Title: Gravitational Physics Lecture

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Collection: Gravitational Physics

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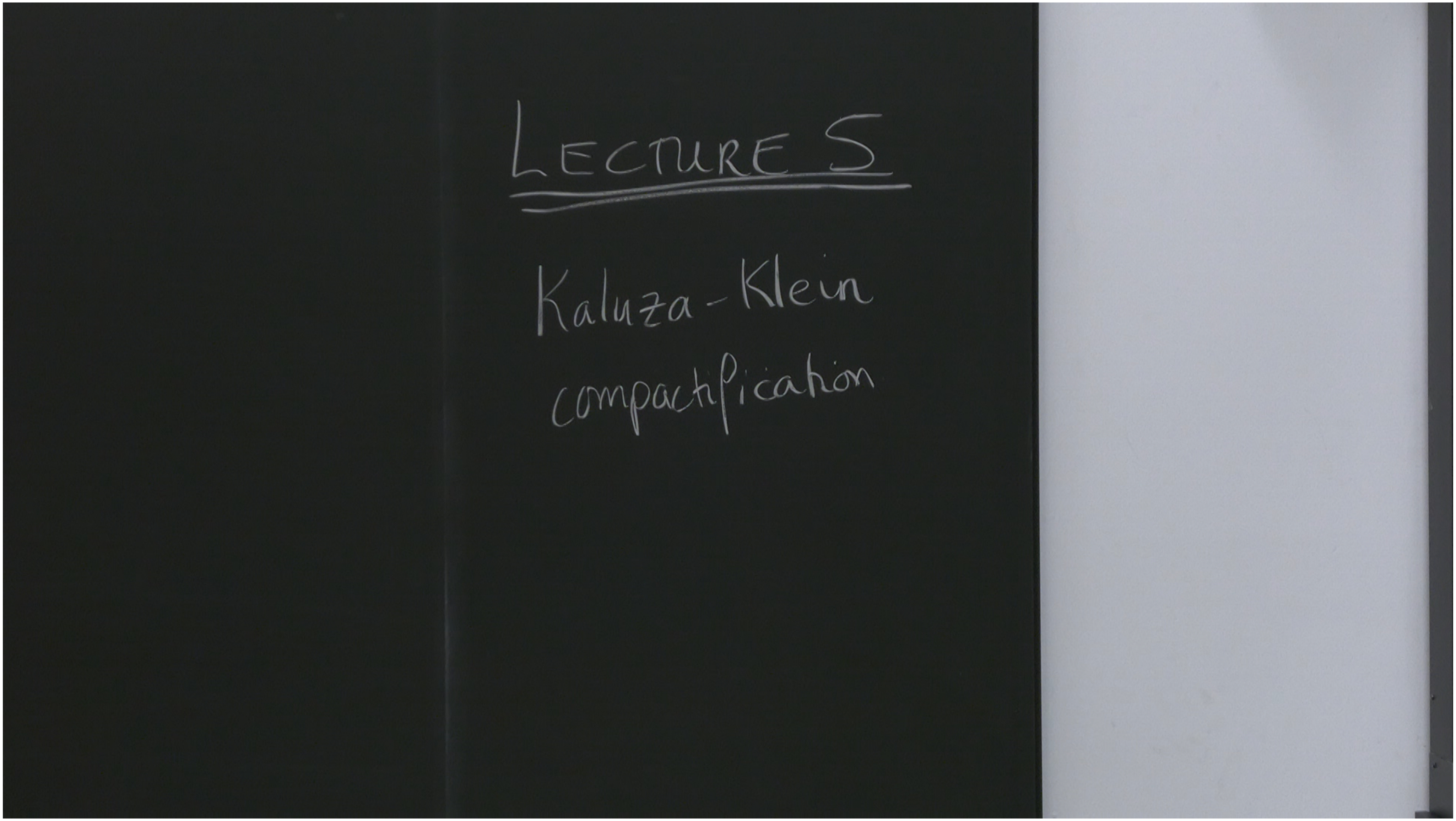
LECTURE 5
Kaluza-Klein
compactification

Introduce an extra dimension;

consider a scalar on $\mathbb{R}^4 \times S^1$

$$\Phi(\underset{4D}{x^\mu}, \underset{\text{extra dim}}{\chi}) \qquad ds^2 = ds_4^2 - L^2 d\chi^2$$

$$\Box_5 \Phi = \Box_4 \Phi - \frac{1}{L^2}\frac{\partial^2 \Phi}{\partial \chi^2}$$

