Title: Gravitational Physics Lecture

Speakers: Ruth Gregory

Collection: Gravitational Physics

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L4: Black Holes & Branes

Last time:

$$ds^2 = A^2(r)\,dt^2 - B^2(r)\,dr^2 - C^2(r)\,d\Omega^2_{D-2}$$

$$R^t{}_t = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB} + (D-2)\frac{A'C'}{AC}\right)$$

$$R^r{}_r = \frac{1}{B^2}\left(\frac{A''}{A} + (D-2)\frac{C''}{C} - \frac{B'}{B}\left(\frac{A'}{A} + (D-2)\frac{C'}{C}\right)\right)$$

$$R^\alpha{}_\beta = \delta^\alpha_\beta\left[\frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB} + \frac{A'C'}{AC} + (D-3)\,C'^2/C^2\right) - \frac{1}{C^2}R^\alpha{}_{\gamma\,\beta}\right]$$

L4: Black Holes & Branes

Last time:

$\alpha, \beta \ldots$

$$ds^2 = A^2(r)\,dt^2 - B^2(r)\,dr^2 - C^2(r)\,d\Omega^2_{D-2}$$

K

$$R^t_t = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB} + (D-2)\frac{A'C'}{AC}\right)$$

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For a subspace of curvature κ,

$$\underbrace{R_{\sigma}{}^{\alpha}{}_{\beta}}_{n} \sim \underbrace{(D-3)}_{n-1}\,\kappa\,\delta^{\alpha}{}_{\beta}$$

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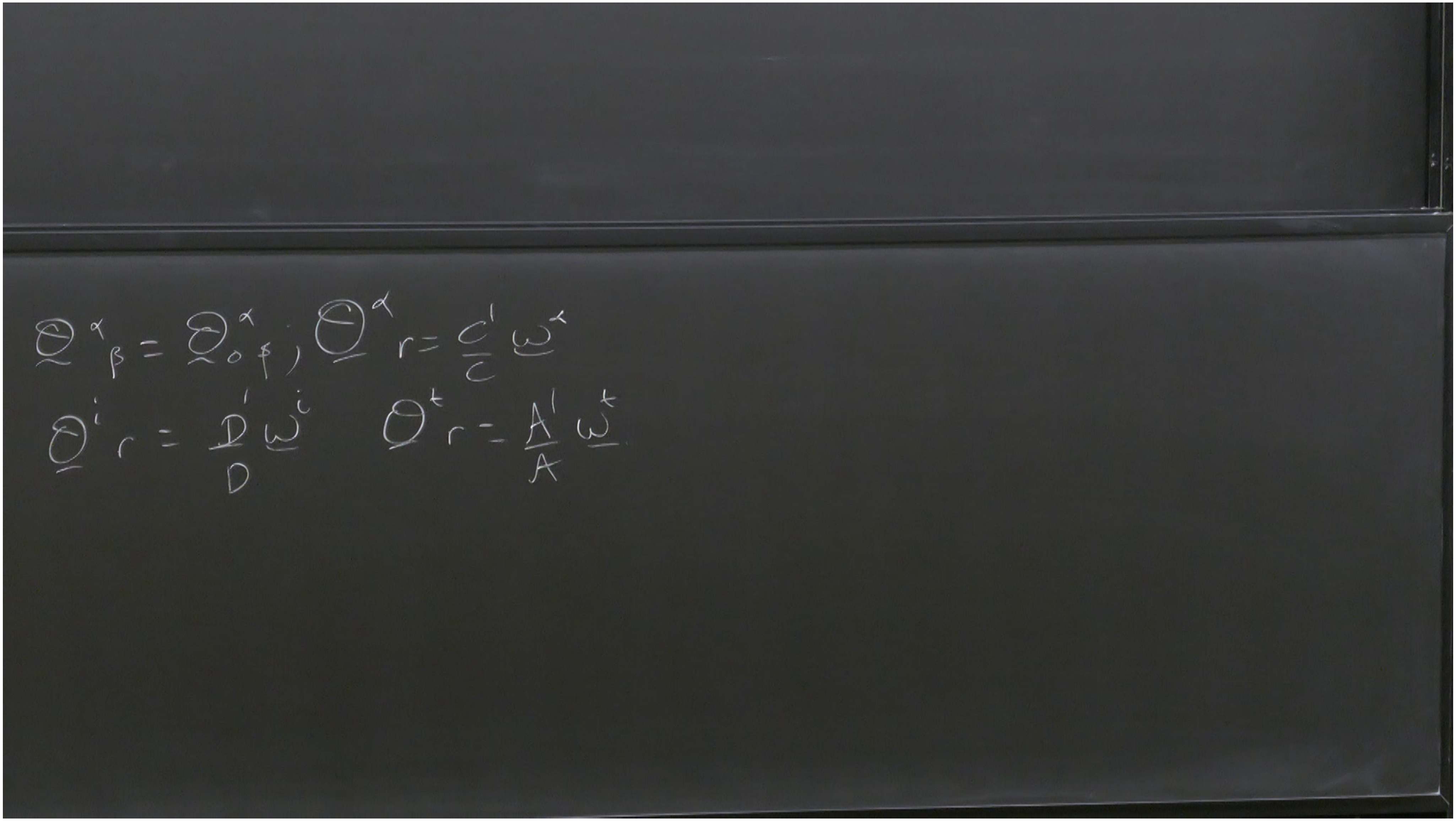
Add extra building blocks

$$ds^2 = A^2dt^2 - B^2dr^2 - C^2d\Omega^2_{\kappa,n} - D^2\overset{\text{FLAT}}{\downarrow}dy_p^2$$

Clear that y & Ω do not mix at connection level.


$\Theta^{\alpha}{}_{\beta} = \Theta_0{}^{\alpha}{}_{\beta}$; $\Theta^{\alpha}{}_{r} = \frac{C'}{C}\omega^{\alpha}$
$\Theta^{i}{}_{r} = \frac{D'}{D}\omega^{i}$ $\Theta^{t}{}_{r} = \frac{A'}{A}\omega^{t}$

$$\underset{\sim}{\Theta}^{\alpha}{}_{\beta} = \underset{\sim}{\Theta}_0{}^{\alpha}{}_{\beta} \; ; \quad \underline{\Theta}^{\alpha}{}_{r} = \frac{C'}{C}\,\underline{\omega}^{\alpha}$$

$$\underline{\Theta}^{i}{}_{r} = \frac{D'}{D}\,\underline{\omega}^{i} \qquad \underline{\Theta}^{t}{}_{r} = \frac{A'}{A}\,\underline{\omega}^{t}$$

2-forms are same or similar;

get extra cross terms:

e.g. $\underline{R}^{t}{}_{i} = \underline{\Theta}^{t}{}_{r} \wedge \underline{\Theta}^{r}{}_{i} = \frac{A'D'}{AD}\,\underline{\omega}^{t} \wedge \underline{\omega}_{i}$

↑ lowered with η.

$$\underset{\sim}{\Theta}^{\alpha}{}_{\beta} = \underset{\sim}{\Theta}_{0}{}^{\alpha}{}_{\beta}\,; \quad \underset{\sim}{\Theta}^{\alpha}{}_{r} = \frac{C'}{C}\,\underset{\sim}{\omega}^{\alpha}$$

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↑ lowered with η.

$$R^{tr}{}_{tr} = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'D'}{AD}\right) \qquad R^{ti}{}_{tj} = \frac{A'D'}{AB^2D}\,\delta^i_j$$

$$R^{ir}{}_{jr} = \frac{1}{B^2}\left(\frac{D''}{D} - \frac{B'D'}{BD}\right)$$

$$R^{i\alpha}{}_{j\beta} = \frac{C'D'}{B^2CD}\,\delta^i_j\,\delta^\alpha_\beta$$

$$R^{ij}{}_{kl} = \frac{D'^2}{D^2}\left(\delta^i_k\delta^j_l - \delta^i_l\delta^j_k\right)$$

plus the same $R^{t\alpha}{}_{t\beta}$, $R^{\alpha\beta}{}_{\gamma\delta}$, $R^{r\alpha}{}_{r\beta}$ from Friday

$\underset{\sim}{\Theta}^{\alpha}{}_{\beta} = \underset{\sim}{\Theta}_0{}^{\alpha}{}_{\beta}$; $\underset{\sim}{\Theta}^{\alpha}{}_{r} = \frac{C'}{C}\underset{\sim}{\omega}^{\alpha}$

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($R = d\Theta + \Theta \wedge \Theta$)

↑ lowered with η.

$R^{tr}{}_{tr} = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'D'}{AD}\right)$ $\quad R^{ti}{}_{tj} = \frac{A'D'}{AB^2D}\delta^i_j$

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$$\Theta^\alpha{}_\beta = \Theta_0{}^\alpha{}_\beta \,; \quad \Theta^\alpha{}_r = \frac{C'}{C}\,\omega^\alpha$$

$$\Theta^i{}_r = \frac{D'}{D}\,\omega^i \qquad \Theta^t{}_r = \frac{A'}{A}\,\omega^t$$

2-forms are same or similar; get extra cross terms:

e.g. $R^t{}_i = \Theta^t{}_r \wedge \Theta^r{}_i = \frac{A'D'}{AD}\,\omega^t \wedge \omega_i$

$(R = d\Theta + \Theta \wedge \Theta)$ ↑ lowered with η.

$$R^{tr}{}_{tr} = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'D'}{AD}\right) \qquad R^{ti}{}_{tj} = \frac{A'D'}{AB^2D}\,\delta^i_j$$

$$R^{ir}{}_{jr} = -\frac{1}{B^2}\left(\frac{D''}{D} - \frac{B'D'}{BD}\right)$$

$$R^{i\alpha}{}_{j\beta} = \frac{C'D'}{B^2CD}\,\delta^i_j\,\delta^\alpha_\beta$$

$$R^{ij}{}_{kl} = \frac{D'^2}{D^2}\left(\delta^i_k\delta^j_l - \delta^i_l\delta^j_k\right)$$

plus the same $R^{t\alpha}{}_{t\beta}$, $R^{\alpha\beta}{}_{\gamma\delta}$, $R^{r\alpha}{}_{r\beta}$ from Friday

$$R^{\alpha\beta}{}_{\gamma\delta} = \frac{1}{C^2}R_0{}^{\alpha\beta}{}_{\gamma\delta} + \frac{C'^2}{C^2B^2}\left(\delta^\alpha_\gamma\delta^\beta_\delta - \delta^\alpha_\delta\delta^\beta_\gamma\right)$$

