Title: Standard Model Lecture

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Collection: Standard Model 2023/24

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Today: $\chi_{PT}$ meson mass spectrum

meson decay / mixing to study CKM

- Flavour problem -

For new physics

Recall: $\mathcal{L}_{SM} \supseteq -\left[ H_I^{\dagger} \bar{d}_R^i Y_D^{ij} Q_L^{j,I} + \tilde{H}_I^{\dagger} \bar{u}_R^i Y_u^{ij} Q_L^j + h.c. \right]$

$\supseteq -\left[ \frac{v}{\sqrt{2}} \bar{d}_R^i Y_D^{ij} d_L^j + \frac{v}{\sqrt{2}} \bar{u}_R^i Y_u^{ij} u_L^j + h.c. \right]$

Rotate to mass eigenstates

SU(3) rotation

$u_L = U(u,L)\, u_L'$ $\qquad u_R = U(u,R)\, u_R'$

$d_L = U(d,L)\, d_L'$ $\qquad d_R = U(d,R)\, d_R'$

$e_L = U(e,L)\, e_L'$ $\qquad e_R = U(e,R)\, e_R'$

weak eigen $\qquad$ mass eigen

$Y_u^{ij} Q_L^j + h.c \, ]$

$u_L^j + h.c \, ]$

tion

$u_R = U(u,R)\, u_R'$

$d_R = U(d,R)\, d_R'$

$e_R = U(e,R)\, e_R'$

eigen

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} = U(u,L) \begin{pmatrix} u_L' \\ V_{CKM}\, d_L' \end{pmatrix}$$

So $V_{CKM} = U^\dagger(u,L)\, U(d,L)$

Shows up in $\bar{Q}_L i \not{D} Q_L \rightarrow \frac{g_2}{\sqrt{2}} W_\mu^+ \bar{u}_L^{i\prime} \gamma_\mu d_L^{j\prime} V_{CKM}^{ij}$

Flavour changing charged currents at 'tree level'

$$V_{CKM} = U^{\dagger}(u,L)\, U(d,L)$$

$$= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

only 4 parameters

$\leftarrow$ not all independent parameters

Define $\lambda \equiv V_{us}$

$A\lambda = V_{cb}$

derive others

Wolfenstein

$$\Rightarrow \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 \\ -\lambda \\ A\lambda^3(1-\rho-i\eta) \end{pmatrix}$$

$+ h.c \,]$

$= U(u,R)\, u_R'$

$= U(d,R)\, d_R'$

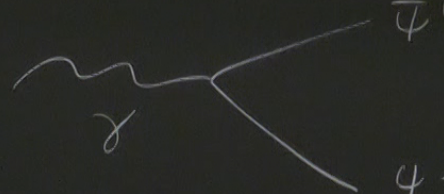
$= U(e,R)\, e_R'$

So $V_{CKM} = U^\dagger(u,L)\, U(d,L)$

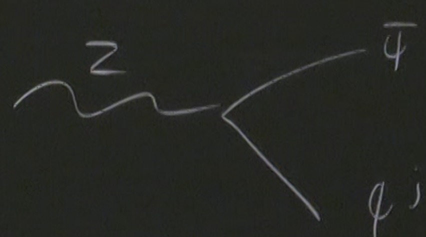
Shows up in $\bar{Q}_L i \not{D} Q_L \rightarrow \frac{g_2}{\sqrt{2}} W^+_\mu \bar{u}_L^{i'} \gamma_\mu d_L^{j'} V_{CKM}^{ij}$

Flavour changing charged currents at 'tree level' weak eigen

What about neutral currents?




$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix}$$




Come from $T_3$, $Y_{\text{hypercharge}}$ generators

in $\bar{\Psi}_L i \not{\partial} \psi_L$ $\bar{\Psi}_R i \not{\partial} \psi_R$

↑ mass eigen

$T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\underset{hyp}{Y} = \underset{\underset{\text{hypercharge}}{\uparrow}}{y} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Diagonal generators

commute with $U(u,R), U(d,R)$

no off diagonal $\bar{u}d$ or $\bar{d}u$ like $W^\pm$

commute with $U(u,L), U(d,L)$

$\Rightarrow$ no flavour changing neutral currents at 'tree level'

However $\sim\sim Z$

- Flavor change exists with neutral curre...

