Title: Standard Model Lecture

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Collection: Standard Model 2023/24

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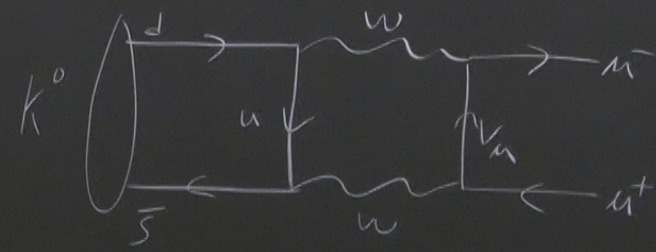
Today: GIM mechanism $K^0 \to \mu^+\mu^-$, meson mixing

Flavor and CP violation

Kaon mixing

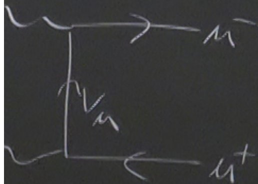
'The flavor problem' for new physics

$K^0 \to \mu^+\mu^-$



How does this work in EFT?

In S.M setting external $\vec{p}_{K^0} = 0$



$$iA = -i\frac{g_2^4}{4} V_{ud}^{\dagger} V_{us} \int \frac{d^4k}{(2\pi)^4} \left[\bar{s}\gamma^{\rho} P_L \frac{(\not{p} + m_u)}{p^2 - m_u^2}\gamma^{\sigma} P_L d\right]\left[\bar{\mu}\gamma_{\sigma}\frac{\not{p}}{p^2}\gamma_{\rho} P_L \mu\right]\frac{1}{(p^2 - m_W^2)^2}$$

$$= -i\frac{g_2^4}{64\pi^2 m_W^2} V_{ud}^{\dagger} V_{us}\left[\bar{s}\gamma^{\rho} P_L d\right]\left[\bar{\mu}\gamma_{\rho} P_L \mu\right]\int_0^{\infty} dx \frac{x}{(1+x)^2}\left[\frac{1}{x + m_u^2/m_W^2}\right]$$

Projector $\Rightarrow 0$

$$= \frac{i g_2^4}{m_W^4} V_{ud}^\dagger V_{us} \int \frac{d^4k}{(2\pi)^4} \frac{[\bar{s}\gamma^\rho P_L (\not{p} + \not{k} + m_u)\gamma_\sigma P_L d][\bar{u}\gamma_\rho P_L \not{k} \gamma^\sigma P_L u]}{[(p+k)^2 - m_m^2][k^2]}$$

In

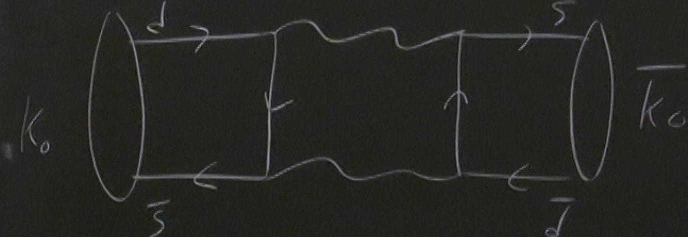
$$\Rightarrow \frac{i g_2^4}{m_W^4} V_{ud}^\dagger V_{us} \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}[(\not{p} + \not{k})\not{k}]}{[(p+k)^2 - m_u^2]k^2} \leftarrow dk^4 \frac{k^2}{k^4} \propto m_m^2$$

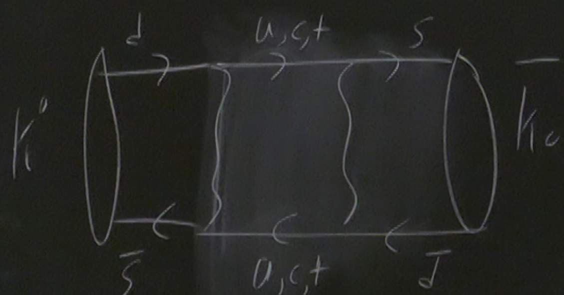
also

So again $\quad iA = -i\dfrac{g_2^2}{m_W^4}\lambda(m_m^2 - m_c^2)[\bar{s}\gamma^\rho P_L d][\bar{u}\gamma_\rho P_L u]$