Hence $$R^t{}_t = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB} + p\frac{A'D'}{AD} + n\frac{A'C'}{AC}\right)$$

$$R^i{}_j = \frac{\delta^i_j}{B^2}\left(\frac{D''}{D} - \frac{D'B'}{DB} + \frac{A'D'}{AD} + n\frac{C'D'}{CD} + (p-1)\frac{D'^2}{D^2}\right)$$

$$R^\alpha{}_\beta = \frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB} + \frac{A'C'}{AC} + p\frac{C'D'}{CD} + (n-1)\frac{C'^2}{C^2}\right) - \frac{1}{C^2}R_0{}^\alpha{}_\beta$$

$$R^r{}_r = \frac{1}{B^2}\left(\frac{A''}{A} + n\frac{C''}{C} + p\frac{D''}{D} - \frac{B'}{B}\left(\frac{A'}{A} + n\frac{C'}{C} + p\frac{D'}{D}\right)\right)$$

Hence $R^t{}_t = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB} + p\frac{A'D'}{AD} + n\frac{A'C'}{AC}\right)$

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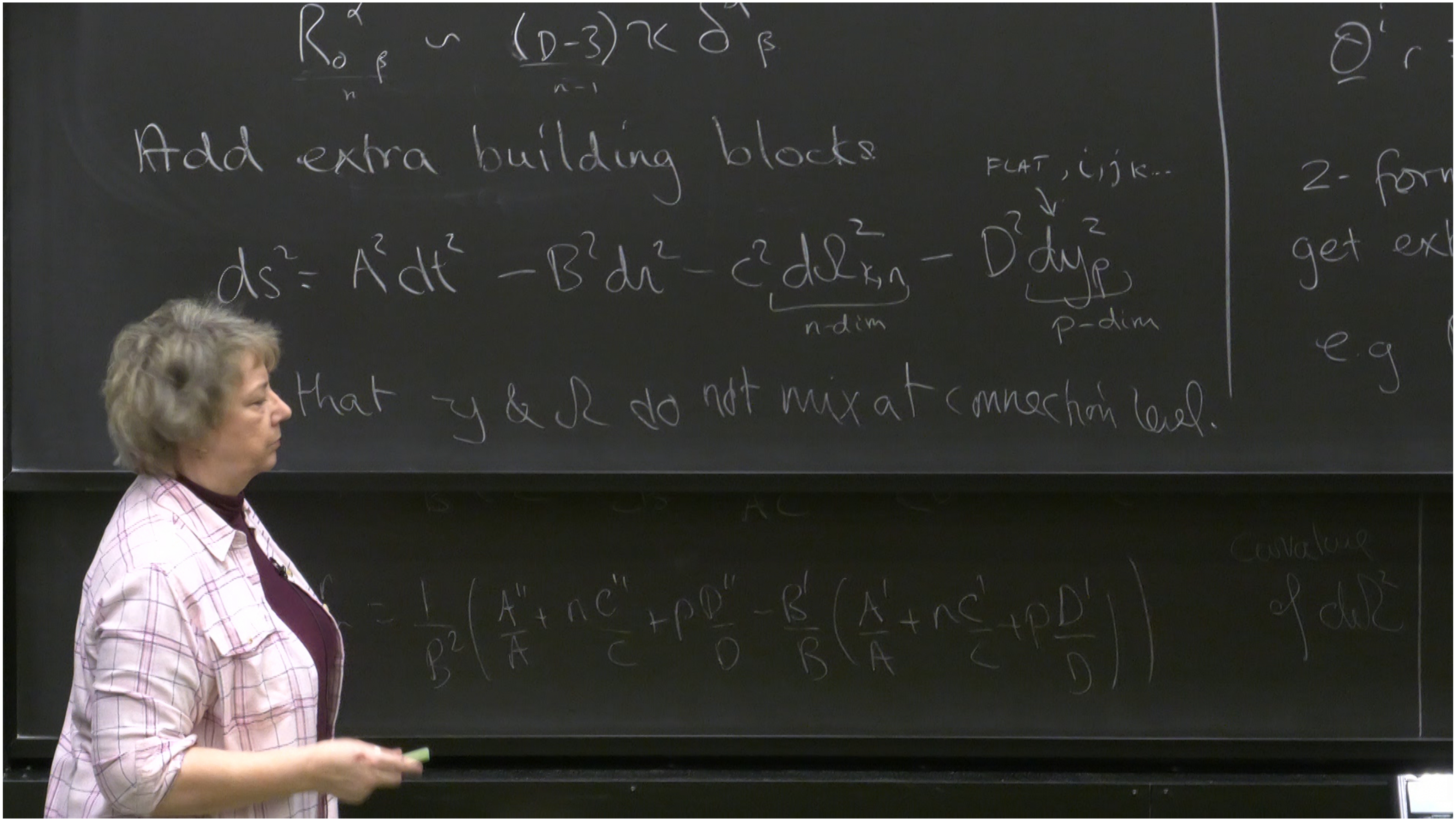
curvature of $d\Omega^2$

$$R^r{}_r = \frac{1}{B^2}\left(\frac{A''}{A} + n\frac{C''}{C} + p\frac{D''}{D} - \frac{B'}{B}\left(\frac{A'}{A} + n\frac{C'}{C} + p\frac{D'}{D}\right)\right)$$


R_0^α_β ~ (D-3) κ δ^α_β
Add extra building blocks
FLAT, i, j k...
ds² = A²dt² − B²dr² − C²dΩ²_{κ,n} − D²dy²_p
n-dim
p-dim
that y & Ω do not mix at connection level.
Curvature
of dΩ²
2- form
get ext
e.g

Examples $p=0$, $n=2$ i.e. $D=4$ Take $C=r$ (areal)

$C'=1$, $C''=0$.

$$G^t{}_t = \frac{1}{r^2} - \frac{1}{B^2}\left(-\frac{2B'}{Br} + \frac{1}{r^2}\right) = \frac{1}{r^2} - \frac{1}{r^2}\left(\frac{r}{B^2}\right)' = 8\pi G T^0_0$$

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$$\Rightarrow \frac{1}{B^2} = 1 - \frac{2G}{r}\int 4\pi r^2 T^0{}_0\, dr.$$

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$$\Rightarrow \frac{1}{B^2} = 1 - \frac{2G}{r}\int \underbrace{4\pi r^2 \overset{\rho}{T^0{}_0}\, dr}_{\text{"energy" between } r \,\&\, r+dr}$$

$$R^r{}_r = \frac{1}{B^2}\left(\frac{A''}{A} + n\frac{C''}{C} + p\frac{D''}{D} - \frac{B'}{B}\left(\frac{A'}{A} + n\frac{C'}{C} + p\frac{D'}{D}\right)\right)$$

of $d\Omega^2$

$$B^{-2} = 1 - \frac{2GM(r)}{r}$$

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of $d\Omega^2$

$$B^{-2} = 1 - \frac{2GM(r)}{r}$$

Next, $R^t{}_t - R^r{}_r = \frac{2}{rB^2}\left(\frac{A'}{A} + \frac{B'}{B}\right)$