Diagonal generators

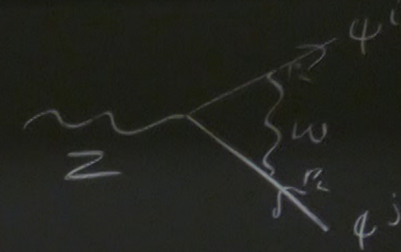
commute with $U(u,R)$, $U(d,R)$

no off diagonal $\bar{u}d$ or $\bar{d}u$ like $W^\pm$

commute with $U(u,L)$, $U(d,L)$

→ no flavour changing neutral currents at "tree level"

However




- Flavor change exists at loop level with neutral currents

$\chi_{SB}$ and meson masses: $\mathcal{L}_{QCD} = -\frac{1}{4} G_A^{\mu\nu} G_{\mu\nu}^A + \bar{q}_i' (i\not{\partial} - m_q^i) q_i' +$ counter

$q' = \begin{pmatrix} u' \\ d' \\ s' \end{pmatrix}$ in $m_q \to 0$ limit $\mathcal{L}_{kin} = \bar{Q}_L i\not{\partial} Q_L + \bar{u}_R i\not{\partial} u_R + \bar{d}_R i\not{\partial} d_R$

$\boxed{m_{q}/\Lambda_{QCD} \to 0}$

$m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$

– SM has a $SU_Q(3,L) \times SU_u(3,R) \times SU_d(3,R)$ flavour symm

– All breaking due to $V_{CKM}^{ij}$ in a highly predictive p

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_A^{\mu\nu} G^A_{\mu\nu} + \bar{q}_i' (i\not{\partial} - m_q^i) q_i' + \text{counterterms}$$

limit $\quad \mathcal{L}_{kin} = \bar{Q}_L i\not{\partial} Q_L + \bar{u}_R i\not{\partial} u_R + \bar{d}_R i\not{\partial} d_R$

$$\underline{U^5(3)}$$

– SM has a $SU_u(3,L) \times SU_u(3,R) \times SU_d(3,R)$ flavour symmetry in quark sector

– All breaking due to $V_{CKM}^{ij}$ in a highly predictive pattern – GIM mechanism

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_A^{\mu\nu} G_{\mu\nu}^A + \bar{q}_i^{\,i} (i\not{\partial} - m_q^i) q_i^{\,i} + \text{counterterms}$$

$\to 0$ limit $\quad \mathcal{L}_{kin} = \bar{Q}_L i\not{\partial} Q_L + \bar{u}_R i\not{\partial} u_R + \bar{d}_R i\not{\partial} d_R$

$\underline{U^5(3)}$

– SM has a $SU_Q(3,L) \times SU_u(3,R) \times SU_d(3,R)$ flavour symetry in quark sector

– All breaking due to $V_{CKM}^{ij}$ in a highly predictive pattern – GIM mechanism

$\langle \bar{q}^i q^j \rangle$ lower energy than free quarks, so $\langle \bar{q}^i q^j \rangle \neq 0$

$\uparrow_{\text{dim } 3}$ so $\equiv v'$

$\uparrow_{\text{dim } 3} \propto \Lambda_{QCD}^3$

Introduce field parameterizing condensate

$$v' \Sigma^{ij}(x) \simeq \langle \bar{q}^i q^j \rangle \simeq v' \delta^{ij}$$

So $\Sigma^{ij}(x)$ dimensionless and diagonal $\langle \Sigma(x) \rangle = \mathbb{1}$

$\Sigma(x) \to L$

$$\Sigma(x) = 1 + 2i\,\frac{\underline{M}(x)}{f} - \frac{4}{f^2}\,\underline{M}(x)\,\underline{M}(x) + \cdots$$

$$\text{So } \frac{f^2}{8}\,\mathrm{Tr}\left[\partial_\mu \Sigma\, \partial_\mu \Sigma^\dagger\right] \rightarrow \sim \frac{f^2}{f^4}\,\mathrm{Tr}[$$