Meson Mixing

$$\mathcal{L}_{sm} = \frac{\hat{G}_F}{\sqrt{2}} \frac{\hat{\alpha}_{ew}}{4\pi \sin^2\hat{\theta}} \sum_i V_{is}^* V_{id} \sum_j V_{js} V_{jd}^* \; E$$





$$\mathcal{L}_{SM} = \frac{\hat{G}_F}{\sqrt{2}} \frac{\hat{\alpha}_{ew}}{4\pi \sin^2\hat{\theta}} \sum_i V_{is}^* V_{id} \sum_j V_{js} V_{jd}^* \, E(x_i, x_j) \, [\bar{s} \gamma_\mu P_L d][\bar{d} \gamma^\mu P_L s] + h.c.$$

$\bar{K}_0$

$$E(x_i, x_j) = -x_i x_j \left[-\frac{3}{4} \frac{1}{(1-x_i)} \frac{1}{(1-x_j)} + \text{log terms} \right]$$

In S.M setting external $\vec{p}_{K^0} = 0$, Gaillard, Lee 74

$-\mu^-$

$-\mu^+$

$$iA = -i\frac{g_2^4}{4} V_{ud}^\dagger V_{us} \int \frac{d^4k}{(2\pi)^4} \left[\bar{s}\gamma^\rho P_L \frac{(\not{k} + m_u)}{k^2 - m_u^2}\gamma^\sigma P_L d\right]\left[\bar{\mu}\gamma_\sigma \frac{\not{k}}{k^2}\gamma_\rho P_L \mu\right] \frac{1}{(k^2-m_w^2)^2}$$

$$= -i \frac{g_2^4 V_{ud}^\dagger V_{us}}{64\pi^2 m_w^2}\left[\bar{s}\gamma^\rho P_L d\right]\left[\bar{\mu} \gamma_\rho P_L \mu\right]\int_0^\infty \frac{dx\, x}{(1+x)^2}\left[\frac{1}{x + m_u^2/m_w^2}\right]$$

And $\int^\infty dx \, \frac{x}{\ldots}\left[\frac{1}{\ldots}\right] = 1 + \frac{m_u^2}{\ldots} + \cdots$

<u>Recall:</u> $$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

So $V_{ud}^{\dagger} V_{us} = \lambda(1-\frac{1}{2}\lambda^2)$

$V_{cd}^{\dagger} V_{cs} = -\lambda(1-\frac{1}{2}\lambda^2)$

$$s = \lambda(1-\tfrac{1}{2}\lambda^2)$$

And $\int_0^\infty dx \, \frac{x}{(1+x)^2}\left[\frac{1}{x+m_u^2/m_w^2}\right] = 1 + \frac{m_u^2}{m_w^2} + \cdots$

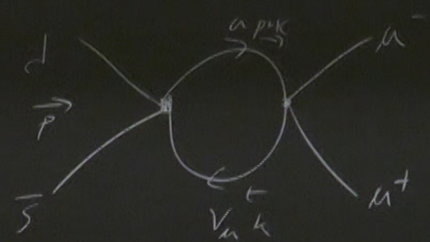
$$s = -\lambda(1-\tfrac{1}{2}\lambda^2)$$

<u>If</u> just u quark $iA_{sm} = \frac{-i g_2^4 \lambda m_u^2}{64\pi^2 m_w^4}[\bar{s}\gamma^\rho P_L d][\bar{u}\gamma_\rho P_L u]$

If u & c quarks $iA_{sm} = \frac{-i g_2^4 \lambda (m_u^2 - m_c^2)}{64\pi^2 m_w^4}[\bar{s}\gamma^\rho P_L d][\bar{u}\gamma_\rho P_L u]$