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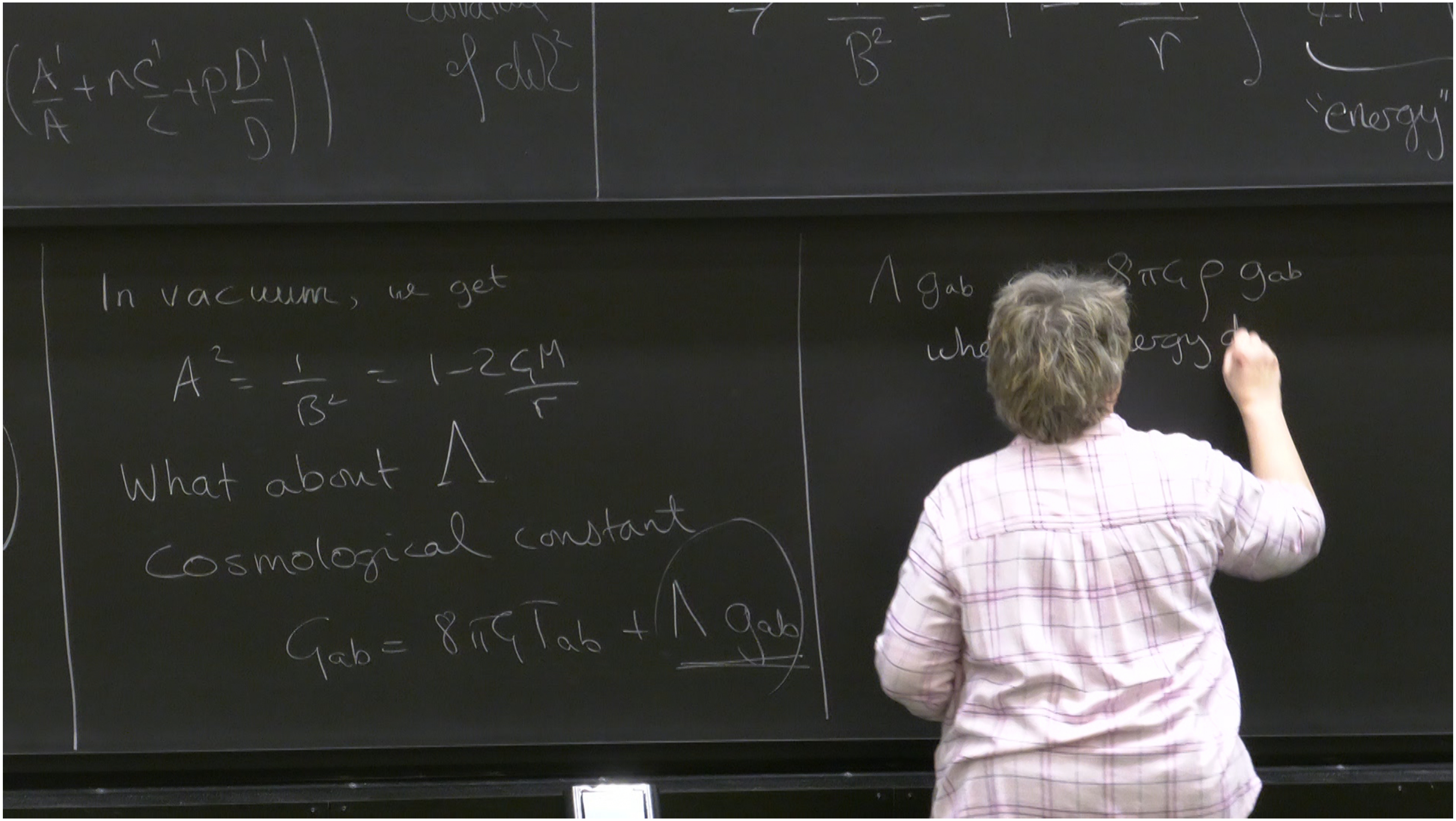
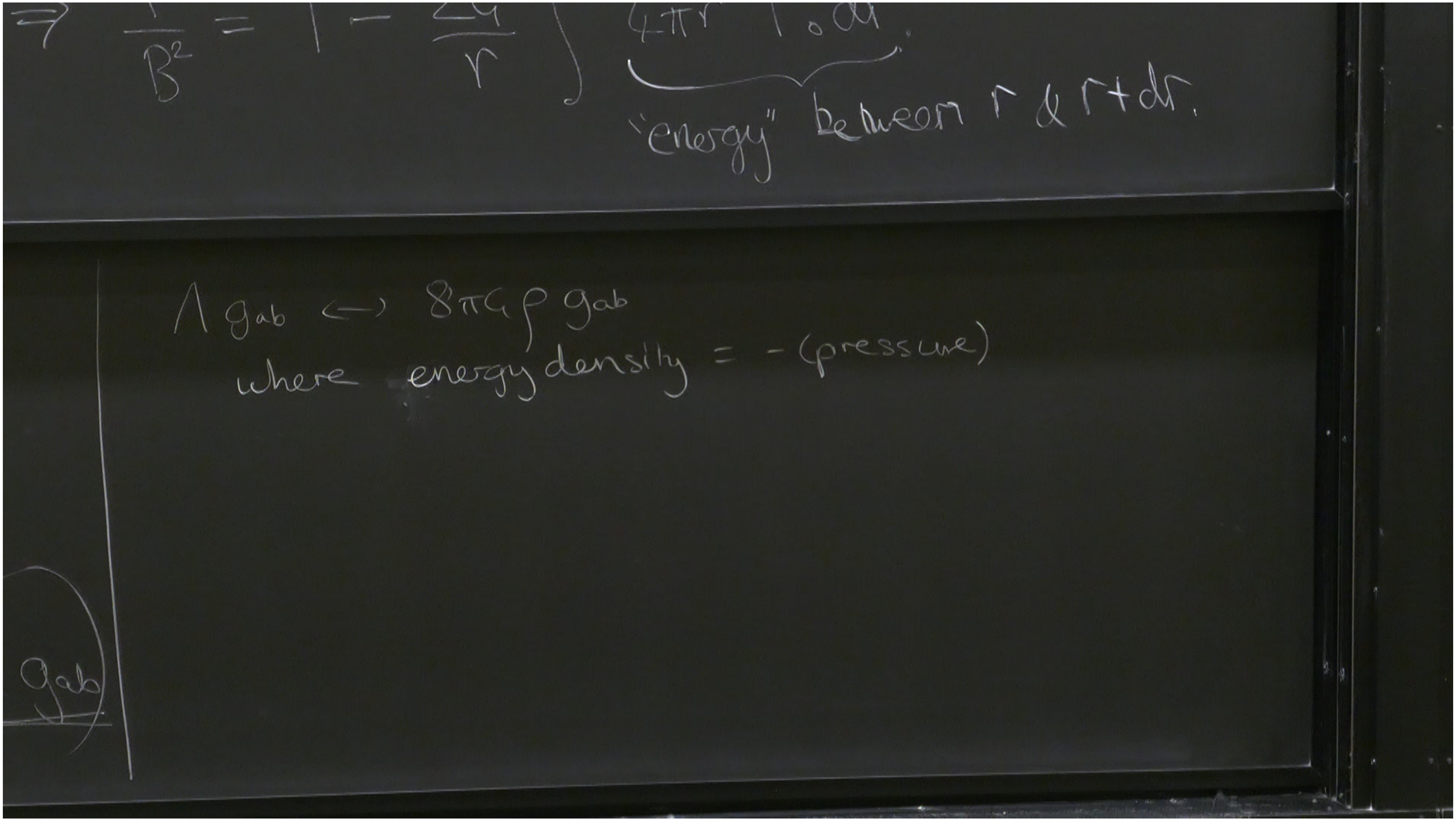
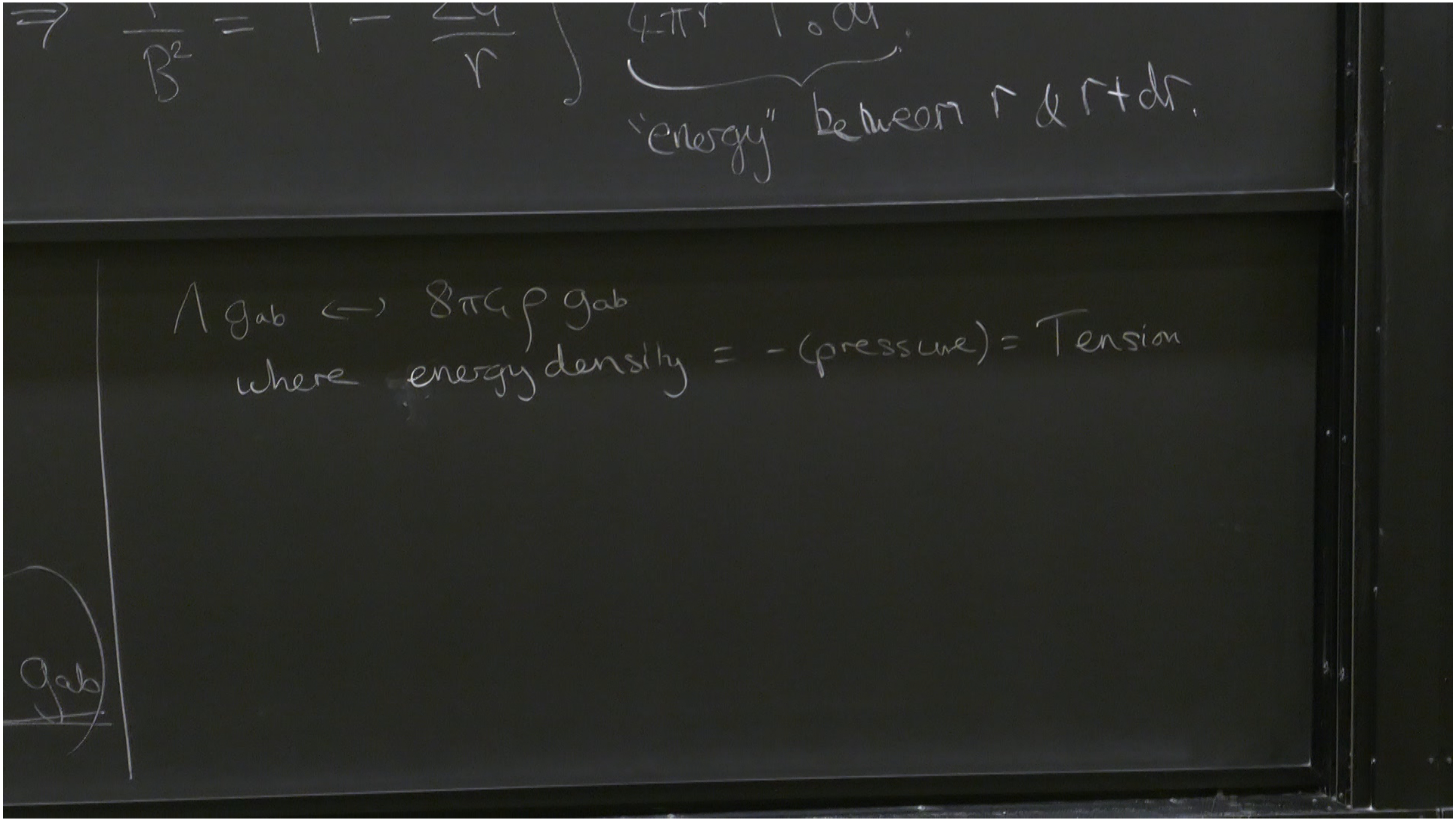
Next, $R^t{}_t - R^r{}_r = \frac{2}{rB^2}\left(\frac{A'}{A} + \frac{B'}{B}\right)$

If $T^0{}_0 = T^r{}_r$, then $A \propto 1/B$


"energy"
In vacuum, we get
$A^2 = \frac{1}{B^2} = 1 - \frac{2GM}{r}$
What about Λ
Cosmological constant
$G_{ab} = 8\pi G T_{ab} + \Lambda g_{ab}$
Λg_{ab}
$8\pi G \rho g_{ab}$


"energy" between r & r+dr.
Λ gab ⟷ 8πGρ gab
where energy density = -(pressure)
gab