$$\sim \frac{1}{f^2}\,\mathrm{Tr}[$$

$$\text{So } p^2/f^2$$

eak | mass eigen
eigen

$$m^2_{K_0} = m^2_{\bar{K}_0} = \frac{4v'}{f^2}(m_d + m_s)$$

not diagonal $\quad m^2_{\eta\pi^0} = \frac{4v'}{\sqrt{2}f^2}(m_u - m_d)$

$$\propto \eta\pi^0 \qquad (m_u - m_d) \ll m_s$$

$$(m_d + m_s)$$

$$= \frac{4v'}{\sqrt{2}f^2}(m_u - m_d)$$

$$\pi^0 \qquad (m_u - m_d) << m_s$$

$$\bar{3} \times 3 = 8 + 1$$

$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

$$\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_8 \end{pmatrix} \qquad \theta' \sim -11^\circ$$

eigen

$$= \frac{4v'}{f^2}(m_u + m_d) \simeq 130\,\text{MeV} \qquad m^2_{K_0} = m^2_{\bar{K}_0} = \frac{4v'}{f^2}(m_d + m_s) \simeq 497\,\text{MeV}$$

$$= \frac{4v'}{f^2}(4m_s + m_u + m_d)$$

<u>not diagonal</u>

$$m^2_{\eta\pi^0} = \frac{4v'}{\sqrt{2}f^2}(m_u - m_d)$$

$$= \frac{4v'}{f^2}(m_u + m_d) \simeq 130\,\text{MeV}$$

$$\propto \eta\pi^0 \qquad (m_u - m_d) \ll m_s$$

$$= \frac{4v'}{f^2}(m_u + m_s) \simeq 490\,\text{MeV}$$

$$\bar{3}\times 3 = 8+1$$

$$= \frac{4v'}{F^2}(m_d+m_s) \quad \sim 497\ \mathrm{MeV}$$

$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u}+d\bar{d}-2s\bar{s}) \qquad m_{\eta'} \simeq 957\ \mathrm{MeV}$$

$$m^2_{\eta\pi^0} = \frac{4v'}{\sqrt{2}F^2}(m_u-m_d)$$

$$\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u}+d\bar{d}+s\bar{s}) \qquad m_\eta \simeq 547\ \mathrm{MeV}$$

$$\eta\pi^0 \qquad (m_u-m_d) \ll m_s$$

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix} = \begin{pmatrix}\cos\theta' & -\sin\theta'\\ \sin\theta' & \cos\theta'\end{pmatrix}\begin{pmatrix}\eta_1\\ \eta_8\end{pmatrix} \qquad \theta' \sim -11^\circ$$

$\boxed{\text{'t Hooft}}$

$\frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$ $\quad m_{\eta'} \simeq 957\,\text{MeV}$

$\frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$ $\quad m_{\eta} \simeq 547\,\text{MeV}$

$\begin{pmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_8 \end{pmatrix}$ $\quad \theta' \simeq -11^\circ$