<u>GIM</u> mechanism

Loop in this case



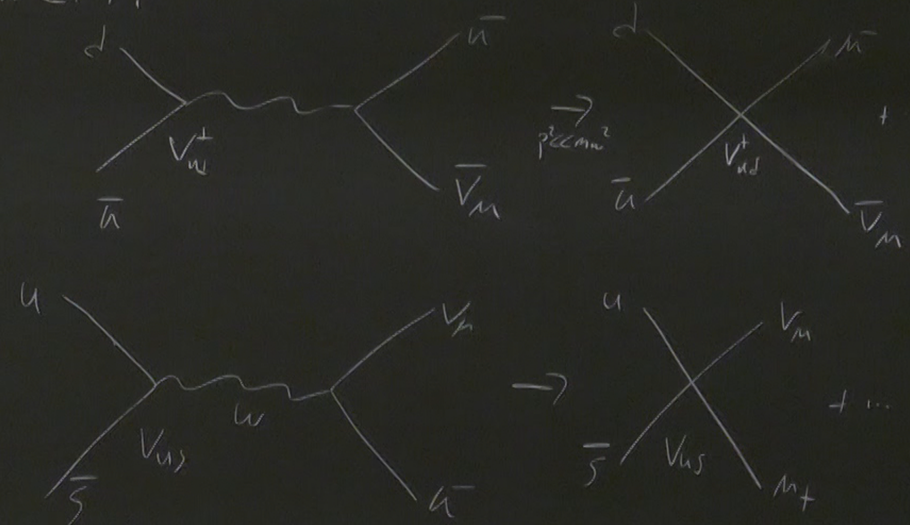
$$\Rightarrow \frac{i g_2^2 V_{ud}^\dagger}{m_W^2} [\bar{u} \gamma^\rho P_L d][\bar{\nu}_\mu \gamma_\rho P_L \mu]$$

$$\Rightarrow \frac{i g_2^2 V_{us}}{m_W^2} [\bar{s} \gamma^\rho P_L u][\bar{u} \gamma_\rho P_L \nu_\mu]$$

$$iA = \frac{i g_2^4}{m_W^4} V_{us} V_{ud}^\dagger [\bar{s} \gamma^\rho P_L u][\bar{u} \gamma_\rho P_L \nu_\mu][\bar{u} \gamma^\sigma P_L d][\bar{\nu}_\mu \gamma_\sigma P_L \mu]$$

How does this work in EFT?

$\mathcal{L}_{SM} \xrightarrow[p^2 \ll m_W^2]{} \mathcal{L}_{\text{weak EFT}}$ or 'LEFT'



$+ \cdots \Rightarrow \dfrac{i g_2^2 V_{ud}^*}{m_W^2} [\bar u \gamma^\rho P_L d][\bar\nu$

$+ \cdots \Rightarrow \dfrac{i g_2^2 V_{us}}{m_W^2} [\bar s \gamma^\rho P_L u][$

Projector $\Rightarrow 0$

$$= \frac{i g_2^4}{m_W^4} V_{ud}^\dagger V_{us} \int \frac{d^4k}{(2\pi)^4} \frac{[\bar{s}\gamma^\rho P_L (\not p + \not k + m_u)\gamma_\sigma P_L d][\bar{u}\gamma_\rho P_L \not k \gamma^\sigma P_L u]}{[(p+k)^2 - m_u^2][k^2]}$$

In SM also

$$\Rightarrow \frac{i g_2^4}{m_W^4} V_{ud}^\dagger V_{us} \int \frac{d^4k}{(2\pi)^4} \frac{Tr[(\not p + \not k)\not k]}{[(p+k)^2 - m_u^2]k^2} \leftarrow dk^4 \frac{k^2}{k^4} \propto m_u^2$$

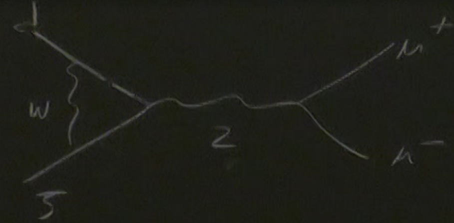
- also has same quark

(C

So again $i\mathcal{A} = -\frac{i g_2^2}{m_W^4} \lambda (m_u^2 - m_c^2) [\bar{s}\gamma^\rho P_L d][\bar{\mu}\gamma_\rho P_L u]$