Fourier decompose: $\Phi = \varphi_n(x^\mu)e^{in\chi}$

$d\chi^2$

$$\Box_5 \Phi = e^{in\chi}\Big[\Box_4 \varphi_n + \underbrace{\frac{n^2}{L^2}}_{\text{mass term}}\varphi_n\Big]$$

From 4D perspective, the Fourier modes have masses $\propto 1/L$. If L is "small", modes that depend on χ are very massive.

Low energy theory indep. of $\chi \leftrightarrow \frac{\partial}{\partial \chi}$ Killing vec

$$g_{ab} \sim \begin{bmatrix} \breve{g}_{\mu\nu} & g_{\mu 5} \\ g_{\nu 5} & g_{55} \end{bmatrix} \qquad g_{ab,\chi} = 0$$

Encode as

$$ds^2 = g_{4,\mu\nu} dx^\mu dx^\nu - e^{2\sigma}\left[d\chi + A_\mu dx^\mu\right]^2$$

uer

. If

end

Low energy theory indep. of $\chi \Leftrightarrow \frac{\partial}{\partial \chi}$ Killing vec.

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$$\text{so } \det g_5 = e^{2\sigma} \det g_4$$

σ, A_μ, $g_{4\,\mu\nu}$ depend only on X^μ.

scalar vector tensor

To compute curvature, use Cartan (handout)

e.g. connection

off diagonality

$$\underset{\sim}{\omega}^{\hat{a}} = e_4{}^{\hat{a}}{}_\mu \, dx^\mu$$

$$\underset{\sim}{\omega}^5 = e^\sigma \left[dX + A_\mu dX^\mu \right]$$

...ut)

...nality

$$d\omega^{\hat a} = d\, e_4{}^{\hat a}{}_\mu \wedge dx^\mu = -\,\Theta_0{}^{\hat a}{}_{\hat b} \wedge \omega^{\hat b}$$

$$= -\,\Theta^{\hat a}{}_{\hat b} \wedge \omega^{b} - \Theta^{\hat a}{}_{5} \wedge \omega^{5}$$

$$d\omega^5 = d\sigma \wedge \omega^5 + e^{\sigma}\, dA$$

$$= \sigma_{,\hat a}\, \omega^{\hat a} \wedge \omega^5 + e^{\sigma} F$$

$$= -\,\Theta^{5}{}_{\hat a} \wedge \omega^{\hat a}$$

$$= -\underset{\sim}{\theta}_0{}^{\hat a}{}_{\hat b} \wedge \underset{\sim}{\omega}^{\hat b}$$

$$\theta^{\hat a}{}_5 \wedge \underset{\sim}{\omega}^5$$

$$d\underset{\sim}{A}$$

$$e^\sigma \underset{\sim}{F}$$

Solving simultaneous eqns:

$$\underset{\sim}{\theta}^5{}_{\hat a} = \sigma_{,\hat a}\,\underset{\sim}{\omega}^5 + \frac{1}{2} e^\sigma F_{\hat a\hat b}\,\underset{\sim}{\omega}^{\hat b}$$

$$\underset{\sim}{\theta}^{\hat a}{}_{\hat b} = \underset{\sim}{\theta}_0{}^{\hat a}{}_{\hat b} + \frac{1}{2} e^\sigma F^{\hat a}{}_{\hat b}\,\underset{\sim}{\omega}^5$$

Eventually, get:

$$R^5{}_5 = -\Box\sigma - (\partial\sigma)^2 - \frac{1}{4} e^{2\sigma} F^2$$

$$R^{\hat a}{}_{\hat b} = R_0{}^{\hat a}{}_{\hat b} + \frac{1}{4} e^{2\sigma} F^{\hat a\hat c} F_{\hat b\hat c} - \nabla^{\hat a}\nabla_{\hat b}\sigma - \sigma^{,\hat a}\sigma_{,\hat b}$$

$$R = R_0 - 2\Box\sigma - 2(\nabla\sigma)^2 + \frac{1}{4}e^{2\sigma}F^2$$

$$\Rightarrow \sqrt{g_5}\,R_5 = \boxed{e^{\sigma}}\sqrt{g_4}\left[R_0 - 2e^{-\sigma}\Box e^{\sigma} + \frac{1}{4}e^{2\sigma}F^2\right]$$