Parameterize

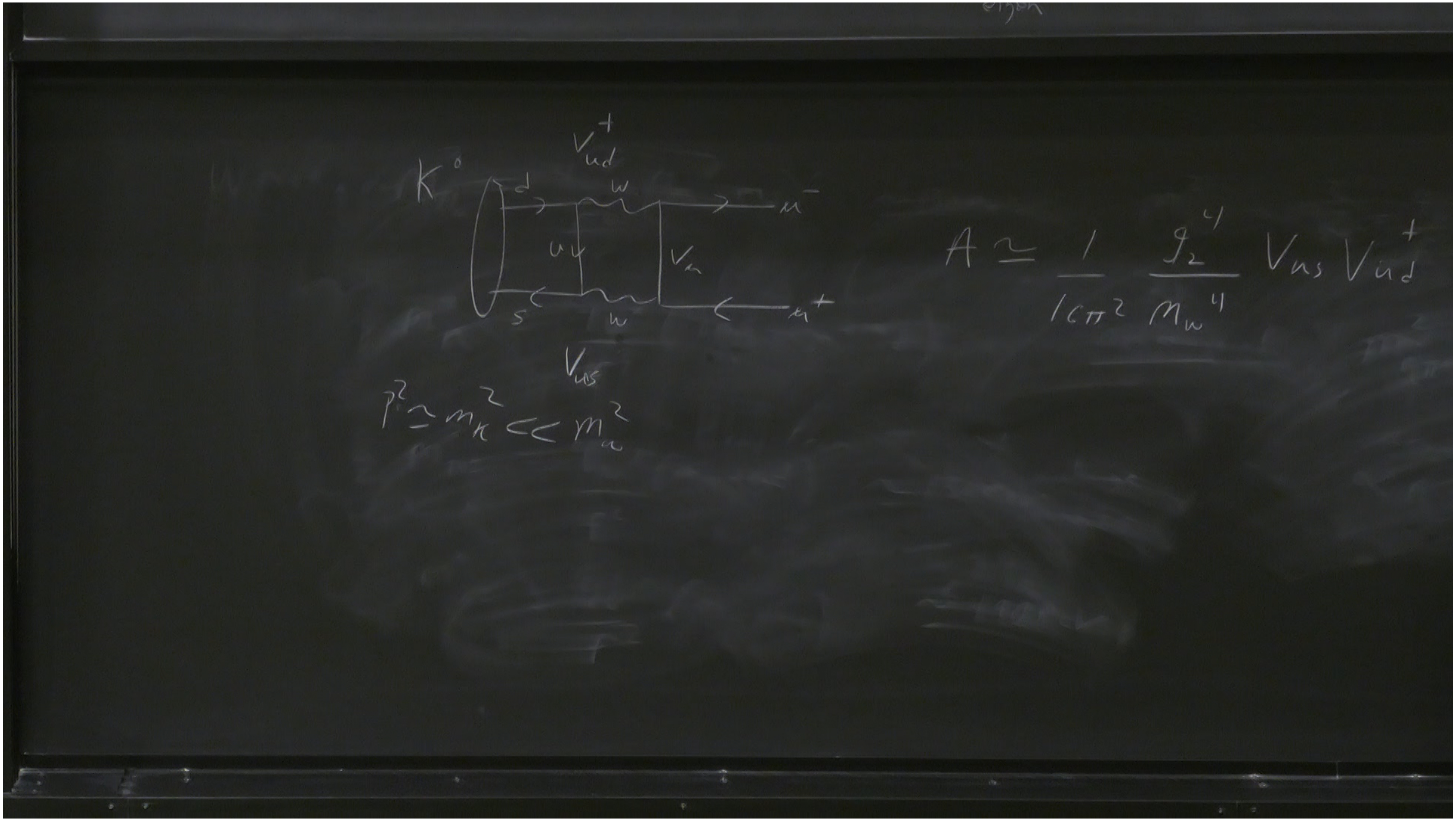
th

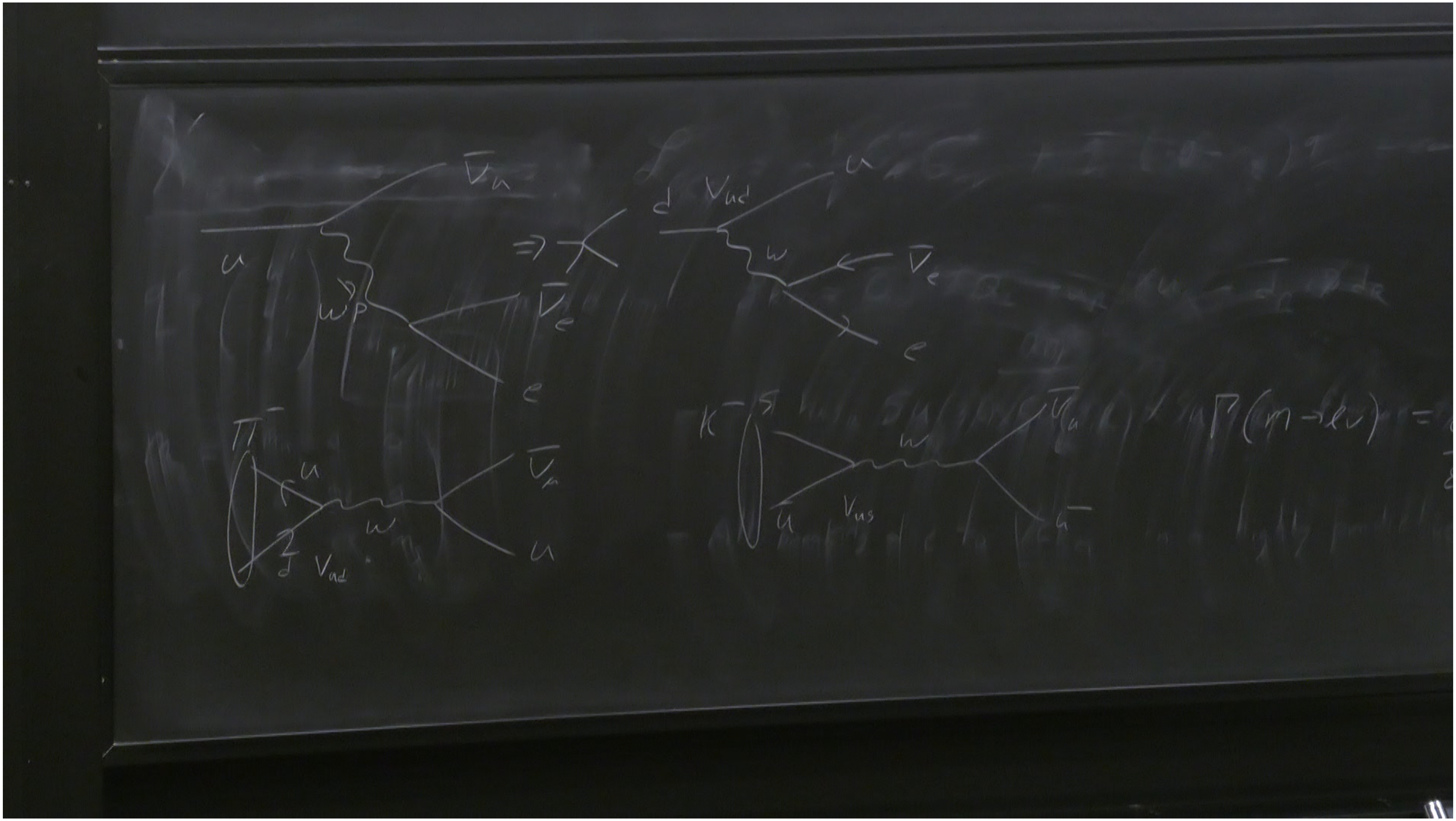
meson decay constant

$$\langle 0 | \bar{q} \gamma_\mu q | M(p) \rangle \equiv i p_\mu f_M \sim 100\,\mathrm{MeV}$$

$$\langle 0 | \bar{q} \gamma_\mu \gamma_5 q | M(p) \rangle$$

$$f_M\, m_\ell^2\, M_{meson} \left(1 - \frac{m_\ell^2}{M_{meson}^2}\right) |V_{q_1 q_2}|^2$$


$V_{ud}^{\dagger}$
$K^0$
$d$
$W$
$\mu^-$
$u$
$V_\mu$
$s$
$W$
$\mu^+$
$V_{us}$
$p^2 \simeq m_K^2 << m_w^2$
$A \simeq \frac{1}{16\pi^2} \frac{g_2^4}{m_W^4} V_{us} V_{ud}^{\dagger}$


$\bar{\nu}_\mu$
$u$
$W$
$\bar{\nu}_e$
$e$
$d$
$V_{ud}$
$u$
$W$
$\bar{\nu}_e$
$e$
$\pi^-$
$u$
$d$
$V_{ud}$
$W$
$\bar{\nu}_\mu$
$\mu$
$K^-$
$s$
$\bar{u}$
$V_{us}$
$W$
$\bar{\nu}_\mu$
$\mu^-$
$\Gamma(m \to \ell\nu) =$

## GIM mechanism

$$A \simeq \frac{1}{16\pi^2}\,\frac{g_2^4\, m_u^2}{m_W^4}\, V_{us} V_{ud}^{\dagger} \qquad \bar{s}_L \gamma_\mu d_L \qquad \bar{\mu}\gamma_\mu P_L \mu$$

$$|A|^2 \propto \frac{(m_u^2)^2}{(m_W^4)^2}$$

$$\Rightarrow\quad m_u^2 V_{us} V_{ud}^{\dagger} + m_c^2 V_{cs} V_{cd}^{\dagger}$$

$$\propto \frac{(m_u^2 - m_c^2)}{m_W^4}$$