SM: <u>Very</u> predicti

In SM also

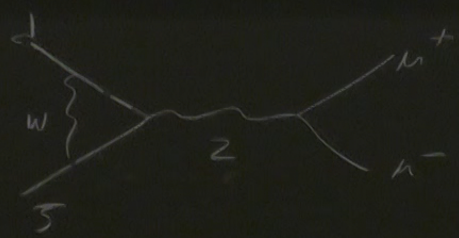


- also has same quark mass, V_{CKM} dependence → exactly cancels axial W^+W^- current contribution (GIM)

$$\langle K^0|\bar{s}\gamma^\rho P_L d|0\rangle = \langle K^0|\bar{s}\frac{\gamma^\rho}{2} d|0\rangle - \frac{\langle K^0|\bar{s}\gamma^\rho\gamma_5 d|0\rangle}{2}$$

SM very predictive in flavor-change

In SM also



– also has same quark mass, V_{CKM} dependence → exactly cancels axial W^+W^- current contribution

(GIM)

$$\langle K^0|\bar{s}\gamma^\rho P_L d|0\rangle = \frac{\langle K^0|\bar{s}\gamma^\rho d|0\rangle}{2} - \frac{\langle K^0|\bar{s}\gamma^\rho\gamma_5 d|0\rangle}{2}$$