"energy" between r & r+dr.
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where energy density = −(pressure) = Tension
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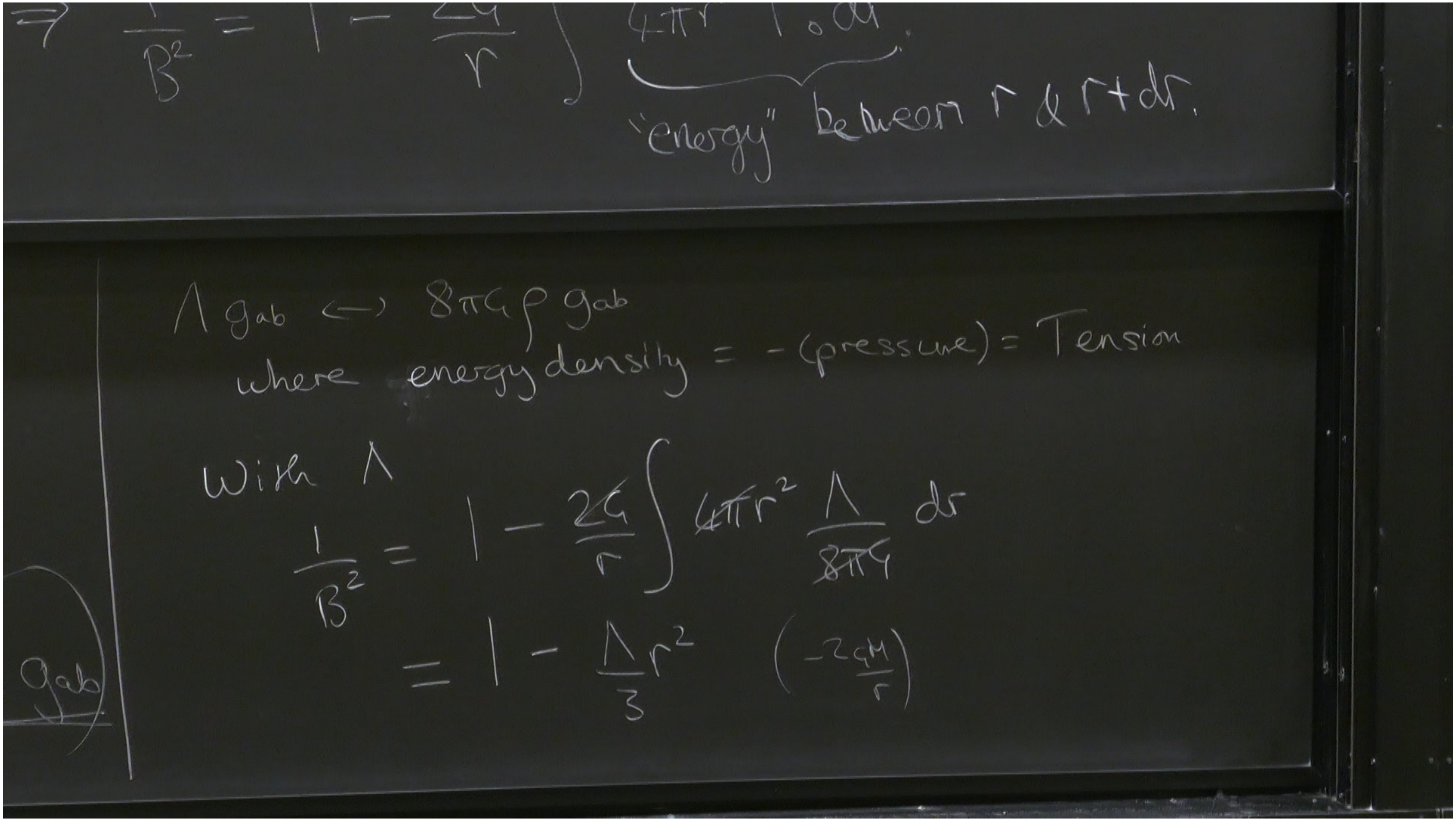
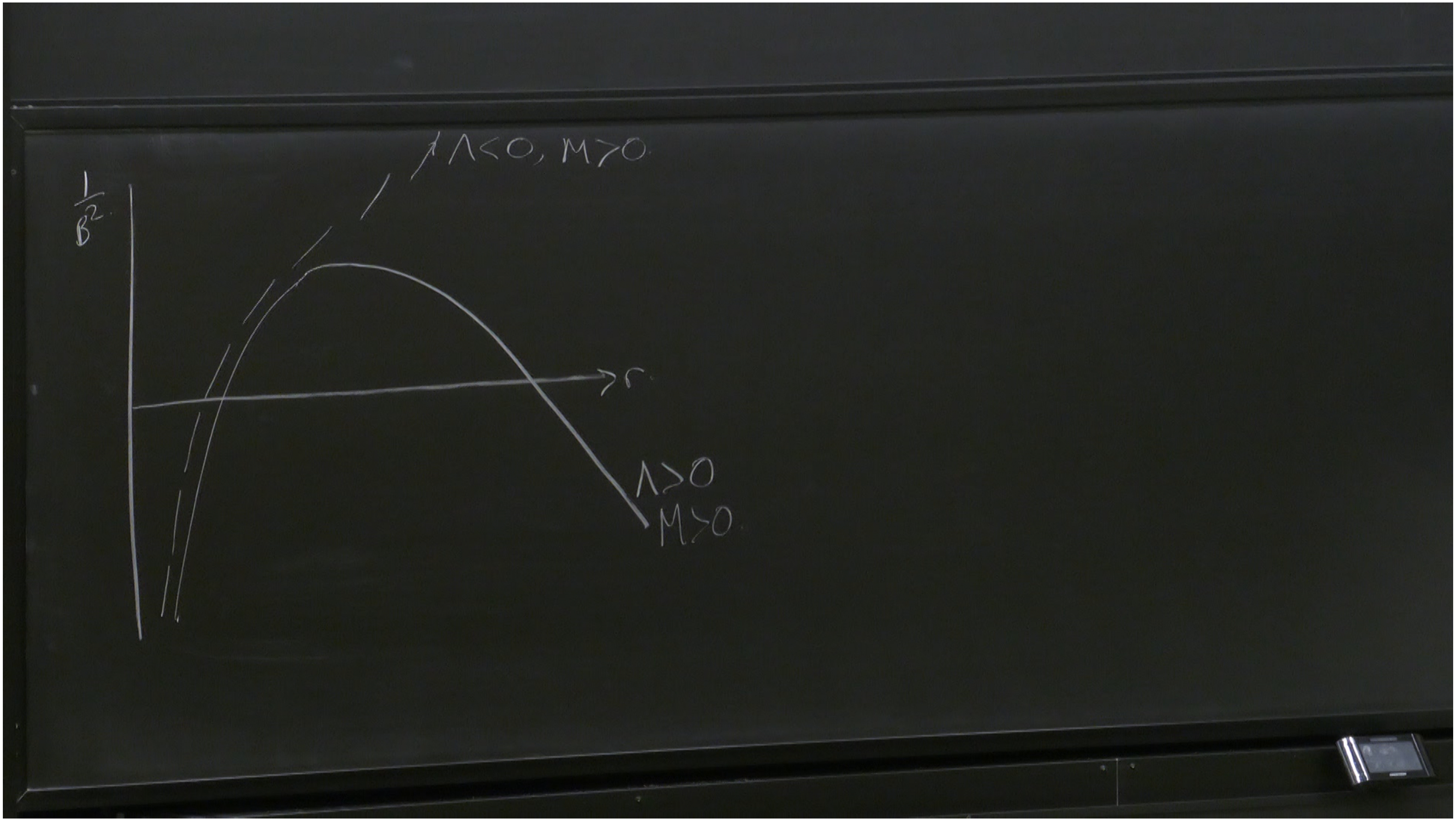


"energy" between r & r+dr.
Λ g_ab ⟷ 8πG ρ g_ab
where energy density = -(pressure) = Tension
With Λ
1/B² = 1 − 2G/r ∫ 4πr² Λ/8πG dr
= 1 − Λ/3 r² (−2GM/r)
g_ab


$\frac{1}{B^2}$
$\Lambda<0, M>0$
r
$\Lambda>0$
$M>0$

"energy" between r & $r+dr$.

nt

$+ \Lambda g_{ab}$

$$\Lambda g_{ab} \leftrightarrow 8\pi G \rho g_{ab}$$

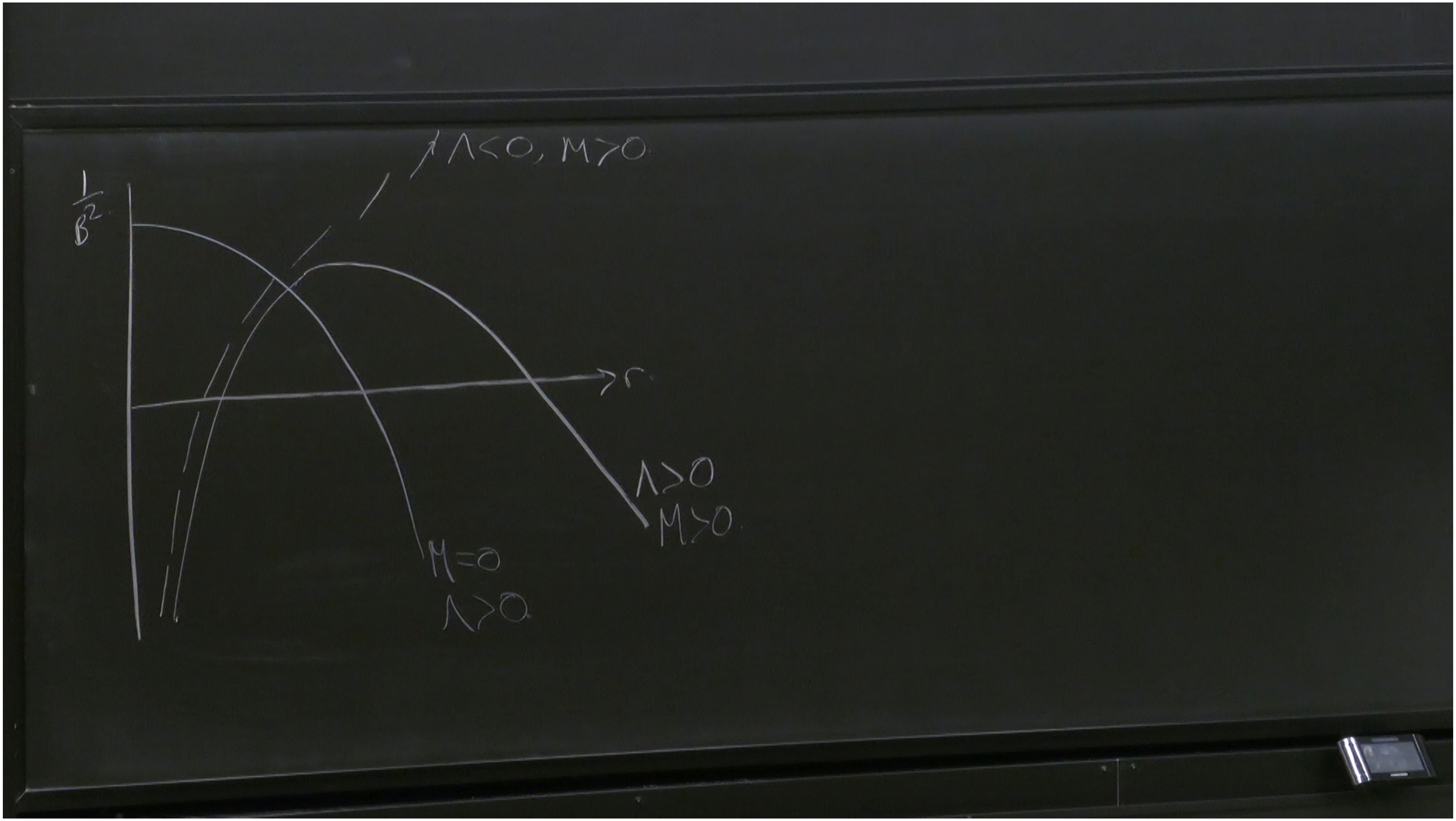
where energy density = −(pressure) = Tension