– this is the gravitational Lagrangian

"Einstein", no kinetic term for σ, F^2

New

σ

$2(\nabla\sigma)^2 + \frac{1}{4} e^{2\sigma} F^2$

$[R_0 - 2e^{-\sigma} \Box e^{\sigma} + \frac{1}{4} \boxed{e^{2\sigma}} F^2]$

itational Lagrangian

c term for σ, F^2

New 4D physics

$\sigma \leftrightarrow$ scalar, "breathing mode"

(gives size of S^1) - modulus

$A_\mu \leftrightarrow$ vector (almost) Maxwell

New 4D physics:

$\sigma \leftrightarrow$ scalar, "breathing mode"

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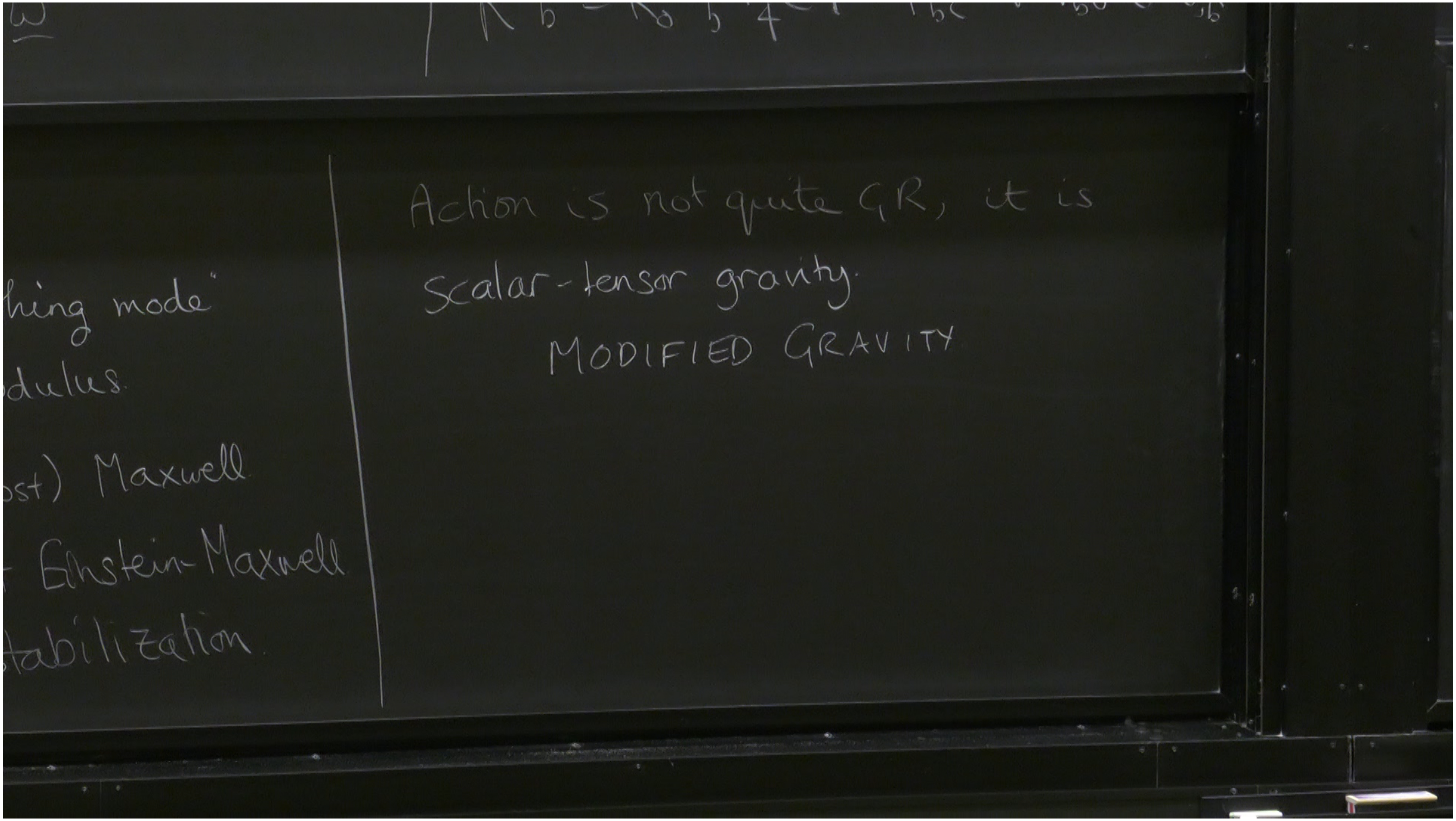
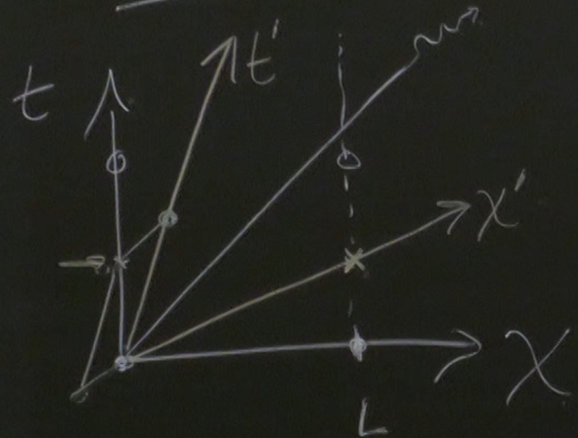
$A_\mu \leftrightarrow$ vector (almost) Maxwell.