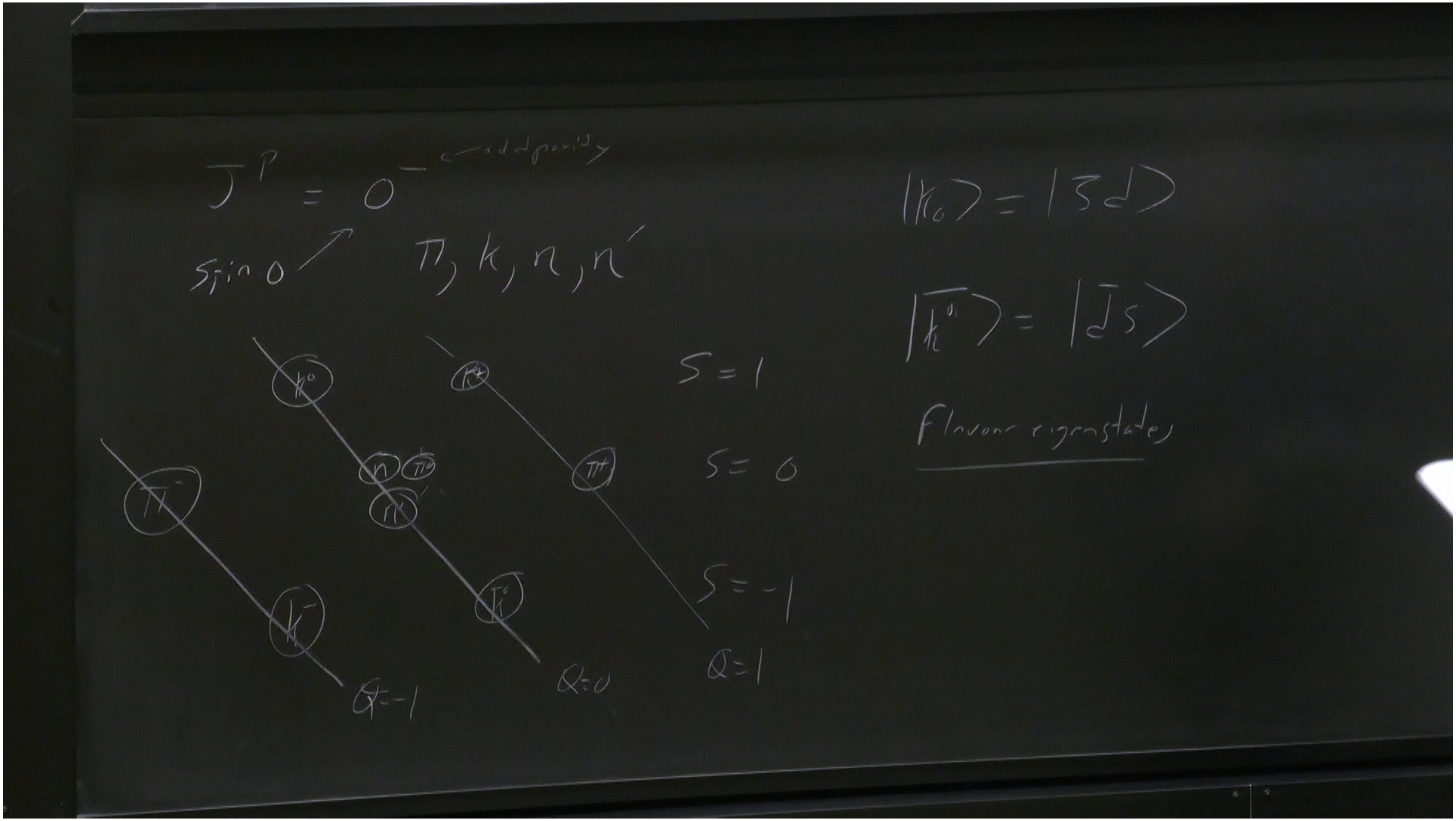
m · Very predictive in flavour change

$$\langle K^0(p)|\bar{\psi}\gamma^\rho\psi|0\rangle = p^\rho f_K$$

$$\mathcal{L}_{SM} = \frac{\hat{G}_F}{\sqrt{2}} \frac{\hat{\alpha}_{ew}}{4\pi \sin^2\theta} \sum_i V_{is}^* V_{id} \sum_j V_{js} V_{jd}^* \, E(x_i, x_j) \, [\bar{s} \gamma_\mu P_L d][\bar{d} \gamma^\mu P_L s] + h.c.$$

$$E(x_i, x_j) = -x_i x_j \left[-\frac{3}{4} \frac{1}{(1-x_i)} \frac{1}{(1-x_j)} + \text{log terms} \right]$$

$$x_i = m_i^2 / m_W^2$$


$J^P = 0^-$ ← odd parity
Spin 0
π, K, η, η'
K^0
K^+
π^-
η
π^0
η'
π^+
K^-
$\bar{K}^0$
S = 1
S = 0
S = -1
Q = -1
Q = 0
Q = 1
$|K_0\rangle = |\bar{s}d\rangle$
$|\bar{K}^0\rangle = |\bar{d}s\rangle$
Flavour eigenstates

$\bar{s}d\rangle$

$|\bar{d}s\rangle$

genstates

CP

$CP|K^0\rangle = -|\bar{K}^0\rangle$

$CP|\bar{K}^0\rangle = -|K^0\rangle$

Hamiltonian $H = m - \frac{i}{2}\Gamma$

$$= \begin{pmatrix} m - \frac{i}{2}\Gamma & m_{12} - \frac{i}{2}\Gamma_{12} \\ m_{12}^* - \frac{i}{2}\Gamma_{12}^* & m - \frac{i}{2}\Gamma \end{pmatrix}$$

$m^\dagger = m$

$\Gamma^\dagger = \Gamma$

CP invarian

$= -|\bar{K}^0\rangle$

$= -|K^0\rangle$

$H = m - \frac{i}{2}\Gamma$

$= \begin{pmatrix} m - i/_2\Gamma & m_{12} - \frac{i}{2}\Gamma_{12} \\ m_{12}^* - \frac{i}{2}\Gamma_{12}^* & m - \frac{i}{2}\Gamma \end{pmatrix}$