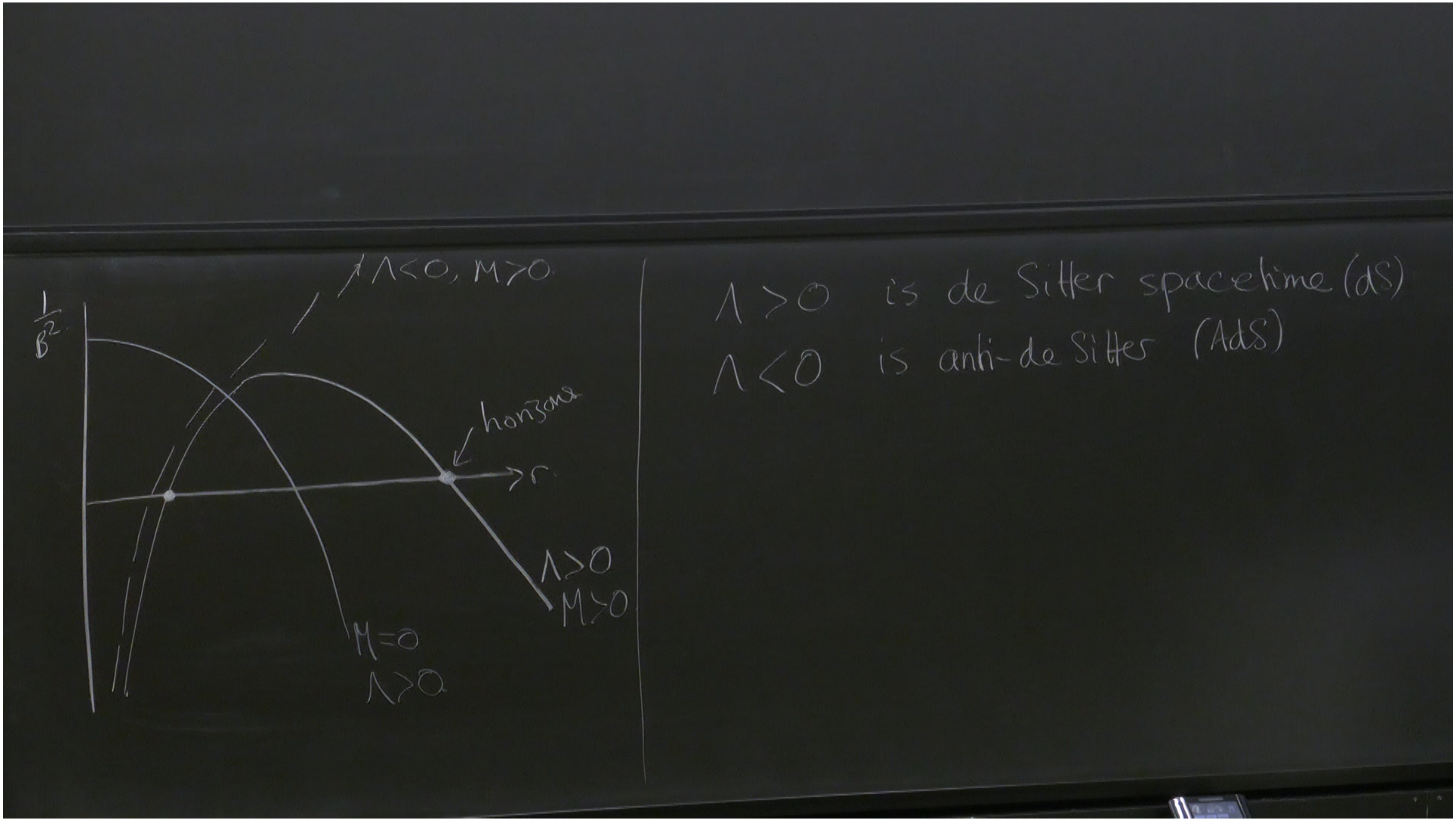
With Λ

$$\frac{1}{B^2} = 1 - \frac{2G}{r}\int 4\pi r^2 \frac{\Lambda}{8\pi G}\, dr$$

$$= 1 - \frac{\Lambda}{3} r^2 \quad \left(-\frac{2GM}{r}\right)$$

$A \propto 1/B$ still.


$\frac{1}{B^2}$
$\Lambda<0,\ M>0$
r
$\Lambda>0$
$M>0$
$M=0$
$\Lambda>0$



$\Lambda > 0$ is de Sitter spacetime (dS)
$\Lambda < 0$ is anti-de Sitter (AdS)
$\frac{1}{B^2}$
$\Lambda < 0, M > 0$
horizons
r
$\Lambda > 0$
$M > 0$
$M = 0$
$\Lambda > 0$

$\Lambda > 0$ is de Sitter spacetime (dS)

$\Lambda < 0$ is anti-de Sitter (AdS)

$$(M=0) \quad ds^2 = \left(1 - \frac{r^2}{L^2}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{r^2}{L^2}\right)} - r^2 d\Omega^2 \quad \text{where } \Lambda = \frac{3}{L^2}$$

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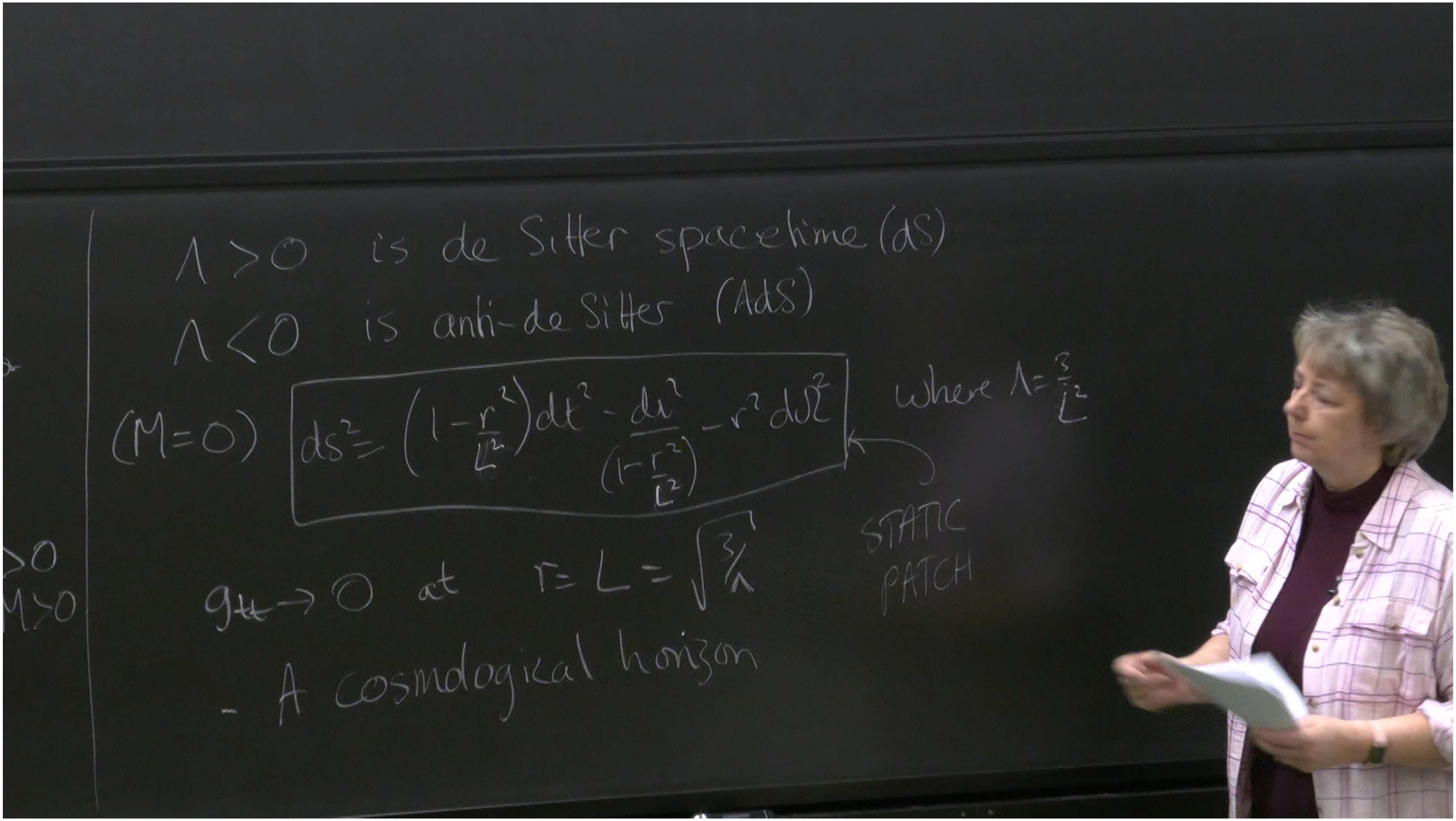
$g_{tt} \to 0$ at $r = L = \sqrt{3/\Lambda}$