Klein fixed σ to get Einstein-Maxwell

Moduli stabilization.


hing mode"
dulus.
st) Maxwell
Einstein-Maxwell
tabilization
Action is not quite GR, it is
scalar-tensor gravity.
MODIFIED GRAVITY

Rest Frame

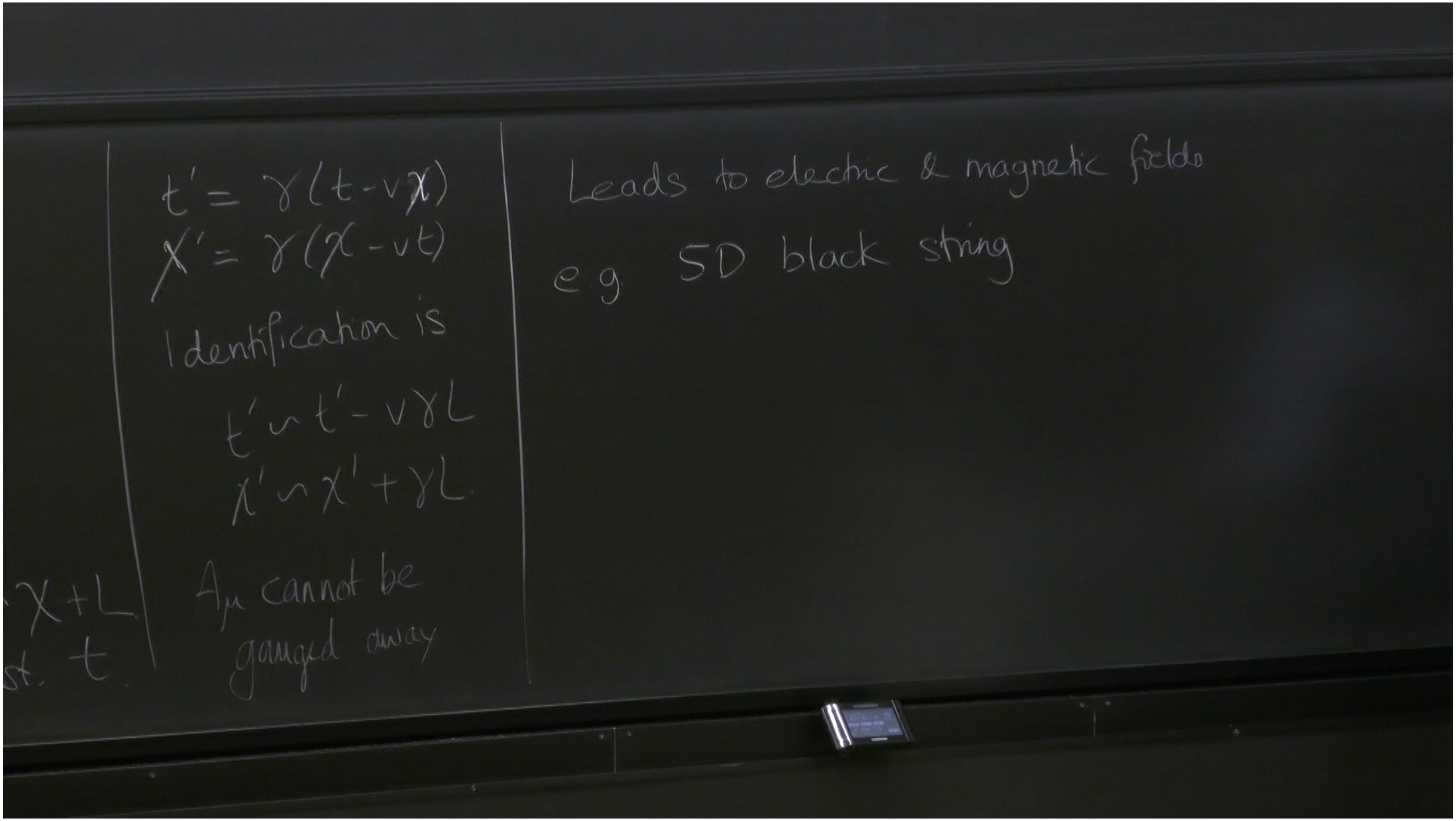


χ compact $\Rightarrow$ $\chi \sim \chi + L$
at const. t

$$t' = \gamma(t - v\chi)$$
$$\chi' = \gamma(\chi - vt)$$

Identification is

$$t' \sim t' - v\gamma L$$
$$\chi' \sim \chi' + \gamma L.$$


$x + L$
st. t
$t' = \gamma(t - vx)$
$x' = \gamma(x - vt)$
Identification is
$t' \sim t' - v\gamma L$
$x' \sim x' + \gamma L$.
A_μ cannot be gauged away
Leads to electric & magnetic fields
e.g. 5D black string

$\gamma(t-vx)$

$\gamma(x-vt)$

...fication is

$\sim t' - v\gamma L$

$\sim x' + \gamma L$.

cannot be

...anged away.

Leads to electric & magnetic fields

e.g. 5D black string

$$ds^2 = \left(1-\frac{2GM}{r}\right)dt^2 - \frac{dr^2}{\left(1-\frac{2GM}{r}\right)} - r^2 d\Omega_{II}^2 - dz^2$$