$m^\dagger = m$

$\Gamma^\dagger = \Gamma$

CP invariance

$m_{12}^* = m_{12}$

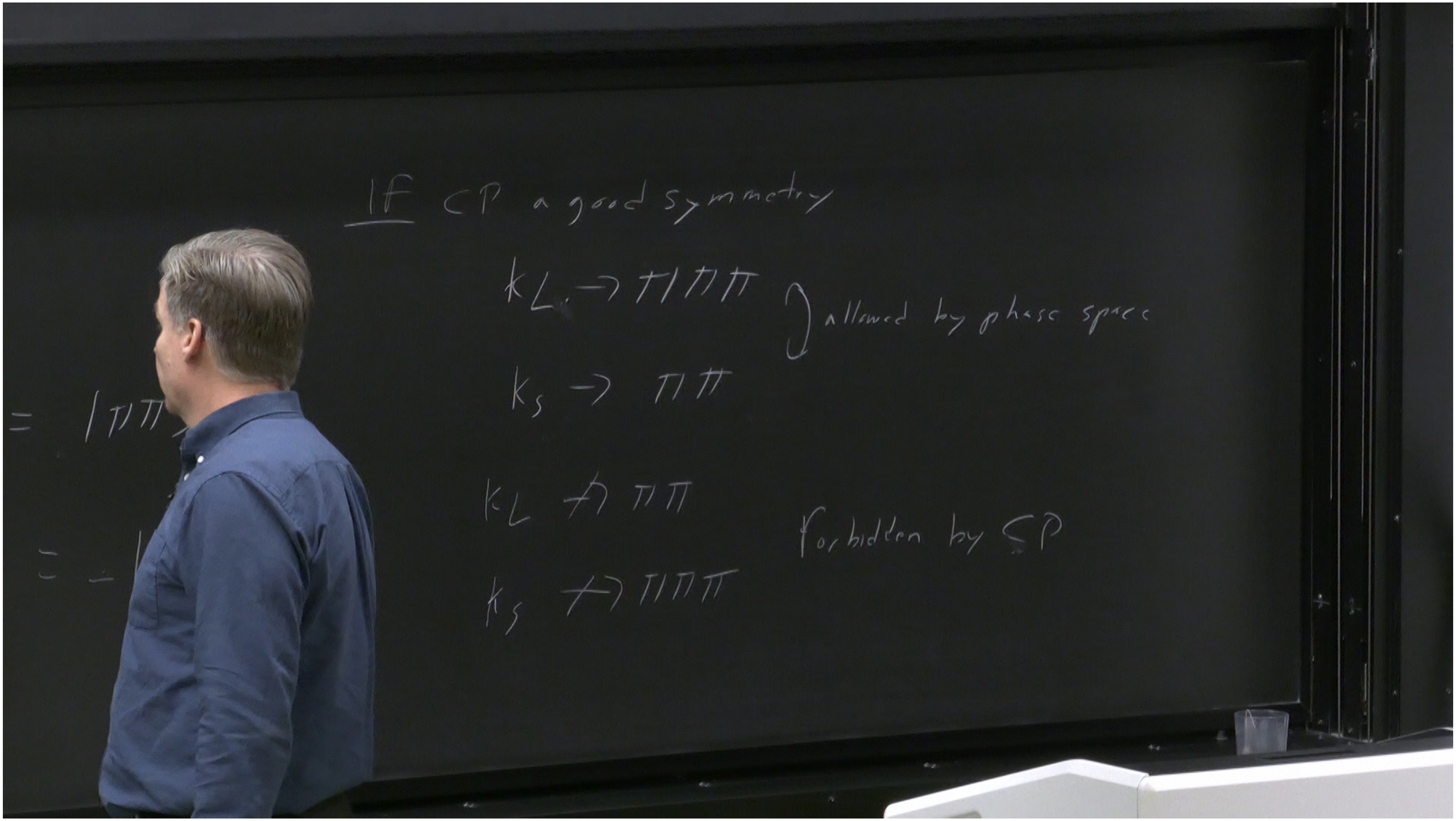
$\Gamma_{12}^* = \Gamma_{12}$

$$|K_{L/S}\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}}\left[(1+\varepsilon)|K^0\rangle \pm (1-\varepsilon)|\bar{K}^0\rangle\right]$$

if $\varepsilon = 0$ CP eigenstates $CP|K_L\rangle = -|K_L\rangle$

$CP|K_S\rangle = |K_S\rangle$

$CP|T$


IF CP a good symmetry
$K_L \to \pi\pi\pi$
allowed by phase space
$K_S \to \pi\pi$
$K_L \not\to \pi\pi$
forbidden by CP
$K_S \not\to \pi\pi\pi$