- A cosmological horizon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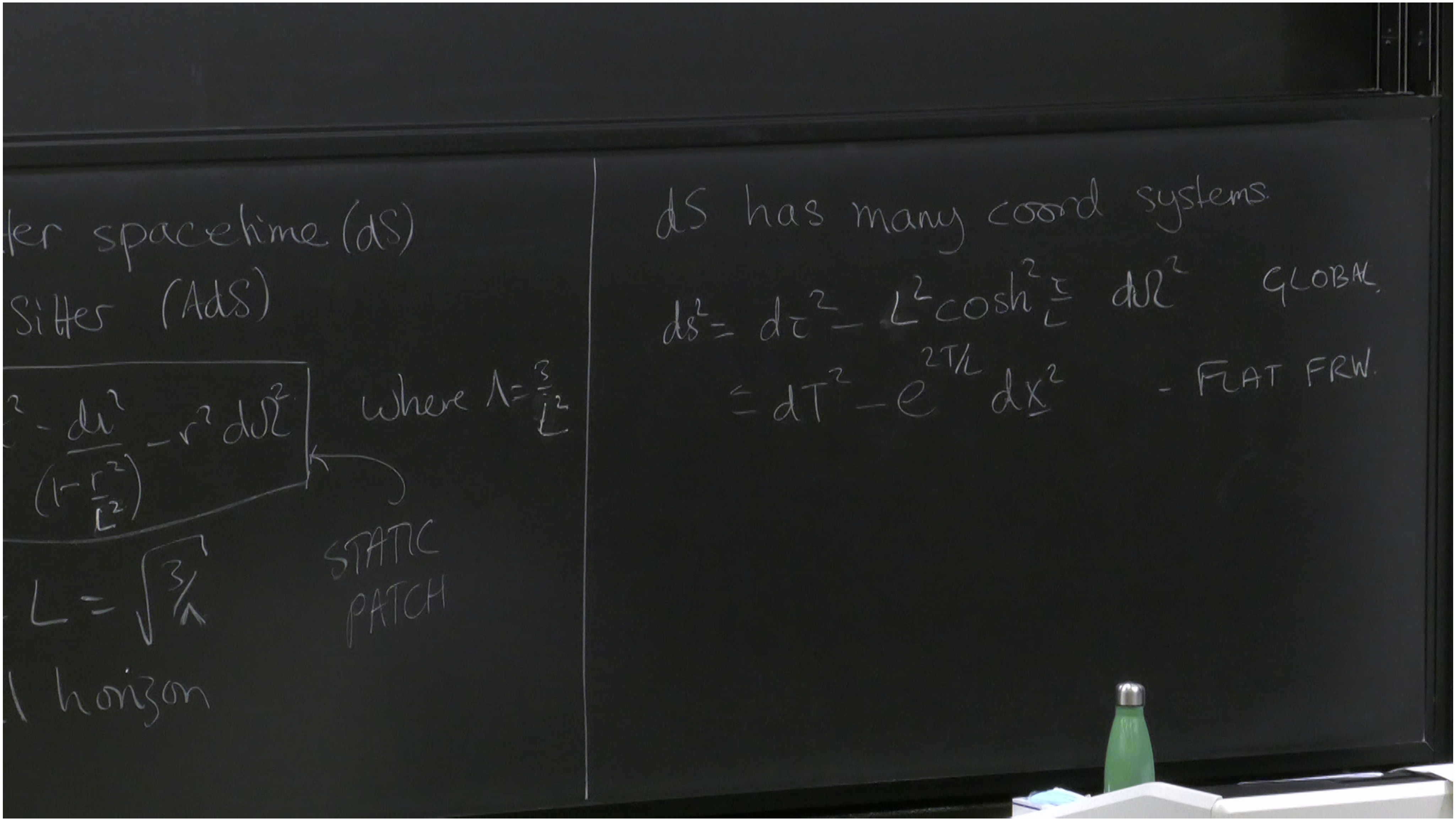
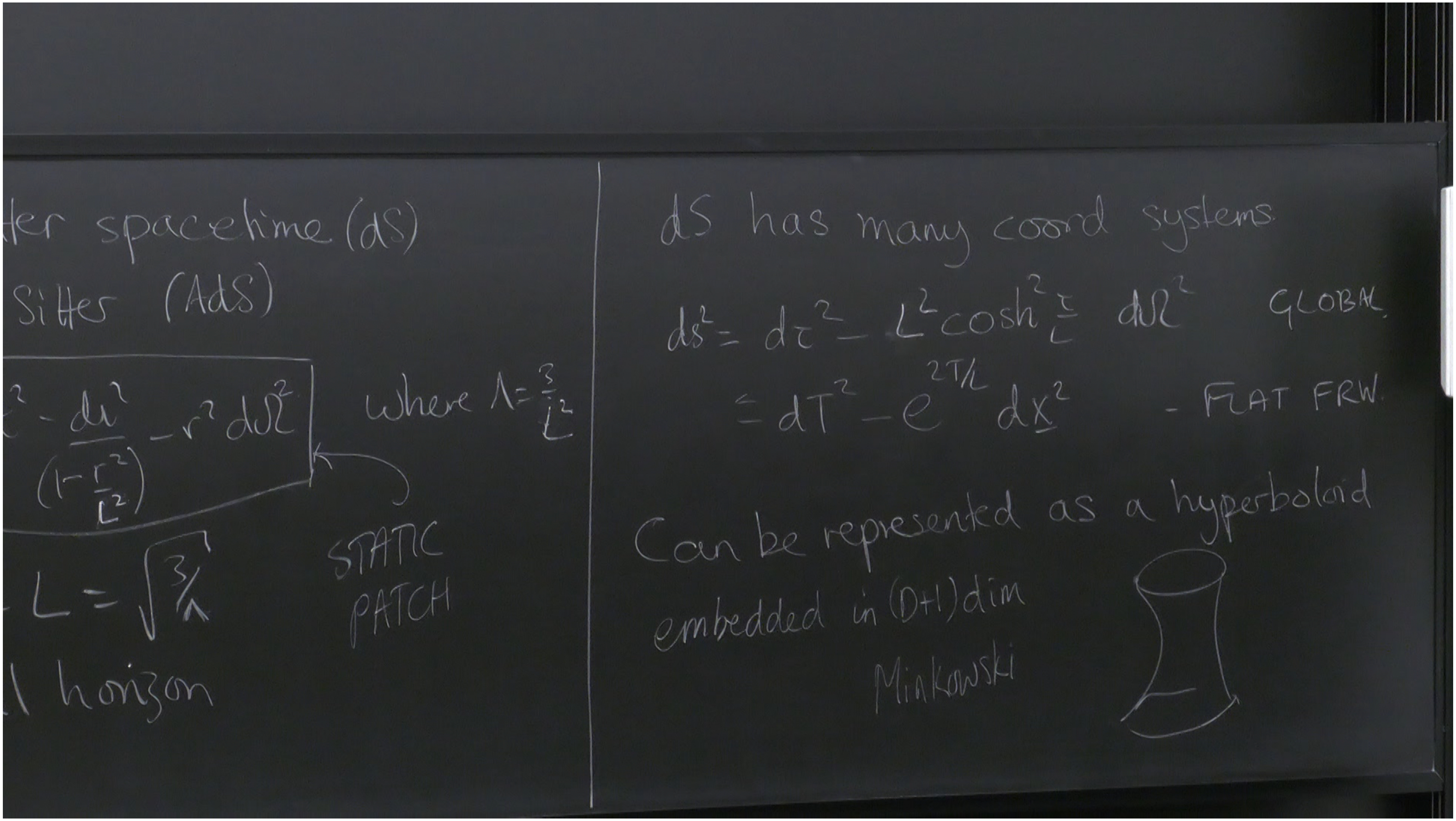


ter spacetime (dS)
Sitter (AdS)
where $\Lambda = \frac{3}{L^2}$
$L = \sqrt{3/\Lambda}$
STATIC PATCH
l horizon
dS has many coord systems.
$ds^2 = d\tau^2 - L^2 \cosh^2 \frac{\tau}{L} \, d\Omega^2$ GLOBAL
$= dT^2 - e^{2T/L} \, d\underline{x}^2$ - FLAT FRW.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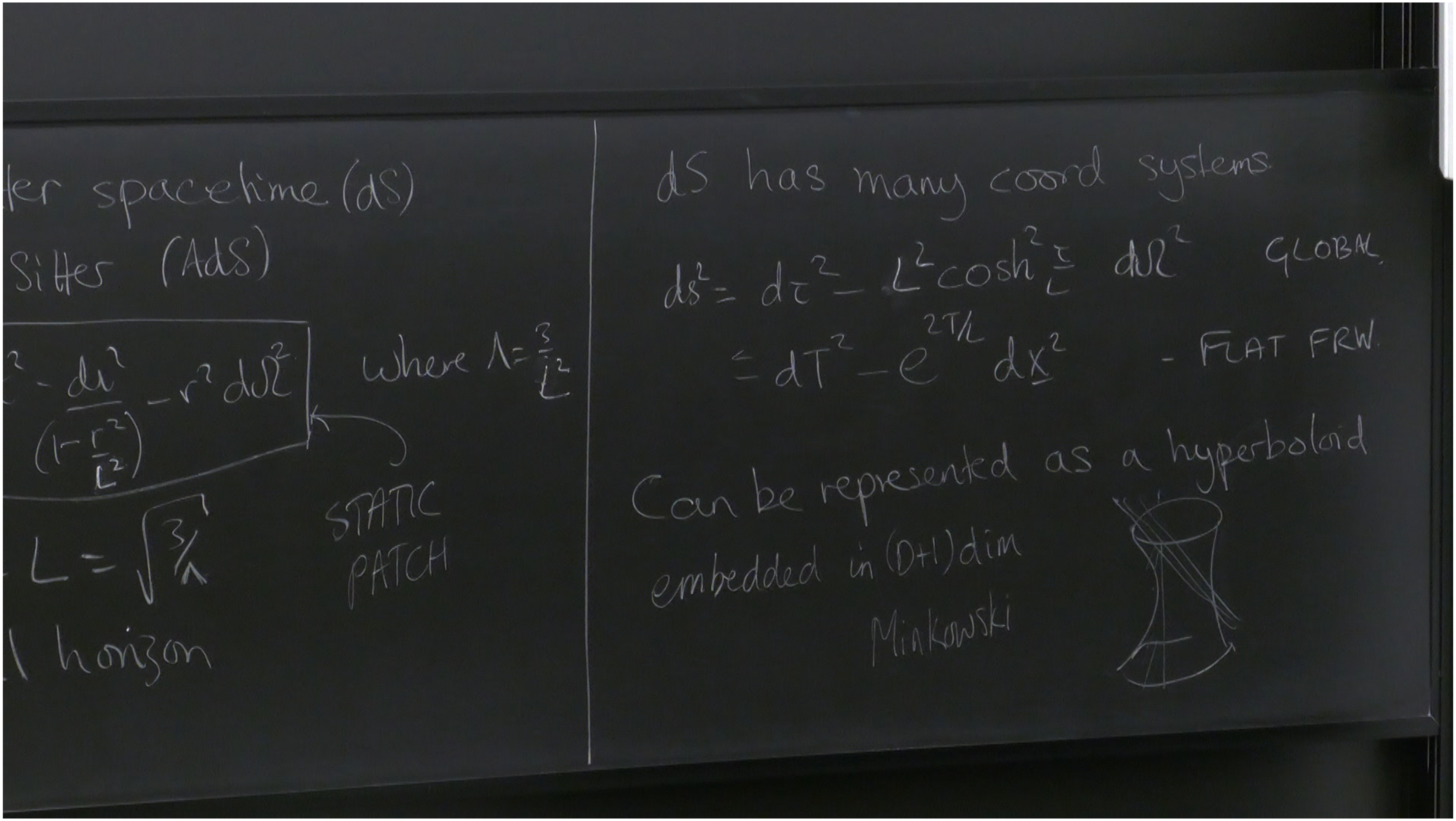
Can be represented as a hyperboloid
embedded in (D+1) dim
Minkowski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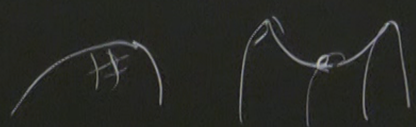
AdS is hyperbolic - negatively curved)

