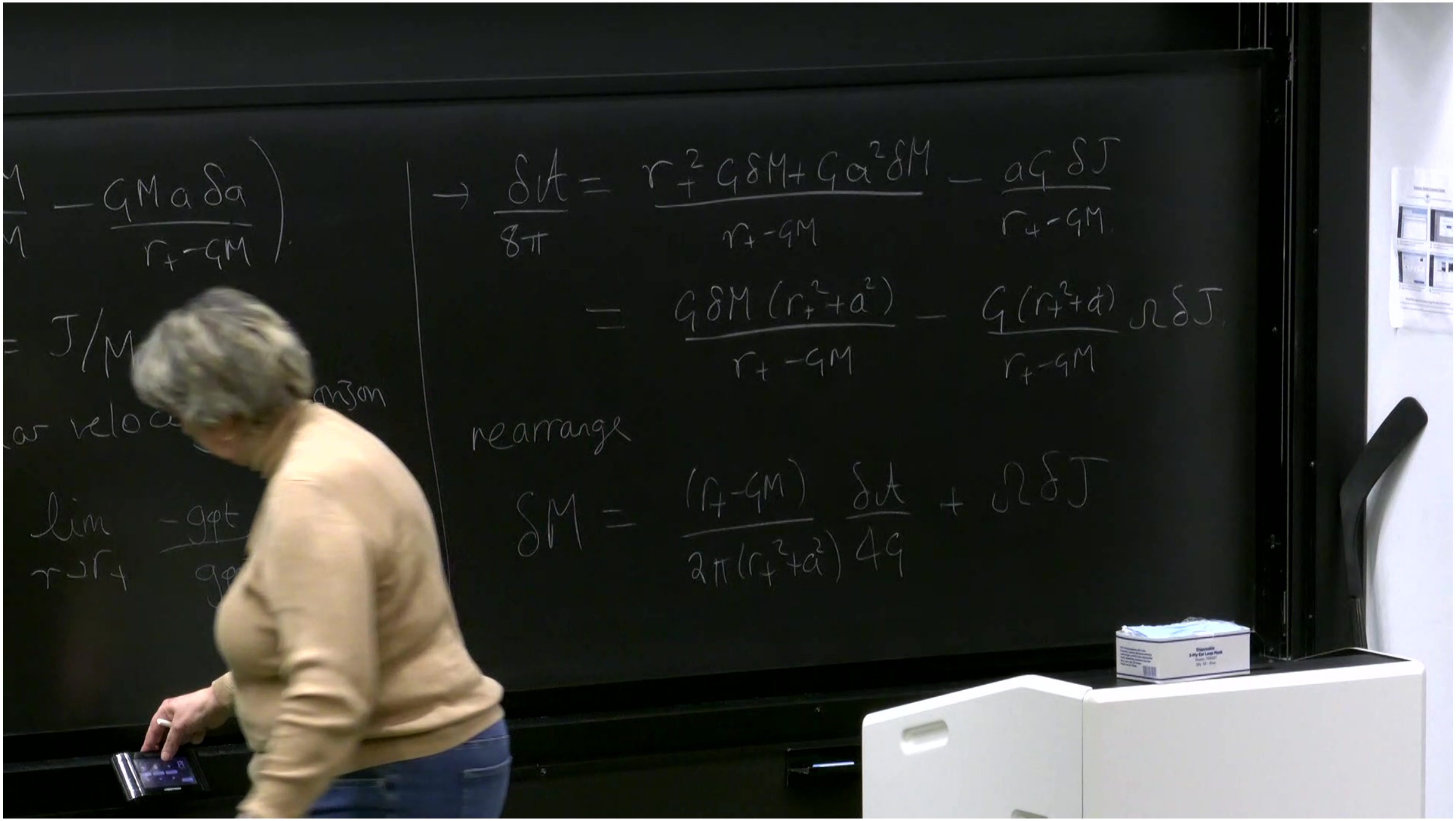
$\pi\left(r_+\left[G\,\delta M + \frac{G^2 M\,\delta M - a\,\delta a}{r_+ - GM}\right] + a\,\delta a\right)$

$$\to 8\pi\left(\frac{r_+^2 G\,\delta M}{r_+ - GM} - \frac{GMa\,\delta a}{r_+ - GM}\right).$$

Note $a = J/M$.

& angular velocity of horizon

$$\Omega = \lim_{r \to r_+} \frac{-g_{\phi t}}{g_{\phi\phi}} = \frac{a}{r_+^2 + a^2}$$



$-\frac{GMa\,\delta a}{r_+ - GM}$
$= J/M$
$\lim_{r \to r_+} \frac{-g_{\varphi t}}{g_{\varphi\varphi}}$
$\to \frac{\delta A}{8\pi} = \frac{r_+^2 G\delta M + Ga^2\delta M}{r_+ - GM} - \frac{aG\,\delta J}{r_+ - GM}$
$= \frac{G\delta M(r_+^2 + a^2)}{r_+ - GM} - \frac{G(r_+^2+a^2)}{r_+ - GM}\Omega\,\delta J$
rearrange
$\delta M = \frac{(r_+ - GM)}{2\pi(r_+^2 + a^2)}\frac{\delta A}{4G} + \Omega\,\delta J$

$$r_+ - GM$$

cf $\quad dU = T\,dS - p\,dV + \mu\,dQ$

$\qquad\qquad\qquad\qquad\quad \times \qquad \Omega \quad J$

$$T \to \hbar\,\frac{r_+ - GM}{2\pi(r_+^2 + a^2)} \qquad dS \to \frac{\delta A}{4G\hbar}$$

$$\to \frac{1}{8\pi G M} \quad \text{SCH}$$