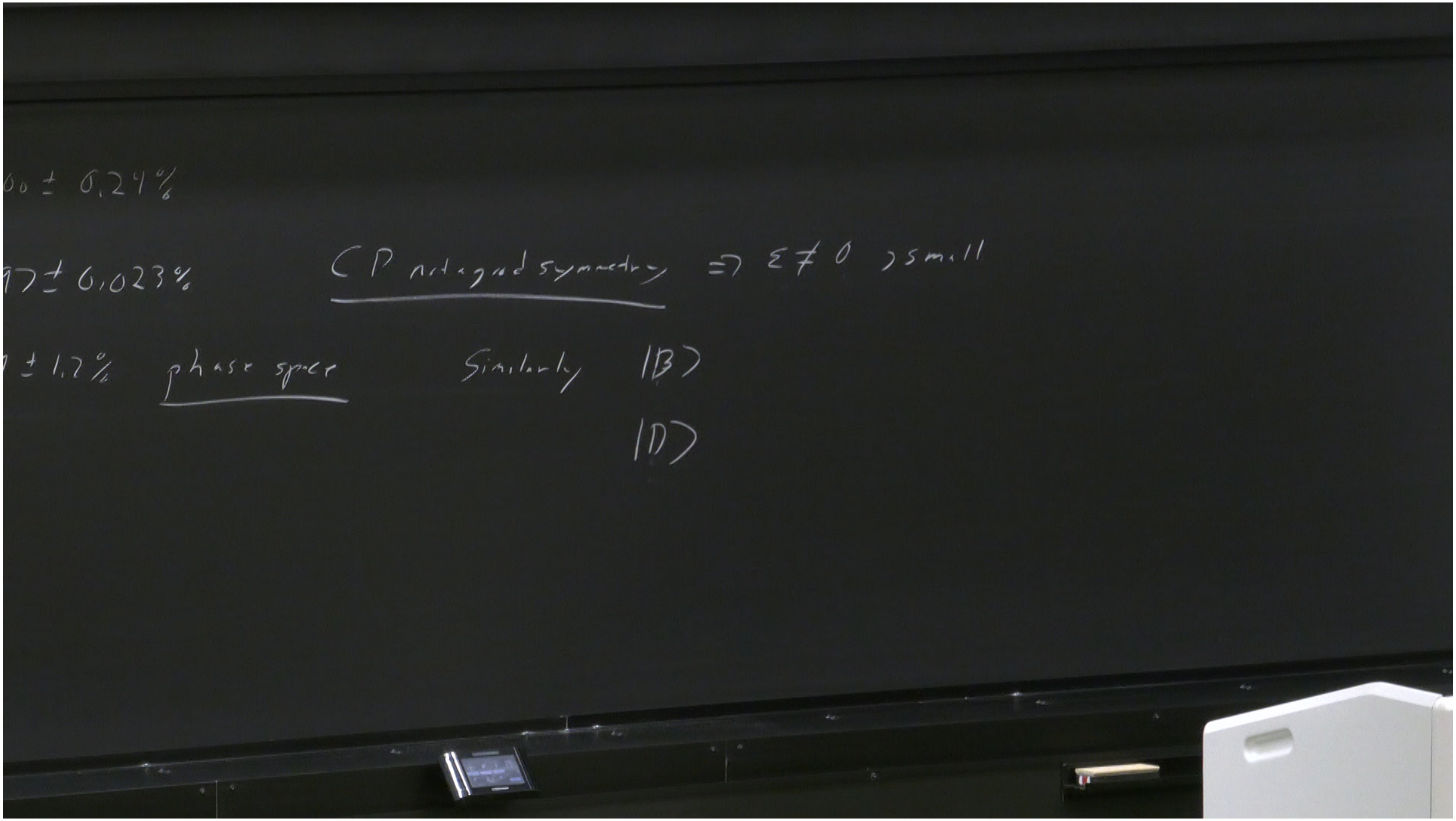
Exp gives $\quad Br(K_S \to \pi\pi) = 100.00 \pm 0.24\%$

$Br(K_L \to \pi\pi) = 0.297 \pm 0.023\%$ $\quad$ <u>CP not a good sy</u>

$Br(K_L \to \pi\pi\pi) = 33.1 \pm 1.2\%$ $\quad$ <u>phase space</u>

$\tau_{K_S} = 0.54 \times 10^{-10}\,s$ `short'

$\tau_{K_L} = 5.18 \times 10^{-8}\,s$ `long'