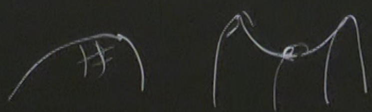


If $\kappa \neq 1$ (ie $d\Omega^2$ not a sphere)

$$A^2 = B^{-2} = \kappa - \frac{\Lambda}{3} r^2 - \frac{2GM}{r}$$

need $\Lambda < 0$, $\kappa = 0, -1$ possible

AdS is hyperbolic - negatively curved



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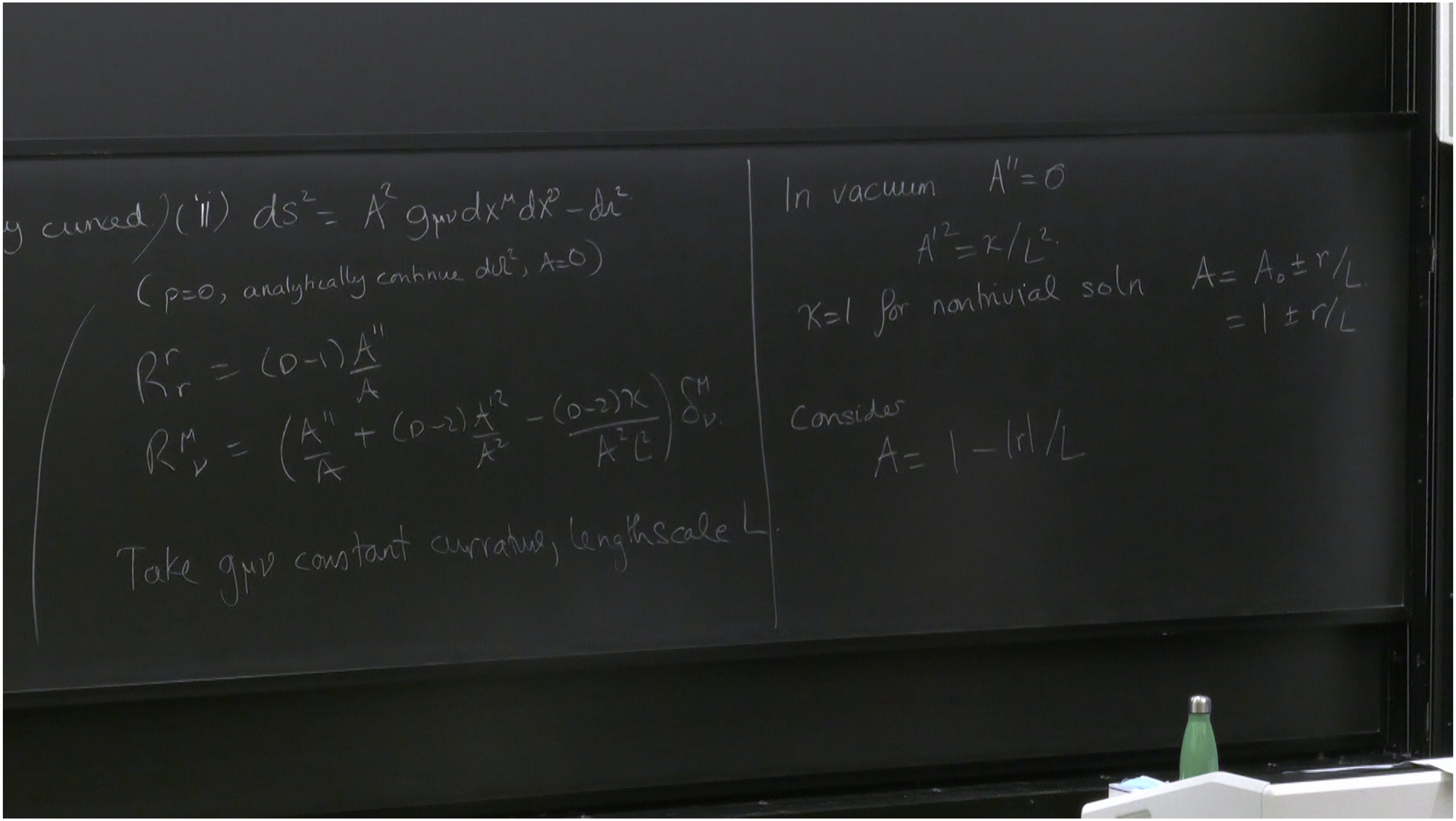
(ii) $ds^2 = A^2 g_{\mu\nu} dx^\mu dx^\nu - dr^2$.

($\rho = 0$, analytically continue $d\Omega^2$, $A=0$)



y curved) (ii) $ds^2 = A^2 g_{\mu\nu} dx^\mu dx^\nu - dr^2$
($p=0$, analytically continue $d\Omega^2$, $A=0$)
$R^r{}_r = (D-1)\frac{A''}{A}$
$R^\mu{}_\nu = \left(\frac{A''}{A} + (D-2)\frac{A'^2}{A^2} - \frac{(D-2)\kappa}{A^2 L^2}\right)\delta^\mu_\nu$
Take $g_{\mu\nu}$ constant curvature, lengthscale L
In vacuum $A''=0$
$A'^2 = \kappa/L^2$
$\kappa=1$ for nontrivial soln $A = A_0 \pm r/L$
$= 1 \pm r/L$
Consider
$A = 1 - |r|/L$

$g_{\mu\nu} dx^\mu dx^\nu - dr^2$.

ontinue $d\Omega^2$, $A=0$)

$(D-2)\frac{A'^2}{A^2} - \frac{(D-2)\kappa}{A^2 L^2}\Big)\delta^\mu_\nu$.

t curvature, lengthscale L.

In vacuum $A''=0$

$A'^2 = \kappa/L^2$.

$\kappa=1$ for nontrivial soln $A = A_0 \pm r/L$.