$$dt^2 \to \frac{(dt+v\,dz)^2}{1-v^2} \qquad dz^2 \to \frac{(dz+v\,dt)^2}{1-v^2}$$

$g_{zz} = -$

$$\left. + v\,dt\right)^2 \over 1 - v^2$$

$$g_{zz} = -\frac{1}{1-v^2}\left[1 + \frac{2GMv^2}{r} - v^2\right] = -\left[1 + \frac{2GMv^2}{(1-v)r}\right]$$

$$ds^2 = \left(1 - \frac{2GM}{(1-v^2)r + 2GMv^2}\right)dt^2 - \frac{dr^2}{1-\frac{2GM}{r}} - r^2 d\Omega_{II}^2$$

$$- \left(1 + \frac{2GMv^2}{(1-v^2)r}\right)\left[dz + \frac{2GMv\,dt}{(1-v^2)r + 2GMv^2}\right]^2$$

A_μ...

$$\hat{r} = r + \frac{2GMv^2}{1-v^2} \qquad Q = \frac{2GMv}{1-v^2}$$

$$\hat{M} = \frac{2GM}{1-v^2} \qquad \underline{A} = \frac{Q}{\hat{r}}\,\underline{dt}$$

ELECTRIC KK B.H.

$$e^{2\sigma} = \frac{\hat{r}}{\hat{r} - vQ}$$

$$ds^2 = \left(1 - \frac{2G\hat{M}}{\hat{r}}\right) dt^2 - \frac{(\hat{r} - vQ)}{(\hat{r} - 2G\hat{M})}\, d\hat{r}^2 - (\hat{r} - vQ)^2 d\Omega^2 - e^{2\sigma}\left[\underline{d}z + \underline{A}\right]^2$$

RIC
H.

Horizon at $2G\hat{M}$,
but area of spheres
not $4\pi\hat{r}^2$. Extremal
limit $v \to 1$

MAGNETIC CHARGE?