00 ± 0.24%
97 ± 0.023%
CP not a good symmetry ⇒ ε ≠ 0, small
± 1.2% phase space
Similarly |B⟩
|D⟩

mixing

on analysis

physics

$$\mathcal{L}^{\text{quark}}_{\text{yuk}} = -\left[H^{\dagger}\, \bar{d}_R^{\,i}\, Y_{D\,i}^{\;j}\, Q_{L\,j} + \tilde{H}\, \bar{u}_R^{\,i}\, Y_{u\,i}^{\;j}\, Q_{L\,j} + \text{h.c} \right]$$

$\mathcal{L}_{SM}$ preserves $SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_{L} \times SU(3)_{e} \Rightarrow U^5(3)$

Spurion analysis $\quad Y_D \sim (1, 3, \bar{3})$

$\qquad\qquad\qquad\quad\; Y_u \sim (3, 1, \bar{3})$

$Y^{\dagger\,k}_{u\;i}\, Y^{\,i}_{u\;j} \to SU$

$Y^{\dagger\,h}_{D\;i}\, Y^{\,i}_{D\;j} \to SU$

$Y_{u\,ij} Q_{L\,j} + h.c \,]$

$\times\, SU(3)_L \times SU(3)_c \to U^5(3)$

flavor violation

$(\bar{Q}_L^i \Gamma Q_L^j)(\bar{Q}^k \Gamma Q_L^l)$

$Y_{u\,ik}^\dagger Y_{u\,ij} \to SU(3)_Q \,(Y_u^\dagger Y_u)\, SU(3)_Q$ and 2 powers of quark mass

$Y_{D\,ik}^\dagger Y_{D\,ij} \to SU(3)_Q \,(Y_D^\dagger Y_D)\, SU(3)_Q$

SM

$$\bar{u}_L \gamma^\mu d_L \xrightarrow{CP} -\bar{d}_L \gamma_\mu u_L$$

$$W^{+\mu} \xrightarrow{CP} -W^{-\mu}$$

so $\bar{u}_L \gamma^\mu d_L W^+ \xrightarrow{CP} \bar{d}_L \gamma_\mu u W^-$ if $V^\dagger = V$ CP a good symmetry of SM

Weak eigenstate $\to U$ mass eigenstates

CKM in SM <u>unitary</u>

$$V_{CKM}^{\dagger} V_{CKM} = \mathbb{1}$$

$$\sum_k |V_{jk}|^2 = \sum_k |V_{kj}|^2 = 1$$

$$\sum_k V_{ik} V_{jk}^{*} = 0$$

Unitarity triangles

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \qquad \frac{V_{ud}V_{u}}{|V_{cd}V_{cb}^*|}$$

$$O(\varepsilon^3) \qquad O(\varepsilon^3) \qquad O(\varepsilon^3)$$

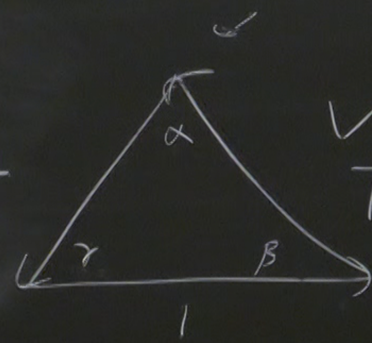
$$|V_{ij}|^2 = 1$$

$$\div \text{ by } |V_{cd}^*V_{cb}| \Rightarrow \frac{V_{ud}V_{ub}^*}{|V_{cd}^*V_{cb}|} - 1 + \frac{V_{td}V_{tb}^*}{|V_{cd}^*V_{cb}|} = 0$$

$$0$$

$B^0 \to \pi\pi, \rho\pi$

$\ldots_b^* + V_{td} V_{tb}^* = 0$ $\quad \frac{V_{ud} V_{ub}^*}{|V_{cd} V_{cb}^*|}$



$\frac{V_{td} V_{tb}^*}{|V_{cd}^* V_{cb}|}$

$O(\varepsilon^3)$

$B \to DK$

$B^0 \to J\psi / K_s$

$\ldots_{ub}^* - 1 + \frac{V_{td} V_{tb}^*}{|V_{cd}^* V_{cb}|} = 0$

0.24%

0.023%

?% phase sp

$K_0 \to \mu^+\mu^-$

$\frac{c}{\Lambda^2}\, \bar{d}\,\Gamma_s\, \mu\,\Gamma\,\mu$