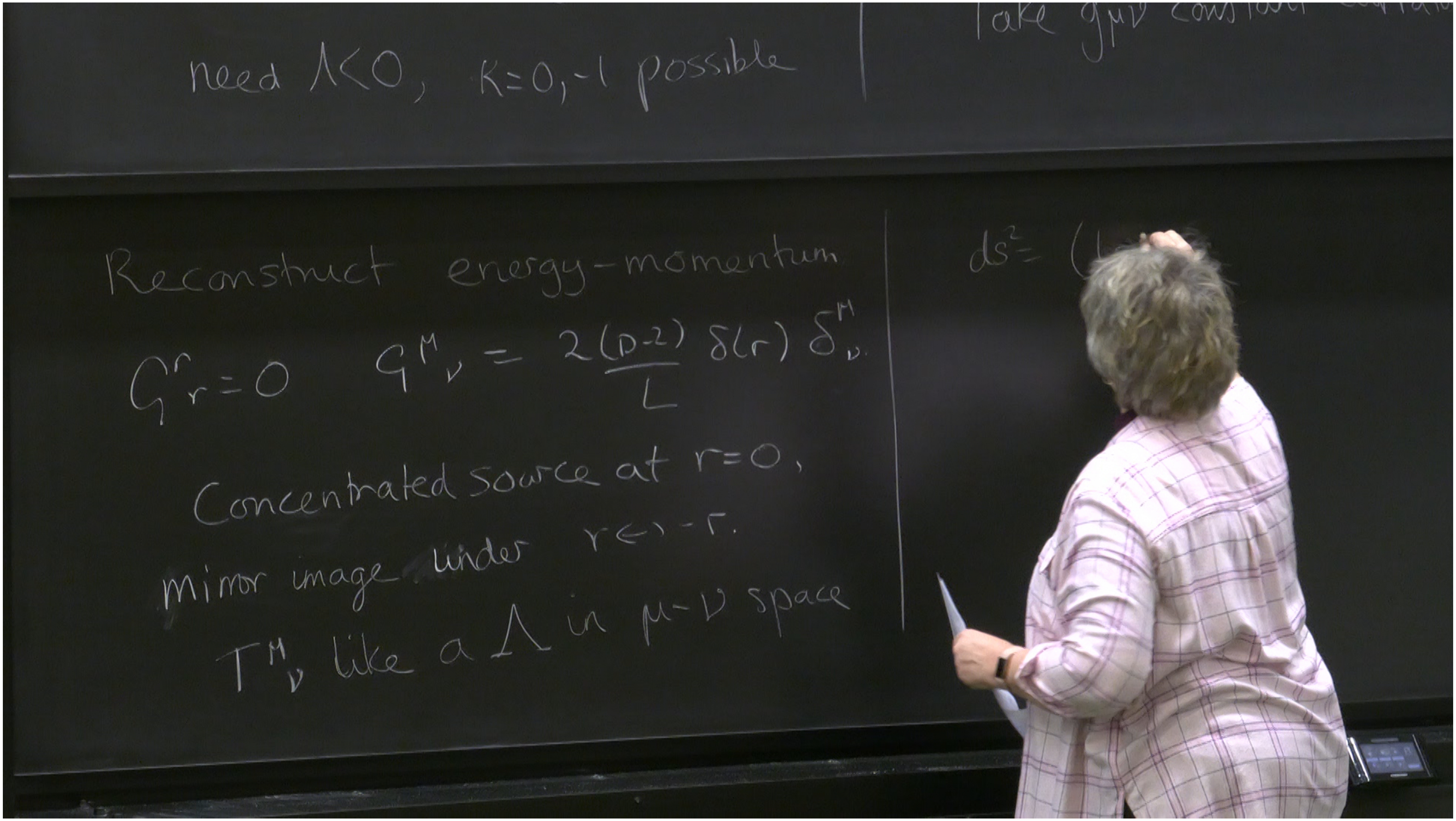
$= 1 \pm r/L$

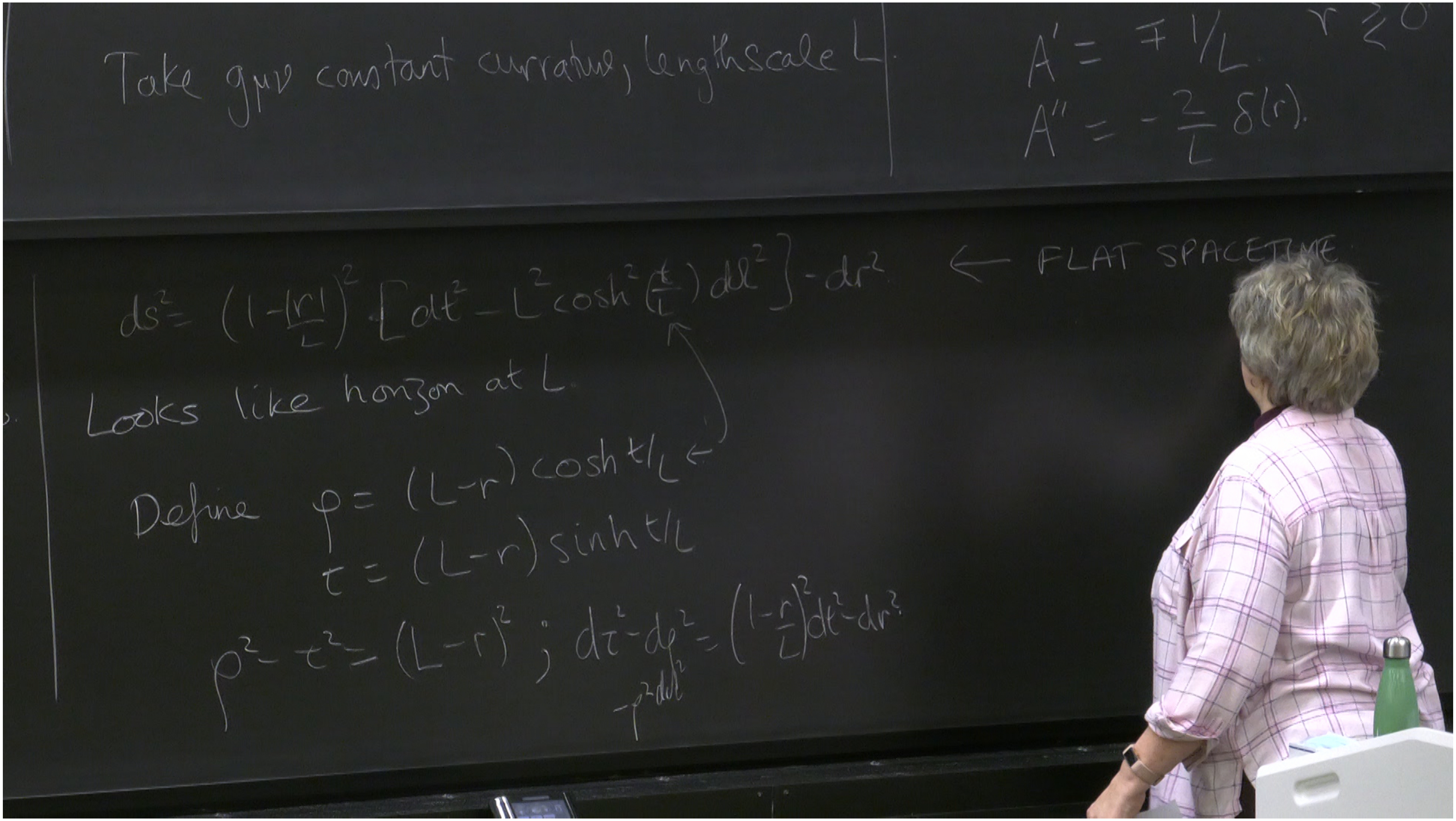
Consider

$A = 1 - |r|/L$

$A' = \mp 1/L$. $r \gtrless 0$

$A'' = -\frac{2}{L}\delta(r)$.


need Λ<0, K=0,-1 possible
Take gμν constant
Reconstruct energy-momentum
G^r_r = 0   G^μ_ν = 2(D-2)/L δ(r) δ^μ_ν.
Concentrated source at r=0,
mirror image under r↔-r.
T^μ_ν like a Λ in μ-ν space
ds² = (

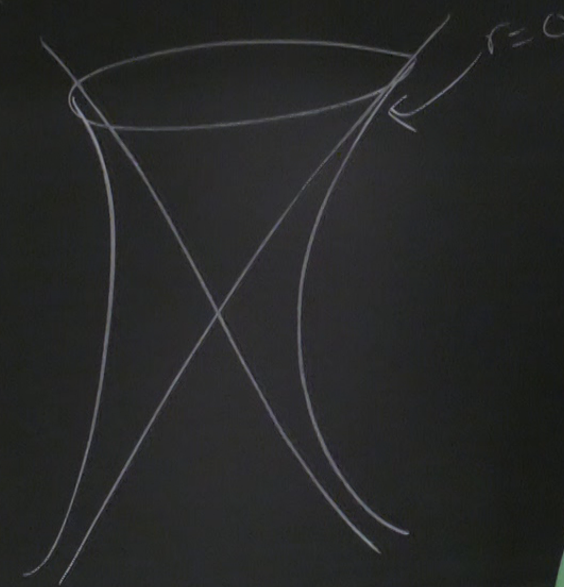

Take gμν constant curvature, lengthscale L.
A' = ∓ 1/L
r ≷ 0
A'' = - 2/L δ(r).
ds² = (1 - |r|/L)² [dt² - L² cosh²(t/L) dΩ²] - dr²
← FLAT SPACETIME
Looks like horizon at L.
Define ρ = (L-r) cosh t/L
τ = (L-r) sinh t/L
ρ² - τ² = (L-r)² ; dτ² - dρ² - ρ²dΩ² = (1 - r/L)² dt² - dr²

Take $g_{\mu\nu}$ constant curvature, lengthscale L.

$A' = \mp 1/L$. $\quad r \gtrless 0$

$A'' = -\frac{2}{L}\delta(r)$.

$$ds^2 = \left(1-\frac{|r|}{L}\right)^2\left[dt^2 - L^2\cosh^2\left(\frac{t}{L}\right)d\Omega^2\right] - dr^2$$ ← FLAT SPACETIME



Looks like horizon at L.

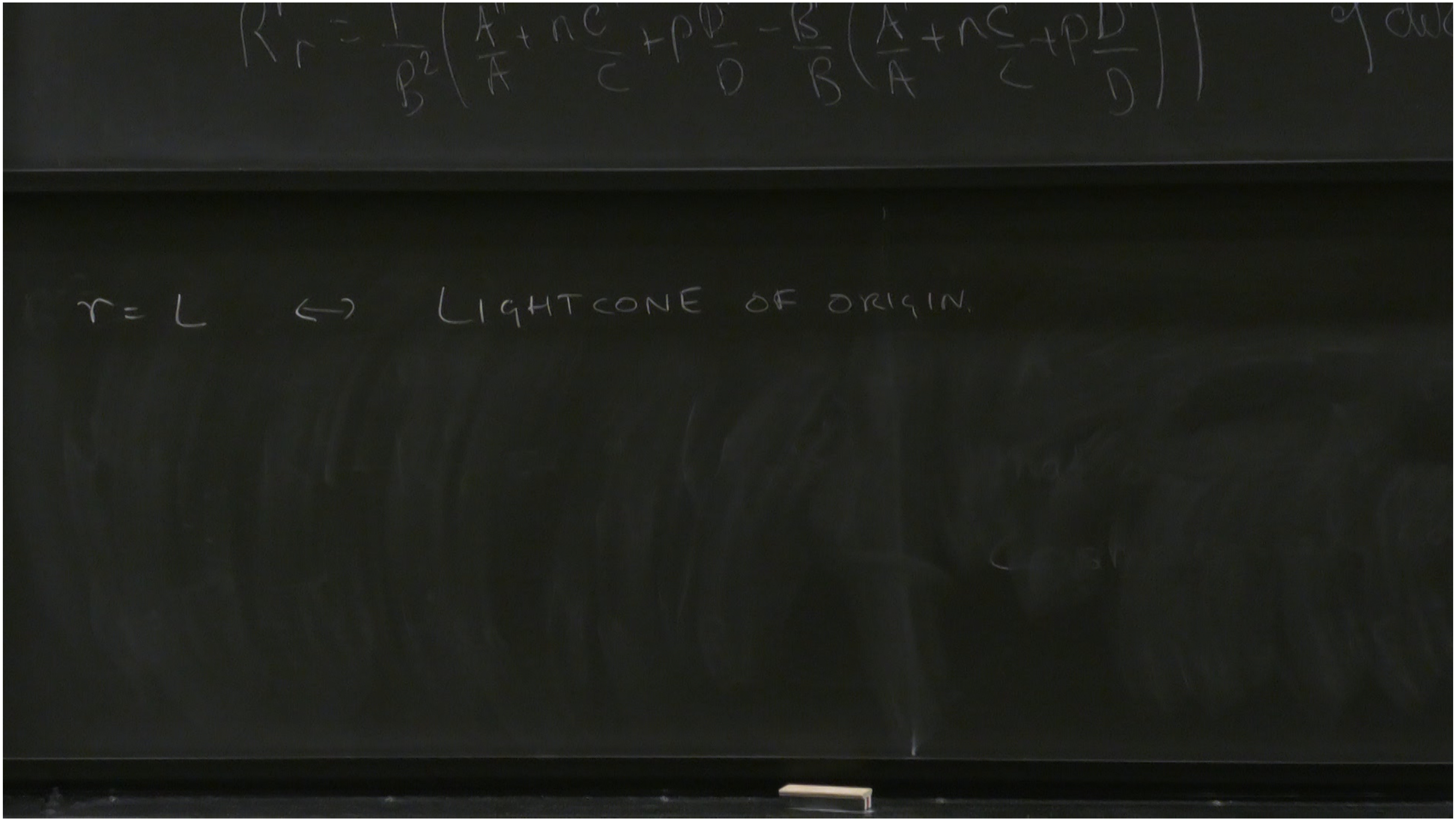
Define $\rho = (L-r)\cosh t/L$

$\tau = (L-r)\sinh t/L$

$$\rho^2 - \tau^2 = (L-r)^2 \; ; \; d\tau^2 - d\rho^2 = \left(1-\frac{r}{L}\right)^2 dt^2 - dr^2 - \rho^2 d\Omega^2$$

$$r = 0 \qquad \rho^2 - \tau^2 = L^2$$

$$r = L \quad t = 0 \longleftrightarrow \rho = \tau = 0$$


r = L ⟷ LIGHTCONE OF ORIGIN.

$$R^r{}_r = \frac{1}{B^2}\left(\frac{A''}{A} + n\frac{C''}{C} + p\frac{D''}{D} - \frac{B'}{B}\left(\frac{A'}{A} + n\frac{C'}{C} + p\frac{D'}{D}\right)\right)$$

of $d\Omega^2$

$r = L \leftrightarrow$ LIGHTCONE OF ORIGIN $\rightarrow$ DOMAIN WALL

$r < 0$ identical

2 copies of interior of hyperboloid

"glued" at hyperboloid.