The electromagnetic e-m tensor

$$T^a{}_b = -F^{ac}F_{bc} + \frac{1}{4}F^2\delta^a_b$$

is invariant under $F \leftrightarrow *F$

$$\frac{Q}{r^2}\, dt \wedge dr \xrightarrow{*} Q\sin\theta\, d\theta \wedge d\varphi$$

MAGNETIC MONOPOLE.

$$d(Q\sin\theta\, d\theta \wedge d\varphi) = 0$$

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MAGNETIC MONOPOLE.

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Is $\underline{F} = \underline{d}\,\underline{A}$?

$\underset{\sim}{A} \sim -Q\cos\theta \, d\varphi$.

Singular at $\theta = 0, \pi$.

$A_N = Q(1-\cos\theta)\, d\varphi$.

$A_S = -Q(1+\cos\theta)\, d\varphi$.

Regular at N,S pole

$\underline{F} = \underline{d}\underline{A}$, but not globally.

2 gauge patches, N & S.

$$\underline{A}_S = \underline{A}_N - 2Q\,\underline{d}\varphi$$

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$$\underline{A}_S = \underline{A}_N - 2Q\,\underline{d}\varphi$$

Insert into KK perspective.

$(\hat{r} - 2q\hat{M})$ $\quad -e^{2\sigma}[\underline{d}z + \underline{A}]^2$

$\underline{A}_-$, but not globally.

gauge patches, N & S.

$$\underline{A}_S = \underline{A}_N - 2Q\,\underline{d}\varphi$$

into KK perspective.

2 patches

$$[d\psi_N + Q(1-\cos\theta)\,d\varphi]$$

& $[d\psi_S - Q(1+\cos\theta)\,d\varphi]$.

$$\psi_S = \psi_N - 2Q\varphi$$

Regular at N, S p

$\theta)\, d\varphi]$

$s\theta)\, d\varphi]$.

The soln is

$$ds^2 = \left(\frac{r-r_+}{r-r_-}\right)dt^2 - \frac{dr^2}{1-\frac{r_+}{r}} - r(r-r_-)\,d\Omega_{II}^2 - \left(1-\frac{r_-}{r}\right)\left[d\psi_N + \overbrace{\sqrt{r_+r_-}}^{Q}\,(1-\cos\theta)\,d\varphi\right]^2$$

For coord transfm on overlap of N & S patches to be well defined

$$\psi_S = \psi_N + 2Q\varphi = \psi_N + 2Q(\varphi + 2\pi)$$

$$= \psi_S + \underline{4\pi Q}$$

$$\omega^5 = e^{\sigma}[d\chi + A_\mu dx^\mu]$$

$$= -\Theta^5 \hat{a}$$

Extremal limit $r_+ = r_-$

$$ds^2 = dt^2 - \frac{dr^2}{1-Q/r} - \left(1-\frac{Q}{r}\right)\left[r^2 d\Omega_{II}^2 + Q^2\left(\underbrace{\frac{d\psi}{Q}}_{4\pi}^{\chi} + (1-\cos\theta)d\varphi\right)^2\right]$$

$$\rho^2 = 4Q(r-Q) \quad d\rho^2 \quad -\frac{\rho^2}{4}\Big[\underbrace{d\theta^2 + \sin^2\theta\, d\varphi^2 + [d\chi + (1-\cos\theta)d\varphi]^2}_{\text{Metric on } S^3}\Big]$$

FLAT SPACETIME.

Kaluza-Klein monopole

$$X + iY = \rho \cos\theta/_2 \, e^{i\chi/2}$$

$$Z + iW = \rho \sin\theta/_2 \, e^{i(\varphi + \chi/_2)}$$

$$X^2 + Y^2 + Z^2 + W^2 = \rho^2$$

Kaluza-Klein monopole

$$(1-\cos\theta)d\varphi)^2]]$$

$$X + iY = \rho \cos\theta/_2 \, e^{i\chi/2}$$

$$Z + iW = \rho \sin\theta/_2 \, e^{i(\varphi + \chi/2)}$$

$$d\chi + (1-\cos\theta)d\varphi)^2]$$

$$X^2+Y^2+Z^2+W^2=\rho^2$$

S^3

Hopf Fibration